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Approximate N^3LO Higgs-boson production cross section using physical-kernel constraints

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Abstract

The single-logarithmic enhancement of the physical kernel for Higgs production by gluon-gluon fusion in the heavy top-quark limit is employed to derive the leading so far unknown contributions, $\ln^{5,4,3}(1-z)$, to the N^3LO coefficient function in the threshold expansion. Also using knowledge from Higgs-exchange DIS to estimate the remaining terms not vanishing for $z = m_H^2/\hat{s} \to 1$, these results are combined with the recently completed soft + virtual contributions to provide an uncertainty band for the complete N^3LO correction. For the 2008 MSTW parton distributions these N^3LO contributions increase the cross section at 14 TeV by $(10 \pm 2)\%$ and $(3 \pm 2.5)\%$ for the standard choices $\mu_R = m_H$ and $\mu_R = m_H/2$ of the renormalization scale. The remaining uncertainty arising from the hard-scattering cross sections can be quantified as no more than 5%, which is smaller than that due to the strong coupling and the parton distributions.

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1 Introduction

After the recent discovery of a new boson by the ATLAS and CMS collaborations [1,2] at the Large Hadron Collider (LHC), precise theoretical predictions are needed in order to determine whether or not this particle is indeed, as it appears so far [3, 4], the Standard Model (SM) Higgs boson. In particular, to study its properties and to be able to distinguish between SM and Beyond-the-SM scenarios, it is important to provide precision calculations of the Higgs production rate.

The main production mechanism for the SM Higgs boson at the LHC is the gluon-gluon fusion process. The radiative corrections in Quantum Chromodynamics (QCD) for the corresponding inclusive cross section have been computed to next-to-next-to-leading order (NNLO) in the effective theory [5–7] based on the limit of a large top-quark mass, $m_t \gg m_H$, and later for $m_H \lesssim 2\,m_t$ in the full theory [8–10]. The large size of the QCD corrections at this and the previous [11–14] order, mainly due to large contributions from the $z \to 1$ limit, where z is the ratio of the Higgs mass m_H to the partonic center-of-mass energy $\sqrt{\hat{s}}$ squared, $z = m_H^2/\hat{s}$, together with the still sizeable scale uncertainty have motivated systematic theory improvements beyond NNLO.

At the next-to-next-to-next-to-leading order (N^{3}LO), all plus-distribution contributions to the partonic cross section in the $\overline{\mathrm{MS}}$ scheme, $[(1-z)^{-1}\ln^k(1-z)]_+$ with $0 \le k \le 5$, i.e., the leading contributions for Higgs boson production at threshold, are known in the large top-mass limit [15]. Recently also the corresponding terms proportional to $\delta(1-z)$ have been computed [16] which include the 3-loop virtual contributions. In Mellin N-space, with N being the conjugate variable of z, the threshold logarithms appear as $\ln^k N$ with $1 \le k \le 2n$ at the n-th order, while the virtual contributions lead to a constant in N. Based on comparisons at the previous orders, the soft-virtual (SV) approximation in N-space (which can be supplemented by an all-order resummation of threshold contributions up to next-to-next-to-next-to-leading logarithmic (N^{3}LL) accuracy [17]) has been shown to yield reliable predictions for the total Higgs production cross section, see, e.g., Refs. [15, 18–21]. Studies in the soft-collinear effective theory (SCET) have reached similar conclusions concerning the validity of an approximation based on threshold logarithms [22, 23].

In this paper we present N^{3}LO and N^{4}LO results beyond the SV approximation. For a scheme-independent description of the hard scattering process one can employ physical evolution kernels (also called physical anomalous dimensions) which arise from standard QCD factorization once the parton densities (PDFs) are eliminated from the evolution equation for the physical cross section. Since the physical evolution kernels exhibit only a single-logarithmic enhancement at large z, see Refs. [24, 25], we are able to establish constraints on the coefficient functions in the $\overline{\mathrm{MS}}$ scheme. In this manner we obtain at N^{3}LO the subleading logarithmic contributions $\ln^k(1-z)$ (or in Mellin space $N^{-1}\ln^k N$) for $k = 5, 4, 3$ to the gluon-gluon partonic cross section. In addition, with the help of results for inclusive deep-inelastic scattering (DIS) by Higgs exchange which are known to N^{3}LO [25], we can also systematically estimate the size of the remaining $O(N^{-1})$ terms.

Based on the SV contributions together with the new subleading double logarithmically enhanced $N^{-1}\ln^k N$ terms, we are then able to provide improved predictions for the yet unknown full N^{3}LO corrections to the gluon-gluon coefficient function for inclusive Higgs production. As an additional uncertainty estimate we study the numerical impact of the N^{4}LO corrections in the SV approximation. Our rigorous analytical results at N^{3}LO can be compared to previous phenomenologically motivated approximations for the third-order cross section [26, 27].

Beyond the $(1-z)^0$ terms in the expansion about $z=1$, the gluon-gluon coefficient function receives 'flavour-singlet' contributions which, unlike for DIS and semi-inclusive e^+e^- annihilation (SIA), cannot be analyzed (so far) in terms of physical kernels for hadron-collider observables. Hence an extension of the above results to all powers of $(1-z)$ along the lines of Ref. [24] can be performed only for the 'non-singlet' $C_A^k n_f^\ell$ contributions. Yet the corresponding terms can, at least, provide useful checks of future Feynman-diagram calculations. Finally we take the opportunity to update the corresponding results for the dominant quark-antiquark annihilation contribution to the Drell-Yan (DY) process to the same accuracy at N^{3}LO and N^{4}LO.

2 Constraints from the physical evolution kernel

For $m_H \simeq 125$ GeV [1, 2] the higher-order corrections can be addressed in the large top-mass approximation, in which the effective coupling of the Higgs to partons is given by the Lagrangian

$$\mathcal{L}_{\rm eff} \; = \; -\frac{1}{4\upsilon} C(\mu_R^2)\, H\, G^a_{\mu\nu} G_a^{\mu\nu} \; , \tag{2.1}$$

where $\upsilon \simeq 246$ GeV is the Higgs vacuum expectation value and $G^a_{\mu\nu}$ denotes the gluon field strength tensor. The matching coefficient $C(\mu_R^2)$ is fully known up to N^{3}LO [28–30]. Standard QCD factorization, here as usual performed in the $\overline{\rm MS}$ scheme, allows to express the inclusive hadronic cross section for Higgs boson production at a center-of-mass energy $E_{cm} = \sqrt{S}$ as

$$\begin{aligned} \sigma(S, m_H^2) \; &= \; \tau \sum_{a,b} \int_0^1 \frac{dx_1}{x_1}\,\frac{dx_2}{x_2}\; f_{a/h_1}(x_1,\mu_F^2)\; f_{b/h_2}(x_2,\mu_F^2) \int_0^1 dz\, \delta\Big(z - \frac{\tau}{x_1 x_2}\Big) \times \\ &\quad \times \widetilde{\sigma}_0\, c_{ab}(z,\, \alpha_{\rm s}(\mu_R^2),\, m_H^2/\mu_R^2,\, m_H^2/\mu_F^2) \; , \end{aligned} \tag{2.2}$$

where $\tau = m_H^2/S$, and μ_F and μ_R are the mass-factorization and renormalization scales, respectively. The PDFs of the colliding hadrons are denoted by $f_{a/h}(x,\mu_F^2)$, the subscripts a,b indicating the type of massless parton. The variable $z = m_H^2/\hat{s}$ is the partonic equivalent of τ, with $\hat{s} = x_1 x_2 S$ being the partonic center-of-mass energy squared. The complete $\alpha_{\rm s}$-expansion of the effective Higgs-gluon vertex is included in $\widetilde{\sigma}_0$, viz

$$\widetilde{\sigma}_0 \; = \; \frac{\pi C(\mu_R^2)^2}{64\upsilon^2} \quad \text{with} \quad C(\mu_R^2) \; = \; -\frac{\alpha_{\rm s}(\mu_R^2)}{3\pi}\left\{1 + 11\,\frac{\alpha_{\rm s}(\mu_R^2)}{4\pi} + \ldots\right\} \; . \tag{2.3}$$

We expand the coefficient functions c_{ab} in powers of the strong coupling with $a_{\rm s} \equiv \alpha_{\rm s}(\mu_R^2)/(4\pi)$,

$$c_{ab}(z,\, \alpha_{\rm s}(\mu_R^2),\, m_H^2/\mu_R^2,\, m_H^2/\mu_F^2) \; = \; \sum_{n=0}^{\infty} a_{\rm s}^n c_{ab}^{(n)}(z,\, m_H^2/\mu_R^2,\, m_H^2/\mu_F^2) \; . \tag{2.4}$$

At leading order (LO) we have $c_{ab}^{(0)} = \delta_{ag}\,\delta_{bg}\,\delta(1-z)$; at $n \geq 1$ the coefficient functions $c_{ab}^{(n)}$ in Eq. (2.4) differ from the quantities Δ_{ab} in Refs. [6,7] by a factor of z^{-1}, cf. Eq. (4.3) of [7]. As mentioned above, the QCD corrections within the large top-mass limit are known up to NNLO [5–7], while at N^{3}LO only the soft and virtual (SV) contributions, i.e., the plus-distributions $\mathcal{D}_k(z) = [(1-z)^{-1}\ln^k(1-z)]_+$ and the $\delta(1-z)$ terms in the gluon-gluon channel are available so far [15,16]. Very recently, also the leading double-logarithmic threshold contribution to the quark-gluon coefficient function $c_{qg}^{(3)}$ has been obtained as part of an all-order result [31].

More information about large-z contributions to the N^{3}LO coefficient function $c_{gg}^{(3)}$ and its higher-order counterparts can be extracted from the physical evolution kernel. To that end, we consider the case $\mu_F = \mu_R = m_H$ (the scale-dependent terms can be reconstructed by renormalization-group arguments) and define dimensionless partonic 'structure functions' $\mathcal{F}_{ab}$

$$\sigma(S, m_H^2) \;=\; \sum_{a,b} \widetilde{\sigma}_0 \, \mathcal{F}_{ab} \;. \tag{2.5}$$

For the sub-dominant $(1-z)^0$ terms we can restrict ourselves to the 'non-singlet' case where only the coefficient function c_{gg} and the splitting function P_{gg} are taken into account; other contributions are suppressed by two powers of $(1-z)$ relative to the leading $(1-z)^{-1}$ terms. Exploiting the evolution equations for α_s and the PDFs one arrives at the expression, cf. Ref. [24],

$$\begin{aligned}
\frac{d}{d\ln m_H^2}\,\mathcal{F}_{gg} \;&=\; \left\{ 2P_{gg}(a_s) + \beta(a_s)\,\frac{dc_{gg}(a_s)}{da_s} \otimes \left(c_{gg}(a_s)\right)^{-1} \right\} \otimes \mathcal{F}_{gg} \\
&\equiv\; K_{gg} \otimes \mathcal{F}_{gg} \;\equiv\; \sum_{\ell=0}^{\infty} a_s^{\ell+1} K_{gg}^{(\ell)} \otimes \mathcal{F}_{gg} \\
&=\; \left\{ 2a_s P_{gg}^{(0)} + \sum_{\ell=1}^{\infty} a_s^{\ell+1} \left(2P_{gg}^{(\ell)} - \sum_{k=0}^{\ell-1} \beta_k \, \tilde{c}_{gg}^{(\ell-k)} \right) \right\} \otimes \mathcal{F}_{gg}
\end{aligned} \tag{2.6}$$

which defines the physical evolution kernel K_{gg} and its perturbative expansion. Here $\otimes$ denotes the usual Mellin convolution, cf. Eq. (2.2), while $\beta(a_s)$ stands for the standard QCD beta function, $\beta(a_s) = -\beta_0 a_s^2 - \ldots$ with $\beta_0 = 11/3\, C_A - 2/3\, n_f$. $P_{gg}^{(\ell)}$ are the $(\ell+1)$-loop gluon-gluon splitting functions, defined analogously to $K_{gg}^{(\ell)}$ in the middle line of Eq. (2.6). Up to N^{4}LO the expansion coefficients $\tilde{c}_{gg}^{(\ell)}$ in the last line are given by [32]

$$\begin{aligned}
\tilde{c}_{gg}^{(1)} \;&=\; c_{gg}^{(1)} \,, \\
\tilde{c}_{gg}^{(2)} \;&=\; 2c_{gg}^{(2)} - c_{gg}^{(1)} \otimes c_{gg}^{(1)} \,, \\
\tilde{c}_{gg}^{(3)} \;&=\; 3c_{gg}^{(3)} - 3c_{gg}^{(2)} \otimes c_{gg}^{(1)} + c_{gg}^{(1)} \otimes c_{gg}^{(1)} \otimes c_{gg}^{(1)} \,, \\
\tilde{c}_{gg}^{(4)} \;&=\; 4c_{gg}^{(4)} - 4c_{gg}^{(3)} \otimes c_{gg}^{(1)} - 2c_{gg}^{(2)} \otimes c_{gg}^{(2)} + 4c_{gg}^{(2)} \otimes c_{gg}^{(1)} \otimes c_{gg}^{(1)} - c_{gg}^{(1)} \otimes c_{gg}^{(1)} \otimes c_{gg}^{(1)} \otimes c_{gg}^{(1)} \,.
\end{aligned} \tag{2.7}$$

The calculation of the physical kernel, given the fact that it contains several convolutions, is best carried out in N-space. The Mellin N-moments are defined as

$$f(N) \;=\; \int_0^1 dz \left(z^{N-1}\{-1\} \right) \, f(z)_{\{+\}} \;, \tag{2.8}$$

where the parts in curly brackets apply to plus-distributions. A useful if approximate dictionary between the logarithms in z-space and N-space is

$$\begin{aligned}
(-1)^k \left(\frac{\ln^{k-1}(1-z)}{1-z} \right)_+ \;&\stackrel{\mathrm{M}}{=}\; \frac{1}{k} \Big([S_{1-}(N)]^k \,+\, \frac{1}{2} k(k-1)\zeta_2 \,[S_{1-}(N)]^{k-2} \,+\, O([S_{1-}(N)]^{k-3}) \Big) \,, \\
(-1)^k \ln^k(1-z) \;&\stackrel{\mathrm{M}}{=}\; \frac{1}{N} \Big(\ln^k \widetilde{N} \,+\, \frac{1}{2} k(k-1)\zeta_2 \ln^{k-2} \widetilde{N} \,+\, O(\ln^{k-3} \widetilde{N}) \Big) \,+\, O\left(\frac{1}{N^2} \right)
\end{aligned} \tag{2.9}$$

with $S_{1-}(N) = \ln\widetilde{N} - 1/(2N) + O(1/N^2)$ and $\widetilde{N} = Ne^{\gamma_e}$, i.e., $\ln\widetilde{N} = \ln N + \gamma_e$ with $\gamma_e \simeq 0.577216$. Here $\stackrel{M}{=}$ indicates that the right-hand-side is the Mellin transform (2.8) of the previous expression. The splitting functions, coefficients functions and their products in Mellin space can be expressed in terms of harmonic sums [33]. These give rise to harmonic polylogarithms [34] in z-space from which one can then extract the large-z and large-N expansions. All these manipulations were carried out using the symbolic manipulation system FORM [35–37].

The crucial feature of the (factorization scheme independent) physical evolution kernels to be exploited here is the fact that they display only a single-logarithmic large-z enhancement. This behaviour is in striking contrast to that of the $\overline{\mathrm{MS}}$ scheme coefficient functions, which do include double-logarithmic contributions, i.e., $\ln^k(1-z)$ with $k > n \geq 1$ at N^{n}LO, at all orders in the expansion around $z = 1$. This behaviour of the physical evolution kernels has been observed at higher orders in perturbative QCD for a variety of observables in DIS, semi-inclusive e^+e^- annihilation (SIA) and DY lepton-pair production [24, 25]. For DIS and SIA it can be derived from properties of the unfactorized partonic cross sections in dimensional regularization, see Refs. [38, 39].

Also the kernel K_{gg} in Eq. (2.6) is single-log enhanced as far as it is known so far, i.e., to NNLO. It is therefore plausible to conjecture this behaviour to all orders in α_s. In particular, requiring the cancellation of the $\ln^5(1-z)$ and $\ln^4(1-z)$ terms in the third line of Eq. (2.7), we can determine the corresponding coefficients of $c_{gg}^{(3)}$. Moreover, we observe that the leading large-N logarithms of K_{gg} take a simple form for the sub-dominant N^{-1} contributions,

$$
\begin{aligned}
K_{gg}^{(1)}\Big|_{N^{-1}} &= -\left(8\beta_0 C_A + 32 C_A^2\right)\ln N + O(1)\,, \\
K_{gg}^{(2)}\Big|_{N^{-1}} &= -\left(16\beta_0^2 C_A + 112\beta_0 C_A^2\right)\ln^2 N + O(\ln N)\,, \\
K_{gg}^{(3)}\Big|_{N^{-1}} &= -\left(32\beta_0^3 C_A + \xi_H^{(3)}\beta_0^2 C_A^2\right)\ln^3 N + O(\ln^2 N)\,,
\end{aligned}
\tag{2.10}
$$

where the first two lines follow from the NLO and NNLO coefficient functions known from the respective diagram calculations in Refs. [11, 12] and [5–7]. The last line is an obvious generalization based on the results for DIS (where the leading-β_0 coefficients can be derived from the large-n_f results in Ref. [40] to all orders) and DY, where the coefficients are the same except for $C_A \to C_F$, see Eq. (6.17) of Ref. [24]; the unknown coefficient $\xi_H^{(3)}$ can be estimated from Eq. (2.10) and its completely known analogue in DIS as about 300 ± 150. This result provides the information about the $\ln^3(1-z)$ term of the N^{3}LO coefficient function. Note that the splitting functions in Eq. (2.6) do not contribute to Eq. (2.10) beyond NLO, as the diagonal quantities and $P_{qq}^{(n)}$ and $P_{gg}^{(n)}$ do not show any logarithmic higher-order enhancement of the N^0 and N^{-1} terms [41–44].

Eqs. (2.6) – (2.10) with $K_{gg}^{(3)}\Big|_{N^{-1}} = O(\ln^4 N)$ lead to the N^{3}LO and N^{4}LO predictions

$$
\begin{aligned}
c_{gg}^{(3)}(z) &= c_{gg}^{(3)}(z)\Big|_{\mathcal{D}_k,\delta(1-z)} - 512 C_A^3 \ln^5(1-z) + \left\{1728 C_A^3 + \frac{640}{3} C_A^2 \Omega\right\}\ln^4(1-z) \\
&\quad + \left\{\left(-\frac{1168}{3} + 3584\zeta_2\right)C_A^3 - \left(\frac{2512}{3} + \frac{1}{3}\xi_H^{(3)}\right)C_A^2\Omega - \frac{64}{3}C_A\beta_0^2\right\}\ln^3(1-z) \\
&\quad + O\Big(\ln^2(1-z)\Big)
\end{aligned}
\tag{2.11}
$$

and

$$c^{(4)}_{gg}(z) \;=\; c^{(4)}_{gg}(z)\Big|_{\mathcal{D}_k,\delta(1-z)} - \frac{4096}{3}C_A^4 \ln^7(1-z) + \left\{\frac{19712}{3}C_A^4 + \frac{3584}{3}C_A^3\,\mathfrak{d}\right\}\ln^6(1-z)$$
$$+ \left\{(-2240+23552\,\zeta_2)C_A^4 - \left(\frac{19136}{3}+\frac{8}{3}\xi_H^{(3)}\right)C_A^3\,\mathfrak{d} - \frac{1024}{3}C_A^2\beta_0^2\right\}\ln^5(1-z)$$
$$+ \mathcal{O}\Big(\ln^4(1-z)\Big) \tag{2.12}$$

at $\mu_R = \mu_F = m_H$, where $c^{(n)}_{gg}(z)|_{\mathcal{D}_k,\delta(1-z)}$ denotes the z-space SV approximation at N^{n}LO. The coefficients for $n=3$ can be found in Eqs. (17) – (22) of Ref. [15] and Eq. (10) of Ref. [16] (where the expansion is in powers of α_s/π instead of our $a_s = \alpha_s/(4\pi)$). The coefficients multiplying leading and next-to-leading $\ln^k(1-z)$ terms in Eq. (2.11) and (2.12) agree with those for DY case in Eqs. (6.24) and (6.25) in Ref. [24] if C_F is replaced by C_A in the latter results. For the third logarithm this is, unsurprisingly, only true for the β_0^2 contribution. The leading $\ln^k(1-z)$ terms in Eq. (2.11) and (2.12) agree with the old conjecture of Ref. [45], i.e., the coefficients of $\ln^{2n-1}(1-z)$ and $\mathcal{D}_{2n-1}$ are the same at N^{n}LO up to a sign. On the other hand, the subleading terms in Eq. (2.11) do not agree with the phenomenological ansatz employed in Refs. [26, 27].

Seven of the eight plus-distributions of the N^{4}LO SV contribution $c^{(4)}_{gg}(z)|_{\mathcal{D}_k,\delta(1-z)}$ in Eq. (2.12) can be obtained by expanding and Mellin inverting the result of the N^{3}LO + N^{3}LL soft-gluon exponentiation. The coefficients of $\mathcal{D}_k$ for $2 \le k \le 7$ can be found in Eq. (16) of Ref. [46] and that of $\mathcal{D}_1$ in Eq. (13) of Ref. [47]. The remaining $\mathcal{D}_0$ and $\delta(1-z)$ terms, on the other hand, require a fourth-order calculation. The $\mathcal{D}_0$ term can be predicted up to two unknown anomalous dimensions at four loops which are usually denoted by $A_{g,4}$ and $D_{g,4}$, see, e.g., Refs. [15, 17], as

$$c^{(4)}_{gg}\Big|_{\mathcal{D}_0} \;=\; D_{g,4} + C_A^4\bigg(-\frac{50096}{9} + \frac{11328416}{729}\zeta_2 + \frac{8392600}{81}\zeta_3 + \frac{1581760}{81}\zeta_2^2 + \frac{3461120}{9}\zeta_5$$
$$- \frac{6894080}{27}\zeta_2\zeta_3 + \frac{372416}{15}\zeta_2^3 - 217184\,\zeta_3^2 - \frac{595616}{15}\zeta_2^2\zeta_3 - 562176\,\zeta_2\zeta_5 + 983040\,\zeta_7\bigg)$$
$$+ C_A^3 n_f\bigg(\frac{191776}{81} - \frac{3613696}{729}\zeta_2 - \frac{2285696}{81}\zeta_3 - \frac{401920}{81}\zeta_2^2 + \frac{492800}{9}\zeta_2\zeta_3$$
$$- \frac{729088}{9}\zeta_5 - \frac{69248}{15}\zeta_2^3 + 30400\,\zeta_3^2\bigg)$$
$$+ C_A^2 n_f^2\bigg(-\frac{17920}{81} + \frac{290816}{729}\zeta_2 + \frac{89344}{81}\zeta_3 + \frac{2560}{9}\zeta_2^2 - \frac{69376}{27}\zeta_2\zeta_3 + \frac{32768}{9}\zeta_5\bigg)$$
$$+ C_A^2 C_F n_f\bigg(\frac{108272}{81} - \frac{62752}{27}\zeta_2 - \frac{340712}{27}\zeta_3 - 256\,\zeta_2^2 + \frac{13312}{9}\zeta_2\zeta_3 + \frac{512}{5}\zeta_2^3 + 9088\,\zeta_3^2\bigg)$$
$$+ C_A C_F n_f^2\bigg(-\frac{15008}{81} + \frac{2144}{9}\zeta_2 + \frac{3584}{27}\zeta_3 - \frac{512}{3}\zeta_2\zeta_3\bigg)\,. \tag{2.13}$$

The derivation of the this result required the extension of the calculations of Ref. [15] to the α_s^4 part of the exponentiation function g_5, see also Refs. [18, 49].

The coefficient $A_{g,4}$ has been estimated by Padé approximants as $A_{g,4} = (17.7, 9.70, 3.49) \cdot 10^3$ for $n_f = 3, 4, 5$ effectively massless flavours. A corresponding estimate for $D_{g,4}$ is

$$D_{g,4}(n_f = 3) \;=\; 12 \cdot 10^5\,, \quad D_{g,4}(n_f = 4) \;=\; 9.3 \cdot 10^5\,, \quad D_{g,4}(n_f = 5) \;=\; 6.8 \cdot 10^5\,, \qquad (2.14)$$

which is less reliable, as due to $D_{g,1} = 0$ only the two coefficients of Refs. [15, 18, 49–51] are available. Corresponding estimates for the quark quantities $A_{q,4}$ and $D_{q,4}$ relevant to the Drell-Yan process can be obtained by multiplying the above results by C_F/C_A.

Using Eqs. (2.9), our new result (2.11) together with the coefficients of $c^{(3)}_{gg}(z)|_{\mathcal{D}_k,\delta(1-z)}$ in Refs. [15, 16] can be employed to rigorously extend the N-space N^{3}LO threshold expansion to

$$\begin{aligned} \kappa_3\, c^{(3)}_{gg}(N) \;\simeq\;& 1.152 \ln^6 N + 5.46171 \ln^5 N + 23.8352 \ln^4 N + 44.9659 \ln^3 N \\ & + 85.6361 \ln^2 N + 60.7085 \ln N + 57.0781 \\ & + N^{-1}\Big\{3.456 \ln^5 N + 19.7023 \ln^4 N + (61.7304 + .0115\, \xi^{(3)}_H) \ln^3 N + O(\ln^2 N)\Big\} \end{aligned} \qquad (2.15)$$

with $\kappa_3 = 1/2000 \simeq 1/(4\pi)^3$. Here we have inserted the QCD values of the group factors, $C_A = 3$ and $C_F = 4/3$, used the physical value of $n_f = 5$ light flavours at scales of order m_H^2, and truncated coefficients including the Riemann ζ-function and the Euler-Mascheroni constant γ_e. The factor κ_3, as κ_4 in Eq. (2.16) below, approximately converts the coefficients to an expansion in α_s.

Note that the N^{-1} coefficients receive contributions from both the plus-distributions and the $\ln^k(1-z)$ terms of Eq. (2.11), hence the z-space and N-space SV approximations lead to different predictions for cross sections. It is clear from Eq. (2.15) that the coefficient $\xi^{(3)}_H$ is not a major source of uncertainty; its contribution to the coefficient of $N^{-1} \ln^3 N$ is expected to be below 10%.

The N^{4}LO result corresponding to Eq.(2.15) reads, with $\kappa_4 = 1/25000 \simeq 1/(4\pi)^4$,

$$\begin{aligned} \kappa_4\, c^{(4)}_{gg}(N) \;\simeq\;& 0.55296 \ln^8 N + 3.96654 \ln^7 N + 21.2587 \ln^6 N + 62.2985 \ln^5 N \\ & + 150.141 \ln^4 N + 212.443 \ln^3 N + (256.373 + 2\,\kappa_4 A_{g,4}) \ln^2 N \\ & + (142.548 + \kappa_4\, [4\gamma_e A_{g,4} - D_{g,4}]) \ln N \;+\; \kappa_4\, g_{0,4} \\ & + N^{-1}\Big\{2.21184 \ln^7 N + 19.6890 \ln^6 N + (86.4493 + 552\,\kappa_4\, \xi^{(3)}_H) \ln^5 N + O(\ln^4 N)\Big\}\;. \end{aligned} \qquad (2.16)$$

Here the coefficient $A_{g,4}$ is practically negligible, its contribution to the $\ln^2 N$ and $\ln N$ coefficients being of the order of 0.1%. The uncertainty of $D_{g,4}$ in Eq. (2.14), conservatively set to 100%, is an effect of order $\pm 20\%$ for the $\ln N$ term. The constant-N contribution $g_{0,4}$, i.e., the fourth-order term of the prefactor of the soft-gluon exponential, see, e.g., Refs. [17, 49] can be estimated by three Padé approximants which yield a fairly wide spread of values suggesting $\kappa_4\, g_{0,4} = 65 \pm 65$. Alternative this quantity can be estimated via a calculation in which the constant-N contributions in the integrals for the soft-gluon exponent are not discarded, leading to much smaller coefficients of the constant-N prefactors of the soft-gluon exponential at NNLO and N^{3}LO. This approach leads to an estimate consistent which the one given above.

Exact SU(N) expressions corresponding to Eq. (2.15) and the $\ln N$ enhanced parts of Eq. (2.16) can be found in the Appendix, together with third- and fourth-order predictions for the respective highest-three logarithms beyond the $(1-z)^0$ terms given in Eqs. (2.11) and (2.12) above.

3 Approximate N^{3}LO phenomenology

Before we address the numerical impact of N^{-1} contributions to the coefficient function, we briefly discuss the soft+virtual (SV) approximation. In z-space this approximation can be defined by keeping only the $\mathcal{D}_k(z)$ and $\delta(1-z)$ terms in the cross section, cf. Eq. (2.11). The soft coefficients in z-space are affected, however, by the artificial presence of factorially-growing subleading terms, originating in the mis-treatment of kinematic constraints such as energy conservation, that spoil the accuracy of the approximation for higher-order predictions at limited logarithmic depth [52].

The natural choice for the soft-gluon enhanced contributions is Mellin N-space, where instead of plus-distributions in z the dominant threshold contributions are given by powers of $\ln N$, and the kinematic constraints are automatically imposed. Consequently the N-space SV approximation is defined by keeping the terms in the coefficient function that do not vanish for $N \to \infty$, cf. Eq. (2.15).

The numerical contributions of the $\ln^k N$ terms, $0 \le k \le 2n$, of the Mellin-transformed coefficient functions $c_{gg}^{(n)}$ in Eq. (2.4) to the cross section (2.2) are illustrated up to N$^{n=3}$LO in Table 1, where all numbers are normalized to the lowest-order result proportional to $[f_{g/p} \otimes f_{g/p}](\tau)$ with $\tau = m_H^2/S$. All these results have been calculated in the heavy-top limits for $m_H = 125$ GeV, $E_{\rm cm} = \sqrt{S} = 14$ TeV, the central gluon distribution $f_{g/p}$ of the 2008 NNLO MSTW set [53] and the corresponding value of the strong coupling, $\alpha_{\rm s}(m_H^2) = 0.1118$, at $\mu_F = \mu_R = m_H$. Also shown is the corresponding normalized expansion of the prefactor function $[C(\mu_R^2 = m_H^2)]^2$ in Eq. (2.3).

All these contributions are positive, as are the $\ln N$ enhanced terms at N^{4}LO, see Eq. (2.16). The same is true for the corresponding coefficient functions for the Drell-Yan process and semi-inclusive e^+e^- annihilation, cf. Table 1 and Eq. (37) of Ref. [54], while for DIS only the $a_s^n \ln^{k \ge n} N$ contributions are positive at $n \le 4$, see Table 1 of Ref. [17]. In all these cases the complete SV result is smoothly approached when the $\ln^k N$ terms are included one by one. This is in contrast to the z-space SV approximation which exhibits large cancellations between the $\mathcal{D}_k(z)$ contributions as illustrated at N^{3}LO for DIS in Fig. 4 of Ref. [17] and for Higgs production in Ref. [16].

Furthermore the formally leading terms, i.e., those with the highest powers of $\ln N$, provide numerically small contributions to the cross section; the dominant part of the threshold corrections arises from the lowest-power logarithms and the constant terms. This is due to the pattern of coefficients in, for example, Eq. (2.15), which is comparable but less pronounced than that in DIS and SIA, and the low value of τ for the production of a 125 GeV Higgs-boson at the LHC, which leads a low effective value of N of $N_{\rm eff} \approx 2$ for the $\ln^k N$ contributions according to Table 1.

Another interesting feature shown in Table 1 is the rather large value of the $\delta(1-z)$ term at N^{3}LO [16] which contributes, for the value of $\alpha_{\rm s}$ given above, about three times as much as its NNLO counterpart. It accounts for 63% of the constant-N contribution at this order, the rest of which arises from the Mellin transform of the $\mathcal{D}_k$ terms, such as the first line of Eq. (2.9) for $k = 2$.

We are now ready to analyze the effect of adding the subdominant N^{-1} contributions to the SV terms. Before turning to N^{3}LO, we compare the resulting approximation to the exact result at NLO and NNLO in Fig. 1. It is clear that including the N^{-1} terms improves the approximation at large N. Interestingly, the exact result lies between the SV and the SV$+N^{-1}$ approximations at $N \gtrsim 2$ at both NLO and NNLO. It is therefore not unreasonable to assume that this behaviour also holds at N^{3}LO; hence one can constrain $c_{gg}^{(3)}(N)$ even in this region in N.

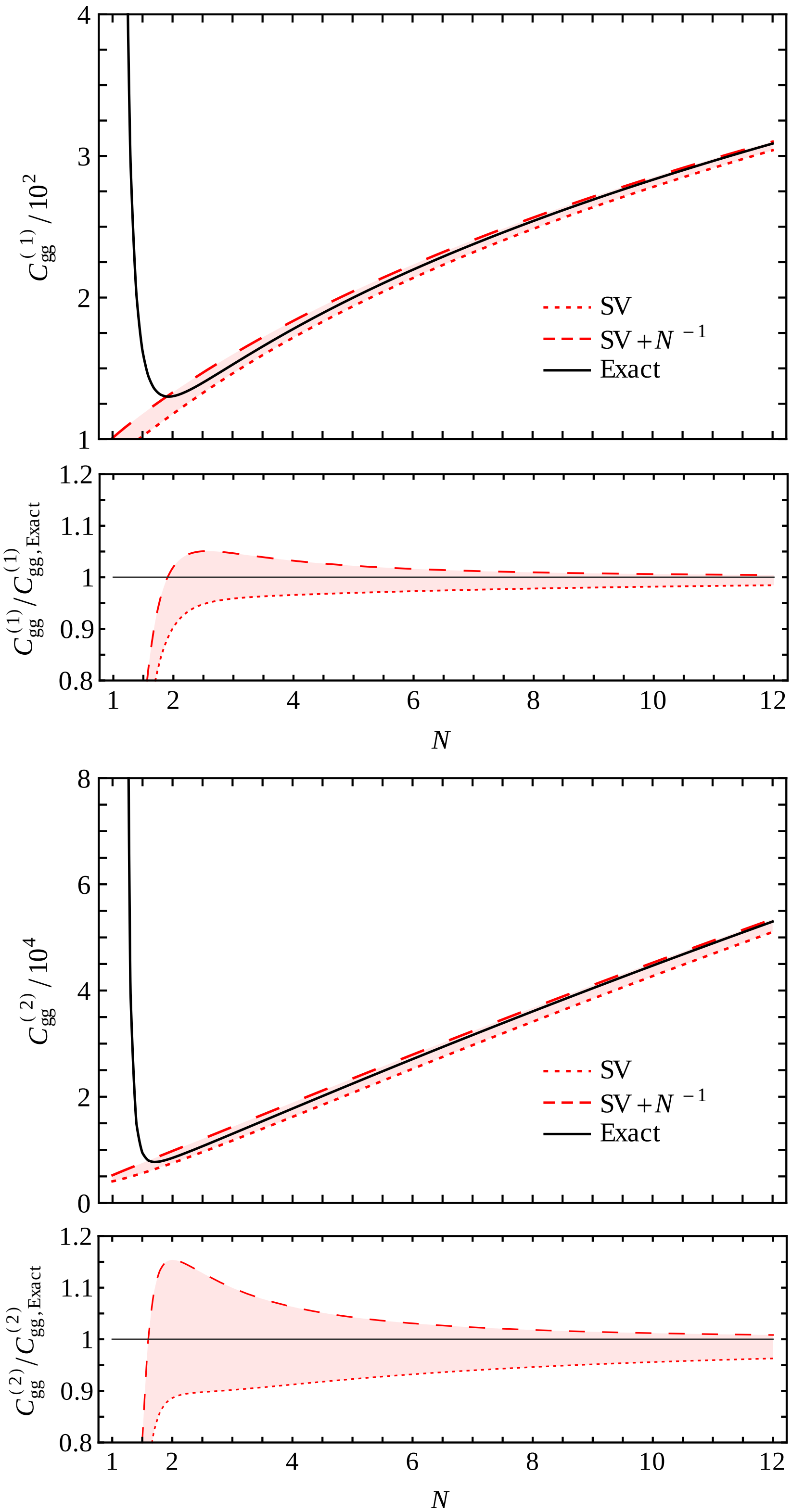

Figure 1: The exact results for the N-space gluon-gluon coefficient functions for $\mu_R = \mu_F = m_H$ at NLO (top) and NNLO (bottom) in the heavy-top limit, together with the corresponding SV approximations (dotted) and the SV terms plus the N^{-1} contributions (dashed). The respective lower panels show the relative positions and widths of the error bands defined by these two approximations.

	LO	NLO	NNLO	N^3LO
constant	100	77.4	32.2	8.04
(delta)	(100)	(35.1)	(1.72)	(5.07)
$\ln N$		14.8	12.0	5.14
$\ln^2 N$		7.16	7.56	4.04
$\ln^3 N$			1.07	1.09
$\ln^4 N$			0.18	0.27
$\ln^5 N$				0.025
$\ln^6 N$				0.002
SV	100	99.4	53.0	18.6
$C^2(m_H^2)$	100	19.6	2.05	0.12

Table 1: The individual contributions of the $\ln^k N$ terms in the N-space coefficient functions $c_{gg}^{(n\leq 3)}$ at $\mu_R = \mu_F = m_H$ to the Higgs production cross section for $m_H = 125$ GeV, $E_{\rm cm} = 14$ TeV, and the central gluon density and five-flavour α_s of Ref. [53]. All results are given as percentages of the LO contribution. Also shown, in the same manner, is the expansion of the prefactor function $[C(\mu_R^2 = m_H^2]^2)$, calculated in the on-shell scheme for the top mass with $m_t^2 = 3.00 \cdot 10^4$ GeV^2.

This situation is, in fact, expected from related studies of the DY process [24] and Higgs-exchange DIS [25]. It is particularly interesting to consider the latter case as the coefficient functions are completely known to N^3LO. Thus, in order to estimate the size of the N^{-1} logarithms not determined in Eq. (2.15), we compare with Ref. [25] and expand the gluon coefficient function $c_{\rm DIS}^{(n)}(N)$ of Higgs-exchange DIS up to $O(N^{-1})$ at both NNLO and N^3LO. We find

$$
\begin{aligned}
c_{\rm DIS}^{(2)}\Big|_{N^{-1}\ln^k N} &\propto \ln^3 N + 5.732 \ln^2 N + 8.244 \ln N - 3.275 \,, \\
c_{\rm DIS}^{(3)}\Big|_{N^{-1}\ln^k N} &\propto \ln^5 N + 12.65 \ln^4 N + 52.56 \ln^3 N + 92.01 \ln^2 N + 18.13 \ln N - 24.30
\end{aligned}
\tag{3.1}
$$

for $C_A = 3$, $C_F = 4/3$ and $n_f = 5$, where we have normalized the expressions such that the coefficient of the leading logarithm is equal to 1. The analogous expressions for Higgs production are

$$
\begin{aligned}
c_{gg}^{(2)}\Big|_{N^{-1}\ln^k N} &\propto \ln^3 N + 2.926 \ln^2 N + 5.970 \ln N + 2.007 \,, \\
c_{gg}^{(3)}\Big|_{N^{-1}\ln^k N} &\propto \ln^5 N + 5.701 \ln^4 N + \left(17.86 + 0.00333\,\xi_H^{(3)}\right) \ln^3 N + O(\ln^2 N) \,.
\end{aligned}
\tag{3.2}
$$

Comparing Eqs. (3.1) and (3.2) an interesting pattern emerges: the size of the coefficients of the non-leading logarithms for Higgs production is always smaller than that of their analogues for Higgs-exchange DIS; the ratio is a factor of about 1/2 or (much) less except for the $\ln^1 N$ terms. Thus we suggest as a conservative estimate of the complete N^{-1} contribution

$$
c_{gg}^{(3)}\Big|_{N^{-1}\ln^k N}^{\rm estimate} \propto \ln^5 N + 5.701 \ln^4 N + 18.9 \ln^3 N + 46 \ln^2 N + 18 \ln N + 9 \,,
\tag{3.3}
$$

where we have used $\xi_H^{(3)} = 300$ as roughly indicated by the physical-kernel coefficients in Ref. [24].

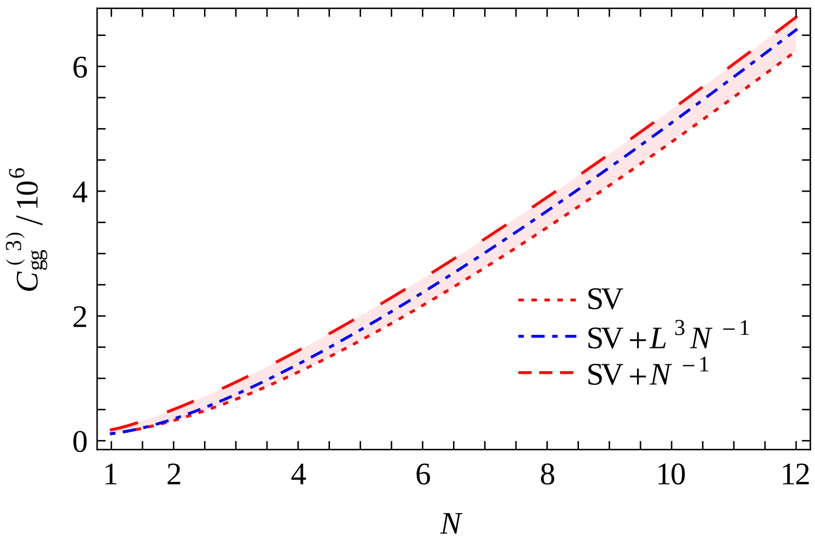

Figure 2: The Mellin-space N^{3}LO coefficient function $c_{gg}^{(3)}(N)$ as approximated, for $N \gtrsim 2$, by the N^0 SV contributions in Eq. (2.15) (dotted), the SV contribution plus the three $N^{-1}\ln^k N$ terms (approximately) known from physical kernels constraints (dash-dotted), and by the SV terms plus the estimated complete N^{-1} contributions in Eq. (3.3) (dashed).

The above equation includes an estimate of the non-logarithmic N^{-1} contribution to $c_{gg}^{(3)}(N)$. The ratio of the corresponding coefficient to that of $N^{-1}\ln N$ is moderate with 0.58 at NLO and 0.34 at NNLO, which may even indicate a trend towards lower values if the order is increased. Hence a ratio of 0.5 at N^{3}LO, as used in Eq. (3.3), appears to be sufficiently conservative (recall that these terms contribute positively to the cross section, so for larger coefficients we have larger contributions from the estimated terms which lead to a wider, i.e., more conservative error band).

Summarizing these constraints, we show in Fig. 2 the coefficient function $c_{gg}^{(3)}(N)$ in the SV approximation, for the SV terms plus the $N^{-1}\ln^k N$ contributions with $k \geq 3$ as in Eq. (2.15), and for the SV terms plus the estimate (3.3) of all N^{-1} contributions. Varying the value of $\xi_H^{(3)}$ by $\pm 50\%$ has a very small impact on the latter two results. Based on the pattern observed at NLO and NNLO, we expect that the exact result falls in the band displayed in the figure for $N \gtrsim 2$.

The consistency of the bands in Fig. 1 with the exact results at $N \gtrsim 2$ does not guarantee the same for the hadronic cross sections at high collider energies $E_{\rm cm}$. Hence we show in Fig. 3 the NLO and NNLO gluon-gluon contributions to the cross section (2.2) for a wide range of $E_{\rm cm}$. Here and below we have used the exact top-quark mass dependence at LO instead of the constant $\widetilde{\sigma}_0$ in Eq. (2.3) but for now, as in Table 1, the NNLO MSTW [53] parton set and its $\alpha_{\rm s}$ value irrespective of the order of the calculation. Also displayed in the figure are the results for the corresponding 'K-factors' at NLO and NNLO, $K_{\rm N^kLO} = \sigma_{\rm N^kLO}/\sigma_{\rm N^{k-1}LO}$, where we show the rather small (but not negligible) negative effect of the quark-gluon and quark-(anti)quark contributions as well.

We observe that the exact results, for both gluon-gluon fusion and all channels, are consistent with the band defined by the SV and SV$+N^{-1}$ approximations for $E_{\rm cm} \lesssim 20$ TeV at NLO (the deviation from it remains small even at higher energies) and at all energies considered at NNLO, where the approximations are applicable down to somewhat lower values of N as shown in Fig 1. The effect of the non-SV gluon-gluon terms is largely compensated by the other channels at NNLO.

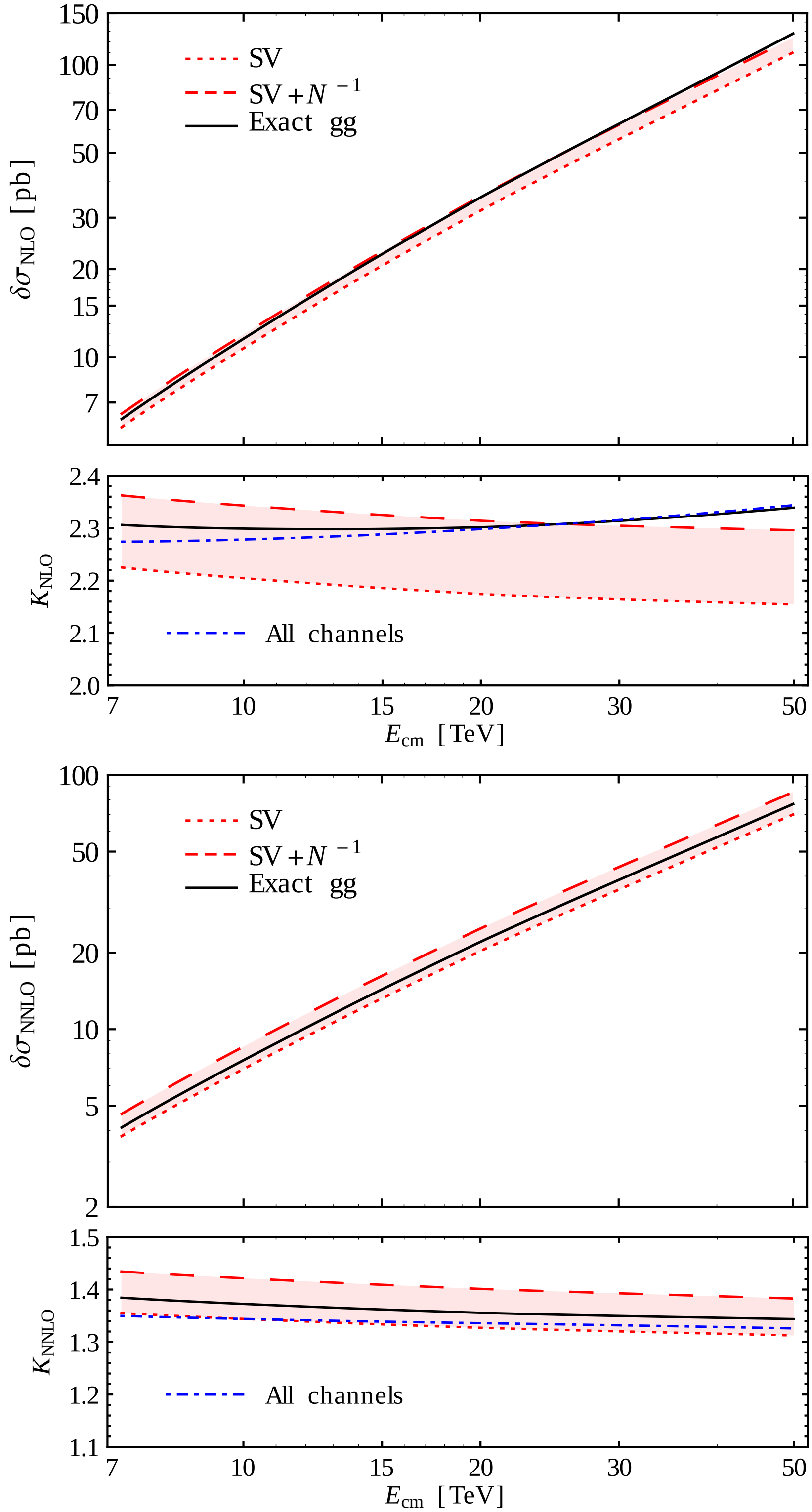

Figure 3: The NLO (top) and NNLO (bottom) gluon-gluon contributions to the Higgs production cross section as a function of the collider energy for the exact coefficient function (solid), the SV approximation (dotted) and the SV terms plus the N^{-1} contributions (dashed). The lower panels show the corresponding K-factors, including the impact of the other partonic subprocesses (dash-dotted). All curves have been calculated using the central NNLO α_s and parton densities of Ref. [53].

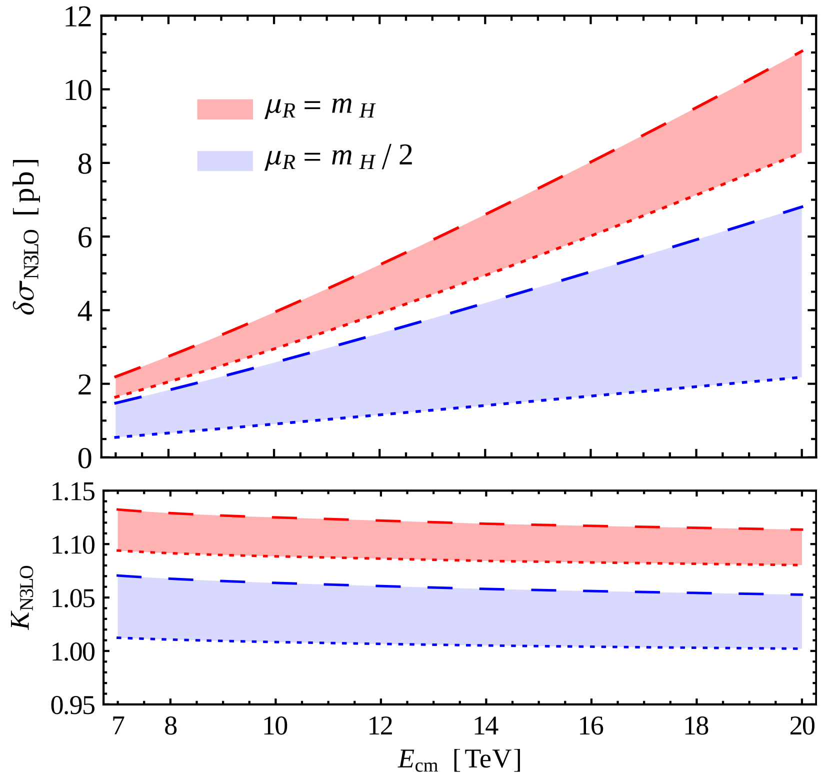

Figure 4: The N^{3}LO contribution to the Higgs production cross section as a function of the collider energy for the SV approximation (dotted) and the SV terms plus the $O(N^{-1})$ contributions in Eq. (3.3) (dashed) at $\mu_R = m_H$ (upper curves) and $\mu_R = 0.5\,m_H$ (lower curves) for the NNLO gluon distribution of Ref. [53] at $\mu_F = m_H$. The lower panel shows the ratio of these N^{3}LO predictions to the complete NNLO result.

In view of these results, we can reliably employ our approximations of $c_{gg}^{(3)}(N)$ to predict the size of the N^{3}LO corrections for $E_{cm} \lesssim 20$ TeV, as shown in Fig. 4. Here all partonic channels are included up to NNLO, while at N^{3}LO we consider only the gluon-gluon process. The N^{3}LO scale dependence of α_s [55, 56] has been used with $\alpha_s(M_Z^2) = 0.1165$ in the latter case with, since there are no PDF parametrizations at this order yet, the NNLO PDFs of Ref. [53] at the scale m_H^2.

Under these conditions, the N^{3}LO cross sections are larger at $\mu_R = m_H$ that their NNLO counterparts by $11.3\% \pm 1.9\%$ at $E_{\rm cm} = 7$ TeV and $9.7\% \pm 1.7\%$ at $E_{\rm cm} = 20$ TeV. At $\mu_R = m_H/2$, which is closer to the point of minimal sensitivity and provides a scale choice that closely reproduces the effect of threshold resummation [19], the corrections are substantially smaller with $4.1\% \pm 2.9\%$ and $2.7\% \pm 2.5\%$, respectively, at 7 TeV and 20 TeV. Hence the size and present uncertainty of the N^{3}LO corrections is only weakly dependent of the collider energy in this range, the latter amounting to about 2-3% at these natural values of μ_R.

Fig. 5 displays the dependence of the total cross section on the renormalization and factorization scales μ_R and μ_F for the successive perturbative orders, now consistently calculated using (where possible) the corresponding values and evolution of α_s and the PDFs, at 14 TeV. As shown in the upper plot, the variation with μ_F for fixed μ_R is small already at LO, despite the PDFs changing considerably over the wide range of scales used in the plots. The dependence on μ_F is, in fact, larger at N^{3}LO than at NNLO; this is due to the (presently unavoidable) use of the NNLO gluon distributions also at this order and the omission of the quark-gluon and quark-(anti) quark channels.

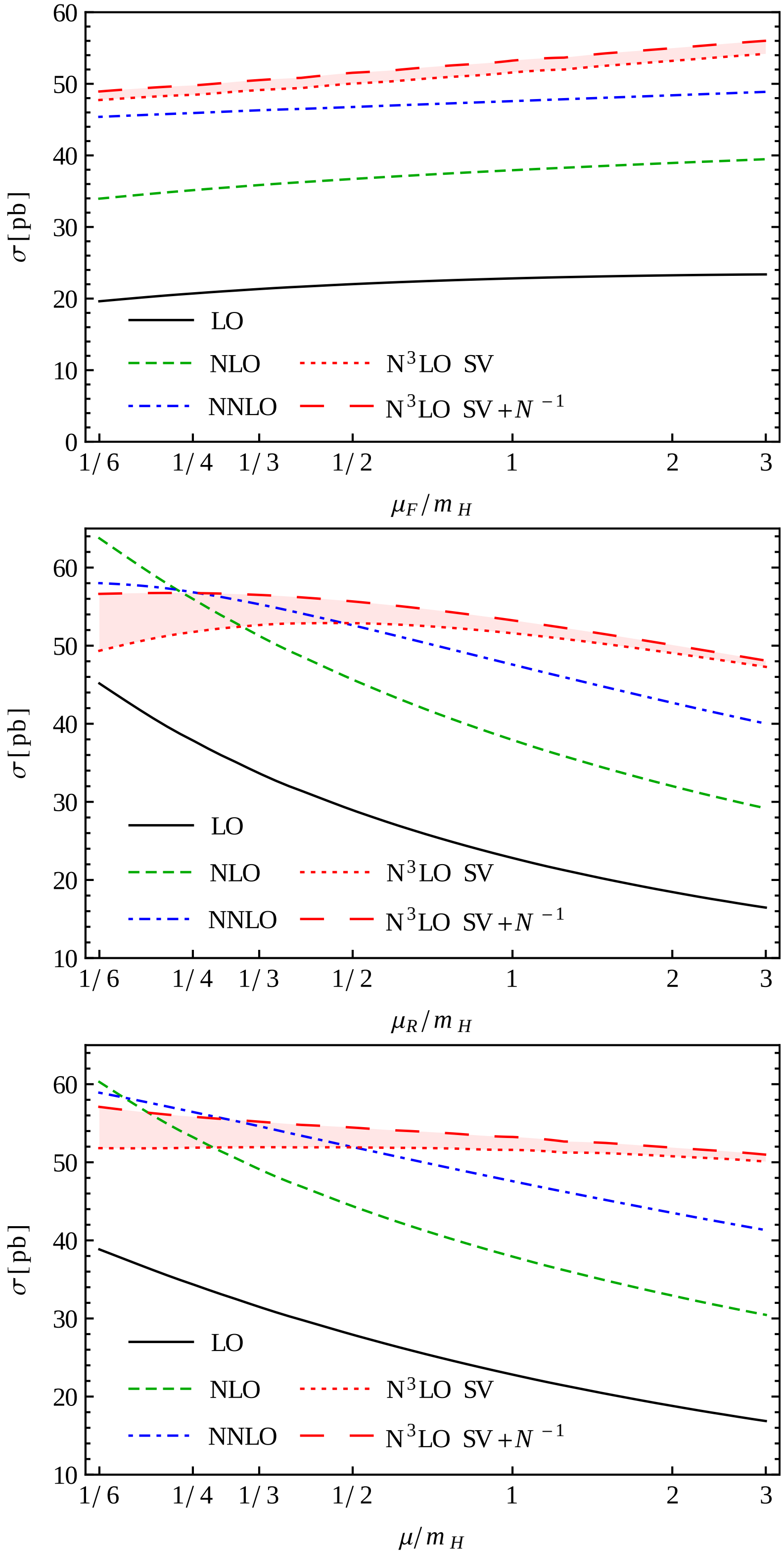

Figure 5: The dependence of the Higgs production cross section on the factorization scale μ_F for $\mu_R = m_H$ (top), the renormalization scale μ_R for $\mu_F = m_H$ (middle), and on $\mu \equiv \mu_F = \mu_R$ (bottom) at $E_{\rm cm} = 14$ TeV. Our N^{3}LO band defined by the SV and SV$+N^{-1}$ approximations for the coefficient function $c^{(3)}_{gg}(N)$ is compared to the LO, NLO and NNLO results for the respective PDFs and α_s values of Ref. [53].

No such caveats apply to the dependence on μ_R for fixed μ_F which at N^{3}LO requires 'only' the four-loop beta function [55, 56] but not the so far unknown fourth-order splitting functions. Using the interval $0.25\,m_H \leq \mu_R \leq 2\,m_H$, the cross section ranges from 32 to 56 pb at NLO, from 42.5 to 57 pb at NNLO, and from 49.5 to 54.5 pb for the center of our N^{3}LO uncertainty band. The respective lower numbers change to 38, 47.5 and 52.5 pb if a more conventional variation by a factor of 2 is used about the apparently preferred scale $m_H/2$. These results indicate an uncertainty due to the truncation of the perturbation series at N^{3}LO of slightly less than $\pm 5\%$.

Finally, in the bottom plot in Fig. 5, μ_F and μ_R are varied together relative to m_H. The resulting scale dependence of the cross sections at LO, NLO and NNLO is similar to, but slightly smaller than, those just discussed. The further improvement at N^{3}LO can not be trusted quantitatively, as the falling trend towards large scales with μ_R is combined with the partly spurious (see above) increase with μ_F shown in the upper plot. Hence it is best, at least for the time being, to use the results for a fixed μ_F for a conservative error estimate.

While often unavoidable, error estimates using scale variations are, of course, not particularly reliable; they summarize rather what is known than what will be added by yet unknown higher orders, and (width of) the scale range considered is somewhat arbitrary. A useful alternative is to estimate, where possible, the size to the next order in the perturbative expansion at a standard scale. In the case at hand this is possible, since the size of the complete SV contribution at N^{4}LO has been determined in terms of two parameters that can be estimated, see Eq. (2.16). In line with the discussion at the end of Section 2, we use $D_{g,4} = 0$ and $\kappa_4\, g_{0,4} = 130$ for a 'large' estimate of the N^{4}LO gluon-gluon coefficient function, and $\kappa_4\, D_{g,4} = 55$, i.e., twice the Padé approximant in Eq. (2.14) and $g_{0,4} = 0$ for a 'small' estimate (recall that $\kappa_4 = 1/25000$ effectively converts the fourth-order quantities to an expansion in α_s).

In principle, the N^{4}LO cross section in the SV limit also involves the α_s^5 contribution to the constant $C(\mu_R^2)$ in Eq. (2.3) which, in fact, is known except for the n_f-dependent part of the five-loop beta-function of QCD [29, 30]. However, as obvious from the last row of Table 1, this contribution can be safely neglected in the present context.

The resulting estimates for the N^{4}LO correction are shown in Fig. 6 in the same manner as the N^{3}LO contributions in Fig. 4. Also here the relative size to the corrections depends weakly on the colliders energy between 7 TeV and 20 TeV, with about 3.0% to 2.5% at $\mu_R = m_H$ and -0.4% to -0.5% at $\mu_R = m_H/2$. At $E_{\rm cm} = 14$ TeV the N^{4}LO SV terms change the respective N^{3}LO cross sections by about 1.5 pb and -0.5 pb. Even if these results were to considerably underestimate the true N^{4}LO correction, the latter would still amount to less than 5%.

In view of these and the above results, a combined perturbation-series uncertainty of about $\pm 5\%$ can be assigned to our present N^{3}LO cross section, which takes into account the approximate character of $c_{gg}^{(3)}(N)$, the omission of the N^{3}LO quark-gluon and quark-(anti)quark contributions and the truncation of the expansion at this point. Calculating all higher-order contributions in the heavy-top approximation but normalizing with the full lowest-order result, this leads to a total cross section of 54.3 ± 2.7 pb at 14 TeV for the NNLO PDFs of Ref. [53] – which should under- or overestimate the corresponding N^{3}LO gluon-gluon luminosity by less than 1% – and $\alpha_s(M_Z^2) = 0.1165$, where the central value refers the choice $\mu_R = m_H/2$ and $\mu_F = m_H$. As all our results, the above cross section does include neither electroweak corrections nor bottom-mass effects.

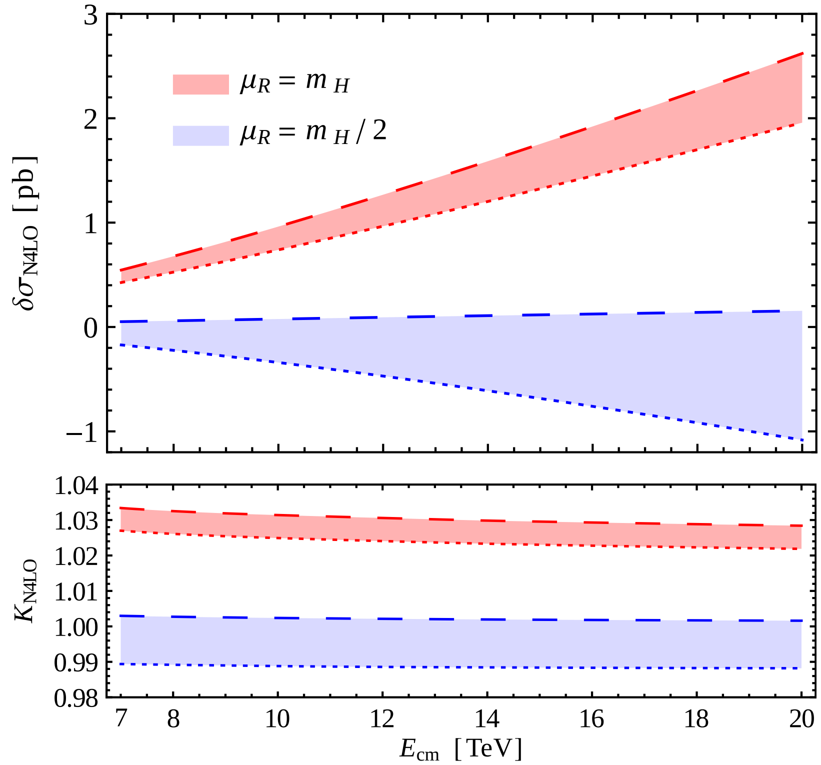

Figure 6: As Fig. 4, but for the N^{4}LO corrections as obtained from the 'large' and 'small' SV estimates of the coefficient function $c_{gg}^{(4)}(N)$ discussed in the text. In the lower panel the N^{4}LO results are shown relative to the corresponding N^{3}LO cross sections in the SV approximation.

Our present result for the N^{3}LO corrections in the SV approximation is larger, by about a factor of two at $\mu_R = m_H$, than that given ten years ago in Ref. [15]. This is due to the recently calculated coefficient of $\delta(1-z)$ [16], which turns out to be almost twice as large as anticipated for the uncertainty estimate in Ref. [15], and the different input parameters, most notably a larger value of $\alpha_s(M_Z^2)$. Our results including the $N^{-1}\ln^k N$ term in Eq. (3.3) can be compared to Refs. [26,27], where an approximate N^{3}LO prediction has been constructed, based on the large and small-N behavior of the partonic cross section (for which the latter has a small effect at LHC energies). As mentioned above, their $N^{-1}\ln^k N$ terms due not agree with our result except for the obvious coefficient of $N^{-1}\ln^5 N$. Nevertheless, the central prediction of Refs. [26, 27] for the N^{3}LO cross section is rather comparable to our result.

Finally, with the perturbative QCD corrections to the coefficient function of the dominant hard scattering process well under control, the largest remaining uncertainties in predictions of the physical cross section originate from the input parameters for α_s and the PDFs, cf. Eq. (2.2). For instance, use of the ABM12 value of α_s and PDFs [57], which were tuned to LHC data, leads to central values for the cross section which are significantly lower, by some 11-14% (depending on the collider energy), than those reported, e.g., in Table 1 and Fig. 5, see Ref. [57]. This is due to a smaller value of $\alpha_s(M_Z^2)$ and a smaller gluon distribution in the relevant z-range for the ABM12 parametrization as compared to MSTW [53]; the origin of these differences has been understood [58, 59]. Very recently, also the NNPDF collaboration has reported new and slightly lower values of the Higgs cross section for the NNPDF 3.0 parton set [60] also tuned to LHC data.

4 Summary and outlook

For almost ten years rigorous results for the total Higgs-production cross section in the heavy top-quark limit have been confined to the exact NNLO coefficient functions [5–7] plus the N^{3}LL soft-gluon resummation [15,50,51] which fixes the highest six threshold logarithms at all higher orders. Earlier this year an N^{3}LO diagram calculation has been completed in the soft + virtual limit [16], adding the coefficient of $\delta(1-z)$ to those of the $[(1-z)^{-1}\ln^k(1-z)]_+$ terms with $0 \le k \le 5$.

Progress has also been made in the past years on resumming sub-dominant large-z logarithms, $(1-z)^a \ln^k(1-z)$ with $a \ge 0$, via physical evolution kernels [24,25] or the structure of unfactorized cross sections in dimensional regularization [38,39]; the latter has been used recently to derive the leading large-z logs for the quark-gluon contribution to Higgs production to all orders [31].

Here we have considered the dominant gluon-gluon channel and extended the calculations of Ref. [24] to Higgs-boson production. Based on the results of Refs. [5–7] we have thus derived the leading sub-SV contributions, $\ln^k(1-z)$ with $k = 5, 4, 3$, the first two completely (unsurprisingly verifying the conjecture of Ref. [45] for the leading logarithm) and the third up to a constant of minor numerical relevance. The corresponding results for $a \ge 1$ can only be derived for the non-C_F terms at this point, consequently only the coefficient of the leading logarithms is complete. These results, included in the Appendix together with their fourth-order counterparts, can provide a non-trivial check on a future complete N^{3}LO calculation.

Switching to Mellin moments for phenomenological considerations, a comparison of the pattern of the coefficients at NLO, NNLO and N^{3}LO with those for Higgs-exchange DIS, where the coefficient function is fully known to N^{3}LO [25], allows to give well-motivated estimates for the remaining $N^{-1}\ln^{2,1,0}N$ third-order contributions to $c_{gg}^{(3)}(N)$. It turns out that both the corresponding coefficient functions at $N \gtrsim 2$ as well as the NLO and NNLO contributions to the cross sections for LHC energies are contained in a band spanned by the respective SV and SV+$O(N^{-1})$ approximations. Assuming the same situation at the third order, we have been able to improve upon previous estimates [15,26] of the size and remaining uncertainty of the N^{3}LO correction.

We have studied the dependence of these approximate N^{3}LO results on the renormalization and factorization scales, as well as the size of the N^{4}LO corrections in the SV approximation. We conclude that the remaining perturbation-series uncertainty amounts to no more than $\pm 5\%$, which includes the effects of approximate character of $c_{gg}^{(3)}(N)$, the omission of the N^{3}LO quark-gluon and quark-(anti)quark contributions and the truncation of the series. Using the central NNLO PDFs of Ref. [53] at $\mu_F = m_H$ and the N^{3}LO strong coupling with $\alpha_s(M_Z^2) = 0.1165$ leads to an increase by $(10 \pm 2)\%$ at $\mu_R = m_H$ and $(3 \pm 2.5)\%$ at $\mu_R = m_H/2$, which appears to be the preferred central scale, over the corresponding NNLO cross sections at a collider energy of 14 TeV.

The perturbative expansion of the hard scattering cross section is, therefore, now quite well under control, rendering the uncertainties of the PDFs and α_s an at least as important source of uncertainties for LHC predictions. Given the progress on the perturbative QCD corrections reported in Ref. [16] and here, together with new global fits of PDFs to LHC data, it appears that the cross section values [61] recommended for use in the ongoing and upcoming ATLAS and CMS Higgs analyses require revision, for Run2 of the LHC, to include the latest theory developments and improvements on the evaluation of the parton distributions and the value of α_s.

A Large-N expansions at N^{3}LO and N^{4}LO

Here we present the general expressions corresponding to Eqs. (2.15) and (2.16). For compactness the results are written in terms of $\ln\widetilde{N} = \ln N + \gamma_e$. The N^0 coefficients at N^{3}LO read

$$
\begin{aligned}
c_{gg}^{(3)}\Big|_{\ln^6\widetilde{N}} &= \frac{256}{3}C_A^3\,, \\
c_{gg}^{(3)}\Big|_{\ln^5\widetilde{N}} &= \frac{1408}{9}C_A^3 - \frac{256}{9}C_A^2 n_f\,, \\
c_{gg}^{(3)}\Big|_{\ln^4\widetilde{N}} &= C_A^3\left[\frac{14800}{27} + 384\zeta_2\right] - \frac{2624}{27}C_A^2 n_f + \frac{64}{27}C_A n_f^2\,, \\
c_{gg}^{(3)}\Big|_{\ln^3\widetilde{N}} &= C_A^3\left[\frac{67264}{81} - 448\zeta_3 + \frac{704}{3}\zeta_2\right] - C_A^2 n_f\left[\frac{14624}{81} + \frac{128}{3}\zeta_2\right] - \frac{32}{3}C_A C_F n_f + \frac{640}{81}C_A n_f^2\,, \\
c_{gg}^{(3)}\Big|_{\ln^2\widetilde{N}} &= C_A^3\left[\frac{122276}{81} + \frac{15008}{9}\zeta_2 - \frac{5632}{9}\zeta_3 + \frac{2752}{5}\zeta_2^2\right] - C_A^2 n_f\left[\frac{33688}{81} + \frac{2240}{9}\zeta_2 + \frac{704}{9}\zeta_3\right] \\
&\quad - C_A C_F n_f\left[252 - 192\zeta_3\right] + \frac{800}{81}C_A n_f^2\,, \\
c_{gg}^{(3)}\Big|_{\ln^1\widetilde{N}} &= C_A^3\left[\frac{594058}{729} + \frac{64784}{81}\zeta_2 - \frac{24656}{27}\zeta_3 - \frac{176}{5}\zeta_2^2 - \frac{2336}{3}\zeta_2\zeta_3 + 384\zeta_5\right] \\
&\quad + C_A^2 n_f\left[-\frac{125252}{729} - \frac{9104}{81}\zeta_2 + \frac{1808}{27}\zeta_3 - \frac{32}{5}\zeta_2^2\right] - C_A C_F n_f\left[\frac{3422}{27} - \frac{608}{9}\zeta_3 - \frac{64}{5}\zeta_2^2\right] \\
&\quad + C_A n_f^2\left[\frac{3712}{729} + \frac{64}{9}\zeta_3\right]\,, \\
c_{gg}^{(3)}\Big|_{\ln^0\widetilde{N}} &= C_A^3\left[\frac{215131}{81} + \frac{186880}{81}\zeta_2 - \frac{130828}{81}\zeta_3 + \frac{119692}{135}\zeta_2^2 - \frac{2024}{3}\zeta_2\zeta_3 + \frac{3476}{9}\zeta_5\right. \\
&\qquad \left. + \frac{3872}{15}\zeta_2^3 + 96\zeta_3^2\right] + C_A n_f^2\left[\frac{2515}{27} - \frac{1328}{81}\zeta_2 + \frac{3344}{81}\zeta_3 - \frac{224}{15}\zeta_2^2\right] \\
&\quad + C_A^2 n_f\left[-\frac{98059}{81} - \frac{38168}{81}\zeta_2 + \frac{296}{81}\zeta_3 - \frac{4696}{135}\zeta_2^2 - \frac{784}{3}\zeta_2\zeta_3 + \frac{808}{9}\zeta_5\right] \\
&\quad + C_A C_F n_f\left[-\frac{63991}{81} - \frac{3404}{9}\zeta_2 + \frac{1184}{3}\zeta_3 + \frac{176}{45}\zeta_2^2 + 384\zeta_2\zeta_3 + 160\zeta_5\right] \\
&\quad + C_F^2 n_f\left[\frac{608}{9} + \frac{592}{3}\zeta_3 - 320\zeta_5\right] + C_F n_f^2\left[\frac{8962}{81} - \frac{184}{9}\zeta_2 - \frac{224}{3}\zeta_3 - \frac{32}{45}\zeta_2^2\right]\,.
\end{aligned}
\tag{A.1}
$$

Except for the $\ln^0\widetilde{N}$ part, these results have been presented before in a different notation, e.g., in Appendix E of Ref. [18]. Our new N^{-1} terms read, with one unknown coefficient $\xi_H^{(3)}$ of Eq. (2.10)

$$
\begin{aligned}
c_{gg}^{(3)}\Big|_{N^{-1}\ln^5\widetilde{N}} &= 256C_A^3\,, \\
c_{gg}^{(3)}\Big|_{N^{-1}\ln^4\widetilde{N}} &= \frac{7552}{9}C_A^3 - \frac{640}{9}C_A^2 n_f\,, \\
c_{gg}^{(3)}\Big|_{N^{-1}\ln^3\widetilde{N}} &= C_A^3\left[\frac{29312}{27} + \frac{11}{9}\xi_H^{(3)} + 768\zeta_2\right] + C_A^2 n_f\left[-\frac{4960}{27} - \frac{2}{9}\xi_H^{(3)}\right] + \frac{128}{27}C_A n_f^2\,.
\end{aligned}
\tag{A.2}
$$

The corresponding N^{4}LO results are given by

$$
\begin{aligned}
c_{gg}^{(4)}\Big|_{\ln^8\widetilde{N}} &= \frac{512}{3}\,C_A^4\,,\\
c_{gg}^{(4)}\Big|_{\ln^7\widetilde{N}} &= \frac{5632}{9}\,C_A^4-\frac{1024}{9}\,C_A^3n_f\,,\\
c_{gg}^{(4)}\Big|_{\ln^6\widetilde{N}} &= C_A^4\left[\frac{216320}{81}+\frac{2560}{3}\,\zeta_2\right]-\frac{45568}{81}\,C_A^3n_f+\frac{2048}{81}\,C_A^2n_f^2\,,\\
c_{gg}^{(4)}\Big|_{\ln^5\widetilde{N}} &= C_A^4\left[\frac{838112}{135}+\frac{14080}{9}\,\zeta_2-1792\,\zeta_3\right]-C_A^3n_f\left[\frac{26048}{15}+\frac{2560}{9}\,\zeta_2\right]-\frac{256}{3}\,C_A^2C_Fn_f\\
&\quad+\frac{17024}{135}\,C_A^2n_f^2-\frac{256}{135}\,C_An_f^3\,,\\
c_{gg}^{(4)}\Big|_{\ln^4\widetilde{N}} &= C_A^4\left[\frac{3450592}{243}-\frac{45056}{9}\,\zeta_3+\frac{250912}{27}\,\zeta_2+\frac{7936}{5}\,\zeta_2^2\right]\\
&\quad+C_A^3n_f\left[-\frac{1084592}{243}-\frac{1024}{9}\,\zeta_3-\frac{41600}{27}\,\zeta_2\right]+C_A^2C_Fn_f\left[-\frac{12592}{9}+1024\,\zeta_3\right]\\
&\quad+C_A^2n_f^2\left[\frac{77152}{243}+\frac{640}{27}\,\zeta_2\right]+\frac{160}{9}\,C_AC_Fn_f^2-\frac{640}{81}\,C_An_f^3\,,\\
c_{gg}^{(4)}\Big|_{\ln^3\widetilde{N}} &= C_A^4\left[\frac{13631360}{729}+\frac{923968}{81}\,\zeta_2-\frac{1125184}{81}\,\zeta_3+\frac{7040}{9}\,\zeta_2^2-\frac{16000}{3}\,\zeta_2\zeta_3+3072\,\zeta_5\right]\\
&\quad+C_A^3n_f\left[-\frac{4591096}{729}-\frac{219904}{81}\,\zeta_2+\frac{116096}{81}\,\zeta_3-\frac{11008}{45}\,\zeta_2^2\right]+\frac{16}{3}\,C_AC_F^2n_f\\
&\quad+C_A^2C_Fn_f\left[-2208-128\,\zeta_2+\frac{3968}{3}\,\zeta_3+\frac{512}{5}\,\zeta_2^2\right]+C_AC_Fn_f^2\left[\frac{5600}{27}-\frac{1280}{9}\,\zeta_3\right]\\
&\quad+C_A^2n_f^2\left[\frac{436760}{729}+\frac{1280}{9}\,\zeta_2+\frac{7424}{81}\,\zeta_3\right]-\frac{3200}{243}\,C_An_f^3\,,\\
c_{gg}^{(4)}\Big|_{\ln^2\widetilde{N}} &= C_A^4\left[\frac{28356478}{729}+\frac{2800672}{81}\,\zeta_2-\frac{799888}{27}\,\zeta_3+\frac{873104}{135}\,\zeta_2^2-\frac{82720}{9}\,\zeta_2\zeta_3\right.\\
&\qquad\left.+\frac{65824}{9}\,\zeta_5+\frac{25792}{15}\,\zeta_2^3+2336\,\zeta_3^2\right]\;+\;C_AC_F^2n_f\left[\frac{4864}{9}-2560\,\zeta_5+\frac{4736}{3}\,\zeta_3\right]\\
&\quad+C_A^3n_f\left[-\frac{12176488}{729}-\frac{661136}{81}\,\zeta_2+3152\,\zeta_3-\frac{32768}{135}\,\zeta_2^2-\frac{19520}{9}\,\zeta_2\zeta_3-\frac{448}{9}\,\zeta_5\right]\\
&\quad+C_A^2C_Fn_f\left[-\frac{751982}{81}-\frac{34576}{9}\,\zeta_2+\frac{15232}{3}\,\zeta_3+\frac{7744}{45}\,\zeta_2^2+3840\,\zeta_2\zeta_3+1280\,\zeta_5\right]\\
&\quad+C_A^2n_f^2\left[\frac{1072784}{729}+\frac{11680}{81}\,\zeta_2+\frac{9760}{27}\,\zeta_3-\frac{320}{3}\,\zeta_2^2\right]+C_An_f^3\left[-\frac{7424}{729}-\frac{128}{9}\,\zeta_3\right]\\
&\quad+C_AC_Fn_f^2\left[\frac{110996}{81}-\frac{2624}{3}\,\zeta_3-\frac{1472}{9}\,\zeta_2-\frac{1408}{45}\,\zeta_2^2\right]\;+\;2A_{g,4}\,,
\end{aligned}
$$

$$
\begin{aligned}
c^{(4)}_{gg}\Big|_{\ln^1\widetilde{N}} \;=\;& C_A^4\left[\frac{50096}{9}+\frac{29565664}{729}\,\zeta_2+\frac{2426936}{243}\,\zeta_3-\frac{1592288}{405}\,\zeta_2^2-\frac{876608}{27}\,\zeta_2\zeta_3\right.\\
&\left.+\frac{4928}{5}\,\zeta_2^3+\frac{17248}{9}\,\zeta_3^2-\frac{9824}{3}\,\zeta_2^2\zeta_3+6144\,\zeta_2\,\zeta_5\right]+16\,\zeta_2\,C_AC_F^2n_f\\
&+C_A^3n_f\left[-\frac{191776}{81}-\frac{10159592}{729}\,\zeta_2-\frac{1819648}{243}\,\zeta_3+\frac{820928}{405}\,\zeta_2^2+\frac{127616}{27}\,\zeta_2\zeta_3\right.\\
&\left.+\frac{4928}{9}\,\zeta_3^2-384\,\zeta_2^3\right]\;+\;C_AC_Fn_f^2\left[\frac{15008}{81}+384\,\zeta_2+\frac{256}{27}\,\zeta_3-256\,\zeta_2\,\zeta_3\right]\\
&+C_A^2C_Fn_f\left[-\frac{108272}{81}-\frac{116096}{27}\,\zeta_2+\frac{38504}{27}\,\zeta_3+128\,\zeta_2^2+\frac{22400}{9}\,\zeta_2\,\zeta_3\right.\\
&\left.+\frac{1024}{5}\,\zeta_2^3-896\,\zeta_3^2\right]\;+\;C_An_f^3\left[-\frac{3200}{81}\,\zeta_2-\frac{5120}{81}\,\zeta_3+\frac{256}{45}\,\zeta_2^2\right]\\
&+C_A^2n_f^2\left[\frac{17920}{81}+\frac{1019464}{729}\,\zeta_2+\frac{349184}{243}\,\zeta_3-\frac{10624}{45}\,\zeta_2^2\right]\;-\;D_{g,4}
\end{aligned}
\tag{A.3}
$$

with the yet unknown fourth-order quantities $A_{g,4}$ and $D_{g,4}$, and

$$
\begin{aligned}
c^{(4)}_{gg}\Big|_{N^{-1}\ln^7\widetilde{N}} \;=\;& \frac{2048}{3}\,C_A^4\;,\\
c^{(4)}_{gg}\Big|_{N^{-1}\ln^6\widetilde{N}} \;=\;& \frac{35840}{9}\,C_A^4-\frac{3584}{9}\,C_A^3n_f\;,\\
c^{(4)}_{gg}\Big|_{N^{-1}\ln^5\widetilde{N}} \;=\;& C_A^4\left[\frac{244736}{27}+2560\,\zeta_2+\frac{88}{9}\,\xi_H^{(3)}\right]-C_A^3n_f\left[\frac{49792}{27}+\frac{16}{9}\,\xi_H^{(3)}\right]+\frac{2048}{27}\,C_A^2n_f^2\;.
\end{aligned}
\tag{A.4}
$$

The corresponding N^0 contributions for the DY process can now be written down at the same accuracy due to the determination of the coefficient of $\delta(1-z)$ at N^{3}LO in Refs. [47, 48]. The DY counterparts of Eqs. (A.2) and (A.4) have been determined in Ref. [24].

B z-space results beyond $(1-z)^0$ for large z

For non-singlet quantities such as the dominant quark-antiquark annihilation contribution to the total cross section for Drell-Yan lepton-pair production, $pp/p\bar{p}\to l^+l^-+X$, the physical kernel is single-log enhanced at all orders in the expansion about $z=1$ [24]. This is also true for the $C_A^k n_f^\ell$ contributions to Higgs production via gluon-gluon fusion in the heavy-top limit, viz

$$
\begin{aligned}
K^{(1)}_{gg}(z) \;=\;& \ln(1-z)\,p_{gg}(z)\left[-16C_A\beta_0-32C_A^2\,\mathrm{H}_0\right]\;+\;O\big(\ln^0(1-z)\big)\;,\\
K^{(2)}_{gg}(z) \;=\;& \ln^2(1-z)\,p_{gg}(z)\left[32C_A\beta_0^2+112C_A^2\,\beta_0\,\mathrm{H}_0+128C_A^3\,\mathrm{H}_{0,0}\right]\;+\;O\big(\ln(1-z)\big)\;,\\
K^{(3)}_{gg}(z) \;=\;& \ln^3(1-z)\,p_{gg}(z)\left[-64C_A\beta_0^3-\xi_H^{(3)}\,C_A^2\,\beta_0^2\,\mathrm{H}_0-\eta_H^{(3)}\,C_A^3\,\beta_0\,\mathrm{H}_{0,0}-\xi_P^{(3)}\,C_A^4\,\mathrm{H}_{0,0,0}\right]\\
&+\;O\big(\ln^2(1-z)\big)
\end{aligned}
\tag{B.1}
$$

at $\mu_R=m_H$ with $\mathrm{H}_0=\ln z$, $\mathrm{H}_{0,0}=1/2\,\ln^2 z$, $\mathrm{H}_{0,0,0}=1/6\,\ln^3 z$ [34] and

$$
p_{gg}(z)\;=\;(1-z)_+^{-1}-2+z^{-1}+z-z^2\;.
$$

The first two lines of Eq. (B.1) are a direct consequence of Refs. [11,12] and [5–7]; their numerical coefficients are the same as for the Drell-Yan case in Eq. (3.27) of Ref. [24], which is based on the results of Refs. [5,62], up to a factor of two due to the different normalizations of p_{gg} here and p_{qq} in Ref. [24]. The N^{3}LO generalization based on the results for DIS, where the corresponding coefficient functions are known [25, 63, 64], involves two presently unknown parameters of the third-order coefficient function, $\xi_H^{(3)}$ already encountered above and $\eta_H^{(3)}$ relevant at $(1-z)^{k\geq 1}$, and one unknown coefficient of the four-loop splitting function $P_{gg}^{(3)}$ which is not relevant here.

Eq. (B.1) together with Eqs. (2.6) and (2.7) above yields the $\mu_F = \mu_R = m_H$ results

$$
\begin{aligned}
4^{-3}\, c_{gg}^{(3)}(z)\Big|_{C_F=0} \;=\;& \Big(\ln^5(1-z)\, 8\, C_A^3 - \ln^4(1-z)\, 10/3\, C_A^2 \Omega + \ln^3(1-z)\, 1/3\, C_A \beta_0^2\Big)\, p_{gg}(z) \\
&+ \ln^4(1-z)\, C_A^3 \Big\{ -27\, \mathrm{H}_0\, p_{gg}(z) - 32\, \mathrm{H}_0\, (1+z) + 59\, (1-z) - 187/3\, \big(z^{-1} - z^2\big) \Big\} \\
&+ \ln^3(1-z)\, C_A^3 \Big\{ \Big[16/3 - 56\, \zeta_2 + \Big(170/3 + \eta_H^{(3)}/96\Big)\, \mathrm{H}_{0,0} \Big]\, p_{gg}(z) \\
&\quad + \Big[4\, \mathrm{H}_{0,0} - 8\, \widetilde{\mathrm{H}}_{-1,0} \Big]\, p_{gg}(-z) - \big(119 - 407/3\, z^{-1} - 205\, z + 605/3\, z^2\big)\, \mathrm{H}_0 \\
&\quad + (76 + 140\, z)\, \mathrm{H}_{0,0} - 128\, (1+z)\, \widetilde{\mathrm{H}}_{1,0} - 721/3 + 2875/12\, z + 2314/9\, \big(z^{-1} - z^2\big) \Big\} \\
&+ \ln^3(1-z)\, C_A^2 \Omega \Big\{ \Big(20/3\, (1 + \mathrm{H}_0) + \xi_H^{(3)}/192\, \mathrm{H}_0 \Big)\, p_{gg}(z) + 10\, (1+z)\, \mathrm{H}_0 - 67/3 \\
&\quad + 271/12\, z + 193/9\, \big(z^{-1} - z^2\big) \Big\} \;+\; O\big(\ln^2(1-z)\big)
\end{aligned}
\tag{B.2}
$$

and

$$
\begin{aligned}
4^{-4}\, c_{gg}^{(4)}(z)\Big|_{C_F=0} \;=\;& \Big(\ln^7(1-z)\, 16/3\, C_A^4 - \ln^6(1-z)\, 14/3\, C_A^3 \Omega + \ln^5(1-z)\, 4/3\, C_A^2 \beta_0^2\Big)\, p_{gg}(z) \\
&+ \ln^6(1-z)\, C_A^4 \Big\{ -77/3\, \mathrm{H}_0\, p_{gg}(z) - 32\, (1+z)\, \mathrm{H}_0 + 166/3\, (1-z) - 550/9\, \big(z^{-1} - z^2\big) \Big\} \\
&+ \ln^5(1-z)\, C_A^4 \Big\{ \Big[8 - 92\, \zeta_2 + \Big(244/3 + \eta_H^{(3)}/96\Big)\, \mathrm{H}_{0,0} \Big]\, p_{gg}(z) \\
&\quad + \Big[4\, \mathrm{H}_{0,0} - 8\, \widetilde{\mathrm{H}}_{-1,0} \Big]\, p_{gg}(-z) - \big(156 - 220\, z^{-1} - 306\, z + 286\, z^2\big)\, \mathrm{H}_0 \\
&\quad + (104 + 232\, z)\, \mathrm{H}_{0,0} - 192\, (1+z)\, \widetilde{\mathrm{H}}_{1,0} - 1265/3 + 5051/12\, z + 3818/9\, \big(z^{-1} - z^2\big) \Big\} \\
&+ \ln^5(1-z)\, C_A^3 \Omega \Big\{ \Big[10 + \Big(91/6 + \xi_H^{(3)}/96 \Big)\, \mathrm{H}_0 \Big]\, p_{gg}(z) + 70/3\, (1+z)\, \mathrm{H}_0 - 265/6 \\
&\quad + 533/12\, z + 93/2\, \big(z^{-1} - z^2\big) \Big\} \;+\; O\big(\ln^4(1-z)\big)\ .
\end{aligned}
\tag{B.3}
$$

Here we have again suppressed the argument z of the harmonic polylogarithms for which we use a partly modified basis in terms of functions that have Taylor expansions about $z = 1$ with rational coefficients [24] including

$$
\begin{aligned}
\widetilde{\mathrm{H}}_{1,0}(z) &= \mathrm{H}_{1,0}(z) + \zeta_2 &&= -\ln z \ln(1-z) - \mathrm{Li}_2(z) + \zeta_2\ , \\
\widetilde{\mathrm{H}}_{-1,0}(z) &= \mathrm{H}_{-1,0}(z) + \zeta_2/2 &&= \ln z \ln(1+z) + \mathrm{Li}_2(-z) + \zeta_2/2\ .
\end{aligned}
$$

Similar to their NNLO analogues [5–7] and the NNLO and N^{3}LO coefficient function for Higgs-exchange DIS [25], the complete coefficient functions corresponding to Eqs. (B.2) and (B.3) will include additional C_F-terns contributing from $(1-z)^1$ beyond the leading logarithms.

The corresponding results for the non-singlet quark-antiquark annihilation contribution to the Drell-Yan process are given by†

$$
\begin{aligned}
4^{-3}\, c^{(3)\,\mathrm{ns}}_{\mathrm{DY}}(z) \;=\; & \Big(\ln^5(1-z)\,4C_F^3 - \ln^4(1-z)\,5/3\,C_F^2\underline{0} + \ln^3(1-z)\,1/6\,C_F\beta_0^2\Big)\, p_{qq}(z) \\
& + \ln^4(1-z)\,C_F^3 \Big\{ -27/2\,\mathrm{H}_0\, p_{qq}(z) + 4\,(1+z)\,\mathrm{H}_0 - 8\,(1-z) \Big\} \\
& + \ln^3(1-z)\,C_F^3 \Big\{ \Big[-16 - 24\,\zeta_2 - 3\,\mathrm{H}_0 - \widetilde{\mathrm{H}}_{1,0} + \Big(79/3 + \eta^{(3)}_{\mathrm{DY}}/192\Big)\,\mathrm{H}_{0,0}\Big]\, p_{qq}(z) \\
& \quad + (17/2 - 73/2\,z)\,\mathrm{H}_0 - 27/2\,(1+z)\,\mathrm{H}_{0,0} + 14\,(1+z)\,\widetilde{\mathrm{H}}_{1,0} + 8 - 17/2\,z \Big\} \\
& + \ln^3(1-z)\,C_F^2\underline{0} \Big\{ \Big[10/3 + \Big(13/3 + \xi^{(3)}_{\mathrm{DY}}/384\Big)\,\mathrm{H}_0\Big]\, p_{qq}(z) - (1+z)\,\mathrm{H}_0 + 4\,(1-z) \Big\} \\
& + \ln^3(1-z)\,C_F^2 C_A \Big\{ \Big(8/3 - 4\,\zeta_2 + \widetilde{\mathrm{H}}_{1,0} + 2\,\mathrm{H}_{0,0}\Big)\, p_{qq}(z) + (1+z)\,(\widetilde{\mathrm{H}}_{1,0} + 2\,\mathrm{H}_0) \\
& \quad + 6 - 11/2\,z \Big\} \;+\; O\big(\ln^2(1-z)\big)
\end{aligned}
\tag{B.4}
$$

and

$$
\begin{aligned}
4^{-4}\, c^{(4)\,\mathrm{ns}}_{\mathrm{DY}}(z) \;=\; & \Big(\ln^7(1-z)\,8/3\,C_F^4 - \ln^6(1-z)\,7/3\,C_F^3\underline{0} + \ln^5(1-z)\,2/3\,C_F^2\beta_0^2\Big)\, p_{qq}(z) \\
& + \ln^6(1-z)\,C_F^4 \Big\{ -77/6\,\mathrm{H}_0\, p_{qq}(z) + 4\,(1+z)\,\mathrm{H}_0 - 8\,(1-z) \Big\} \\
& + \ln^5(1-z)\,C_F^4 \Big\{ \Big[-16 - 40\,\zeta_2 - 3\,\mathrm{H}_0 - \widetilde{\mathrm{H}}_{1,0} + \Big(116/3 + \eta^{(3)}_{\mathrm{DY}}/192\Big)\,\mathrm{H}_{0,0}\Big]\, p_{qq}(z) \\
& \quad + (16 - 52\,z)\,\mathrm{H}_0 - 21\,(1+z)\,\mathrm{H}_{0,0} + 22\,(1+z)\,\widetilde{\mathrm{H}}_{1,0} + 16 - 33/2\,z \Big\} \\
& + \ln^5(1-z)\,C_F^3\underline{0} \Big\{ \Big[5 + \Big(103/12 + \xi^{(3)}_{\mathrm{DY}}/192\Big)\,\mathrm{H}_0\Big]\, p_{qq}(z) - 8/3\,(1+z)\,\mathrm{H}_0 + 22/3\,(1-z) \Big\} \\
& + \ln^5(1-z)\,C_F^3 C_A \Big\{ \Big(4 - 6\,\zeta_2 + \widetilde{\mathrm{H}}_{1,0} + 2\,\mathrm{H}_{0,0}\Big)\, p_{qq}(z) + (1+z)\,(\widetilde{\mathrm{H}}_{1,0} + 2\,\mathrm{H}_0) \\
& \quad + 6 - 11/2\,z \Big\} \;+\; O\big(\ln^4(1-z)\big)
\end{aligned}
\tag{B.5}
$$

with

$$
p_{qq}(z) \;=\; 2\,(1-z)_+^{-1} - 1 - z\;.
$$

The $\ln^3(1-z)$ term in Eq. (B.4) and the $\ln^5(1-z)$ contribution in Eq. (B.5) include the unknown third-order coefficients $\xi^{(3)}_{\mathrm{DY}}$ and $\eta^{(3)}_{\mathrm{DY}}$ which we definitely expect to be equal to their counterparts for Higgs-boson production in Eqs. (B.1) – (B.3). Hence an extension of either Refs. [5–7] or Refs. [5, 62] to N^{3}LO will fix also the third-highest power of $\ln(1-z)$ at N^{4}LO and all higher orders for both processes.

†The $\ln^4(1-z)$ and $\ln^5(1-z)$ contributions to $c^{(3)\,\mathrm{ns}}_{\mathrm{DY}}(z)$ have been presented before in Eq. (6.29) of Ref. [24] where, unfortunately, all coefficients are too small by a factor 3/4.

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