

Prediction of the light \mathcal{CP} -even Higgs-Boson Mass of the MSSM: Towards the ILC Precision*

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Abstract

The signal discovered in the Higgs searches at the LHC can be interpreted as the Higgs boson of the Standard Model as well as the light \mathcal{CP} -even Higgs boson of the Minimal Supersymmetric Standard Model (MSSM). In this context the measured mass value, having already reached the level of a precision observable with an experimental accuracy of about 500 MeV, plays an important role. This precision can be improved substantially below the level of ~ 50 MeV at the future International Linear Collider (ILC). Within the MSSM the mass of the light \mathcal{CP} -even Higgs boson, M_h , can directly be predicted from the other parameters of the model. The accuracy of this prediction should match the one of the experimental measurements. The relatively high experimentally observed value of the mass of about 125.6 GeV has led to many investigations where the supersymmetric (SUSY) partners of the top quark have masses in the multi-TeV range. We review the recent improvements for the prediction for M_h in the MSSM for large scalar top masses. They were obtained by combining the existing fixed-order result, comprising the full one-loop and leading and subleading two-loop corrections, with a resummation of the leading and subleading logarithmic contributions from the scalar top sector to all orders. In this way for the first time a high-precision prediction for the mass of the light \mathcal{CP} -even Higgs boson in the MSSM is possible all the way up to the multi-TeV region of the relevant supersymmetric particles. However, substantial further improvements will be needed to reach the ILC precision. The newly obtained corrections to M_h are included into the code `FeynHiggs`.

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1 Introduction

After the spectacular discovery of a signal in the Higgs-boson searches at the LHC by ATLAS and CMS [1, 2], now exploration of the properties of the observed particle is in the main focus. In particular, the observation in the $\gamma\gamma$ and the $ZZ^{(*)} \rightarrow 4\ell$ channels made it possible to determine its mass with already a remarkable precision. Currently, the combined mass measurement from ATLAS is $125.5 \pm 0.2 \pm 0.6$ GeV [3], and the one from CMS is $125.7 \pm 0.3 \pm 0.3$ GeV [4]. This leads to the naive average of

$$M_H^{\text{LHC, today}} = 125.6 \pm 0.35 \text{ GeV} . \quad (1)$$

At the (planned) future International e^+e^- Linear Collider (ILC), using the Z -recoil method a precision of [5]

$$\delta M_H^{\text{ILC}} \lesssim 50 \text{ MeV} \quad (2)$$

is currently anticipated.

The other properties that have been determined so far, in particular the coupling strength modifiers [6, 7], as well as spin, are compatible with the minimal realisation of the Higgs sector within the Standard Model (SM) [8]. However, a large variety of other interpretations is possible as well, corresponding to very different underlying physics. While within the SM the Higgs-boson mass is just a free parameter, in theories beyond the SM (BSM) the mass of the particle that is identified with the signal at about 125.6 GeV can often be directly predicted, providing an important test of the model. One of the most popular BSM models is the Minimal Supersymmetric Standard Model (MSSM) [9]. In this model the Higgs sector consists of two scalar doublets accommodating five physical Higgs bosons. In lowest order these are the light and heavy \mathcal{CP} -even h and H , the \mathcal{CP} -odd A , and the charged Higgs bosons H^\pm .

The parameters characterising the MSSM Higgs sector at lowest order are the gauge couplings, the mass of the \mathcal{CP} -odd Higgs boson, M_A , and $\tan\beta \equiv v_2/v_1$, the ratio of the two vacuum expectation values. Accordingly, all other masses and mixing angles can be predicted in terms of those parameters, leading to the famous tree-level upper bound for the mass of the light \mathcal{CP} -even Higgs boson, $M_h \leq M_Z$, determined by the mass of the Z boson, M_Z . This tree-level upper bound, which arises from the gauge sector, receives large corrections from the Yukawa sector of the theory, which can amount up to $\mathcal{O}(50\%)$ (depending on the model parameters) upon incorporating the full one-loop and the dominant two-loop contributions [10–12].

The prediction for the light \mathcal{CP} -even Higgs-boson mass in the MSSM is affected by two kinds of theoretical uncertainties. First, the parametric uncertainties induced by the experimental errors of the input parameters. Here the dominant source of parametric uncertainty is the experimental error on the top-quark mass, m_t . Very roughly, the impact of the experimental error on m_t on the prediction for M_h scales like [13]

$$\delta M_h^{\text{para}, m_t} / \delta m_t^{\text{exp}} \sim 1 . \quad (3)$$

As a consequence, high-precision top-physics providing an accuracy on m_t much below the GeV-level is a crucial ingredient for precision physics in the Higgs sector [13]. The second type

of uncertainties are the intrinsic theoretical uncertainties that are due to unknown higher-order corrections. Concerning the SM input parameters, an overall estimate of for the lightest \mathcal{CP} -even Higgs mass of $\delta M_h^{\text{intr}} \sim 3$ GeV had been given in Refs. [10, 12] (the more recent inclusion of the leading $\mathcal{O}(\alpha_t \alpha_s^2)$ 3-loop corrections [14], see below, has slightly reduced this estimated uncertainty by few times $\mathcal{O}(100$ MeV)). It was pointed out that a more detailed estimate needs to take into account the dependence on the considered parameter region of the model. In particular, the uncertainty of this fixed-order prediction is somewhat larger for scalar top masses in the multi-TeV range.

The MSSM parameter space with scalar top masses in the multi-TeV range has received considerable attention recently, partly because of the relatively high value of $M_h \approx 125.6$ GeV, which generically requires either large stop masses or large mixing in the scalar top sector, and partly because of the limits from searches for supersymmetric (SUSY) particles at the LHC. While within the general MSSM the lighter scalar superpartner of the top quark is allowed to be relatively light (down to values even as low as m_t), both with respect to the direct searches and with respect to the prediction for M_h (see e.g. Ref. [15]), the situation is different in more constrained models. For instance, global fits in the Constrained MSSM (CMSSM) prefer scalar top masses in the multi-TeV range [16–18].

Here we review the significantly improved prediction for the mass of the light \mathcal{CP} -even Higgs boson in the MSSM [19], which is expected to have an important impact on the phenomenology in the region of large squark masses and on its confrontation with the experimental results. We briefly review the relevant sectors and the new, improved prediction for M_h . The numerical analysis focuses on the effects of heavy scalar top masses. The feasibility of reaching the anticipated ILC precision will be briefly discussed.

2 The Higgs and scalar top sectors of the MSSM

In the MSSM with real parameters (we restrict to this case for simplicity; for the treatment of complex parameters see Refs. [20, 21] and references therein), using the Feynman diagrammatic (FD) approach, the higher-order corrected \mathcal{CP} -even Higgs-boson masses are derived by finding the poles of the (h, H) -propagator matrix. The inverse of this matrix is given by

$$-i \begin{pmatrix} p^2 - m_{h,\text{tree}}^2 + \hat{\Sigma}_{hh}(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_{H,\text{tree}}^2 + \hat{\Sigma}_{HH}(p^2) \end{pmatrix}, \quad (4)$$

where $m_{h,H,\text{tree}}$ denote the tree-level masses, and $\hat{\Sigma}_{hh,HH,hH}(p^2)$ are the renormalized Higgs boson self-energies evaluated at the squared external momentum p^2 . For the computation of the leading contributions to those self-energies it is convenient to use the basis of the fields ϕ_1, ϕ_2 , which are related to h, H via the (tree-level) mixing angle α :

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} -\sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \quad (5)$$

The new higher-order corrections reviewed here originate in the top/stop sector of the MSSM. The bilinear part of the top-squark Lagrangian,

$$\mathcal{L}_{\tilde{t},\text{mass}} = - (\tilde{t}_L^\dagger, \tilde{t}_R^\dagger) \mathbf{M}_{\tilde{t}} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \quad (6)$$

contains the stop mass matrix, $\mathbf{M}_{\tilde{t}}$, given by

$$\mathbf{M}_{\tilde{t}} = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_t^2 + M_Z^2 \cos 2\beta (T_t^3 - Q_t s_w^2) & m_t X_t \\ m_t X_t & M_{\tilde{t}_R}^2 + m_t^2 + M_Z^2 \cos 2\beta Q_t s_w^2 \end{pmatrix}, \quad (7)$$

with

$$X_t = A_t - \mu \cot \beta. \quad (8)$$

Q_t and T_t^3 denote charge and isospin of the top quark, A_t is the trilinear coupling between the Higgs bosons and the scalar top quarks, and μ is the Higgs mixing parameter. The mass matrix can be diagonalized with the help of a unitary transformation $\mathbf{U}_{\tilde{t}}$, yielding the two stop mass eigenvalues, $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$.

For the MSSM with real parameters the status of higher-order corrections to the masses and mixing angles in the neutral Higgs sector is quite advanced. The complete one-loop result within the MSSM is known [22–25]. The by far dominant one-loop contribution is the $\mathcal{O}(\alpha_t)$ term due to top and stop loops ($\alpha_t \equiv h_t^2/(4\pi)$, h_t being the top-quark Yukawa coupling). The computation of the two-loop corrections has meanwhile reached a stage where all the presumably dominant contributions are available [26–40]. In particular, the $\mathcal{O}(\alpha_t \alpha_s)$ contributions to the self-energies – evaluated in the Feynman-diagrammatic (FD) as well as in the effective potential (EP) method – as well as the $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_b \alpha_s)$, $\mathcal{O}(\alpha_t \alpha_b)$ and $\mathcal{O}(\alpha_b^2)$ contributions – evaluated in the EP approach – are known. (For latest corrections to the charged Higgs boson mass, see Ref. [41].)

The public code `FeynHiggs` [10, 19, 20, 27, 42] includes all of the above corrections, where the on-shell (OS) scheme for the renormalization of the scalar quark sector has been used (another public code, based on the Renormalization Group (RG) improved Effective Potential, is `CPsuperH` [43]). A full 2-loop effective potential calculation (supplemented by the momentum dependence for the leading pieces and the leading 3-loop corrections) has been published [44, 45]. However, no computer code is publicly available. Most recently another leading 3-loop calculation at $\mathcal{O}(\alpha_t \alpha_s^2)$ became available (based on a $\overline{\text{DR}}$ or a “hybrid” renormalisation scheme for the scalar top sector), where the numerical evaluation depends on the various SUSY mass hierarchies [14], resulting in the code `H3m` (which adds the 3-loop corrections to the `FeynHiggs` result).

3 Improved calculation of M_h

We review here the improved prediction for M_h where we combine the fixed-order result obtained in the OS scheme with an all-order resummation of the leading and subleading contributions from the scalar top sector. We have obtained the latter from an analysis of the RG Equations (RGEs) at the two-loop level [46]. Assuming a common mass scale $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ ($M_S \gg M_Z$) for all relevant SUSY mass parameters, the quartic Higgs coupling λ can be evolved via SM RGEs from M_S to the scale Q (we choose $Q = m_t$ in the following) where M_h^2 is to be evaluated (see, for instance, Ref. [30] and references therein),

$$M_h^2 = 2\lambda(m_t)v^2. \quad (9)$$

Here $v \sim 174$ GeV denotes the vacuum expectation value of the SM. Three coupled RGEs, the ones for

$$\lambda, h_t, g_s \tag{10}$$

are relevant for this evolution, with the strong coupling constant given as $\alpha_s = g_s^2/(4\pi)$. Since SM RGEs are used, the relevant parameters are given in the $\overline{\text{MS}}$ scheme. We incorporate the one-loop threshold corrections to $\lambda(M_S)$ as given in Ref. [30],

$$\lambda(M_S) = \frac{3 h_t^4(M_S)}{8 \pi^2} \frac{X_t^2}{M_S^2} \left[1 - \frac{1}{12} \frac{X_t^2}{M_S^2} \right], \tag{11}$$

where as mentioned above X_t is an $\overline{\text{MS}}$ parameter. Furthermore, in Eq. (11) we have set the SM gauge couplings to $g = g' = 0$, ensuring that Eq. (9) consists of the ‘‘pure loop correction’’ and will be denoted $(\Delta M_h^2)^{\text{RGE}}$ below. Using RGEs at two-loop order [46], including fermionic contributions from the top sector only, leads to a prediction for the corrections to M_h^2 including leading and subleading logarithmic contributions at n -loop order,

$$L^n \text{ and } L^{(n-1)}, \quad L \equiv \ln \left(\frac{M_S}{m_t} \right), \tag{12}$$

originating from the top/stop sector of the MSSM.

We have obtained both analytic solutions of the RGEs up to the 7-loop level as well as a numerical solution incorporating the leading and subleading logarithmic contributions up to all orders. In a similar way in Ref. [45] the leading logarithms at 3- and 4-loop order have been evaluated analytically. Most recently a calculation using 3-loop SM RGEs appeared in Ref. [47].

A particular complication arises in the combination of the higher-order logarithmic contributions obtained from solving the RGEs with the fixed-order FD result implemented in `FeynHiggs` comprising corrections up to the two-loop level in the OS scheme. We have used the parametrisation of the FD result in terms of the running top-quark mass at the scale m_t ,

$$\overline{m}_t = \frac{m_t^{\text{pole}}}{1 + \frac{4}{3\pi} \alpha_s(m_t^{\text{pole}}) - \frac{1}{2\pi} \alpha_t(m_t^{\text{pole}})}, \tag{13}$$

where m_t^{pole} denotes the top-quark pole mass. Avoiding double counting of the logarithmic contributions up to the two-loop level and consistently taking into account the different schemes employed in the FD and the RGE approach, the correction ΔM_h^2 takes the form

$$\begin{aligned} \Delta M_h^2 &= (\Delta M_h^2)^{\text{RGE}}(X_t^{\overline{\text{MS}}}) - (\Delta M_h^2)^{\text{FD,LL1,LL2}}(X_t^{\text{OS}}), \\ M_h^2 &= (M_h^2)^{\text{FD}} + \Delta M_h^2. \end{aligned} \tag{14}$$

Here $(M_h^2)^{\text{FD}}$ denotes the fixed-order FD result, $(\Delta M_h^2)^{\text{FD,LL1,LL2}}$ are the logarithmic contributions up to the two-loop level obtained with the FD approach in the OS scheme, while $(\Delta M_h^2)^{\text{RGE}}$ are the leading and sub-leading logarithmic contributions (either up to a certain loop order or summed to all orders) obtained in the RGE approach, as evaluated via

Eq. (9). In all terms of Eq. (14) the top-quark mass is parametrised in terms of \overline{m}_t ; the relation between $X_t^{\overline{\text{MS}}}$ and X_t^{OS} is given by

$$X_t^{\overline{\text{MS}}} = X_t^{\text{OS}} \left[1 + 2L \left(\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi} \right) \right] \quad (15)$$

up to non-logarithmic terms, and there are no logarithmic contributions in the relation between $M_S^{\overline{\text{MS}}}$ and M_S^{OS} .

Since our higher-order contributions beyond 2-loop have been derived under the assumption $M_A \gg M_Z$, to a good approximation these corrections can be incorporated as a shift in the prediction for the $\phi_2\phi_2$ self-energy (where ΔM_h^2 enters with a coefficient $1/\sin^2\beta$). In this way the new higher-order contributions enter not only the prediction for M_h , but also consistently the ones all other Higgs sector observables that are evaluated in `FeynHiggs`, such as the effective mixing angle α_{eff} or the finite field renormalization constant matrix \mathbf{Z}_n [20].

The latest version of the code, `FeynHiggs 2.10.0`, which is available at `feynhiggs.de`, contains those improved predictions as well as a refined estimate of the theoretical uncertainties from unknown higher-order corrections. Taking into account the leading and subleading logarithmic contributions in higher orders reduces the uncertainty of the remaining unknown higher-order corrections. Accordingly, the estimate of the uncertainties arising from corrections beyond two-loop order in the top/stop sector is adjusted such that the impact of replacing the running top-quark mass by the pole mass (see Ref. [10]) is evaluated only for the non-logarithmic corrections rather than for the full two-loop contributions implemented in `FeynHiggs`. First investigations using this new uncertainty estimate can be found in Refs. [17, 48].

Further refinements of the RGE resummed result are possible, in particular

- extending the result to the case of a large splitting between the left- and right-handed soft SUSY-breaking terms in the scalar top sector [36],
- extending the result to the case of small M_A or μ (close to M_Z),
- including the corresponding contributions from the (s)bottom sector.

Some details in these directions can be found in Ref. [47]. We leave those refinements for future work.

4 Numerical analysis

In this section we review the analysis of the phenomenological implications of the improved M_h prediction for large stop mass scales, as evaluated with `FeynHiggs 2.10.0`. Here and in the following X_t denotes X_t^{OS} (for M_S the difference in the two schemes is negligible). The other parameters are $M_A = M_2 = \mu = 1000$ GeV, $m_{\tilde{g}} = 1600$ GeV (where M_2 is the SU(2) gaugino mass term, μ the Higgsino mass parameter and $m_{\tilde{g}}$ the gluino mass) and $\tan\beta = 10$.

In Fig. 1 we show $M_h(X_t/M_S)$ for various values of M_S (as indicated by different colors), evaluated with the full new result as implemented into `FeynHiggs 2.10.0`. It can be seen

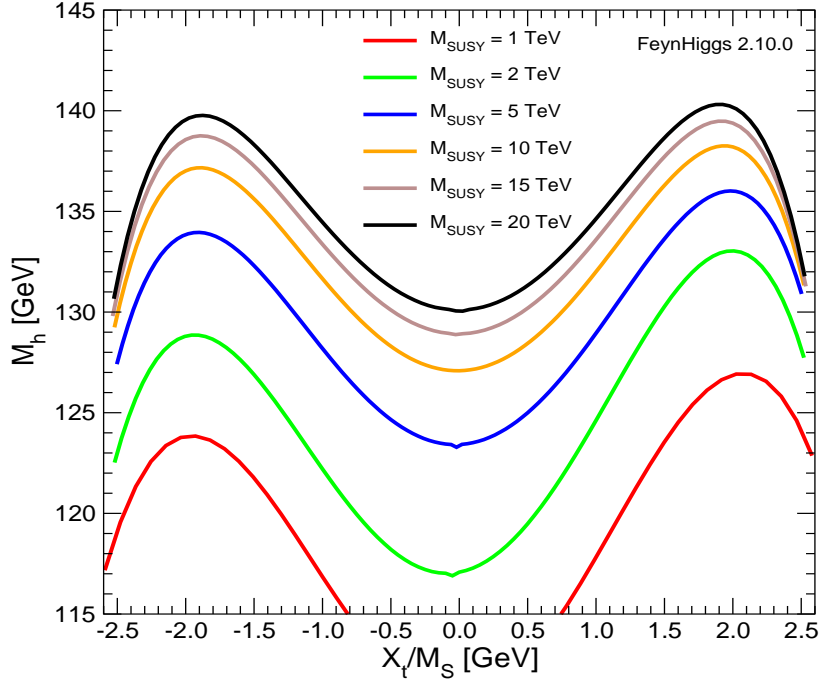


Figure 1: M_h as a function of X_t/M_S for various values of M_S , obtained using the full result as implemented into FeynHiggs 2.10.0.

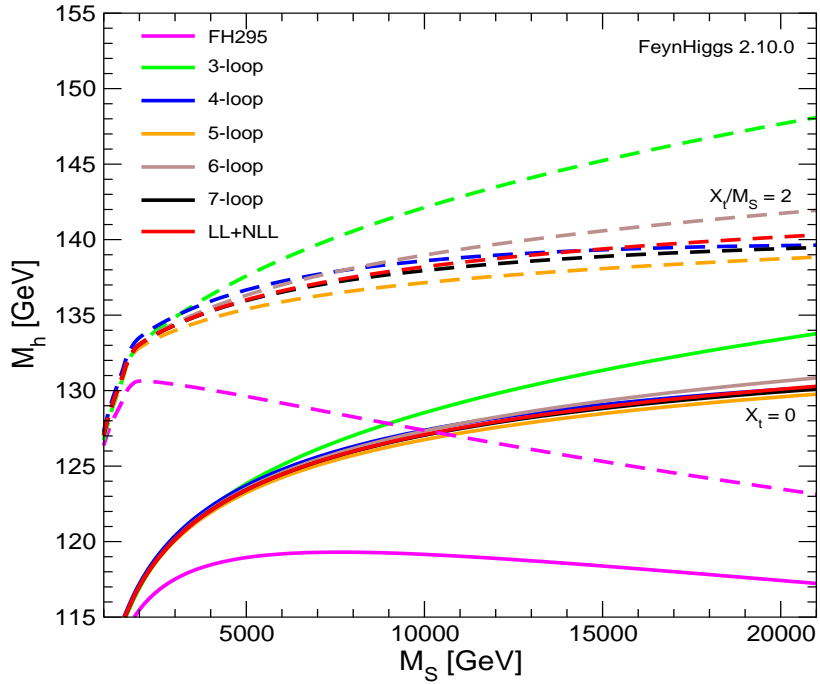


Figure 2: M_h as a function of M_S for $X_t = 0$ (solid) and $X_t/M_S = 2$ (dashed). The full result (“LL+NLL”) is compared with results containing the logarithmic contributions up to the 3-loop, ... 7-loop level and with the fixed-order FD result (“FH295”).

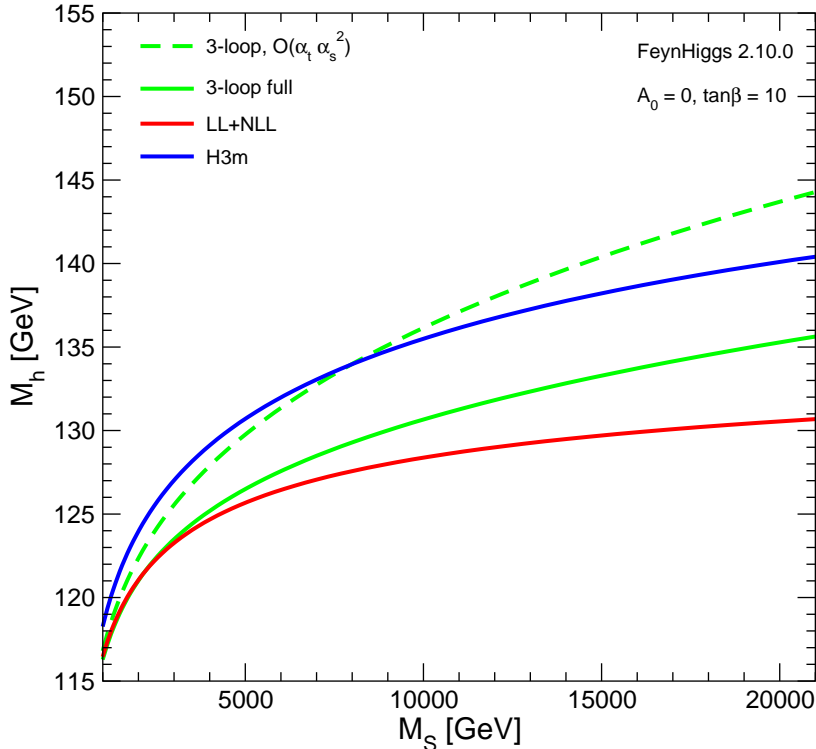


Figure 3: M_h as a function of M_S . Comparison of `FeynHiggs` (red) with `H3m` (blue). In green we show the `FeynHiggs` 3-loop result at $\mathcal{O}(\alpha_t \alpha_s^2)$ ($\mathcal{O}(\alpha_t \alpha_s^2, \alpha_t^2 \alpha_s, \alpha_t^3)$) as dashed (solid) line.

that local maxima are reached for $X_t/M_S \approx \pm 2$, where the M_h values at $X_t/M_S \approx +2$ are slightly larger than the ones at $X_t/M_S \approx -2$. Local minima are reached around $X_t/M_S \approx 0$. This feature was well known for the results at the 2-loop level, and now are shown to persist for the inclusion of the resummed leading and subleading logarithmic corrections. It can furthermore be seen that the current experimental value of $M_h = (125.6 \pm 0.35)$ GeV can be reached for many combinations of X_t and M_S (for the other parameters fixed as given above), but on the other hand, that many of such combinations can also be ruled out by the Higgs-boson mass constraint. For a more detailed analysis the theoretical uncertainties have to be taken into account properly, see the discussion below.

The results of Fig. 1 motivate the choice of parameters used in Fig. 2. Here we show M_h as a function of M_S for $X_t = 0$ and $X_t/M_S = +2$, corresponding to the local minimum and the maximum value of M_h as a function of X_t/M_S , respectively. The plot shows for the two values of X_t/M_S the fixed-order FD result containing corrections up to the two-loop level (labelled as “FH295”, which refers to the previous version of the code `FeynHiggs`) as well as the latter result supplemented with the analytic solution of the RGEs up to the 3-loop, . . . 7-loop level (labelled as “3-loop” . . . “7-loop”). The curve labelled as “LL+NLL” represents our full result, where the FD contribution is supplemented by the leading and next-to-leading logarithms summed to all orders. One can see that the impact of the higher-order logarithmic contributions is relatively small for $M_S = \mathcal{O}(1 \text{ TeV})$, while large differences between the fixed-order result and the improved results occur for large values of M_S . The

3-loop logarithmic contribution is found to have the largest impact in this context, but for $M_S \gtrsim 2500(6000)$ GeV for $X_t/M_S = 2(0)$ also contributions beyond 3-loop are important. A convergence of the higher-order logarithmic contributions towards the full resummed result is clearly visible. At $M_S = 20$ TeV the difference between the 7-loop result and the full resummed result is around 900(200) MeV for $X_t/M_S = 2(0)$. The corresponding deviations stay below 100 MeV for $M_S \lesssim 10$ TeV. The plot furthermore shows that for $M_S \approx 7$ TeV (and the value of $\tan\beta = 10$ chosen here) a predicted value of M_h of about 125.6 GeV is obtained even for the case of vanishing mixing in the scalar top sector ($X_t = 0$). Since the predicted value of M_h grows further with increasing M_S it becomes apparent that the measured mass of the observed signal, when interpreted as M_h , can be used (within the current experimental and theoretical uncertainties) to derive an *upper bound* on the mass scale M_S in the scalar top sector, see also Ref. [49]. However, more robust statements in this direction will require a careful analysis of still present intrinsic as well as the parametric uncertainties.

Finally, in Fig. 3 we compare our result with the prediction obtained from the code `H3m` [14]. The comparison was performed in the CMSSM with the parameters set to $m_0 = m_{1/2} = 200$ GeV...15000 GeV, $A_0 = 0$, $\tan\beta = 10$ and $\mu > 0$. The spectra were generated with `SoftSusy 3.3.10` [50]. The `H3m` result shown as blue line, containing the terms in $\mathcal{O}(\alpha_t\alpha_s^2)\times\mathcal{O}(L^3, L^2, L^1, L^0)$, can be compared with the `FeynHiggs` 3-loop result, $\mathcal{O}(L^3, L^2)$, but restricted to $\mathcal{O}(\alpha_t\alpha_s^2)$ (green dashed). We find that the latter result agrees rather well with `H3m`, with maximal deviation of $\mathcal{O}(1$ GeV) for $M_S \lesssim 10$ TeV. The observed deviations can be attributed to the terms of $\mathcal{O}(L^1, L^0)$ included in `H3m`, to the SUSY mass hierarchies taken into account in `H3m`, and to the use of different scalar top renormalization schemes employed in the two codes (where the latter effect is already expected to be at the GeV-level). Further investigations will be needed to explore the source of the main differences. However, adding also the 3-loop $\mathcal{O}(\alpha_t^2\alpha_s, \alpha_t^3)\times\mathcal{O}(L^3, L^2)$ terms (solid green), as included in the `FeynHiggs` result, leads to a strong reduction of M_h by ~ 5 GeV for $M_S = 10$ TeV (see also Ref. [45]). Going to the full resummed `FeynHiggs` result (red) exhibits a further, but smaller reduction of M_h of about 2 GeV for $M_S = 10$ TeV, even larger changes are found for $M_S > 10$ TeV. Consequently, 3-loop corrections at $\mathcal{O}(\alpha_t^2\alpha_s, \alpha_t^3)$ as well as corrections beyond 3-loop are clearly important for a precise M_h prediction.

In view of the anticipated future accuracy at the ILC, as given in Eq. (2), the remaining theory uncertainties in the current status of the M_h evaluations will have to be re-analyzed carefully. It can be expected, see also Ref. [48], that for scalar top mass scales below the few-TeV level the intrinsic uncertainty is now, i.e. including the resummed contributions, at or below the level of ~ 2 GeV. However, still substantial further refinements will be needed to reach the sub-GeV level. On the other hand, no investigation of the size of the intrinsic uncertainties has been performed for scalar top masses in the multi-TeV range, as explored here. Consequently, the prospects of reaching the sub-GeV level in δM_h^{intr} are so far unclear. Turning to the parametric uncertainties, the ILC precision for m_t with $\delta m_t^{\text{ILC}} \lesssim 100$ MeV [5] will be crucial to lower the m_t -induced uncertainty to the level of the anticipated ILC accuracy for M_h . Being stuck with a hadron collider precision on m_t will always induce a parametric uncertainty in the M_h prediction substantially larger than even the current experimental uncertainty as given in Eq. (1).

5 Conclusions

We have reviewed the improved prediction for the light \mathcal{CP} -even Higgs-boson mass in the MSSM, obtained by combining the FD result at the one- and two-loop level with an all-order resummation of the leading and subleading logarithmic contributions from the top/stop sector obtained from solving the two-loop RGEs. Particular care has been taken to consistently match these two different types of corrections. The result, providing the most precise prediction for M_h in the presence of large masses of the scalar partners of the top quark, has been implemented into the public code `FeynHiggs` and can be obtained at `feynhiggs.de`. We have found a sizable effect of the higher-order logarithmic contributions for $M_S \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \gtrsim 2$ TeV which grows with increasing M_S . In comparison with `H3m`, which calculates the $\mathcal{O}(\alpha_t \alpha_s^2)$ corrections to M_h we find that both, corrections at 3-loop at $\mathcal{O}(\alpha_t^2 \alpha_s, \alpha_t^3)$ as well as corrections beyond 3-loop are important for a precise M_h prediction; for $M_S = 10$ TeV differences of ~ 7 GeV are found between `H3m` and `FeynHiggs` (for the parameters used in our numerical analysis). Finally, we have shown that for sufficiently high M_S the predicted values of M_h reaches about 125.6 GeV even for vanishing mixing in the scalar top sector. As a consequence, even higher M_S values are disfavoured by the measured mass value of the Higgs signal.

Reaching an intrinsic uncertainty in the M_h prediction at the sub-GeV level will require the inclusion of substantially more higher-order corrections. This accuracy will be crucial even to meet the current LHC precision. Reaching the future LHC precision or even the ILC precision of M_h will be correspondingly more demanding.

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References

- [1] G. Aad et al. [ATLAS Collaboration], *Phys. Lett. B* **716** (2012) 1 [arXiv:1207.7214 [hep-ex]].
- [2] S. Chatrchyan et al. [CMS Collaboration], *Phys. Lett. B* **716** (2012) 30 [arXiv:1207.7235 [hep-ex]].
- [3] ATLAS Collaboration, ATLAS-CONF-2013-014, ATLAS-CONF-2013-025.
- [4] CMS Collaboration, CMS-PAS-HIG-13-005.
- [5] H. Baer et al., arXiv:1306.6352 [hep-ph].
- [6] S. Heinemeyer et al. [LHC Higgs Cross Section Working Group Collaboration], arXiv:1307.1347 [hep-ph].

- [7] A. David et al. [LHC Higgs Cross Section Working Group Collaboration], arXiv:1209.0040 [hep-ph].
- [8] E. Gross, talk given at “Rencontres de Moriond EW 2014”, see <https://indico.in2p3.fr/materialDisplay.py?contribId=218&sessionId=8&materialId=slides&confId=9116> ,
E. Musella, talk given at “Rencontres de Moriond EW 2014”, see <https://indico.in2p3.fr/materialDisplay.py?contribId=219&sessionId=8&materialId=slides&confId=9116> .
- [9] H. Nilles, *Phys. Rept.* **110** (1984) 1;
H. Haber and G. Kane, *Phys. Rept.* **117** (1985) 75;
R. Barbieri, *Riv. Nuovo Cim.* **11** (1988) 1.
- [10] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, *Eur. Phys. J. C* **28** (2003) 133 [arXiv:hep-ph/0212020].
- [11] A. Djouadi, *Phys. Rept.* **459** (2008) 1 [arXiv:hep-ph/0503173];
S. Heinemeyer, *Int. J. Mod. Phys. A* **21** (2006) 2659 [arXiv:hep-ph/0407244].
- [12] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rept.* **425** (2006) 265 [arXiv:hep-ph/0412214].
- [13] S. Heinemeyer, S. Kraml, W. Porod and G. Weiglein, *JHEP* **0309** (2003) 075 [arXiv:hep-ph/0306181].
- [14] R. Harlander, P. Kant, L. Mihaila and M. Steinhauser, *Phys. Rev. Lett.* **100** (2008) 191602 [*Phys. Rev. Lett.* **101** (2008) 039901] [arXiv:0803.0672 [hep-ph]]; *JHEP* **1008** (2010) 104 [arXiv:1005.5709 [hep-ph]].
- [15] S. Heinemeyer, O. Stål and G. Weiglein, *Phys. Lett. B* **710** (2012) 201 [arXiv:1112.3026 [hep-ph]];
L. Hall, D. Pinner and J. Ruderman, *JHEP* **1204** (2012) 131 [arXiv:1112.2703 [hep-ph]];
H. Baer, V. Barger and A. Mustafayev, *Phys. Rev. D* **85** (2012) 075010 [arXiv:1112.3017 [hep-ph]];
A. Arbey, M. Battaglia, A. Djouadi, F. Mahmoudi and J. Quevillon, *Phys. Lett. B* **708** (2012) 162 [arXiv:1112.3028 [hep-ph]];
P. Draper, P. Meade, M. Reece and D. Shih, *Phys. Rev. D* **85** (2012) 095007 [arXiv:1112.3068 [hep-ph]].
- [16] O. Buchmueller et al., *Eur. Phys. J. C* **72** (2012) 2243 [arXiv:1207.7315 [hep-ph]].
- [17] O. Buchmueller et al., arXiv:1312.5250 [hep-ph].
- [18] P. Bechtle et al., arXiv:1310.3045 [hep-ph].
- [19] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, to appear in *Phys. Rev. Lett.*, arXiv:1312.4937 [hep-ph].

- [20] M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *JHEP* **0702** (2007) 047 [arXiv:hep-ph/0611326].
- [21] S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *Phys. Lett.* **B 652** (2007) 300 [arXiv:0705.0746 [hep-ph]].
- [22] J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **B 257** (1991) 83;
Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1;
H. Haber and R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815.
- [23] A. Brignole, *Phys. Lett.* **B 281** (1992) 284.
- [24] P. Chankowski, S. Pokorski and J. Rosiek, *Phys. Lett.* **B 286** (1992) 307; *Nucl. Phys.* **B 423** (1994) 437 [arXiv:hep-ph/9303309].
- [25] A. Dabelstein, *Nucl. Phys.* **B 456** (1995) 25 [arXiv:hep-ph/9503443]; *Z. Phys.* **C 67** (1995) 495 [arXiv:hep-ph/9409375].
- [26] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rev.* **D 58** (1998) 091701 [arXiv:hep-ph/9803277]; *Phys. Lett.* **B 440** (1998) 296 [arXiv:hep-ph/9807423].
- [27] S. Heinemeyer, W. Hollik and G. Weiglein, *Eur. Phys. J.* **C 9** (1999) 343 [arXiv:hep-ph/9812472].
- [28] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Lett.* **B 455** (1999) 179 [arXiv:hep-ph/9903404].
- [29] S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *Eur. Phys. J.* **C 39** (2005) 465 [arXiv:hep-ph/0411114].
- [30] M. Carena, H. Haber, S. Heinemeyer, W. Hollik, C. Wagner, and G. Weiglein, *Nucl. Phys.* **B 580** (2000) 29 [arXiv:hep-ph/0001002].
- [31] R. Zhang, *Phys. Lett.* **B 447** (1999) 89 [arXiv:hep-ph/9808299];
J. Espinosa and R. Zhang, *JHEP* **0003** (2000) 026 [arXiv:hep-ph/9912236].
- [32] G. Degrassi, P. Slavich and F. Zwirner, *Nucl. Phys.* **B 611** (2001) 403 [arXiv:hep-ph/0105096].
- [33] R. Hempfling and A. Hoang, *Phys. Lett.* **B 331** (1994) 99 [arXiv:hep-ph/9401219].
- [34] A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, *Nucl. Phys.* **B 631** (2002) 195 [arXiv:hep-ph/0112177].
- [35] J. Espinosa and R. Zhang, *Nucl. Phys.* **B 586** (2000) 3 [arXiv:hep-ph/0003246].
- [36] J. Espinosa and I. Navarro, *Nucl. Phys.* **B 615** (2001) 82 [arXiv:hep-ph/0104047].
- [37] A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, *Nucl. Phys.* **B 643** (2002) 79 [arXiv:hep-ph/0206101].

- [38] G. Degrassi, A. Dedes and P. Slavich, *Nucl. Phys. B* **672** (2003) 144 [arXiv:hep-ph/0305127].
- [39] M. Carena, J. Espinosa, M. Quirós and C. Wagner, *Phys. Lett. B* **355** (1995) 209 [arXiv:hep-ph/9504316];
M. Carena, M. Quirós and C. Wagner, *Nucl. Phys. B* **461** (1996) 407 [arXiv:hep-ph/9508343].
- [40] J. Casas, J. Espinosa, M. Quirós and A. Riotto, *Nucl. Phys. B* **436** (1995) 3, [Erratum-
ibid. **B 439** (1995) 466] [arXiv:hep-ph/9407389].
- [41] M. Frank et al., *Phys. Rev. D* **88** (2013) 055013 [arXiv:1306.1156 [hep-ph]].
- [42] S. Heinemeyer, W. Hollik and G. Weiglein, *Comput. Phys. Commun.* **124** (2000) 76 [arXiv:hep-ph/9812320];
T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *Comput. Phys. Commun.* **180** (2009) 1426; see: www.feynhiggs.de .
- [43] J. Lee et al., *Comput. Phys. Commun.* **156** (2004) 283; J. Lee et al., *Comput. Phys. Commun.* **180** (2009) 312; *Comput. Phys. Commun.* **184** (2013) 1220.
- [44] S. Martin, *Phys. Rev. D* **65** (2002) 116003 [arXiv:hep-ph/0111209]; *Phys. Rev. D* **66** (2002) 096001 [arXiv:hep-ph/0206136]; *Phys. Rev. D* **67** (2003) 095012 [arXiv:hep-ph/0211366]; *Phys. Rev. D* **68** (2003) 075002 [arXiv:hep-ph/0307101]; *Phys. Rev. D* **70** (2004) 016005 [arXiv:hep-ph/0312092]; *Phys. Rev. D* **71** (2005) 116004 [arXiv:hep-ph/0502168]; S. Martin and D. Robertson, *Comput. Phys. Commun.* **174** (2006) 133 [arXiv:hep-ph/0501132].
- [45] S. Martin, *Phys. Rev. D* **75** (2007) 055005 [arXiv:hep-ph/0701051].
- [46] J. Espinosa and M. Quiros, *Phys. Lett. B* **266** (1991) 389; H. Arason et al., *Phys. Rev. D* **46** (1992) 3945.
- [47] P. Draper, G. Lee and C. Wagner, arXiv:1312.5743 [hep-ph].
- [48] O. Buchmueller et al., to appear in *Eur. Phys. J. C*, arXiv:1312.5233 [hep-ph].
- [49] M. Cabrera, J. Casas and A. Delgado, *Phys. Rev. Lett.* **108** (2012) 021802 [arXiv:1108.3867 [hep-ph]];
G. Degrassi, S. Di Vita, J. Elias-Miro, J. Espinosa, G. Giudice, G. Isidori and A. Strumia, *JHEP* **1208** (2012) 098 [arXiv:1205.6497 [hep-ph]].
- [50] B. Allanach, *Comput. Phys. Commun.* **143** (2002) 305. [arXiv:hep-ph/0104145].