TMDlib and TMDplotter:
library and plotting tools for
Transverse Momentum Dependent parton distributions
Version 1.1.00

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Abstract

Transverse-momentum-dependent distributions (TMDs) play a crucial role in high-energy physics from the theoretical and phenomenological point of view. The library TMDlib of transverse-momentum-dependent parton distribution functions (TMD PDFs) and fragmentation functions (TMD FFs) is here introduced together with the online plotting tool TMDplotter. We give a description of the program components with emphasis on the different physical frameworks the user can access via the available parametrizations.

PROGRAM SUMMARY

Title of Program: TMDlib 1.1.00

Computer for which the program is designed and others on which it is operable: any with standard C++, tested on SGI, HP-UX, SUN, PC, MAC

Programming Language used: C++

High-speed storage required: No

Separate documentation available: No

Keywords: QCD, small-x physics, TMD factorization, high-energy factorization, TMD PDFs, TMD FFs, unintegrated PDFs.

Other Program used: ROOT for plotting the result.

Download of the program: `http://tmdlib.hepforge.org`

Unusual features of the program: None

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Citation policy: please cite the current version of the manual and the paper(s) related to the parametrization(s).

1 Introduction

The Quantum Chromodynamics (QCD) interpretation of high-energy particle reactions requires a simultaneous treatment of processes at different energy scales. Factorization theorems provide the mathematical framework to properly separate the physical regimes. For instance, when two protons collide in a Drell-Yan (DY) event, we describe the high-energy partonic cross section with the tools provided by perturbative QCD and the soft physics underlying the structure of the hadrons with parton distribution functions. The same picture applies to other (semi-)inclusive processes, including Deep-Inelastic lepton-hadron Scattering (DIS), Semi-Inclusive DIS (SIDIS), e^+e^- annihilation into hadrons. In the last three cases, fragmentation functions also enter the cross section expressions to describe the transition from the partonic states to the observed hadrons.

Transverse-momentum-dependent (TMD) factorization has been proven for SIDIS, DY and e^+e^- annihilation: this has required the introduction of TMD distribution functions (TMD PDFs) and fragmentation functions (TMD FFs), collectively called TMDs [1–3]. For particular processes in hadron-hadron scattering, like heavy flavor or vector boson (and Higgs) production, TMD factorization has also been proven in the high-energy (small x) limit [4–6]. In this case, the functions encoding the hadronic structure are referred to as *unintegrated* parton distribution functions (uPDFs), see *e.g.* Refs. [7–10].

The different theoretical frameworks for factorization and evolution in which TMDs and uPDFs are defined make a comparison still difficult. In this paper we describe a tool to collect the different ansatz and parametrizations into a single library: TMDlib and the online plotter tool TMDplotter. Provided that the user takes into account all the possible differences, collecting parametrizations for both the objects in TMDlib and TMDplotter will also make phenomenological comparisons easier. The paper is organized as follows. In Sec. 2, we review the theoretical framework for both TMD and high-energy factorization and evolution. In Sec. 3, we present a condensed documentation of the TMDlib library and TMDplotter tool, discussing the basic procedure to readily use them.

2 Theoretical framework

In this Section, we briefly review two different frameworks for factorization and evolution of parton distributions. In particular, we compare TMD and high-energy factorization theorems and evolution equations.

2.1 TMD factorization and evolution

When one hard scale enters a high-energy process (like the invariant mass of the exchanged virtual photon in DIS) and the relevant transverse momenta are integrated over, we apply *collinear* factorization to separate the hard partonic physics from the soft hadronic one. When multiple scales are involved in the process, we need to go beyond the collinear framework in order to separate the physical regimes. For example, this is the case of processes with

obsverved transverse momenta in the final states, like SIDIS and DY lepton pair production at low transverse momentum. In these cases the low transverse momentum sets the second relevant physical scale and TMD factorization introduces a richer physical structure with respect to the collinear case. If the observable transverse momenta are comparable to the hard scale Q, there is no reason for introducing another physical region and standard integrated (collinear) factorization still applies.

Feynman rules allow for a decomposition of the cross section as a contraction of hadronic and leptonic tensors. Where applicable, factorization theorems separate hard and soft contributions within the hadronic tensor, introducing distribution and fragmentation correlators. Depending on the twist order, the partonic polarization and the polarization of the target, TMD distribution and fragmentation functions are introduced. For example, in fully unpolarized SIDIS at leading twist the hadronic tensor is factorized in a convolution of one unpolarized TMD PDF (for the incoming target hadron) and one unpolarized TMD FF (for the final state hadron):

$$W^{\mu\nu} \sim \mathcal{H}^{\mu\nu}(Q;\mu) \int d^2 b_\perp e^{-iq_\perp \cdot b_\perp} f_1^{a,T}(x, b_\perp; \zeta, \mu) \, D_1^{a \to h}(z, b_\perp; \zeta, \mu) \, + \mathcal{O}(q_\perp/Q) \,, \quad (1)$$

where $\mathcal{H}$ is the hard part, a is the flavor of the struck parton, T is the target hadron, h is the detected hadron, x and z are the light-cone momentum fractions, $b_\perp$ is the impact parameter (*i.e.* the Fourier-conjugated of the transverse momentum $q_\perp$) and the last terms quantifies corrections to the expression valid for low transverse momenta. Taking into account all the possible combinations of polarization (parton, target and detected hadron), there are nine TMD PDFs and eight TMD FFs at leading-twist [11] and the expression of the hadronic tensor modifies accordingly [12].

TMD parton distributions or fragmentation functions depend on two scales, ζ an μ and they satisfy evolution equations with respect to both of them. The evolution in μ is given by the standard renormalization group equations (DGLAP), while the evolution with respect to the energy variable ζ is determined by a process-independent soft factor [1,2].

2.2 High-energy factorization and evolution

In the framework of high-energy factorization [4,13,14] the DIS cross section can be written as a convolution in both longitudinal and transverse momenta of the unintegrated parton density function $\mathcal{A}(x, k_t, \mu)$ with off-shell partonic matrix elements

$$\sigma_j(x, Q^2) = \int_x^1 dz \int d^2 k_t \, \hat{\sigma}_j(x, Q^2, z, k_t) \, \mathcal{A}(z, k_t, \mu) \,. \quad (2)$$

with the DIS cross sections σ_j, $(j = 2, L)$ related to the structure functions F_2 and F_L. The hard-scattering kernels $\hat{\sigma}_j$ of Eq. (2) are k_t-dependent and the evolution of the gluon uPDF $\mathcal{A}$ is obtained by combining the resummation of small-x logarithmic contributions [15–17] with medium- and large-x contributions to parton splitting [18–20], according to the CCFM evolution equations [21–24].

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The factorization formula, Eq. (2), allows for resummation of logarithmically enhanced $x \to 0$ contributions to all orders in perturbation theory, both in the hard-scattering coefficients and in the parton evolution, taking into account the full dependence on the factorization scale μ and on the factorization scheme [25, 26].

The cross section σ_j, $(j = 2, L)$ is usually computed in a Fixed Flavour Number (FFN) scheme, where only the photon-gluon fusion process ($\gamma^* g^* \to q\bar{q}$) is included. The masses of the quarks are explicitly included with the light and heavy quark masses being free parameters. In addition to $\gamma^* g^* \to q\bar{q}$, the contribution from valence quarks is included via $\gamma^* q \to q$ by using CCFM evolution of valence quarks [27, 28]. A fit of CCFM uPDFs to DIS has been recently presented in Ref. [28, 29] using the CCFM evolution as given in [30], while the gluon unintegrated distribution has been obtained in Ref. [31] by means of a saturation ansatz.

3   TMDlib documentation

In this Section, we provide the user with a condensed documentation of the TMDlib library, discussing the basic procedure to readily install and use it.

The source code of TMDlib is available from the following web page:

http://tmdlib.hepforge.org/

and can be installed using the *standard* `autotools` sequence

```
1    ./configure
2    make
3    make install
```

By default and if the user has root privileges, the installation path is `usr/local/` and the files with the available TMD/uPDF distributions will also be installed there. A different installation path can be specified as

```
1    ./configure ——prefix=/path/to/the/installation/folder
```

but, in this case, it should be added to the environmental variable LD_LIBRARY_PATH.

The TMDlib library requires the LHAPDF PDF library and the ROOT data analysis framework library to be installed.[1] The path to the LHAPDF library must be specified during the installation as an option of the `./configure` command as follows

```
1    ./configure ——with—lhapdf=/path/to/lhapdf
```

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[1] The current release of TMDlib assumes that LHAPDF5.9.0 or a more recent version has been previously installed. For details about the installation of LHAPDF, please see the corresponding hepforge web page [32].

parton	uPDF/TMD set	iset	$\Lambda_{qcd}^{(4)}$	k_t^{cut} [GeV]	Q_0 [GeV]	Ref.
gluon	ccfm-JS-2001	101000	0.25	0.25	1.4	[29]
	ccfm-setA0	101010	0.25	1.3	1.3	[29]
	ccfm-setA0+	101011	0.25	1.3	1.3	[29]
	ccfm-setA0-	101012	0.25	1.3	1.3	[29]
	ccfm-setA1	101013	0.25	1.3	1.3	[29]
	ccfm-setB0	101020	0.25	0.25	1.3	[29]
	ccfm-setB0+	101021	0.25	0.25	1.3	[29]
	ccfm-setB0-	101022	0.25	0.25	1.3	[29]
	ccfm-setB1	101023	0.25	0.25	1.3	[29]
	ccfm-JH-set 1	101001	0.25	1.33	1.33	[33]
	ccfm-JH-set 2	101002	0.25	1.18	1.18	[33]
	ccfm-JH-set 3	101003	0.25	1.35	1.35	[33]
	ccfm-JH-2013-set1	101201	0.2	2.2	2.2	[28]
	JH 2013 set2	101301	0.2	2.2	2.2	[28]
	GBWlight	200005	–	–	–	[31]
	GBWcharm	200006	–	–	–	[31]
quark	ccfm-setA0	–	0.25	1.3	1.3	
	ccfm-JH-2013-set1	–	0.2	2.2	2.2	[28]
	ccfm-JH-2013-set2	–	0.2	2.2	2.2	[28]
	SBRS-2013-TMDPDFs	300005	–	–	1.55	[34]

Table 1: Available uPDF/TMD parton sets in TMDlib.

Useful information about the installation and the determination of compiler flags in custom `makefiles` is provided by executing the `TMDlib-config` script.

Once TMDlib has been properly compiled and installed, the user has at her/his disposal a set of functions that can be called from a main program. We refrain from giving here a detailed description of all the functions available in TMDlib, since the library is supposed to be frequently updated as new features will be developed. The up-to-date list of these functions and their description are provided at `http://tmdlib.hepforge.org/pdfset.html`. Here, we restrict ourselves to discuss the basic steps required to handle the available uPDF/TMD distributions within TMDlib.

1. **INITIALIZATION.** The first step consists in initialising the desired uPDF/TMD set. This will assign the chosen uPDF/TMD set, specified by its name, an identifying number proper to that set. This number is stored into memory and called every time the identification of the uPDF/TMD set is needed by any TMDlib internal function.[2] If available Monte Carlo replica or uncertainty sets can be initialised. Also at this stage, the user can specify whether grid or parametrization (if available) should be used for uPDF/TMD distributions. The complete list of uPDF/TMD sets available in TMDlib 1.1.00 , with the corresponding name, identifying number, kinematic coverage, and reference, is given in Tab. 1.

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[2]Note that so far only one set of uPDF/TMD at a time can be called.

2. **CALL TO THE DISTRIBUTION.** The second step consists in calling the desired function. At this stage, the user must specify some input variables, typically the light-cone momentum fractions x^+ and x^-, carried by the parton, the parton transverse momentum k_t (in GeV), the energy scale μ (in GeV) and the flavor code identifying the target (*e.g.* proton or antiproton). The value for x times the uPDF/TMD is then returned, either for individual flavors, when available, or for a combination of them.

Both the initialisation and the call to the uPDF/TMD parton set are performed via function overloading of, respectively

```
1    TMDinit;
2    TMDpdf;
```

where the several methods and input/output variables for each of these functions are summarized in Tabs. 2-3. Additional useful code, including collection of methods and examples, can be found in the `src` folder. In particular:

- `TMDhandler.cc`: the code to handle the call for TMDlib, according to the two steps described above and the methods summarized in Tabs. 2-3;

- `TMDutils.cc`: a collection of methods used in TMDlib, including functions to get details about the initialised uPDF/TMD set (like α_s, Λ_{QCD}, number of flavors) as listed in Tab. 4;

- `TMD_test.cc`: an example program to handle uPDF/TMD distributions;

- `TMDplotter.cc`: a `ROOT`-based script to plot uPDF/TMD distributions as obtained from TMDlib.

The TMDlib library is released together with the online plotter platform TMDplotter, also available at

http://tmdplotter.desy.de/

This includes the same uPDF/TMD sets available in TMDlib (for details see Tab. 1) and allows for on-line plotting of distributions. In particular, they can be displayed at a given energy scale either as functions of the momentum fraction x or the transverse momentum k_t, respectively at fixed k_t or x values. Distributions integrated over k_t can also be plotted. Two snapshots from a typical usage of TMDplotter are shown in Fig. 1: the gluon from the `ccfm-JH-2013-set1` set is compared to the `GBW` as a function of k_t and x.

4 Conclusions and feedback

We have presented TMDlib and TMDplotter, respectively a `C++` library to handle different parametrizations of uPDF/TMD and the corresponding online plotting tool. Our aim is to

Method	Usage
`TMDinit(name)`	To initialise the uPDF/TMD set specified by its name `name`. A complete list of uPDF/TMD sets available in the current version of TMDlib with the corresponding name is provided in Tab. 1.
`TMDinit(name,irep)`	To initialise a given `irep` replica in a Monte Carlo uPDF/TMD set specified by its name `name`.
`TMDinit(name,irep,imode)`	To initialise the uncertainty sets with `irep` or to initialise a given `irep` replica in a Monte Carlo uPDF/TMD set specified by its name `name` and `imode`: • `imode=0`: the value obtained from the analytic form of the distribution is returned • `imode=1`: the value obtained as a polynomial interpolation on a numerical grid is returned • `imode=2`: the value obtained from the analytic form of the Fourier transform (b-space distribution)

Table 2: The function overload for `TMDinit` used to initialise uPDF/TMD parton sets.

Method	Usage
`TMDpdf(x,xbar,kt,mu, uval,dval,s,c,b,glu)`	To return the value of $xF(x, \bar{x}, k_t, \mu)$ (F is the initialised uPDF/TMD) for valence u-quarks `uval`, valence d-quarks `dval`, light sea-quarks `s`, charm-quarks `c`, bottom-quarks `b`, and gluons `glu` inside a proton. The input variables `x` and `xbar` are the light-come momentum fractions x^+ and x^- (in some parametrizations the latter is set to zero), `kt` is the parton transverse momentum k_t, and `mu` is the energy scale mu (in GeV).
`TMDpdf(kf,x,xbar,kt,mu, uval,dval,s,c,b,glu)`	As the function above, but for hadron with flavor code `kf` (`kf` = 2212 for proton and `kf` =-2212 for antiproton)
`TMDpdf(x,xbar,kt,mu)`	Void-type function returning the value of $xF(x, \bar{x}, k_t, \mu)$ (F is the initialised uPDF/TMD) as an array `xpq[13]`: at index $0, \ldots, 6$ is $\bar{t}, \ldots, \bar{d}$, at index 7 is the gluon, and at index $8, \ldots, 13$ is $d, \ldots, t$ densities for a proton target.
`TMDpdf(kf,x,xbar,kt,mu)`	As the function above, but for hadron with flavor code `kf` (`kf` = 2212 for proton and `kf` =-2212 for antiproton)
`TMDpdf(x,xbar,kt,mu,xpq)`	Double-type function returning the value of $xF(x, \bar{x}, k_t, \mu)$ (F is the initialised uPDF/TMD) as an array `xpq[13]`: at index $0, \ldots, 6$ is $\bar{t}, \ldots, \bar{d}$, at index 7 is the gluon, and at index $8, \ldots, 13$ is $d, \ldots, t$ densities for a proton target.
`TMDpdf(kf,x,xbar,kt,mu, xpq)`	As the function above, but for hadron with flavor code `kf` (`kf` = 2212 for proton and `kf` =-2212 for antiproton)

Table 3: The function overload for `TMDpdf` used to call uPDF/TMD parton sets.

Method	Usage
`TMDalphas(mu)`	Return α_s used in the set initialized by `TMDinit(name)`.
`TMDgetLam4( )`	Return the value of Λ_{QCD} at $N_f = 4$.
`TMDgetNf( )`	Return the number of flavors, N_f, used for the computation of Λ_{QCD}.
`TMDgetOrderAlphaS( )`	Return the perturbative order of α_s used in the evolution of the TMD/uPDF set initialized by `TMDinit(name)`.
`TMDgetOrderPDF( )`	Return the perturbative order of the evolution of the TMD/uPDF set initialized by `TMDinit(name)`.
`TMDgetXmin()`	Return the minimum value of the momentum fraction x for which the TMD/uPDF set initialized by `TMDinit(name)` was determined.
`TMDgetXmax()`	Return the maximum value of the momentum fraction x for which the TMD/uPDF set initialized by `TMDinit(name)` was determined.
`TMDgetQ2min()`	Return the minimum value of the energy scale μ (in GeV) for which the TMD/uPDF set initialized by `TMDinit(name)` was determined.
`TMDgetQ2max()`	Return the maximum value of the energy scale μ (in GeV) for which the TMD/uPDF set initialized by `TMDinit(name)` was determined.
`TMDnumberPDF(name)`	Return the number `iset` associated with the TMD/uPDF set initialized by `TMDinit(name)`.

Table 4: The list of methods included in the `TMDutils.cc` file.

update these tools with more uPDF/TMD parton sets and new features, as they will become available and will be developed. Future releases will contain parametrizations of TMD FFs too. We encourage discussions, feedback and comments about the TMDlib and TMDplotter projects, which can be addressed through the mailing list of the "TMDlib" project

http://tmdlib.hepforge.org
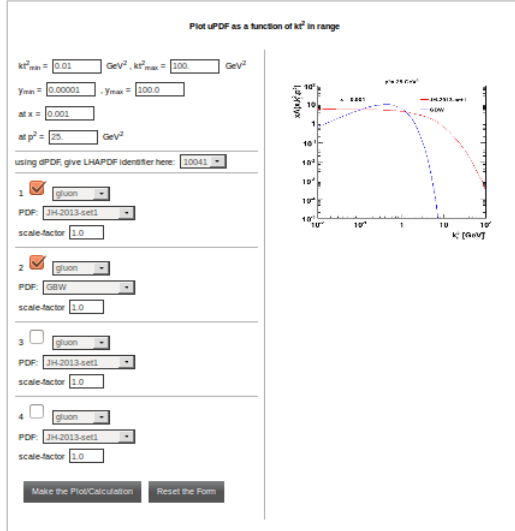
or directly to the contact authors.

Acknowledgments

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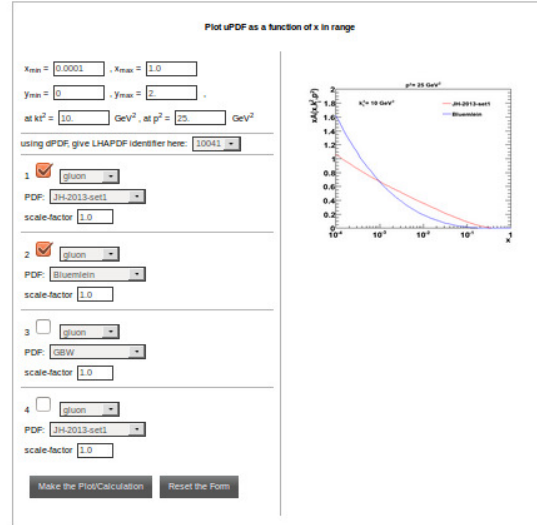
Figure 1: Two snapshots from the online portal TMDplotter for plotting uPDF/TMD distributions: the gluon from the `ccfm-JH-2013-set1` set compared to the `GBW` as a function of k_t (left) and x (right).

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