

The three-loop cusp anomalous dimension in QCD

Andrey Grozin

*Budker Institute of Nuclear Physics SB RAS, Novosibirsk, Russia
Novosibirsk State University, Novosibirsk, Russia**

Johannes M. Henn

Institute for Advanced Study, Princeton, NJ 08540, USA[†]

Gregory P. Korchemsky

Institut de Physique Théorique, CEA Saclay, 91191 Gif-sur-Yvette Cedex, France[‡]

Peter Marquard

Deutsches Elektronen-Synchrotron, DESY, Platanenallee 6, D15738 Zeuthen, Germany[§]

(Dated: September 2, 2014)

We present the full analytic result for the three-loop angle-dependent cusp anomalous dimension in QCD. With this result, infrared divergences of planar scattering processes with massive particles can be predicted to that order. Moreover, we define a closely related quantity in terms of an effective coupling defined by the light-like cusp anomalous dimension. We find evidence that this quantity is universal for any gauge theory, and use this observation to predict the non-planar n_f -dependent terms of the four-loop cusp anomalous dimension.

PACS numbers: 12.39.Hg, 11.15.-q

The cusp anomalous dimension is an ubiquitous quantity in four-dimensional gauge theories. It governs the dependence of a cusped Wilson loop on the ultraviolet cut-off [1] and appears in many physical quantities. In particular, it controls the infrared asymptotics of scattering amplitudes and form factors involving massive particles [2, 3], see e.g. [4] for a recent application to top quark pair production. The two-loop result for this fundamental quantity has been known for more than 25 years [5]. Here we report on the full result for the cusp anomalous dimension in QCD at three loops.

To compute the cusp anomalous dimension, we consider the vacuum expectation value of the Wilson line operator

$$W = \frac{1}{N} \langle 0 | \text{tr} \left[P \exp \left(i \oint_C dx \cdot A(x) \right) \right] | 0 \rangle, \quad (1)$$

with $A_\mu(x) = A_\mu^a(x) T^a$ and T^a being the generators of the fundamental representation of the $SU(N)$ gauge group. Here the integration contour C is formed by two segments along space-like directions v_1^μ and v_2^μ (with $v_1^2 = v_2^2 = 1$), with (Euclidean space) cusp angle ϕ ,

$$\cos \phi = v_1 \cdot v_2, \quad (2)$$

cf. Fig. 1. Perturbative corrections to the Wilson loop (1) contain both ultraviolet (cusp) and infrared divergences. We employ dimensional regularization with $D = 4 - 2\epsilon$ to regularize the former and introduce an infrared cut-off using the heavy quark effective theory (HQET) framework. The cusp anomalous dimension Γ_{cusp} is extracted as the residue at the simple pole $1/\epsilon$ in the corresponding renormalization factor.

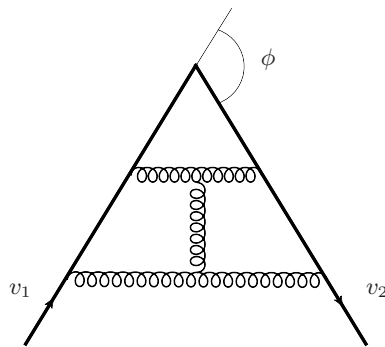


FIG. 1: Sample Feynman diagram producing a contribution to the three-loop cusp anomalous dimension in QCD. Thick lines denote two semi-infinite segments forming a cusp of angle ϕ , and wavy lines represent gauge fields.

It depends on the cusp angle ϕ , the strong coupling constant $\alpha_s = g_{\text{YM}}^2/(4\pi)$, and on $SU(N)$ color factors. It is convenient to introduce the complex variable

$$x = e^{i\phi}, \quad 2 \cos \phi = x + 1/x. \quad (3)$$

In Euclidean space $|x| = 1$, whereas for Minkowskian angles $\phi = i\theta$ (with θ real) the variable x can take arbitrary nonnegative values. Due to the symmetry $x \rightarrow 1/x$ of the definition (3), we can assume $0 < x < 1$ without loss of generality.

We chose to perform the calculation in momentum space. We generated all Feynman diagrams contributing to W up to three loops, in an arbitrary covariant gauge. This was done with the help of the computer programs QGRAF and FORM [6]. Using integration by parts re-

lations [7], we found that a total of 71 master integrals was required. We derived differential equations for them in the complex variable x defined in (3). Switching to a basis of master integrals $\vec{f}(x, \epsilon)$ as suggested in ref. [8], we found the expected canonical form of the differential equations,

$$\partial_x \vec{f}(x, \epsilon) = \epsilon \left[\frac{a}{x} + \frac{b}{x+1} + \frac{c}{x-1} \right] \vec{f}(x, \epsilon), \quad (4)$$

with constant (x - and ϵ -independent) matrices a, b, c .

Eq. (4) has four regular singular points in x , namely $0, 1, -1$, and ∞ . Thanks to the $x \rightarrow 1/x$ symmetry of the definition (3), only the first three are independent. They correspond, in turn, to the light-like limit (infinite

Minkowski angle), to the zero angle limit, and to the antiparallel lines limit. Requiring that the integrals be nonsingular in the straight-line case $x = 1$ allowed us to fix all except one boundary conditions, and we obtained the remaining one from ref. [9].

It follows from (4) that the solution for \vec{f} in the ϵ -expansion can be written in terms of iterated integrals with integration kernels $dx/x, dx/(x-1), dx/(x+1)$. The latter integrals are known as harmonic polylogarithms $H_{n_1 \dots n_k}(x)$ [10]. The indices n_i can take values $0, 1, -1$, corresponding to the three integration kernels, respectively.

To express our results up to three loops, we introduce the following functions [23],

$$\begin{aligned} A_1(x) &= \xi \frac{1}{2} H_1(y), & A_2(x) &= \left[\frac{\pi^2}{3} + \frac{1}{2} H_{1,1}(y) \right] + \xi \left[-H_{0,1}(y) - \frac{1}{2} H_{1,1}(y) \right], \\ A_3(x) &= \xi \left[-\frac{\pi^2}{6} H_1(y) - \frac{1}{4} H_{1,1,1}(y) \right] + \xi^2 \left[\frac{1}{2} H_{1,0,1}(y) + \frac{1}{4} H_{1,1,1}(y) \right], \\ A_4(x) &= \left[-\frac{\pi^2}{6} H_{1,1}(y) - \frac{1}{4} H_{1,1,1,1}(y) \right] + \\ &+ \xi \left[\frac{\pi^2}{3} H_{0,1}(y) + \frac{\pi^2}{6} H_{1,1}(y) + 2H_{1,1,0,1}(y) + \frac{3}{2} H_{0,1,1,1}(y) + \frac{7}{4} H_{1,1,1,1}(y) + 3\zeta_3 H_1(y) \right] \\ &+ \xi^2 \left[-2H_{1,0,0,1}(y) - 2H_{0,1,0,1}(y) - 2H_{1,1,0,1}(y) - H_{1,0,1,1}(y) - H_{0,1,1,1}(y) - \frac{3}{2} H_{1,1,1,1}(y) \right], \\ A_5(x) &= \xi \left[\frac{\pi^4}{12} H_1(y) + \frac{\pi^2}{4} H_{1,1,1}(y) + \frac{5}{8} H_{1,1,1,1,1}(y) \right] + \xi^2 \left[-\frac{\pi^2}{6} H_{1,0,1}(y) - \frac{\pi^2}{3} H_{0,1,1}(y) - \frac{\pi^2}{4} H_{1,1,1}(y) \right. \\ &\quad \left. - H_{1,1,1,0,1}(y) - \frac{3}{4} H_{1,0,1,1,1}(y) - H_{0,1,1,1,1}(y) - \frac{11}{8} H_{1,1,1,1,1}(y) - \frac{3}{2} \zeta_3 H_{1,1}(y) \right] \\ &+ \xi^3 \left[H_{1,1,0,0,1}(y) + H_{1,0,1,0,1}(y) + H_{1,1,1,0,1}(y) + \frac{1}{2} H_{1,1,0,1,1}(y) + \frac{1}{2} H_{1,0,1,1,1}(y) + \frac{3}{4} H_{1,1,1,1,1}(y) \right], \\ B_3(x) &= \left[-H_{1,0,1}(y) + \frac{1}{2} H_{0,1,1}(y) - \frac{1}{4} H_{1,1,1}(y) \right] + \xi \left[2H_{0,0,1}(y) + H_{1,0,1}(y) + H_{0,1,1}(y) + \frac{1}{4} H_{1,1,1}(y) \right], \\ B_5(x) &= \frac{x}{1-x^2} \left[-\frac{\pi^4}{60} H_{-1}(x) - \frac{\pi^4}{60} H_1(x) - 4H_{-1,0,-1,0,0}(x) + 4H_{-1,0,1,0,0}(x) - 4H_{1,0,-1,0,0}(x) \right. \\ &\quad \left. + 4H_{1,0,1,0,0}(x) + 4H_{-1,0,0,0,0}(x) + 4H_{1,0,0,0,0}(x) + 2\zeta_3 H_{-1,0}(x) + 2\zeta_3 H_{1,0}(x) \right], \end{aligned} \quad (5)$$

where $\xi = (1+x^2)/(1-x^2)$ and $y = 1-x^2$. The subscript of A indicates the (transcendental) weight of the functions. Moreover, we introduce the abbreviation $\tilde{A}_i = A_i(x) - A_i(1)$, and similarly for \tilde{B}_i .

Performing the three-loop computation, we reproduced the expected structure of UV divergences of W in the $\overline{\text{MS}}$ scheme, as well as the HQET wavefunction renormalization [9], for arbitrary values of the gauge parameter in the covariant gauge. As yet another check, the dependence on the gauge parameter disappeared for the cusp anomalous dimension.

Let us write the expansion in the coupling constant as

$$\Gamma_{\text{cusp}}(\alpha_s, x) = \sum_{k \geq 1} \left(\frac{\alpha_s}{\pi} \right)^k \Gamma_{\text{cusp}}^{(k)}(x). \quad (6)$$

The previously known one- and two-loop [5] results can

be written as

$$\Gamma_{\text{cusp}}^{(1)} = C_F \tilde{A}_1, \quad (7)$$

$$\Gamma_{\text{cusp}}^{(2)} = \frac{1}{2} C_F C_A \left[\tilde{A}_3 + \tilde{A}_2 \right] + \left(\frac{67}{36} C_F C_A - \frac{5}{9} C_F T_f n_f \right) \tilde{A}_1. \quad (8)$$

At three loops we find

$$\Gamma_{\text{cusp}}^{(3)} = c_1 C_F C_A^2 + c_2 C_F (T_f n_f)^2 + c_3 C_F^2 T_f n_f + c_4 C_F C_A T_f n_f, \quad (9)$$

with

$$c_1 = \frac{1}{4} \left[\tilde{A}_5 + \tilde{A}_4 + \tilde{B}_5 + \tilde{B}_3 \right] + \frac{67}{36} \tilde{A}_3 + \frac{29}{18} \tilde{A}_2 + \left(\frac{245}{96} + \frac{11}{24} \zeta_3 \right) \tilde{A}_1, \quad (10)$$

$$c_2 = -\frac{1}{27} \tilde{A}_1, \quad c_3 = \left(\zeta_3 - \frac{55}{48} \right) \tilde{A}_1, \quad (11)$$

$$c_4 = -\frac{5}{9} \left[\tilde{A}_3 + \tilde{A}_2 \right] - \frac{1}{6} \left(7\zeta_3 + \frac{209}{36} \right) \tilde{A}_1. \quad (12)$$

Here $C_F = (N^2 - 1)/(2N)$ and $C_A = N$ are the quadratic Casimir operators of the $SU(N)$ gauge group in the fundamental and adjoint representation, respectively, n_f is the number of quark flavors, and $T_f = 1/2$.

The following comments are in order. The cusp anomalous dimension has a branch cut for x lying on the negative real axis. The results given in (9) are valid for $0 < x < 1$ and can be analytically continued to other regions according to this choice of branch cuts [24].

The leading n_f^2 term in (9) is in agreement with the known result [11]. We reported on the n_f -dependent part of (9) in [12]. The expression for the coefficient c_1 is new.

As a check of our result, we can consider Minkowskian angles and take the light-like limit, $x = e^{-\theta}$ with $\theta \rightarrow \infty$, of eq. (9), where one expects the behavior [13]

$$\Gamma_{\text{cusp}}(\alpha_s, x) \stackrel{x \rightarrow 0}{\sim} K(\alpha_s) \log(1/x) + \mathcal{O}(x^0), \quad (13)$$

with $K(\alpha_s)$ being the light-like cusp anomalous dimension. To three loops, it is given by [14]

$$\begin{aligned} K^{(1)} &= C_F, \\ K^{(2)} &= C_A C_F \left(\frac{67}{36} - \frac{\pi^2}{12} \right) - \frac{5}{9} n_f T_f C_F, \\ K^{(3)} &= C_A^2 C_F \left(\frac{245}{96} - \frac{67\pi^2}{216} + \frac{11\pi^4}{720} + \frac{11}{24} \zeta_3 \right) \\ &\quad + C_A C_F n_f T_f \left(-\frac{209}{216} + \frac{5\pi^2}{54} - \frac{7}{6} \zeta_3 \right) \\ &\quad + C_F^2 n_f T_f \left(\zeta_3 - \frac{55}{48} \right) - \frac{1}{27} C_F (n_f T_f)^2, \end{aligned} \quad (14)$$

where $K(\alpha_s) = \sum_{m \geq 1} (\alpha_s/\pi)^m K^{(m)}$. We found perfect agreement for all terms.

Finally, in the antiparallel lines limit $\phi = \pi - \delta$, $\delta \rightarrow 0$, one expects to recover the quark antiquark potential [15] (at one loop order lower compared to Γ_{cusp}). Starting from eq. (9) we indeed find perfect agreement with the result quoted in the second ref. of [16], up to terms proportional to the QCD β -function.

Our result for the cusp anomalous dimension is valid in the $\overline{\text{MS}}$ (dimensional regularisation) scheme. Going to the $\overline{\text{DR}}$ (dimensional reduction) scheme amounts to a finite renormalisation of the coupling constant. We can introduce a quantity Ω which is the same in both schemes by switching from α_s to an ‘‘effective coupling’’ a ,

$$\Omega(a, x) := \Gamma_{\text{cusp}}(\alpha_s, x), \quad a := \pi/C_F K(\alpha_s), \quad (15)$$

where Γ_{cusp} and $K(\alpha_s)$ are evaluated in the same scheme (and for the same theory). By construction, Ω has the universal limit

$$\Omega(a, x) \stackrel{x \rightarrow 0}{\sim} \frac{a}{\pi} C_F \log(1/x) + \mathcal{O}(x^0), \quad (16)$$

as one can easily verify by comparing to eq. (13).

Using the results up to three loops given in eqs. (7), (8), (9) and (14), and expanding both sides of the first relation in (15) to third order in α_s , we find

$$\begin{aligned} \Omega(a, x) &= \frac{a}{\pi} C_F \tilde{A}_1 + \left(\frac{a}{\pi} \right)^2 \frac{C_A C_F}{2} \left[\tilde{A}_3 + \tilde{A}_2 + \frac{\pi^2}{6} \tilde{A}_1 \right] \\ &\quad + \left(\frac{a}{\pi} \right)^3 \frac{C_F C_A^2}{4} \left[\tilde{A}_5 + \tilde{A}_4 - \tilde{A}_2 + \tilde{B}_5 + \tilde{B}_3 \right. \\ &\quad \left. + \frac{\pi^2}{3} \tilde{A}_3 + \frac{\pi^2}{3} \tilde{A}_2 - \frac{\pi^4}{180} \tilde{A}_1 \right] + \mathcal{O}(a^4). \end{aligned} \quad (17)$$

Remarkably, this quantity is independent of n_f to three loops! Comparing to eq. (15) we see that this means that e.g. all n_f dependent terms in $\Gamma_{\text{cusp}}^{(3)}$ are generated from lower-loop terms, when expanding $K(\alpha_s)$ in α_s .

In Fig. 2 we plot the one-, two- and three-loop coefficients of Ω in an expansion of a/π , for Minkowskian angles θ , i.e. $x = e^{-\theta}$ for the range $\theta \in [0, 4]$, and with the number of colors set to $N = 3$. Note that the n_f -dependence in QCD can be obtained from eq. (15), and amounts to a rescaling of the coupling. At large θ , the one-loop contribution displays the linear behavior of eq. (16), while the two- and three-loop contributions go to a constant, as expected. In the small angle region, we have,

$$\begin{aligned} \Omega(a, e^{-\theta}) &= C_F \left[\left(\frac{a}{\pi} \right) \frac{1}{3} + \left(\frac{a}{\pi} \right)^2 \frac{C_A}{4} \left(1 - \frac{\pi^2}{9} \right) \right. \\ &\quad \left. + \left(\frac{a}{\pi} \right)^3 \frac{C_A^2}{12} \left(-\frac{5}{3} - \frac{\pi^2}{6} + \frac{\pi^4}{20} - \zeta_3 \right) + \mathcal{O}(a^4) \right] \theta^2 + \mathcal{O}(\theta^4). \end{aligned} \quad (18)$$

The observed n_f -independence of $\Omega(a, x)$ leads us to conjecture that the latter quantity is universal in gauge

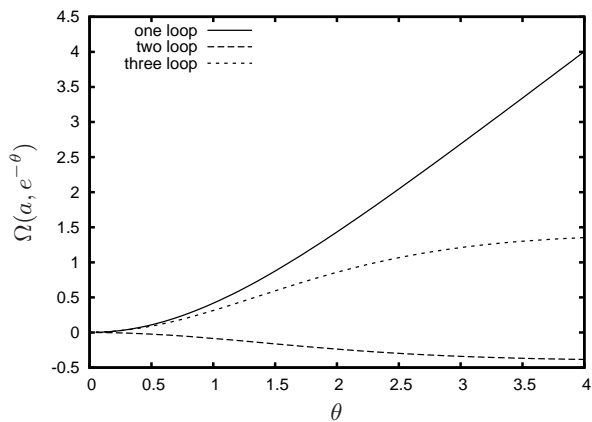


FIG. 2: θ dependence of the cusp anomalous dimension $\Omega(a, e^{-\theta})$ at one (solid), two (dashed), and three (dotted) loops.

theories, i.e. independent of the specific particle content of the theory. Assuming this conjecture leads to a number of non-trivial predictions.

First, let us recall the known value for K in $\mathcal{N} = 4$ super Yang-Mills (in the $\overline{\text{DR}}$ scheme) [17],

$$K_{\mathcal{N}=4}(\alpha_s) = C_F \left[\left(\frac{\alpha_s}{\pi} \right) - \frac{\pi^2}{12} C_A \left(\frac{\alpha_s}{\pi} \right)^2 + \frac{11}{720} \pi^4 C_A^2 \left(\frac{\alpha_s}{\pi} \right)^3 + \mathcal{O}(\alpha_s^4) \right]. \quad (19)$$

Plugging this formula and the result for Ω given in eq. (17) into eq. (15) then gives the previously unknown three-loop result for the cusp anomalous dimension for the Wilson loop operator of eq. (1) in that theory,

$$\Gamma_{\mathcal{N}=4}(\alpha_s, x) = \frac{\alpha_s}{\pi} C_F \tilde{A}_1 + \frac{C_A C_F}{2} \left(\frac{\alpha_s}{\pi} \right)^2 \left[\tilde{A}_3 + \tilde{A}_2 \right] + \frac{C_F C_A^2}{4} \left(\frac{\alpha_s}{\pi} \right)^3 \left[\tilde{A}_5 + \tilde{A}_4 - \tilde{A}_2 + \tilde{B}_5 + \tilde{B}_3 \right] + \mathcal{O}(\alpha_s^4). \quad (20)$$

The two-loop terms agree with ref. [12]. As a test of the three-loop prediction, we take the antiparallel lines limit and obtain

$$\Gamma_{\mathcal{N}=4}(\alpha_s, x) \stackrel{\delta \rightarrow 0}{\equiv} - \frac{C_F \alpha_s}{\delta} \left\{ 1 - \left(\frac{\alpha_s}{\pi} \right) C_A + \left(\frac{\alpha_s}{\pi} \right)^2 C_A^2 \left[\frac{5}{4} + \frac{\pi^2}{4} - \frac{\pi^4}{64} \right] + \mathcal{O}(\alpha_s^3) \right\} + \mathcal{O}(\delta^0), \quad (21)$$

as expected from the direct calculation of the quark anti-quark potential [18].

Second, the conjecture of the n_f -independence of Ω can be used to predict the form of the non-planar n_f corrections that can first appear at four loops. The latter involve quartic Casimir operators of $SU(N)$, whose contribution we abbreviate by $C_4 = d_F^{abcd} d_F^{abcd} / N_A = (18 - 6N^2 + N^4) / (96N^2)$ (with N_A the number of the

$SU(N)$ generators) [19]. Consider a term in $\Gamma_{\text{cusp}}(\alpha_s, x)$ of the form $n_f (\alpha_s / \pi)^4 g(x) C_F C_4 / 64$, for some $g(x)$. Assuming that Ω defined in eq. (15) is independent of n_f then implies $g(x) = g_0 \tilde{A}_1$. Moreover, we can determine g_0 by comparing to the antiparallel lines limit. The expected relation to the known quark anti-quark potential computed (numerically) in ref. [20] then yields $g_0 = -56.83(1)$.

Finally, if Ω is the same in any gauge theory, it is likely that it will be determined in the foreseeable future, at least in the planar limit, by calculations based on integrability in $\mathcal{N} = 4$ super Yang-Mills.

Acknowledgments

A.G.'s work was supported by RFBR grant 12-02-00106-a and by the Russian Ministry of Education and Science. J.M.H. is supported in part by the DOE grant de-sc0009988, the Marvin L. Goldberger fund, and the Munich Institute for Astro- and Particle Physics (MIAPP) of the DFG cluster of excellence "Origin and Structure of the Universe". G.P.K. is supported in part by the French National Agency for Research (ANR) under contract StrongInt (BLANC-SIMI-4-2011). P.M. was supported in part by the DFG through the SFB/TR 9 "Computational Particle Physics" and the EU Networks LHCPHENonet PITN-GA-2010-264564 and HIGGSTOOLS PITN-GA-2012-316704. We thank J. Blümlein and J. Ablinger for making the "Harmonic-Sums" computer code based on [21] available to us.

* Electronic address: A.G.Grozin@inp.nsk.su

† Electronic address: jmhenn@ias.edu

‡ Electronic address: Gregory.Korchemsky@cea.fr

§ Electronic address: peter.marquard@desy.de

- [1] A. M. Polyakov, Nucl. Phys. B **164**, (1980) 171; R. A. Brandt, F. Neri and M. Sato, Phys. Rev. D **24** (1981) 879.
- [2] G. P. Korchemsky and A. V. Radyushkin, Phys. Lett. B **171** (1986) 459; Phys. Lett. B **279** (1992) 359.
- [3] M. Neubert, Phys. Rept. **245** (1994) 259; A. V. Manohar and M. B. Wise, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **10** (2000) 1; A. G. Grozin, Springer Tracts Mod. Phys. **201** (2004) 1.
- [4] M. Czakon, A. Mitov and G. F. Sterman, Phys. Rev. D **80** (2009) 074017.
- [5] G. P. Korchemsky and A. V. Radyushkin, Nucl. Phys. B **283** (1987) 342; N. Kidonakis, Phys. Rev. Lett. **102** (2009) 232003;
- [6] P. Nogueira, J. Comput. Phys. **105** (1993) 279; J. A. M. Vermaseren, math-ph/0010025.
- [7] K. G. Chetyrkin and F. V. Tkachov, Nucl. Phys. B **192** (1981) 159; A. V. Smirnov, JHEP **0810** (2008) 107; R. N. Lee, arXiv:1212.2685 [hep-ph]; P. Marquard, D. Seidel, unpublished.
- [8] J. M. Henn, Phys. Rev. Lett. **110** (2013) no. 25, 251601.
- [9] A. G. Grozin, JHEP **0003** (2000) 013; K. G. Chetyrkin and A. G. Grozin, Nucl. Phys. B **666** (2003) 289.

- [10] E. Remiddi and J. A. M. Vermaseren, *Int. J. Mod. Phys. A* **15** (2000) 725; D. Maître, *Comput. Phys. Commun.* **174** (2006) 222.
- [11] M. Beneke and V. M. Braun, *Nucl. Phys. B* **454** (1995) 253.
- [12] A. Grozin, J. M. Henn, G. P. Korchemsky and P. Marquard, arXiv:1406.7828 [hep-th].
- [13] G. P. Korchemsky, *Mod. Phys. Lett. A* **4** (1989) 1257; G. P. Korchemsky and G. Marchesini, *Nucl. Phys. B* **406** (1993) 225.
- [14] A. Vogt, *Phys. Lett. B* **497** (2001) 228; C. F. Berger, *Phys. Rev. D* **66** (2002) 116002; S. Moch, J. A. M. Vermaseren and A. Vogt, *Phys. Lett. B* **625** (2005) 245.
- [15] W. Kilian, T. Mannel and T. Ohl, *Phys. Lett. B* **304** (1993) 311.
- [16] M. Peter, *Phys. Rev. Lett.* **78** (1997) 602; Y. Schröder, *Phys. Lett. B* **447** (1999) 321.
- [17] A. V. Kotikov, L. N. Lipatov, A. I. Onishchenko and V. N. Velizhanin, *Phys. Lett. B* **595** (2004) 521 [Erratum-*ibid.* B **632** (2006) 754].
- [18] M. Prausa and M. Steinhauser, *Phys. Rev. D* **88** (2013) 2, 025029.
- [19] T. van Ritbergen, J. A. M. Vermaseren and S. A. Larin, *Phys. Lett. B* **400** (1997) 379.
- [20] A. V. Smirnov, V. A. Smirnov and M. Steinhauser, *Phys. Lett. B* **668** (2008) 293.
- [21] J. Ablinger, arXiv:1011.1176 [math-ph]; arXiv:1305.0687 [math-ph]; J. Ablinger, J. Blümlein and C. Schneider, *J. Math. Phys.* **52** (2011) 102301; *J. Math. Phys.* **54** (2013) 082301.
- [22] D. Correa, J. Henn, J. Maldacena and A. Sever, *JHEP* **1205** (2012) 098;
- [23] The function A_5 (and A_1 and A_3) appeared previously in the result for the cusp anomalous dimension in $\mathcal{N} = 4$ super Yang-Mills (SYM) for a locally supersymmetric Wilson line operator [22].
- [24] Note that $\Gamma_{\text{cusp}}(\alpha_s, -x)$ is expected to be related to $\Gamma_{\text{cusp}}(\alpha_s, x)$ by crossing symmetry, i.e. the two functions should be equal, up to terms picked up by the analytic continuation. It turns out that all functions except B_5 contain rational factors that are symmetric under $x \rightarrow -x$, and therefore the harmonic polylogarithms in these expressions can be written with argument $1 - x^2$, and positive indices only. In contrast, the factor $x/(1-x^2)$ contained in B_5 is antisymmetric under $x \rightarrow -x$, and so is the transcendental function multiplying it (up to terms coming from branch cuts).