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Relativistic corrections to prompt J/ψ photo- and hadroproduction

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Abstract

We systematically calculate the relativistic corrections to prompt J/ψ photoproduction and hadroproduction using the factorization formalism of nonrelativistic QCD (NRQCD). Specifically, we include the ${}^3S_1^{[1]}$ and ${}^3P_J^{[1]}$ color-singlet and the ${}^3S_1^{[8]}$, ${}^1S_0^{[8]}$, and ${}^3P_J^{[8]}$ color-octet channels as well as the effects due to the mixing between the ${}^3S_1^{[8]}$ and ${}^3D_1^{[8]}$ channels. We provide all the squared hard-scattering amplitudes in analytic form. Assuming the NRQCD long-distance matrix elements to satisfy the velocity scaling rules, we find the relativistic corrections to be appreciable, except in the ${}^3S_1^{[1]}$ color-singlet channel of hadroproduction. We also observe significant differences in the lineshapes of the relativistic corrections between photoproduction and hadroproduction.

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I. INTRODUCTION

The production of heavy quarkonia, the QCD bound states of heavy-quark pairs ($Q\bar{Q}$), serves a an ideal laboratory to probe both the perturbative and nonperturbative aspects of QCD. The effective quantum field theory of nonrelativistic QCD (NRQCD) [1] endowed with the factorization formalism introduced by Bodwin, Brateen, and Lapage [2] is nowadays the most favorable theoretical approach to study heavy-quarkonium production and decay. In this framework, the theoretical predictions are separated into process-dependent short-distance coefficients (SDCs) and supposedly universal long-distance matrix elements (LDMEs). The SDCs may be calculated perturbatively as expansions in the strong-coupling constant α_s, while the LDMEs are predicted to scale with definite powers of the relative velocity v of the heavy quarks in the quarkonium rest frame [3]. In this way, the theoretical calculations are organized as double expansions in α_s and v. In contrast to the color-singlet (CS) model, in which the $Q\bar{Q}$ pair must be in the CS state that shares the spin S, orbital angular momentum L, and total angular momentum J with the considered quarkonium, NRQCD accommodates all possible Fock states $n = {}^{2S+1}L_J^{[a]}$, where $a = 1, 8$ stands for CS and color octet (CO), respectively.

During the past two decades, tests of NRQCD factorization and the universality of the LDMEs were performed in a vast number of experimental and theoretical works. Despite numerous great successes, there are still some challenges in J/ψ hadroproduction. As for the prompt yield of unpolarized J/ψ mesons, the next-to-leading order (NLO) QCD corrections were calculated for all the relevant channels, including ${}^3S_1^{[1]}$ [4], ${}^3S_1^{[8]}$ and ${}^1S_0^{[8]}$ [5], and ${}^3P_J^{[8]}$ [6, 7] for direct production and the feed-down from ψ' mesons, and ${}^3P_J^{[1]}$ [8, 9] for the feed-down from χ_{cJ} mesons. As for the LDMEs, different ways of fitting lead to different results. By a global fit to the world's J/ψ data from hadroproduction, photoproduction, two-photo scattering, and e^+e^- annihilation, the three CO LDMEs relevant for direct production were successfully pinned down [10] in a way compatible with the velocity scaling rules [3], which greatly supported their universality. Using these LDMEs, the J/ψ polarization in hadroproduction was predicted to be largely transversal [11]. By fitting to the CDF Run II measurements of J/ψ yield and polarization for transverse momenta $p_T > 7$ GeV, the authors of Ref. [12] obtained two linear combinations of the three LDMEs, which led to an almost unpolarized prediction for the LHC. Fitting to the prompt J/ψ yield measured for

$p_T > 7$ GeV by CDF II and LHCb, including also the feed-down contributions from the χ_{cJ} and ψ' mesons, a third set of LDMEs was obtained, which resulted in a moderately transverse J/ψ polarization at the LHC [13]. All of these three LDMEs sets can describe well the J/ψ yield at the LHC. Unfortunately, none of the resulting predictions for J/ψ polarization are consistent with the latest measurements by CMS [14] and LHCb [15]. The above results indicate that the NRQCD prediction of prompt J/ψ polarization strongly depends on the choice of LDMEs. In order to test the universality of the LDMEs in a meaningful way and to clarify the J/ψ polarization puzzle, it is useful to investigate some other effects in the determination of the LDMEs, such as the higher-order relativistic corrections.

In the heavy-quarkonium system, we actually have $v^2 \sim \alpha_s(2m_Q)$, which is not very small. In some cases, it was found that the higher-order v^2 corrections are even as important as the higher-order α_s corrections. For example, the relativistic corrections played an important role in resolving both the double-charmonia [16] and $J/\psi + X_{\rm non-cc}$ production problems at the B factories [17, 18]. In J/ψ hadroproduction, the v^2 corrections in the CO channels ${}^1S_0^{[8]}$ and ${}^3S_1^{[8]}$ were found to be significant in the large-p_T region [19], although they are tiny in the CS ${}^3S_1^{[1]}$ channel [20]. In double-quarkonium hadroproduction, the v^2 corrections also turned out to be significant [21], especially in the CO channels [22]. In the test of the hypothesis $X(3872) = \chi'_{c1}$ in hadroproduction, appreciable v^2 corrections were encountered in the ${}^3P_1^{[1]}$ channel [9]. All these observations provide a strong motivation for us to systematically study the v^2 corrections to the cross sections of prompt J/ψ photoproduction and hadroproduction. This will allow us to render global fits of the contributing LDMEs more reliable and to deepen our understanding of their universality. While the SDCs of direct J/ψ production immediately carry over to the feed-down from the ψ' mesons, the feed-down from the χ_{cJ} mesons require a separate calculation.

The remainder of this paper is organized as follows. In Sec. II, we explain how we calculate all the relevant SDCs. Our numerical results are presented in Sec. III. Our conclusions are contained in Sec. IV. Our analytic results are listed in the Appendix.

II. NRQCD FACTORIZATION FORMULA

Invoking the Weizsäcker-Williams approximation and the factorization theorem of the QCD parton model, the cross sections for the photoproduction or hadroproduction of the hadron $H = J/\psi, \chi_{cJ}, \psi'$ may be written as [10]

$$\sigma(AB \to H + X) = \sum_{i,j,k} \int dx_1 dy_1 dx_2\, f_{i/A}(x_1) f_{j/i}(y_1) f_{k/B}(x_2) \hat{\sigma}(jk \to H + X), \qquad (1)$$

where $f_{i/A}(x)$ is the parton distribution function (PDF) of the parton i in the hadron $A = p, \bar{p}$ or the flux function of the photon $i = \gamma$ in the charged lepton $A = e^-, e^+$, $f_{j/i}(y_1)$ is $\delta_{ij}\delta(1 - y_1)$ or the PDF of the parton j in the resolved photon $i = \gamma$, and $\hat{\sigma}(jk \to H + X)$ is the partonic cross section. In NRQCD through relative order v^2, the latter is factorized as [2]

$$\hat{\sigma}(ij \to H + X) = \sum_n \left(\frac{F_{ij}(n)}{m_c^{d_n-4}} \langle \mathcal{O}^H(n) \rangle + \frac{G_{ij}(n)}{m_c^{d_n-4}} \langle \mathcal{P}^H(n) \rangle \right), \qquad (2)$$

where $\mathcal{O}^H(n)$ is the four-quark operator pertaining to the transition $n \to H$ at LO in v, $\mathcal{P}^H(n)$ is related to its v^2 correction and carries a mass dimension d_n increased by two units, and $F_{ij}(n)$ and $G_{ij}(n)$ are the appropriate SDCs of the partonic subprocesses $i + j \to c\bar{c}(n) + X$. Working in the fixed-flavor-number scheme, the parton i runs over the gluon g and the light quarks $q = u, d, s$ and anti-quarks $\bar{q}$.

According to the velocity scaling rules [3], the leading contributions to direct J/ψ and ψ' production are due to the ${}^3S_1^{[1]}$, ${}^3S_1^{[8]}$, ${}^1S_0^{[8]}$, and ${}^3P_J^{[8]}$ channels, and those to direct χ_{cJ} production are due to the ${}^3P_J^{[1]}$ and ${}^3S_1^{[8]}$ channels. Accordingly, prompt J/ψ photoproduction and hadrproduction proceeds at LO through the partonic subprocesses

$$\begin{aligned}
g + \gamma &\to c\bar{c}({}^3S_1^{[1,8]}, {}^1S_0^{[8]}, {}^3P_J^{[8]}) + g, \\
q(\bar{q}) + \gamma &\to c\bar{c}({}^3S_1^{[8]}, {}^1S_0^{[8]}, {}^3P_J^{[8]}) + q(\bar{q}), \\
g + g &\to c\bar{c}({}^3S_1^{[1,8]}, {}^1S_0^{[8]}, {}^3P_J^{[1,8]}) + g, \\
q(\bar{q}) + g &\to c\bar{c}({}^3S_1^{[8]}, {}^1S_0^{[8]}, {}^3P_J^{[1,8]}) + q(\bar{q}), \\
\bar{q} + q &\to c\bar{c}({}^3S_1^{[8]}, {}^1S_0^{[8]}, {}^3P_J^{[1,8]}) + g. \qquad (3)
\end{aligned}$$

We adopt the definitions of the relevant four-quark operators $\mathcal{O}^H(n)$ from the literature

[2, 23] and define the corresponding four-quark operators $\mathcal{P}^H(n)$ as:

$$\begin{aligned}
\mathcal{P}^{J/\psi}({}^3S_1^{[1]}) &= \chi^\dagger \boldsymbol{\sigma}^i \psi (a^\dagger_{J/\psi} a_{J/\psi}) \psi^\dagger \boldsymbol{\sigma}^i \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\right)^2 \chi + \text{H.c.},\\
\mathcal{P}^{J/\psi}({}^1S_0^{[8]}) &= \chi^\dagger T_a \psi (a^\dagger_{J/\psi} a_{J/\psi}) \psi^\dagger T_a \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\right)^2 \chi + \text{H.c.},\\
\mathcal{P}^{J/\psi}({}^3P_J^{[8]}) &= \chi^\dagger \boldsymbol{\sigma}^i \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}^j\right) T_a \psi (a^\dagger_{J/\psi} a_{J/\psi}) \psi^\dagger \boldsymbol{\sigma}^i T_a \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}^j\right)\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\right)^2 \chi + \text{H.c.},\\
\mathcal{P}^{\chi_{c0}}({}^3P_0^{[1]}) &= \frac{1}{3}\chi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right) \psi (a^\dagger_{\chi_{c0}} a_{\chi_{c0}}) \psi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\right)^2 \chi + \text{H.c.},\\
\mathcal{P}^{\chi_{c1}}({}^3P_1^{[1]}) &= \frac{1}{2}\chi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\times\boldsymbol{\sigma}\right) \psi (a^\dagger_{\chi_{c1}} a_{\chi_{c1}}) \psi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\times\boldsymbol{\sigma}\right)\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\right)^2 \chi + \text{H.c.},\\
\mathcal{P}^{\chi_{c2}}({}^3P_2^{[1]}) &= \frac{1}{2}\chi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}^{(i}\boldsymbol{\sigma}^{j)}\right) \psi (a^\dagger_{\chi_{c2}} a_{\chi_{c2}}) \psi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}^{(i}\boldsymbol{\sigma}^{j)}\right)\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\right)^2 \chi + \text{H.c.},\\
\mathcal{P}^{H}({}^3S_1^{[8]}) &= \chi^\dagger \boldsymbol{\sigma}^i T_a \psi (a^\dagger_{H} a_{H}) \psi^\dagger \boldsymbol{\sigma}^i T_a \left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\right)^2 \chi + \text{H.c.},\\
\mathcal{P}^{H}({}^3S_1^{[8]},{}^3D_1^{[8]}) &= \sqrt{\frac{3}{5}}\chi^\dagger \sigma^i T_a \psi (a^\dagger_{H} a_{H}) \psi^\dagger \boldsymbol{\sigma}^j \boldsymbol{K}^{ij} T_a \chi + \text{H.c.},
\end{aligned} \tag{4}$$

where $\overleftrightarrow{\boldsymbol{D}}^{(i}\boldsymbol{\sigma}^{j)} = (\overleftrightarrow{\boldsymbol{D}}^i\boldsymbol{\sigma}^j + \overleftrightarrow{\boldsymbol{D}}^j\boldsymbol{\sigma}^i)/2 - \overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\delta^{ij}/3$ and $\boldsymbol{K}^{ij} = (-i/2)^2(\overleftrightarrow{\boldsymbol{D}}^i\overleftrightarrow{\boldsymbol{D}}^j - \overleftrightarrow{\boldsymbol{D}}^2\delta^{ij}/3)$. Our definition of the S-D mixing operator $\mathcal{P}^H({}^3S_1^{[8]},{}^3D_1^{[8]})$ differs from that in Refs. [2, 23], where a linear combination of $\mathcal{P}^H({}^3S_1^{[8]})$ and $\mathcal{P}^H({}^3S_1^{[8]},{}^3D_1^{[8]})$ in Eq. (4) is used instead. At order v^2, there are no heavy-quark spin symmetries among the $\mathcal{O}^H(n)$ operators, but they still hold among the $\mathcal{P}^H(n)$ operators, yielding the relationships

$$\begin{aligned}
\langle\mathcal{P}^{J/\psi}({}^3P_0^{[8]})\rangle &= \frac{1}{2J+1}\langle\mathcal{P}^{J/\psi}({}^3P_J^{[8]})\rangle,\\
\langle\mathcal{P}^{\chi_{c0}}({}^3S_1^{[8]})\rangle &= \frac{1}{2J+1}\langle\mathcal{P}^{\chi_{cJ}}({}^3S_1^{[8]})\rangle,\\
\langle\mathcal{P}^{\chi_{c0}}({}^3P_0^{[1]})\rangle &= \frac{1}{2J+1}\langle\mathcal{P}^{\chi_{cJ}}({}^3P_J^{[1]})\rangle,\\
\langle\mathcal{P}^{\chi_{c0}}({}^3S_1^{[8]},{}^3D_1^{[8]})\rangle &= -\frac{2}{3}\langle\mathcal{P}^{\chi_{c1}}({}^3S_1^{[8]},{}^3D_1^{[8]})\rangle = 2\langle\mathcal{P}^{\chi_{c2}}({}^3S_1^{[8]},{}^3D_1^{[8]})\rangle.
\end{aligned} \tag{5}$$

The SDCs $F_{ij}(n)$ and $G_{ij}(n)$ may be obtained perturbatively by matching the QCD and NRQCD calculations via the condition

$$\sigma(c\bar{c})|_{\text{pert QCD}} = \sum_n \frac{F_n(\Lambda)}{m_c^{d_n-4}}\langle 0|\mathcal{O}_n^{c\bar{c}}(\Lambda)|0\rangle|_{\text{pert NRQCD}} + \sum_n \frac{G_n(\Lambda)}{m_c^{d_n-4}}\langle 0|\mathcal{P}_n^{c\bar{c}}(\Lambda)|0\rangle|_{\text{pert NRQCD}}. \tag{6}$$

The left-hand side of Eq. (6) may be computed directly using the spinor projection method developed in Ref. [24], by which the product of Dirac spinors $v(P/2-q)\bar{u}(P/2+q)$ is projected onto the considered ${}^{2S+1}L_J$ state in a Lorentz-covariant form. In an arbitrary reference frame, the four-momenta $P/2+q$ and $P/2-q$ of the heavy quark and antiquark may be related to those in the quarkonium rest frame as

$$\frac{P}{2}+q=L\left(\frac{P_r}{2}+\boldsymbol{q}\right),\qquad \frac{P}{2}-q=L\left(\frac{P_r}{2}-\boldsymbol{q}\right),\tag{7}$$

where $P_r^\mu=(2E_q,\boldsymbol{0})$, $E_q=\sqrt{m_c^2+\boldsymbol{q}^2}$, $2\boldsymbol{q}$ is the relative three-momentum between the two quarks in the quarkonium rest frame, and $L^\mu{}_\nu$ is the Lorentz transformation matrix for the boost from the quarkonium rest frame to the considered reference frame. To all orders in v^2, the projectors onto the spin-singlet ($S=0$) and spin-triplet ($S=1$) states in the quarkonium rest frame read [25]:

$$\begin{aligned}\sum_{\lambda_1,\lambda_2} v(-\boldsymbol{q},\lambda_2)\bar{u}(\boldsymbol{q},\lambda_1)\left\langle\frac{1}{2},\lambda_1;\frac{1}{2},\lambda_2\middle|0,0\right\rangle &= \frac{E_q+m_c}{\sqrt{2}}\left(1-\frac{\boldsymbol{\alpha}\cdot\boldsymbol{q}}{E_q+m_c}\right)\\ &\quad\times\gamma^5\frac{1+\gamma^0}{2}\left(1+\frac{\boldsymbol{\alpha}\cdot\boldsymbol{q}}{E_q+m_c}\right)\gamma^0,\\ \sum_{\lambda_1,\lambda_2} v(-\boldsymbol{q},\lambda_2)\bar{u}(\boldsymbol{q},\lambda_1)\left\langle\frac{1}{2},\lambda_1;\frac{1}{2},\lambda_2\middle|1,\boldsymbol{\epsilon}\right\rangle &= \frac{E_q+m_c}{\sqrt{2}}\left(1-\frac{\boldsymbol{\alpha}\cdot\boldsymbol{q}}{E_q+m_c}\right)\\ &\quad\times\boldsymbol{\alpha}\cdot\boldsymbol{\epsilon}\frac{1+\gamma^0}{2}\left(1+\frac{\boldsymbol{\alpha}\cdot\boldsymbol{q}}{E_q+m_c}\right)\gamma^0.\end{aligned}\tag{8}$$

In an arbitrary reference frame, they become

$$\begin{aligned}\sum_{\lambda_1,\lambda_2} v(-q,\lambda_2)\bar{u}(q,\lambda_1)\left\langle\frac{1}{2},\lambda_1;\frac{1}{2},\lambda_2\middle|0,0\right\rangle &= \frac{-1}{2\sqrt{2}(E_q+m_c)}\left(\frac{\not{P}}{2}-\not{q}-m_c\right)\\ &\quad\times\gamma^5\frac{\not{P}+2E_q}{2E_q}\left(\frac{\not{P}}{2}+\not{q}+m_c\right),\\ \sum_{\lambda_1,\lambda_2} v(-q,\lambda_2)\bar{u}(q,\lambda_1)\left\langle\frac{1}{2},\lambda_1;\frac{1}{2},\lambda_2\middle|1,\epsilon\right\rangle &= \frac{-1}{2\sqrt{2}(E_q+m_c)}\left(\frac{\not{P}}{2}-\not{q}-m_c\right)\\ &\quad\times\not{\epsilon}\frac{\not{P}+2E_q}{2E_q}\left(\frac{\not{P}}{2}+\not{q}+m_c\right).\end{aligned}\tag{9}$$

Note that the normalization of the Dirac spinors is $\bar{u}u=-\bar{v}v=m_c^2$. With the help of the spinor projection method, the partonic scattering amplitude $M(ij\to c\bar{c}(n)+X)$ may then be expanded in the relative momentum q. To this end, we write

$$M(ij\to c\bar{c}(n)+X)=\sqrt{\frac{m_c}{E_q}}A(q),\tag{10}$$

where the factor $\sqrt{m_c/E_q}$ stems from the relativistic normalization of the $c\bar{c}(n)$ state and

$$\begin{aligned} A(q) &= \sum_{\lambda_1,\lambda_2}\sum_{k,l}\left\langle \frac{1}{2},\lambda_1;\frac{1}{2},\lambda_2|S,S_z\right\rangle \langle 3,k;\bar{3},l|1(8,c)\rangle \\ &\quad \times \mathcal{A}\left(ij\to c_{\lambda_1,k}\left(\frac{P}{2}+q\right)\bar{c}_{\lambda_2,l}\left(\frac{P}{2}-q\right)+X\right). \end{aligned} \tag{11}$$

Here, $\langle 3,k;\bar{3},l|1\rangle = \delta_{kl}/\sqrt{N_c}$ and $\langle 3,k;\bar{3},l|8,a\rangle = \sqrt{2}\,T^a_{kl}$ are the color-SU(3) Clebsch-Gordan coefficients for the $c\bar{c}(n)$ pair projected onto CS and CO states, respectively, and $\mathcal{A}(ij\to c_{\lambda_1,k}(P/2+q)\bar{c}_{\lambda_2,l}(P/2-q)+X)$ is the standard Feynman amplitude. Defining

$$A_{\alpha_1\cdots\alpha_N}(0) = \left.\frac{\partial^N A(q)}{\partial q^{\alpha_1}\cdots\partial q^{\alpha_N}}\right|_{q=0}, \tag{12}$$

we may write the expansion of Eq. (11) in q as

$$A(q) = A(0) + q^{\alpha_1}A_{\alpha_1}(0) + \frac{1}{2}q^{\alpha_1}q^{\alpha_2}A_{\alpha_1\alpha_2}(0) + \frac{1}{6}q^{\alpha_1}q^{\alpha_2}q^{\alpha_3}A_{\alpha_1\alpha_2\alpha_3}(0) + \cdots . \tag{13}$$

For S- and D-wave states, only the terms with even powers in q contribute, while for P-wave states it is the other way round. To calculate the relativistic corrections for the production of S- and P-wave states, we need to decompose the higher-rank tensor products of q factors in Eq. (13) into their irreducible representations and to retain the $L=S$ and $L=P$ terms, respectively. Through order v^2, we thus obtain, for $n = {}^3S_1^{[1]}, {}^3S_1^{[8]}, {}^1S_0^{[8]}$,

$$M(ij\to c\bar{c}(n)+X) = \sqrt{\frac{m_c}{E_q}}\left[A(0) + \frac{|\boldsymbol{q}|^2}{6}\Pi^{\alpha_1\alpha_2}A_{\alpha_1\alpha_2}(0)\right], \tag{14}$$

and, for $n = {}^3P_J^{[1]}, {}^3P_J^{[8]}$,

$$M(ij\to c\bar{c}(n)+X) = \sqrt{\frac{m_c}{E_q}}q^{\alpha_1}\left[A_{\alpha_1}(0) - \frac{|\boldsymbol{q}|^2}{30}\Pi_{\alpha_1}{}^{\alpha_2\alpha_3\alpha_4}A_{\alpha_2\alpha_3\alpha_4}(0)\right], \tag{15}$$

where $\Pi^{\alpha_1\alpha_2} = -g^{\alpha_1\alpha_2} + P^{\alpha_1}P^{\alpha_2}/(4E_q^2)$ and $\Pi^{\alpha_1\alpha_2\alpha_3\alpha_4} = (\Pi^{\alpha_1\alpha_2}\Pi^{\alpha_3\alpha_4} + \Pi^{\alpha_1\alpha_3}\Pi^{\alpha_2\alpha_4} + \Pi^{\alpha_2\alpha_3}\Pi^{\alpha_1\alpha_4})$. In the D-wave case, we only need the amplitude at LO in v^2, which is

$$M(ij\to c\bar{c}({}^3D_J^{[8]})+X) = \sqrt{\frac{m_c}{E_q}}\frac{1}{2}q^{\alpha_1}q^{\alpha_2}A_{\alpha_1\alpha_2}(0). \tag{16}$$

We are now in a position to perform the matching between the calculations in NRQCD and full QCD. We thus obtain

$$\begin{aligned} \frac{F_{ij}(n)}{m_c^{d_n-4}} &= \frac{1}{2s}\int d\text{LIPS}|\overline{M}_{ij}(n)|^2, \\ \frac{G_{ij}(n)}{m_c^{d_n-4}} &= \frac{1}{2s}\int d\text{LIPS}\left(K|\overline{M}_{ij}(n)|^2 + |\overline{N}_{ij}(n)|^2\right), \end{aligned} \tag{17}$$

where $\sqrt{s}$ is the invariant mass of the incoming partons and

$$\begin{aligned} |\overline{M}_{ij}(n)|^2 &= \overline{\sum_{L_z}} |A(0)|^2 \bigg|_{\boldsymbol{q}^2=0}, \\ |\overline{N}_{ij}(n)|^2 &= \overline{\sum_{L_z}} \left\{ \frac{\partial}{\partial \boldsymbol{q}^2} \left[\frac{m_c}{E_q} |A(0)|^2 \right] + \frac{1}{3} \Pi^{\alpha_1\alpha_2} \operatorname{Re}[A^*(0) A_{\alpha_1\alpha_2}(0)] \right\} \bigg|_{\boldsymbol{q}^2=0}, \end{aligned} \tag{18}$$

for $n = {}^3S_1^{[1]}, {}^3S_1^{[8]}, {}^1S_0^{[8]}$,

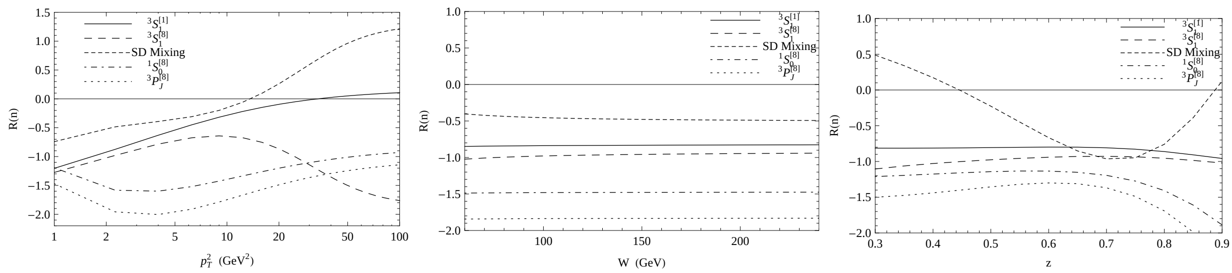
$$\begin{aligned} |\overline{M}_{ij}(n)|^2 &= \overline{\sum_{S_z,L_z,J_z}} |\langle 1, L_z; 1, S_z | J, J_z \rangle \epsilon^{*\alpha}_{L_z} A_\alpha(0)|^2 \bigg|_{\boldsymbol{q}^2=0}, \\ |\overline{N}_{ij}(n)|^2 &= \left\{ \overline{\sum_{S_z,L_z,J_z}} \frac{\partial}{\partial \boldsymbol{q}^2} \left[\frac{m_c}{E_q} |\langle 1, L_z; 1, S_z | J, J_z \rangle \epsilon^{*\alpha}_{L_z} A_\alpha(0)|^2 \right] \right. \\ &\quad - \frac{1}{15} \overline{\sum_{S_z,L_z,J_z}} \langle 1, L_z; 1, S_z | J, J_z \rangle \overline{\sum_{S'_z,L'_z,J'_z}} \langle 1, L'_z; 1, S'_z | J, J'_z \rangle \\ &\quad \left. \times\, \Pi_\alpha{}^{\alpha_1\alpha_2\alpha_3} \operatorname{Re}[\epsilon^{*\alpha}_{L_z} \epsilon^{\beta}_{L'_z} A^*_\beta(0) A_{\alpha_1\alpha_2\alpha_3}(0)] \right\} \bigg|_{\boldsymbol{q}^2=0}, \end{aligned} \tag{19}$$

for $n = {}^3P_J^{[1]}, {}^3P_J^{[8]}$, and

$$|\overline{N}_{ij}(n)|^2 = \overline{\sum_{S_z,L_z,J_z}} \langle 2, L_z; 1, S_z | 1, J_z \rangle \operatorname{Re}[\epsilon^{*\alpha\beta}_{L_z} A^*(0) A_{\alpha\beta}(0)] \bigg|_{\boldsymbol{q}^2=0}, \tag{20}$$

for the ${}^3S_1^{[8]}$-${}^3D_1^{[8]}$ mixing term. Here, $\epsilon^\alpha_{L_z}$ ($\epsilon^{\alpha\beta}_{L_z}$) is the polarization four-vector (four-tensor) for $L = P$ (D), the symbol $\sum$ also implies the summation over the polarizations of the other external partons, and the bar implies the average over the spins and colors of the incoming partons and those of the $c\bar{c}(n)$ state. The factor K in Eq. (17) contains the v^2 corrections to the phase space and depends on the kinematic variables that we are interested in. In the cases under consideration here, we have $K = -4/(s - 4m_c^2)$. We generate the Feynman diagrams using the FeynArts package [26] and compute the amplitude squares using the FeynCalc package [27]. In the Appendix, we present our results for $|\overline{N}_{ij}(n)|^2$.

We reproduce the well-known results for $|\overline{M}_{ij}(n)|^2$, which were called $|\mathcal{A}|^2$ in Ref. [28] and $|\mathcal{M}'|^2$ in Ref. [29]. We may also compare some of our results for $|\overline{N}_{ij}(n)|^2$ with the literature. The v^2 corrections to direct J/ψ photoproduction were first studied in Ref. [30] within relativistic quark model, which accounts for the CS contributions. We reproduce Eq. (23) in Ref. [30] if we do not expand E_q, but set $E_q = m_{J/\psi}/2$. The relativistic corrections to direct J/ψ hadroproduction were considered in Ref. [19] within NRQCD. We find agreement



(a) (b) (c)

FIG. 1: Ratios $R(n)$ for J/ψ direct photoproduction under HERA II kinematic conditions as functions of (a) p_T^2, (b) W, and (c) z.

with Ref. [19], except for some typographical errors in Eq. (A4) therein, which corresponds to $|\overline{N}_{gg}({}^3S_1^{[8]})|^2$ in our notation. Furthermore, the ${}^3S_1^{[8]}$-${}^3D_1^{[8]}$ mixing contribution was not considered there. Apart from correcting the misprints in Eq. (A4) of Ref. [19], our paper reaches beyond the available literature by studying the CO contributions to direct J/ψ photoproduction, the ${}^3S_1^{[8]}$-${}^3D_1^{[8]}$ mixing contribution to direct J/ψ hadroproduction, and the feed-down contributions to J/ψ hadroproduction.

III. PHENOMENOLOGICAL RESULTS

We are now in a position to investigate the phenomenological significance of the v^2 corrections in prompt J/ψ photoproduction and hadroproduction. In our numerical analysis, we use $m_c = 1.5$ GeV, $\alpha = 1/137.036$, the LO formula for $\alpha_s^{(n_f)}(\mu_r)$ with $n_f = 4$ active quark flavors and asymptotic scale parameter $\Lambda_{\mathrm{QCD}}^{(4)} = 215$ MeV [31], the CTEQ6L1 set for proton PDFs [31], the photon flux function given in Eq. (5) of Ref. [32] with $Q^2_{\mathrm{max}} = 2.5$ GeV^2 [33], and the choice $\mu_r = \mu_f = \sqrt{p_t^2 + 4m_c^2}$ for the renormalization and factorization scales. According to Eqs. (1) and (2), the hadronic cross sections of the direct photoproduction and hadroproduction of the charmonia $H = J/\psi, \chi_{cJ}, \psi'$, differential in some observable x, may be generically written as

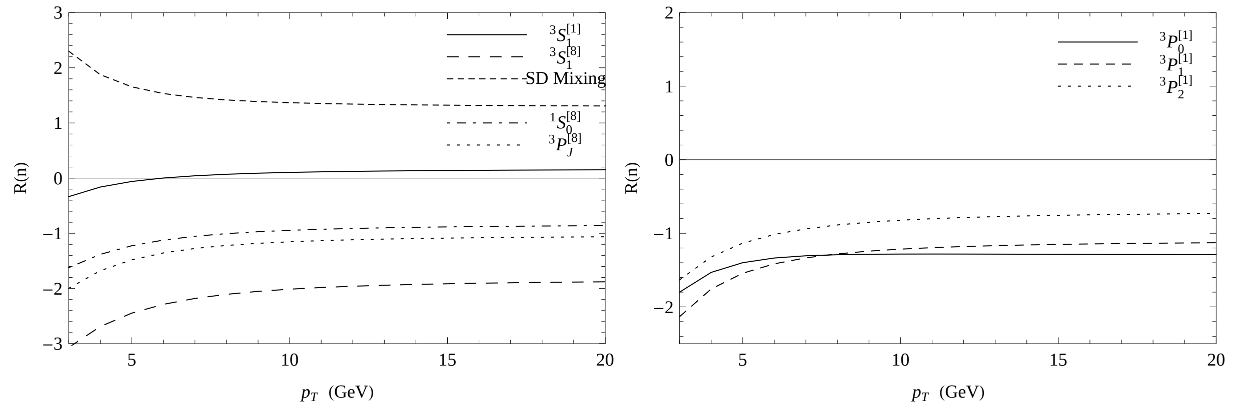
$$\frac{d\sigma}{dx} = \sum_n \left(\frac{dF(n)}{dx} \frac{\langle \mathcal{O}^H(n) \rangle}{m_c^{d_n-4}} + \frac{dG(n)}{dx} \frac{\langle \mathcal{P}^H(n) \rangle}{m_c^{d_n-4}} \right), \tag{21}$$

where it is understood that $dF(n)/dx = 0$ if n stands for ${}^3S_1^{[8]}$-${}^3D_1^{[8]}$ mixing. For all other channels n, the relative v^2 corrections are $R(n)\langle \mathcal{P}^H(n) \rangle / (m_c^2 \langle \mathcal{O}^H(n) \rangle)$, where

$R(n) = dG(n)/dx/(dF(n)/dx)$ is a dimensionless ratio. To standardize the numerical discussion, we also introduce the quantity $R(n) = dG(n)/dx/(dF(^3S_1^{[8]})/dx)$ for the case when n stands for $^3S_1^{[8]}$-$^3D_1^{[8]}$ mixing. According to the velocity scaling rules [3], the weight $\langle\mathcal{P}^H(n)\rangle/(m_c^2\langle\mathcal{O}^H(n)\rangle)$ of $R(n)$ is of order v^2, which is approximately 0.23 for charmonium [34, 35]. For definiteness, we ignore these weights in the following and concentrate on the SDC ratios $R(n)$ instead. Furthermore, we limit ourselves to the direct production of J/ψ mesons and their production via the feed-down from the χ_{cJ} mesons. In the latter case, the branching fractions $B(\chi_{cJ} \to J/\psi + X)$ drop out in the ratios $R(n)$. However, there remains a kinematic effect on the transverse momentum. Since $p_T \equiv p_T^{J/\psi} \gg M_{\chi_{cJ}} - M_{J/\psi}$, we may approximate $p_T^{J/\psi} = p_T^{\chi_{cJ}} M_{J/\psi}/M_{\chi_{cJ}}$, with $M_{J/\psi} = 3.097$ GeV, $M_{\chi_{c0}} = 3.415$ GeV, $M_{\chi_{c1}} = 3.511$ GeV, and $M_{\chi_{c2}} = 3.556$ GeV [36].

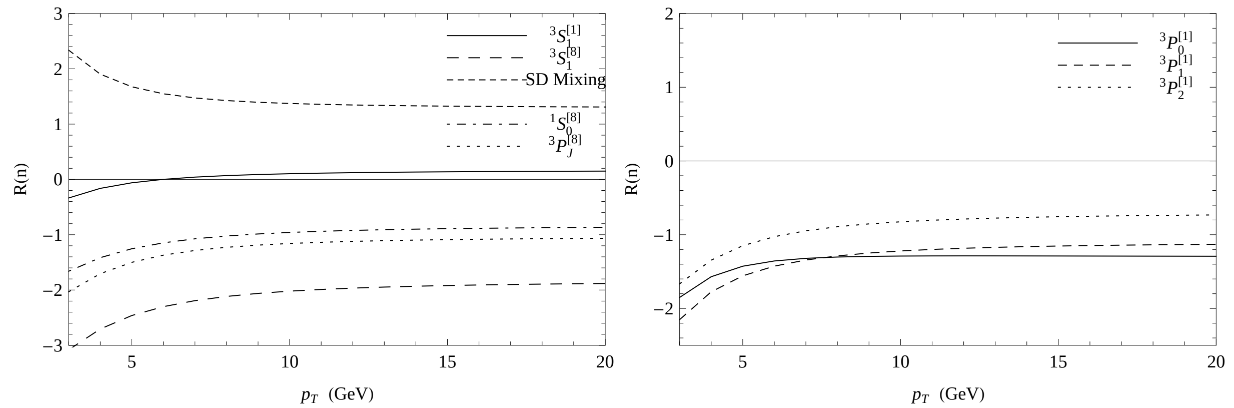
We consider three typical experimental environments, namely, Run II at HERA, Run I at the Tevatron, and the LHCb setup at the LHC. At HERA II, the cross section of prompt J/ψ photoproduction was measured at center-of-mass energy $\sqrt{S} = 319$ GeV differential in p_T^2, $W = \sqrt{(p_p + p_\gamma)^2}$, and $z = p_{J/\psi} \cdot p_p / p_\gamma \cdot p_p$ [33], where p_p, p_γ, and $p_{J/\psi}$ are the four-momenta of the proton, photon, and J/ψ meson, respectively, imposing in turn two of the acceptance cuts 1 GeV$^2 < p_T^2$, 60 GeV $< W <$ 240 GeV, and $0.3 < z < 0.9$. In Figs. 1(a)–(c), the p_T^2, W, and z distributions of $R(n)$ are shown for $n = {}^3S_1^{[1]}, {}^3S_1^{[8]}, {}^1S_0^{[8]}, {}^3P_J^{[8]}$, and $^3S_1^{[8]}$-$^3D_1^{[8]}$ mixing. We recall that $n = {}^3P_J^{[1]}$ is prohibited at this order. For simplicity, we ignore resolved photoproduction, the SDCs of which also contribute to hadroproduction to be studied below. In Run I at the Tevatron, the p_T distribution of prompt J/ψ hadroproduction was measured at $\sqrt{S} = 1.8$ TeV in the pseudorapidity range $|\eta_{J/\psi}| < 0.6$ [37], and it was measured by LHCb at $\sqrt{S} = 7$ TeV in the rapidity range $2.0 < y < 4.5$ [38]. In the latter two cases, we exclude the region $p_T < 3$ GeV, where the application of fixed-order perturbation theory is problematic. In Figs. 2 and 3, the p_T distributions of $R(n)$ are shown for the Tevatron and the LHC, respectively. In each figure, part (a) refers to $n = {}^3S_1^{[1]}, {}^3S_1^{[8]}, {}^1S_0^{[8]}, {}^3P_J^{[8]}$, and $^3S_1^{[8]}$-$^3D_1^{[8]}$ mixing in direct J/ψ production, and part (b) to $n = {}^3P_J^{[1]}$ in the feed-down from the respective χ_{cJ} meson.

We observe from Figs. 1–3 that the ratios $R(n)$ may be of either sign and typically have a magnitude of order unity or larger. In fact, $R(^3S_1^{[8]})$ for hadroproduction reaches the value -3 at $p_T = 3$ GeV, as may be seen in Figs. 2 and 3. An exception to this rule is $R(^3S_1^{[1]})$ for hadroproduction, which is roughly one order of magnitude smaller. Because of



(a) (b)

FIG. 2: Ratios $R(n)$ for J/ψ hadroproduction (a) in the direct mode and (b) via the feed-down from χ_{cJ} mesons under Tevatron Run I kinematic conditions as functions of p_T.



(a) (b)

FIG. 3: Ratios $R(n)$ for J/ψ hadroproduction (a) in the direct mode and (b) via the feed-down from χ_{cJ} mesons under LHCb kinematic conditions as functions of p_T.

the above estimations of the weights $\langle \mathcal{P}^H(n)\rangle/(m_c^2\langle \mathcal{O}^H(n)\rangle)$ and v^2, $R(n) = 2$ is likely to imply a v^2 correction of some 50%, which could have a significant impact on state-of-the-art determinations of CO LDMEs through fits to J/ψ production data and might improve the goodness of such fits. Comparing Figs. 1–3, we observe that the relative importance and the p_T dependencies of the various ratios $R(n)$ greatly differs between photoproduction and hadroproduction. Therefore, the inclusion of v^2 corrections is likely to have a nontrivial effect on global data fits.

IV. CONCLUSIONS

We performed a systematic study of the v^2 corrections to the yields of prompt J/ψ photoproduction and hadroproduction, providing the relevant SDCs in analytic form. Specifically, this includes the partonic subprocesses listed in Eq. (3) in combination with the $c\bar{c}$ Fock states $n = {}^3S_1^{[1]}, {}^3P_J^{[1]}, {}^3S_1^{[8]}, {}^1S_0^{[8]}, {}^3P_J^{[8]}$, and ${}^3S_1^{[8]}$-${}^3D_1^{[8]}$ mixing. We compared our results with the literature as far as the latter goes. We assessed the phenomenological significance of the v^2 corrections in the various channels by studying their ratios with respect to corresponding LO results. Assuming the relevant LDMEs $\langle \mathcal{O}^H(n) \rangle$ and $\langle \mathcal{P}^H(n) \rangle$ to obey the hierarchy predicted by the velocity scaling rules [3], we found that v^2 corrections of up to 50% are realistic. We thus conclude that it is indispensable to include v^2 corrections in determinations of CO LDMEs by global data fits, the more so as the v^2 corrections in the various channels greatly differ between photoproduction and hadroproduction.

Acknowledgments

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V. APPENDIX

In this appendix, we present analytic expressions for the nonvanishing SDCs $|\overline{N}_{ij}(n)|^2$ appropriate for prompt J/ψ photoporduction and hadroproduction evaluated according to Eqs. (18)–(20). For the partonic subprocess $i(p_1) + j(p_2) \to c\bar{c}(P) + X$, the Mandelstam variables are defined as $s = (p_1 + p_2)^2$, $t = (p_1 - P)^2|_{\vec{q}=0}$, and $u = (p_2 - P)^2|_{\vec{q}=0}$ and satisfy $s + t + u = 4m_c^2$. Note that our results for hadroproduction can also be applied to resolved photoproduction.

A. Photoproduction

$g + \gamma \to c\bar{c}({}^3S_1^{[1]}) + g$:

$$|\overline{N}_{g\gamma}(^3S_1^{[1]})|^2 = \frac{8192\pi^3\alpha\alpha_s^2}{729m_c\left(s-4m_c^2\right)^3\left(4m_c^2-t\right)^3(s+t)^3}$$
$$\times\Big[2048m_c^{10}\left(3s^2+2st+3t^2\right)-256m_c^8\left(5s^3-8s^2t+4st^2+5t^3\right)-64m_c^6(15s^4+68s^3t$$
$$+62s^2t^2+32st^3+15t^4)+16m_c^4\left(21s^5+87s^4t+130s^3t^2+106s^2t^3+51st^4+21t^5\right)$$
$$-4m_c^2\left(7s^6+30s^5t+59s^4t^2+64s^3t^3+47s^2t^4+18st^5+7t^6\right)-st(s+t)\left(s^2+st+t^2\right)^2\Big] \tag{22}$$

$g+\gamma\to c\bar{c}(^3S_1^{[8]})+g$:

$$|\overline{N}_{g\gamma}(^3S_1^{[8]})|^2 = \frac{15}{8}|\overline{N}_{g\gamma}(^3S_1^{[1]})|^2 \tag{23}$$

$g+\gamma\to c\bar{c}(^1S_0^{[8]})+g$:

$$|\overline{N}_{g\gamma}(^1S_0^{[8]})|^2 = \frac{128\pi^3\alpha\alpha_s^2 st}{9m_c^3\left(4m_c^2-s\right)^3\left(4m_c^2-t\right)^3(s+t)^3\left(-4m_c^2+s+t\right)}$$
$$\times\Big[131072m_c^{14}-4096m_c^{12}(23s+11t)+1024m_c^{10}\left(33s^2+34st-3t^2\right)-256m_c^8(s+t)$$
$$\left(7s^2-11st-29t^2\right)-64m_c^6\left(13s^4+94s^3t+170s^2t^2+142st^3+37t^4\right)+16m_c^4(23s^5$$
$$+112s^4t+214s^3t^2+226s^2t^3+112st^4+23t^5)-4m_c^2(9s^6+50s^5t+116s^4t^2+138s^3t^3$$
$$+104s^2t^4+38st^5+9t^6)+5st(s+t)\left(s^2+st+t^2\right)^2\Big] \tag{24}$$

$g+\gamma\to c\bar{c}(^3P_J^{[8]})+g$:

$$|\overline{N}_{g\gamma}({}^3P_J^{[8]})|^2 = \frac{128\pi^3\alpha\alpha_s^2}{15m_c^5\left(s-4m_c^2\right)^4\left(t-4m_c^2\right)^4(s+t)^4\left(-4m_c^2+s+t\right)}$$
$$\times\Big[3145728m_c^{18}(s+t)\left(s^2+8st+t^2\right)-65536m_c^{16}(18s^4+113s^3t+370s^2t^2-27st^3$$
$$+18t^4)-32768m_c^{14}\left(39s^5+318s^4t+459s^3t^2+929s^2t^3+548st^4+39t^5\right)+16384m_c^{12}$$
$$\left(63s^6+519s^5t+1200s^4t^2+2043s^3t^3+1805s^2t^4+629st^5+63t^6\right)-2048m_c^{10}(144s^7$$
$$+1231s^6t+3673s^5t^2+7074s^4t^3+7874s^3t^4+4843s^2t^5+1301st^6+144t^7)+512m_c^8$$
$$\left(75s^8+662s^7t+2626s^6t^2+5981s^5t^3+7990s^4t^4+6371s^3t^5+3206s^2t^6+642st^7+75t^8\right)$$
$$-128m_c^6(15s^9+162s^8t+964s^7t^2+2725s^6t^3+4364s^5t^4+4034s^4t^5+2635s^3t^6+1114s^2t^7$$
$$+132st^8+15t^9)+16m_c^4st(27s^8+502s^7t+1968s^6t^2+3774s^5t^3+4338s^4t^4+3114s^3t^5$$
$$+1668s^2t^6+502st^7+27t^8)-16m_c^2s^2t^2(s+t)(25s^6+118s^5t+259s^4t^2+310s^3t^3$$
$$+244s^2t^4+103st^5+25t^6)+31s^3t^3(s+t)^2\left(s^2+st+t^2\right)^2\Big] \tag{25}$$

$g+\gamma\to c\bar{c}({}^3S_1^{[8]},{}^3D_1^{[8]})+g$:

$$|\overline{N}_{g\gamma}({}^3S_1^{[8]},{}^3D_1^{[8]})|^2 = \frac{2048\sqrt{5/3}\pi^3\alpha\alpha_s^2}{81m_c\left(s-4m_c^2\right)^4\left(t-4m_c^2\right)^4(s+t)^4}\Big[32768m_c^{14}(s+t)(3s^2$$
$$+5st+3t^2)-4096m_c^{12}\left(s^2+5st+t^2\right)\left(11s^2+18st+11t^2\right)-1024m_c^{10}(s+t)(10s^4$$
$$-109s^3t-202s^2t^2-109st^3+10t^4)+256m_c^8(36s^6-27s^5t-395s^4t^2-640s^3t^3$$
$$-395s^2t^4-27st^5+36t^6)-64m_c^6(s+t)(28s^6-3s^5t-250s^4t^2-400s^3t^3-250s^2t^4$$
$$-3st^5+28t^6)+16m_c^4(7s^8+4s^7t-100s^6t^2-323s^5t^3-440s^4t^4-323s^3t^5-100s^2t^6$$
$$+4st^7+7t^8)+4m_c^2st(s+t)\left(6s^6+22s^5t+51s^4t^2+66s^3t^3+51s^2t^4+22st^5+6t^6\right)$$
$$-s^2t^2(s+t)^2\left(s^2+st+t^2\right)^2\Big] \tag{26}$$

$q(\bar{q})+\gamma\to c\bar{c}({}^3S_1^{[8]})+q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})\gamma}({}^3S_1^{[8]})|^2 = \frac{8\pi^3\alpha\alpha_s^2e_q^2}{27m_c^5st\left(4m_c^2-s\right)}$$
$$\times\Big[640m_c^6-160m_c^4(2s+t)+4m_c^2\left(27s^2+10st+5t^2\right)-11s\left(s^2+t^2\right)\Big] \tag{27}$$

$q(\bar{q})+\gamma\to c\bar{c}({}^1S_0^{[8]})+q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})\gamma}(^1S_0^{[8]})|^2 = \frac{256\pi^3\alpha\alpha_s^2\left[m_c^2\left(44s^3+92s^2t-4st^2+44t^3\right)-5s(s+t)\left(s^2+t^2\right)\right]}{81m_c^3\left(s-4m_c^2\right)(s+t)^3\left(-4m_c^2+s+t\right)} \tag{28}$$

$q(\bar{q})+\gamma \to c\bar{c}(^3P_J^{[8]})+q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})\gamma}(^3P_J^{[8]})|^2 = \frac{256\pi^3\alpha\alpha_s^2}{135m_c^5\left(s-4m_c^2\right)(s+t)^4\left(-4m_c^2+s+t\right)}$$
$$\times\Big[512m_c^6\left(5s^2+26st+25t^2\right)+64m_c^4\left(s^3-23s^2t-111st^2-19t^3\right)+4m_c^2(s+t)$$
$$\left(57s^3+169s^2t-3st^2+61t^3\right)-31s(s+t)^2\left(s^2+t^2\right)\Big] \tag{29}$$

$q(\bar{q})+\gamma \to c\bar{c}(^3S_1^{[8]}, {}^3D_1^{[8]})+q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})\gamma}(^3S_1^{[8]}, {}^3D_1^{[8]})|^2 = -\frac{16\sqrt{5/3}\pi^3\alpha\alpha_s^2e_q^2\left[32m_c^4-8m_c^2(s+t)+s^2+t^2\right]}{9m_c^5st} \tag{30}$$

B. Hadroproduction

$g+g \to c\bar{c}(^3S_1^{[1]})+g$:

$$|\overline{N}_{gg}(^3S_1^{[1]})|^2 = \frac{15\alpha_s}{128\alpha}|\overline{N}_{g\gamma}(^3S_1^{[1]})|^2 \tag{31}$$

$g+g \to c\bar{c}(^3P_0^{[1]})+g$:

$$
\begin{aligned}
|\overline{N}_{gg}(^3P_0^{[1]})|^2 =& \frac{8\pi^3\alpha_s^3}{45m_c^5 st\left(4m_c^2-s\right)^5\left(4m_c^2-t\right)^5(s+t)^5\left(-4m_c^2+s+t\right)} \\
&\times\Big[251658240(s+t)^3\left(11s^2+30ts+11t^2\right)m_c^{24}-4194304(s+t)^2(5s+3t)\big(231s^3+847ts^2 \\
&+921t^2s+385t^3\big)m_c^{22}+1048576(s+t)^2\big(4155s^5+19680ts^4+36066t^2s^3+34246t^3s^2 \\
&+17160t^4s+3795t^5\big)m_c^{20}-262144\big(9735s^8+69804ts^7+219507t^2s^6+399776t^3s^5 \\
&+466128t^4s^4+359956t^5s^3+181147t^6s^2+54864t^7s+7755t^8\big)m_c^{18}+65536\big(15750s^9 \\
&+119943ts^8+411033t^2s^7+838246t^3s^6+1131460t^4s^5+1057300t^5s^4+688706t^6s^3 \\
&+304133t^7s^2+83403t^8s+10890t^9\big)m_c^{16}-32768\big(8955s^{10}+72307ts^9+267272t^2s^8 \\
&+600188t^3s^7+913993t^4s^6+992452t^5s^5+782713t^6s^4+445538t^7s^3+177082t^8s^2 \\
&+44767t^9s+5445t^{10}\big)m_c^{14}+4096\big(14235s^{11}+121663ts^{10}+482477t^2s^9+1181531t^3s^8 \\
&+1997582t^4s^7+2459484t^5s^6+2258884t^6s^5+1553862t^7s^4+791411t^8s^3+287757t^9s^2 \\
&+67783t^{10}s+7755t^{11}\big)m_c^{12}-1024\big(7575s^{12}+69080ts^{11}+295378t^2s^{10}+788288t^3s^9 \\
&+1469633t^4s^8+2024628t^5s^7+2117532t^6s^6+1697668t^7s^5+1040113t^8s^4+480268t^9s^3 \\
&+160638t^{10}s^2+35180t^{11}s+3795t^{12}\big)m_c^{10}+256\big(2415s^{13}+24464ts^{12}+115885t^2s^{11} \\
&+342339t^3s^{10}+708615t^4s^9+1090700t^5s^8+1287226t^6s^7+1180426t^7s^6+842220t^8s^5 \\
&+464375t^9s^4+194239t^{10}s^3+58985t^{11}s^2+11684t^{12}s+1155t^{13}\big)m_c^8-64\big(345s^{14} \\
&+4528ts^{13}+25895t^2s^{12}+89188t^3s^{11}+211442t^4s^{10}+369630t^5s^9+495006t^6s^8+517164t^7s^7 \\
&+424126t^8s^6+271810t^9s^5+134322t^{10}s^4+49688t^{11}s^3+12995t^{12}s^2+2128t^{13}s+165t^{14}\big)m_c^6 \\
&+16st(s+t)\big(259s^{12}+2323ts^{11}+9736t^2s^{10}+25649t^3s^9+47816t^4s^8+66583t^5s^7+71232t^6s^6 \\
&+58883t^7s^5+37456t^8s^4+17829t^9s^3+6056t^{10}s^2+1303t^{11}s+139t^{12}\big)m_c^4-4s^2t^2(s+t)^2 \\
&\left(s^2+ts+t^2\right)^2\left(87s^6+372ts^5+746t^2s^4+854t^3s^3+666t^4s^2+292t^5s+67t^6\right)m_c^2 \\
&+13s^3t^3(s+t)^3\left(s^2+ts+t^2\right)^4\Big] \qquad (32)
\end{aligned}
$$

$g+g\to c\bar{c}(^3P_1^{[1]})+g$:

$$|\overline{N}_{gg}(^3P_1^{[1]})|^2 = \frac{16\pi^3\alpha_s^3}{45m_c^5\left(s-4m_c^2\right)^5\left(4m_c^2-t\right)^5\left(s+t\right)^5}$$

$$\times\Big[2097152m_c^{20}(s+t)^2\left(s^2+t^2\right) - 131072m_c^{18}\left(11s^5-35s^4t-40s^3t^2+5st^4+11t^5\right)$$

$$-32768m_c^{16}\left(125s^6+864s^5t+1419s^4t^2+1164s^3t^3+739s^2t^4+404st^5+85t^6\right)$$

$$+8192m_c^{14}(818s^7+4579s^6t+9054s^5t^2+9591s^4t^3+7291s^3t^4+4914s^2t^5+2379st^6$$

$$+538t^7) - 2048m_c^{12}(2106s^8+11940s^7t+27483s^6t^2+35428s^5t^3+31746s^4t^4+23628s^3t^5$$

$$+14863s^2t^6+6300st^7+1306t^8) + 512m_c^{10}(2905s^9+17675s^8t+46718s^7t^2+71799s^6t^3$$

$$+75989s^5t^4+63429s^4t^5+44939s^3t^6+25058s^2t^7+9275st^8+1705t^9)$$

$$-128m_c^8(2265s^{10}+15280s^9t+46309s^8t^2+84008s^7t^3+105380s^6t^4+101476s^5t^5$$

$$+79720s^4t^6+51188s^3t^7+24729s^2t^8+7940st^9+1265t^{10}) + 32m_c^6(944s^{11}+7381s^{10}t$$

$$+26060s^9t^2+56013s^8t^3+83898s^7t^4+95046s^6t^5+85066s^5t^6+61038s^4t^7+34133s^3t^8$$

$$+14020s^2t^9+3821st^{10}+504t^{11}) - 8m_c^4(164s^{12}+1700s^{11}t+7455s^{10}t^2+19660s^9t^3$$

$$+35909s^8t^4+48760s^7t^5+51084s^6t^6+41660s^5t^7+26329s^4t^8+12440s^3t^9+4175s^2t^{10}$$

$$+900st^{11}+84t^{12}) + 2m_c^2st(s+t)\left(s^2+st+t^2\right)^2(106s^6+490s^5t+981s^4t^2+1092s^3t^3$$

$$+821s^2t^4+330st^5+66t^6) - 11s^2t^2(s+t)^2\left(s^2+st+t^2\right)^4\Big] \tag{33}$$

$g+g\to c\bar{c}(^3P_2^{[1]})+g$:

$$
\begin{aligned}
|\overline{N}_{gg}(^3P_2^{[1]})|^2 =& \frac{16\pi^3\alpha_s^3}{225 m_c^5 st\left(4m_c^2-s\right)^5\left(4m_c^2-t\right)^5(s+t)^5\left(-4m_c^2+s+t\right)} \\
&\times\Big[100663296(s+t)^3\left(15s^2+38ts+15t^2\right)m_c^{24}-8388608(s+t)^2(315s^4+1312ts^3 \\
&+2010t^2s^2+1252t^3s+315t^4)m_c^{22}+524288(s+t)(4620s^6+25287ts^5+58998t^2s^4 \\
&+73262t^3s^3+53438t^4s^2+21927t^5s+4140t^6)m_c^{20}-262144(5550s^8+36807ts^7 \\
&+109272t^2s^6+189017t^3s^5+212562t^4s^4+162877t^5s^3+84442t^6s^2+27387t^7s \\
&+4230t^8)m_c^{18}+32768(18360s^9+126189ts^8+389652t^2s^7+714467t^3s^6+879782t^4s^5 \\
&+782522t^5s^4+517507t^6s^3+248752t^7s^2+78429t^8s+11880t^9)m_c^{16}-8192(21240s^{10} \\
&+155344ts^9+504167t^2s^8+967724t^3s^7+1252087t^4s^6+1199524t^5s^5+911047t^6s^4 \\
&+558524t^7s^3+262447t^8s^2+81544t^9s+11880t^{10})m_c^{14}+2048(17100s^{11}+136829ts^{10} \\
&+477883t^2s^9+972667t^3s^8+1311058t^4s^7+1293789t^5s^6+1038389t^6s^5+732018t^7s^4 \\
&+446587t^8s^3+210243t^9s^2+63149t^{10}s+8460t^{11})m_c^{12}-512(9180s^{12}+82966ts^{11} \\
&+321992t^2s^{10}+716914t^3s^9+1033387t^4s^8+1055592t^5s^7+855762t^6s^6+634352t^7s^5 \\
&+454727t^8s^4+283274t^9s^3+129252t^{10}s^2+35566t^{11}s+4140t^{12})m_c^{10}+128(2940s^{13} \\
&+31855ts^{12}+143846t^2s^{11}+365607t^3s^{10}+592170t^4s^9+663334t^5s^8+565754t^6s^7 \\
&+427154t^7s^6+326754t^8s^5+239410t^9s^4+141047t^{10}s^3+57346t^{11}s^2+13375t^{12}s \\
&+1260t^{13})m_c^8-32(420s^{14}+6548ts^{13}+37651t^2s^{12}+116096t^3s^{11}+224575t^4s^{10} \\
&+299040t^5s^9+299220t^6s^8+251484t^7s^7+199880t^8s^6+152480t^9s^5+99555t^{10}s^4 \\
&+48976t^{11}s^3+16051t^{12}s^2+2828t^{13}s+180t^{14})m_c^6+8st(s+t)(464s^{12}+4070ts^{11} \\
&+15035t^2s^{10}+32905t^3s^9+49483t^4s^8+56786t^5s^7+54168t^6s^6+44806t^7s^5+32183t^8s^4 \\
&+18405t^9s^3+7675t^{10}s^2+2030t^{11}s+224t^{12})m_c^4-2s^2t^2(s+t)^2\left(s^2+ts+t^2\right)^2(138s^6 \\
&+552ts^5+943t^2s^4+922t^3s^3+783t^4s^2+392t^5s+98t^6)m_c^2+7s^3t^3(s+t)^3\left(s^2+ts+t^2\right)^4\Big]
\end{aligned} \tag{34}
$$

$g+g\to c\bar{c}(^3S_1^{[8]})+g$:

$$|\overline{N}_{gg}(^3S_1^{[8]})|^2 = \frac{\pi^3\alpha_s^3}{54m_c^5\left(s-4m_c^2\right)^3\left(4m_c^2-t\right)^3(s+t)^3}$$
$$\times\Big[16384m_c^{14}\left(87s^2+22st+87t^2\right)-4096m_c^{12}\left(14s^3-449s^2t-221st^2+14t^3\right)$$
$$-2048m_c^{10}\left(480s^4+1969s^3t+2182s^2t^2+1384st^3+399t^4\right)+256m_c^8(2910s^5$$
$$+11616s^4t+17509s^3t^2+15433s^2t^3+8178st^4+2100t^5)-64m_c^6(4048s^6+17148s^5t$$
$$+31739s^4t^2+35722s^3t^3+25841s^2t^4+11412st^5+2590t^6)+16m_c^4(2754s^7+12968s^6t$$
$$+28779s^5t^2+39352s^4t^3+35950s^3t^4+22137s^2t^5+8270st^6+1620t^7)$$
$$-108m_c^2\left(s^2+st+t^2\right)\left(27s^6+131s^5t+269s^4t^2+302s^3t^3+227s^2t^4+89st^5+15t^6\right)$$
$$+297st(s+t)\left(s^2+st+t^2\right)^3\Big] \tag{35}$$

$g+g\to c\bar{c}(^1S_0^{[8]})+g$:

$$|\overline{N}_{gg}(^1S_0^{[8]})|^2 = \frac{5\pi^3\alpha_s^3}{3m_c^3st\left(4m_c^2-s\right)^3\left(4m_c^2-t\right)^3(s+t)^3\left(-4m_c^2+s+t\right)}$$
$$\times\Big[65536m_c^{16}\left(5s^3+20s^2t+8st^2+5t^3\right)-16384m_c^{14}(25s^4+116s^3t+110s^2t^2+56st^3$$
$$+25t^4)+12288m_c^{12}\left(28s^5+129s^4t+189s^3t^2+137s^2t^3+65st^4+20t^5\right)$$
$$-2048m_c^{10}\left(87s^6+426s^5t+801s^4t^2+806s^3t^3+501s^2t^4+204st^5+45t^6\right)$$
$$+256m_c^8\left(222s^7+1182s^6t+2653s^5t^2+3385s^4t^3+2749s^3t^4+1501s^2t^5+522st^6+90t^7\right)$$
$$-64m_c^6(180s^8+1020s^7t+2591s^6t^2+3964s^5t^3+4006s^4t^4+2812s^3t^5+1355s^2t^6$$
$$+408st^7+60t^8)+16m_c^4(85s^9+516s^8t+1456s^7t^2+2561s^6t^3+3096s^5t^4+2688s^4t^5$$
$$+1673s^3t^6+724s^2t^7+192st^8+25t^9)-4m_c^2\left(s^2+st+t^2\right)^2(17s^6+88s^5t+167s^4t^2$$
$$+168s^3t^3+119s^2t^4+40st^5+5t^6)+5st(s+t)\left(s^2+st+t^2\right)^4\Big] \tag{36}$$

$g+g\to c\bar{c}(^3P_J^{[8]})+g$:

$$|\overline{N}_{gg}({}^3P_J^{[8]})|^2 = \frac{\pi^3\alpha_s^3}{m_c^5 st\left(s-4m_c^2\right)^4\left(t-4m_c^2\right)^4(s+t)^4\left(-4m_c^2+s+t\right)}$$

$$\times\Big[1048576m_c^{20}\left(115s^4+435s^3t+564s^2t^2+295st^3+115t^4\right)-524288m_c^{18}(345s^5$$
$$+1571s^4t+2683s^3t^2+2148s^2t^3+1156st^4+345t^5)+65536m_c^{16}(2235s^6+11102s^5t$$
$$+22740s^4t^2+23720s^3t^3+16400s^2t^4+7932st^5+1955t^6)-16384m_c^{14}(4710s^7$$
$$+24932s^6t+57177s^5t^2+71731s^4t^3+60491s^3t^4+37287s^2t^5+16102st^6+3450t^7)$$
$$+4096m_c^{12}(6660s^8+38018s^7t+94927s^6t^2+134978s^5t^3+132094s^4t^4+97708s^3t^5$$
$$+54647s^2t^6+21708st^7+4140t^8)-1024m_c^{10}(6390s^9+39787s^8t+107560s^7t^2$$
$$+168509s^6t^3+183028s^5t^4+154598s^4t^5+104319s^3t^6+54540s^2t^7+20307st^8+3450t^9)$$
$$+256m_c^8(4055s^{10}+27855s^9t+82334s^8t^2+142320s^7t^3+169197s^6t^4+156936s^5t^5$$
$$+120977s^4t^6+77050s^3t^7+38034s^2t^8+13095st^9+1955t^{10})-64m_c^6(1530s^{11}$$
$$+12009s^{10}t+40145s^9t^2+79031s^8t^3+106482s^7t^4+110003s^6t^5+93903s^5t^6+68322s^4t^7$$
$$+40531s^3t^8+18105s^2t^9+5449st^{10}+690t^{11})+16m_c^4(255s^{12}+2610s^{11}t+10798s^{10}t^2$$
$$+26205s^9t^3+43547s^8t^4+54320s^7t^5+53906s^6t^6+43520s^5t^7+28607s^4t^8+14485s^3t^9$$
$$+5278s^2t^{10}+1230st^{11}+115t^{12})-4m_c^2st(s+t)\left(s^2+st+t^2\right)^2(150s^6+696s^5t$$
$$+1299s^4t^2+1358s^3t^3+1059s^2t^4+456st^5+90t^6)+31s^2t^2(s+t)^2\left(s^2+st+t^2\right)^4\Big] \quad (37)$$

$g+g\to c\bar{c}({}^3S_1^{[8]},{}^3D_1^{[8]})+g$:

$$|\overline{N}_{gg}({}^3S_1^{[8]},{}^3D_1^{[8]})|^2 = -\frac{\pi^3\alpha_s^3}{108\sqrt{15}m_c^5\,(s-4m_c^2)^4\,(t-4m_c^2)^4\,(s+t)^4}$$
$$\times\Big[-524288m_c^{20}\left(4463s^2+25840st+5057t^2\right)+65536m_c^{18}(s+t)(7767s^2+145760st$$
$$+13689t^2)+16384m_c^{16}\left(40922s^4+83203s^3t-78401s^2t^2+46753st^3+25451t^4\right)$$
$$-4096m_c^{14}(s+t)\left(41543s^4+579922s^3t+620206s^2t^2+525220st^3+20753t^4\right)-3072m_c^{12}$$
$$\left(40919s^6-153453s^5t-767319s^4t^2-1075115s^3t^3-737820s^2t^4-133812st^5+45008t^6\right)$$
$$+256m_c^{10}(s+t)(302824s^6+248112s^5t-1111219s^4t^2-1766115s^3t^3-1102093s^2t^4$$
$$+250353st^5+304390t^6)-64m_c^8(287151s^8+1147982s^7t+1476785s^6t^2+462324s^5t^3$$
$$-288201s^4t^4+391881s^3t^5+1418006s^2t^6+1132457st^7+286503t^8)+16m_c^6(s+t)$$
$$(136080s^8+645678s^7t+1174264s^6t^2+1234125s^5t^3+1166106s^4t^4+1190394s^3t^5$$
$$+1143268s^2t^6+640305st^7+136080t^8)-4m_c^4(27216s^{10}+226800s^9t+774349s^8t^2$$
$$+1531962s^7t^3+2158036s^6t^4+2377567s^5t^5+2135518s^4t^6+1512603s^3t^7+768949s^2t^8$$
$$+226800st^9+27216t^{10})+m_c^2st(s+t)(18144s^8+104976s^7t+274390s^6t^2+449444s^5t^3$$
$$+528929s^4t^4+449390s^3t^5+274363s^2t^6+104976st^7+18144t^8)$$
$$-1620s^2t^2(s+t)^2\left(s^2+st+t^2\right)^3\Big] \tag{38}$$

$q(\bar{q})+g\to c\bar{c}({}^3P_0^{[1]})+q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})g}({}^3P_0^{[1]})|^2 = \frac{16\pi^3\alpha_s^3}{405m_c^5t\,(4m_c^2-s)\,(4m_c^2-t)^5}\left(12m_c^2-t\right)\Big[87040m_c^{10}-256m_c^8$$
$$(285s+278t)+256m_c^6\left(115s^2+166st+81t^2\right)-32m_c^4\left(115s^3+311s^2t+243st^2+77t^3\right)$$
$$+4m_c^2t\left(96s^3+188s^2t+120st^2+23t^3\right)-13st^2\left(2s^2+2st+t^2\right)\Big] \tag{39}$$

$q(\bar{q})+g\to c\bar{c}({}^3P_1^{[1]})+q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})g}({}^3P_1^{[1]})|^2 = \frac{32\pi^3\alpha_s^3}{405m_c^5\,(4m_c^2-s)\,(4m_c^2-t)^5}\Big[256m_c^8(124s+21t)-64m_c^6$$
$$(288s^2+341st+63t^2)+16m_c^4(164s^3+456s^2t+321st^2+63t^3)-4m_c^2t(106s^3$$
$$+190s^2t+115st^2+21t^3)+11st^2\left(2s^2+2st+t^2\right)\Big] \tag{40}$$

$q(\bar{q})+g\to c\bar{c}({}^3P_2^{[1]})+q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})g}({}^3P_2^{[1]})|^2 = \frac{32\pi^3\alpha_s^3}{2025m_c^5t\,(4m_c^2-s)\,(4m_c^2-t)^5}\Big[614400m_c^{12} - 34816m_c^{10}(15s$$
$$+13t) + 256m_c^8\left(840s^2+1386st+443t^2\right) - 64m_c^6\left(420s^3+1708s^2t+1315st^2+177t^3\right)$$
$$+16m_c^4t\left(464s^3+1060s^2t+519st^2+43t^3\right) - 4m_c^2t^2\left(138s^3+206s^2t+87st^2+17t^3\right)$$
$$+7st^3\left(2s^2+2st+t^2\right)\Big] \tag{41}$$

$q(\bar{q}) + g \rightarrow c\bar{c}({}^3S_1^{[8]}) + q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})g}({}^3S_1^{[8]})|^2 = \frac{\pi^3\alpha_s^3}{81m_c^5st\,(4m_c^2-s)\,(s+t)^3}\Big[128m_c^6\left(20s^3+69s^2t-39st^2+20t^3\right)$$
$$-32m_c^4\left(40s^4+113s^3t+27s^2t^2+10st^3+20t^4\right) + 4m_c^2(108s^5+193s^4t+41s^3t^2$$
$$+225s^2t^3+st^4+20t^5) - 11s\left(4s^5+3s^4t+7s^3t^2+7s^2t^3+3st^4+4t^5\right)\Big] \tag{42}$$

$q(\bar{q}) + g \rightarrow c\bar{c}({}^1S_0^{[8]}) + q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})g}({}^1S_0^{[8]})|^2 = \frac{10\pi^3\alpha_s^3\left[m_c^2\left(44s^3+92s^2t-4st^2+44t^3\right) - 5s\left(s^3+s^2t+st^2+t^3\right)\right]}{27m_c^3\,(s-4m_c^2)\,(s+t)^3\,(-4m_c^2+s+t)} \tag{43}$$

$q(\bar{q}) + g \rightarrow c\bar{c}({}^3P_J^{[8]}) + q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})g}({}^3P_J^{[8]})|^2 = \frac{2\pi^3\alpha_s^3}{9m_c^5\,(s-4m_c^2)\,(s+t)^4\,(-4m_c^2+s+t)}\Big[512m_c^6\left(5s^2+26st+25t^2\right)$$
$$+64m_c^4\left(s^3-23s^2t-111st^2-19t^3\right) + 4m_c^2(s+t)\left(57s^3+169s^2t-3st^2+61t^3\right)$$
$$-31s(s+t)^2\left(s^2+t^2\right)\Big] \tag{44}$$

$q(\bar{q}) + g \rightarrow c\bar{c}({}^3S_1^{[8]}, {}^3D_1^{[8]}) + q(\bar{q})$:

$$|\overline{N}_{q(\bar{q})g}({}^3S_1^{[8]}, {}^3D_1^{[8]})|^2 = -\frac{2\pi^3\alpha_s^3}{27\sqrt{15}m_c^5st(s+t)^4}\Big[128m_c^4\left(5s^4+2s^3t+21s^2t^2+2st^3+5t^4\right)$$
$$-32m_c^2\left(5s^5+16s^4t+14s^3t^2+14s^2t^3+16st^4+5t^5\right)$$
$$+5(s+t)^2\left(4s^4-s^3t+8s^2t^2-st^3+4t^4\right)\Big] \tag{45}$$

$\bar{q} + q \rightarrow c\bar{c}({}^3P_0^{[1]}) + g$:

$$|\overline{N}_{\bar{q}q}(^3P_0^{[1]})|^2 = -\frac{128\pi^3\alpha_s^3\left(12m_c^2-s\right)\left(880m_c^4-112m_c^2s+13s^2\right)}{3645m_c^5s\left(4m_c^2-s\right)^5}$$
$$\times\left[2t\left(s-4m_c^2\right)+\left(s-4m_c^2\right)^2+2t^2\right] \tag{46}$$

$\bar{q}+q\to c\bar{c}(^3P_1^{[1]})+g$:

$$|\overline{N}_{\bar{q}q}(^3P_1^{[1]})|^2 = \frac{256\pi^3\alpha_s^3}{1215m_c^5\left(s-4m_c^2\right)^5}\Big[64m_c^6(31s+84t)-16m_c^4(73s^2+150st$$
$$+84t^2)+4m_c^2s\left(53s^2+88st+66t^2\right)-11s^2\left(s^2+2st+2t^2\right)\Big] \tag{47}$$

$\bar{q}+q\to c\bar{c}(^3P_2^{[1]})+g$:

$$|\overline{N}_{\bar{q}q}(^3P_2^{[1]})|^2 = \frac{256\pi^3\alpha_s^3}{6075m_c^5s\left(s-4m_c^2\right)^5}\Big[92160m_c^{10}-512m_c^8(71s+90t)+64m_c^6$$
$$(33s^2+404st+180t^2)-112m_c^4s\left(s^2+46st+32t^2\right)+4m_c^2s^2\left(33s^2+112st+98t^2\right)$$
$$-7s^3\left(s^2+2st+2t^2\right)\Big] \tag{48}$$

$\bar{q}+q\to c\bar{c}(^3S_1^{[8]})+g$:

$$|\overline{N}_{\bar{q}q}(^3S_1^{[8]})|^2 = \frac{8\pi^3\alpha_s^3}{243m_c^5t\left(4m_c^2-s\right)^3\left(-4m_c^2+s+t\right)}\Big[45056m_c^{10}$$
$$-256m_c^8(84s+205t)+64m_c^6\left(128s^2+496st+475t^2\right)-16m_c^4(224s^3+780s^2t$$
$$+1029st^2+540t^3)+4m_c^2\left(180s^4+676s^3t+1155s^2t^2+936st^3+270t^4\right)$$
$$-11s\left(4s^4+17s^3t+35s^2t^2+36st^3+18t^4\right)\Big] \tag{49}$$

$\bar{q}+q\to c\bar{c}(^1S_0^{[8]})+g$:

$$|\overline{N}_{\bar{q}q}(^1S_0^{[8]})|^2 = -\frac{400\pi^3\alpha_s^3\left[16m_c^4-8m_c^2(s+t)+s^2+2st+2t^2\right]}{81m_c^3s\left(s-4m_c^2\right)^2} \tag{50}$$

$\bar{q}+q\to c\bar{c}(^3P_J^{[8]})+g$:

$$|\overline{N}_{\bar{q}q}(^3P_J^{[8]})|^2 = \frac{-16\pi^3\alpha_s^3}{27m_c^5s\left(s-4m_c^2\right)^4}\Big[29440m_c^8-128m_c^6(68s+115t)+32m_c^4(59s^2$$
$$+205st+115t^2)-8m_c^2s\left(64s^2+121st+90t^2\right)+31s^2\left(s^2+2st+2t^2\right)\Big] \tag{51}$$

$\bar{q}+q \to c\bar{c}({}^3S_1^{[8]},{}^3D_1^{[8]})+g$:

$$|\overline{N}_{\bar{q}q}({}^3S_1^{[8]},{}^3D_1^{[8]})|^2 = -\frac{16\pi^3\alpha_s^3}{81\sqrt{15}m_c^5 t\left(s-4m_c^2\right)^4\left(-4m_c^2+s+t\right)}\Big[81920m_c^{12} -1024m_c^{10}(80s+157t)+256m_c^8\left(140s^2+569st+535t^2\right)-128m_c^6(80s^3+425s^2t +710st^2+378t^3)+32m_c^4\left(70s^4+353s^3t+705s^2t^2+630st^3+189t^4\right)-4m_c^2s(80s^4 +353s^3t+700s^2t^2+684st^3+252t^4)+5s^2\left(4s^4+17s^3t+35s^2t^2+36st^3+18t^4\right)\Big] \quad (52)$$

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