DESY-14-162

High-Gradient Operators in the $\mathfrak{psl}(2|2)$ Gross-Neveu Model

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20th October 2014

Abstract

It has been observed more than 25 years ago that sigma model perturbation theory suffers from strongly RG-relevant high-gradient operators. The phenomenon was first seen in 1-loop calculations for the O(N) vector model and it is known to persist at least to two loops. More recently, Ryu et al. suggested that a certain deformation of the $\mathfrak{psl}(N|N)$ WZNW-model at level $k = 1$, or equivalently the $\mathfrak{psl}(N|N)$ Gross-Neveu model, could be free of RG-relevant high-gradient operators and they tested their suggestion to leading order in perturbation theory. In this note we establish the absence of strongly RG-relevant high-gradient operators in the $\mathfrak{psl}(2|2)$ Gross-Neveu model to all loops. In addition, we determine the spectrum for a large subsector of the model at infinite coupling and observe that all scaling weights become half-integer. Evidence for a conjectured relation with the $\mathbb{CP}^{1|2}$ sigma model is not found.

1 Introduction

Non-linear sigma models (NLSM), with and without WZ term, play an important role in the description of condensed matter systems as well as string compactifications. It has long been known [1] that NLSM generically suffer from the strong RG-relevance of certain high-gradient operators. At zero coupling the dimension of an operator in a NLSM is determined by the number of derivatives, making high-gradient operators highly irrelevant. The usual assumption in perturbation theory is that corrections to the scaling weight, the anomalous dimensions, remain small as long as the coupling does. However, it has been shown [1] that this assumption fails to hold for certain invariant high-gradient operators in the $O(N)$-vector model and that in fact these operators can become relevant even at infinitesimal values of the coupling. Similar results hold for NLSM defined on a wide variety of compact and non-compact target (super-)spaces, see [2] and references therein.

The generation of relevant operators represents a puzzling instability of the UV fixed point. One might hope that higher orders in perturbation theory correct this issue. But unfortunately, the problem has been shown [3] to become even worse at two loops. On the other hand, lattice simulations and other approaches have never shown signs of an instability, which adds to the long-standing puzzle. In the existing literature on the subject, the predominant attitude is to consider the RG-relevance of high-gradient operators as a bug, though Polyakov has argued [4] that it could be a desirable feature in order to establish a general pattern of dualities between NLSMs and WZNW models. In any case, the question remains whether the strong RG-relevance of high-gradient operators is corrected by higher loop or non-perturbative effects.

The related WZNW models, when perturbed by a current-current deformation, suffer from a similar issue [2], even if the perturbation preserves conformal symmetry as it does for a number of target supergroups. A few years ago, Ryu et al. suggested [2] that WZNW-models with affine $\mathfrak{psl}(N|N)$ symmetry at level $k = 1$ could be free from strongly RG-relevant high-gradient operators. They also tested this idea to leading order in perturbation theory, at least for certain classes of high-gradient operators. In this note, we will exploit previously obtained results on the perturbation theory of WZNW models and the representation theory of $\mathfrak{psl}(2|2)$ to prove that the deformed

$\mathfrak{psl}(2|2)$ WZNW model at $k = 1$ is indeed free of strongly RG-relevant $\mathfrak{psl}(2|2)$ invariant operators in any order of perturbation theory. In addition, we evaluate the spectrum of all fields that transform in maximally atypical ("$\frac{1}{2}$ BPS") representations of the target space symmetry $\mathfrak{psl}(2|2)$ up to scaling weight $\Delta \leq 5$. Very remarkably, the spectrum at infinite coupling turns out to assume half-integer values only, nurturing hopes it might be described by a dual free field theory. In fact, it has been argued that such a dual model is provided by the $\mathbb{CP}^{1|2}$ NLSM [5]. Our results, however, do not provide evidence for a duality with this sigma model.

The plan of this short note is as follows. In the next section we review some relevant results on all-loop anomalous dimensions in perturbed WZNW models from [8]. In order to apply these to the $\mathfrak{psl}(2|2)$ Gross-Neveu model, we need some background from representation theory which is collected in section 3. All-loop stability is established in section 4 before we compute the low lying spectrum at infinite coupling in section 5.

2 Results from WZNW perturbation theory

In this section we review recent results [8] on the anomalous dimensions of WZNW models with Lie superalgebra symmetry. If the Lie superalgebra $\mathfrak{g}$ has vanishing dual Coxeter number $g^\vee = 0$, the current-current interaction

$$\Omega(z, \bar{z}) = J^\mu(z)\bar{J}_\mu(\bar{z}) \tag{1}$$

is exactly marginal. Here, the index μ labels basis elements of the Lie superalgebra $\mathfrak{g}$ and we sum over all its allowed values. The interaction (1) clearly breaks the affine $\hat{\mathfrak{g}}$-symmetry. Moreover, it also breaks the chiral $\mathfrak{g}$-symmetries, since the operator Ω does not commute with the zero modes J_0^a and $\bar{J}_0^b$ of the chiral currents. However, Ω does commute with the sum of the zero modes $J_0^a + \bar{J}_0^a$, thereby leaving the diagonal $\mathfrak{g}$-symmetry intact.

The authors of [8] were able to obtain an all-order result for the anomalous dimension of special operators in such Ω perturbed conformal field theories.

It applies to all operators that transform in a maximally atypical representation under the diagonal action of the superalgebra $\mathfrak{g}$. Maximally atypical (or $\frac{1}{2}$BPS) representations are indecomposables that contain a subrepresentation with non-vanishing superdimension. The anomalous dimension δ_g for such operators turns out to only depend on the representation labels and the level k of the affine superalgebra,

$$\delta_g = \frac{g}{2(1-k^2g^2)}\big(\mathrm{Cas}^D - (1-kg)(\mathrm{Cas}^L + \mathrm{Cas}^R)\big), \tag{2}$$

where Cas is the quadratic Casimir operator and the superscripts D, L and R indicate the diagonal, left and right action of the algebra, respectively.

Let us now specialize to the case $\mathfrak{g} = \mathfrak{psl}(2|2)$. The superalgebra $\mathfrak{psl}(2|2)$ has only one atypicality condition and the quadratic Casimir vanishes on all atypical representations. Thus, eq. (2) simplifies to

$$\delta_g = -\frac{g}{2(1+kg)}(\mathrm{Cas}^L + \mathrm{Cas}^R). \tag{3}$$

We are particularly interested in operators that are invariant under the diagonal action of the symmetry algebra since such operators could be used to generate a $\mathfrak{g}$ preserving perturbation. The assumption of $\mathfrak{g}$ invariance does not simplify our formula (3) any further but it restricts it to operators for which the tensor product of left and right action contains the trivial representation. The finite-dimensional representation theory of $\mathfrak{psl}(2|2)$ has been worked out in detail in [9]. The results imply that the only way to obtain an invariant with $\Lambda_R = \Lambda$ is to tensor with the same representation $\Lambda_L = \Lambda$.

3 Review of $\mathfrak{psl}(2|2)$ representation theory

In this section we give a brief review of the pertinent facts regarding the Lie superalgebra $\mathfrak{psl}(2|2)$ and its finite dimensional representation theory. The algebra $\mathfrak{psl}(2|2)$ has rank two and its even subalgebra is $\mathfrak{g}^{(0)} \simeq \mathfrak{sl}(2) \oplus \mathfrak{sl}(2)$. Consequently, all finite dimensional representations are uniquely characterized by a pair of $\mathfrak{sl}(2)$ weights $j, l \in \frac{1}{2}\mathbb{Z}$. Representations of $\mathfrak{psl}(2|2)$ can satisfy one shortening, or atypicality, condition which is simply given by $j = l$. We will denote typical representations of $\mathfrak{psl}(2|2)$ by $[j, l]$ and atypical irreducibles by $[j]$. Irreducible representations of the even subalgebra will be denoted by (j, l).

Upon restriction to the even subalgebra $\mathfrak{g}^{(0)}$ the irreducible representations decompose as

$$[j]\big|_{\mathfrak{g}^{(0)}} \simeq (j+\tfrac{1}{2}, j-\tfrac{1}{2}) \oplus 2(j,j) \oplus (j-\tfrac{1}{2}, j+\tfrac{1}{2}) \qquad \text{(for } j > 0\text{)} \tag{4}$$

$$[j,l]\big|_{\mathfrak{g}^{(0)}} \simeq (j,l) \otimes \big[2(0,0) \oplus 2(\tfrac{1}{2},\tfrac{1}{2}) \oplus (0,1) \oplus (1,0)\big] \tag{5}$$

and $[0]$ is the trivial representation. Let us also remark that $[\frac{1}{2}]$ corresponds to the adjoint representation. Atypical irreducibles of Lie superalgebras can form indecomposables. If one is not interested in the precise form in which such indecomposables are built from their constituents, all tensor products of finite dimensional $\mathfrak{g}$ representations may be determined by restricting the factors to the even subalgebra, tensoring the associated $\mathfrak{g}^{(0)}$ representations and combining the resulting products back into representations of $\mathfrak{g}$. The first and last step require no more than our decomposition formulas (4) and (5). The tensor products of irreducibles, including the indecomposable structures, have been worked out in [9].

We will also need to know the eigenvalue of the quadratic Casimir invariant Cas. It is given in terms of the highest weights by

$$\begin{aligned} \mathrm{Cas}\big([j,l]\big) &= -j(j+1) + l(l+1) \\ \mathrm{Cas}\big([j]\big) &= 0. \end{aligned} \tag{6}$$

Note that its value in atypical representations is given by evaluating the Casimir for typicals on weights which satisfy the shortening condition $j = l$.

Let us conclude with a few scattered comments on the notation we are about to use. As we mentioned above, atypical irreducibles can combine to form complicated indecomposables. We will not concern ourselves with this indecomposable structure of the spectrum and simply look at the constituent irreducible representations. For this reason, we shall not use the symbol $\oplus$ in our formulas but simply write $+$ instead. Many of the sums of representations we are about to see are in fact not direct. Since traces are blind to the indecomposable structures, our formulas for representations encode true identities among their characters χ_Λ in which $+$ and tensor products are ordinary sums and products of characters.

4 Absence of relevant high-gradient operators

The spectrum of WZNW models on type I supergroups is quite well understood, see [10]. Almost all of these models give rise to logarithmic conformal field theories, see also [11, 12], and hence their Hamiltonian (generator of dilations) is not diagonalizable. In our analysis of the spectrum we shall only be concerned with the generalized eigenvalues of the dilation operator. This information is encoded in the partition function of the WZNW model. The latter decomposes into a sum of products of characters for representations of the left- and right moving chiral algebra. This does not mean that these models experience holomorphic factorization – they do not. But the trace we take when we compute the partition function cannot see the intricate coupling between left and right movers.

The representations of the affine $\widehat{\mathfrak{psl}}(2|2)_k$ algebra along with their characters have been worked out for arbitrary level k in [9]. When $k = 1$, the theory contains a single sector which is based on the vacuum representation of the current algebra. Using the results of [9] one can obtain the branching functions for the decomposition of the affine modules into irreducible representations of the zero-mode subalgebra $\mathfrak{psl}(2|2)$. In case of the vacuum representation of the affine $\mathfrak{psl}(2|2)$ at level $k = 1$ the branching functions into representations (j,l) of the even subalgebra $\mathfrak{g}^{(0)}$ read

$$\psi^{(0)}_{(j,l)} = \frac{q^{\frac{1}{12}}}{\phi(q)^4} \sum_{s\in\mathbb{Z}} \sum_{m,n=0}^{\infty} (-1)^{m+n}\, q^{\frac{m(m+1)+n(n+1)}{2}+s(s+m-n)-j(m+n+1)} \times \left(1-q^{-(m+n+1)}\right)\left(1-q^{2l+1}\right) q^{l^2}, \tag{7}$$

where $j,l\in\mathbb{Z}$ and with $s\to s+1/2$ for $j,l\in\mathbb{Z}+1/2$. From these formulas one can determine the branching functions into represention of the superalgebra $\mathfrak{g}$ with the help of eqs. (4) and (5). For the first few levels, the resulting decomposition of the vacuum character $\hat{\chi}_0$ reads

$$\begin{aligned}\hat{\chi}_0(q,x,y) = q^{\frac{1}{12}}\big(&q^0\chi_{[0]} + q^1\chi_{[\frac{1}{2}]} + q^2(\chi_{[1,0]} + \chi_{[\frac{1}{2}]} + \chi_{[0]}) \\ &+ q^3(\chi_{[2,0]} + \chi_{[1,0]} + 2\chi_{[1]} + 3\chi_{[\frac{1}{2}]} + 4\chi_{[0]}) \\ &+ q^4(\chi_{[3,0]} + \chi_{[2,0]} + 3\chi_{[1,0]} + \chi_{[0,1]} + 2\chi_{[\frac{3}{2},\frac{1}{2}]} \\ &+ 2\chi_{[1]} + 4\chi_{[\frac{1}{2}]} + 5\chi_{[0]})\big) + \mathcal{O}(q^5)\ .\end{aligned} \tag{8}$$

Here, we expanded the vacuum character of the affine $\mathfrak{psl}(2|2)$ at level $k=1$ into characters $\chi_\Lambda = \chi_\Lambda(x,y)$ of the zero mode algebra $\mathfrak{psl}(2|2)$. The arguments x,y keep track of the $\mathfrak{psl}(2|2)$ weights while q is associated with the eigenvalues of L_0, i.e. with the conformal weight h, as usual. In the partition function, $\hat{\chi}_0$ gets multiplied with an identical contribution from the anti-holomorphic sector, only that we need to replace q by $\bar{q}$.

From (7) it follows that the smallest conformal weight at which a bosonic module $(j,l)\neq(0,0)$ appears is given by

$$h^{\min}_{g=0}(j,l) = \begin{cases} j+l^2 & j,l\in\mathbb{Z} \\ j+l^2+\dfrac{1}{4} & j,l\in\mathbb{Z}+\frac{1}{2}. \end{cases} \tag{9}$$

In addition we note that modules with $j,l\in\mathbb{Z}+\frac{1}{2}$ always appear with multiplicity two at their lowest weight. From the decomposition (5) of typical irreducible modules we can now deduce that the minimal weight $h^{\min}_{g=0}([j,l])$ of a module $[j,l]$, $j\neq l$, is given by the minimal weight $h^{\min}_{g=0}(j,l+1)$ of the bosonic module $(j, l+1)$,

$$h_{g=0}^{\min}\big([j,l]\big) = \begin{cases} j + (l+1)^2 & j,l \in \mathbb{Z} \\ j + (l+1)^2 + \dfrac{1}{4} & j,l \in \mathbb{Z} + \frac{1}{2}, \end{cases} \tag{10}$$

for typical $[j, l]$. With the help of the decomposition (4) one can find a similar result for atypical representations,

$$h_{g=0}^{\min}\big([j]\big) = \begin{cases} j^2 + 2j & j \in \mathbb{Z} \\ j^2 + 2j - \dfrac{1}{4} & j \in \mathbb{Z} + \frac{1}{2}. \end{cases} \tag{11}$$

Given the values (6) of the quadratic Casimir it is clear that if we take $g \leq 0$, high-gradient operators become relevant for arbitrarily small values of the coupling, since their engineering dimension grows linearly in j, while the anomalous dimension grows like $-j^2$. So this direction of the perturbation cannot lead to a stable theory.

Let us therefore turn to the case $g \geq 0$. From [9] we know that operators that are invariant under the diagonal action of $\mathfrak{psl}(2|2)$ must transform in the same representation $\Lambda_L = \Lambda_R$ with respect to the left and right action. Eq. (3) implies that the only invariant operators that become more relevant as we increase the coupling g must sit in multiplets $\Lambda_L = [j, l] = \Lambda_R$ with $l > j$. Among those, the lowest lying ones at $g = 0$, namely those with $j = 0$, are also those that receive the largest correction to their conformal weights. From eq. (3), the anomalous dimension $\delta_g\big([0, l]\big)$ of invariant operators with $\Lambda_L = [0, l] = \Lambda_R$ is given by

$$\delta_g\big([0,l]\big) = -\frac{g}{1+g} l(l+1). \tag{12}$$

Comparing with eq. (10), we infer that these operators remain irrelevant for all finite values of the coupling. In conclusion, the models with $g \geq 0$ actually contain no RG-relevant invariant operators. Thereby, we have extended the 1-loop result of [2] to all loops and all invariant operators.

5 The spectrum at infinite coupling

The limiting point $g = \infty$ is obviously of special interest. Let us therefore describe its spectrum in some more detail. From the above discussion we can conclude that there are no relevant invariant operators in the spectrum for any positive value of the coupling. Moreover, we see that as $g \to \infty$ the spectrum of operators in atypical ($\frac{1}{2}$BPS) representations under the diagonal action of $\mathfrak{psl}(2|2)$ is half-integer valued, i.e.

$$h_g := h_{g=0} + \delta_g \quad \text{satisfies} \quad h_\infty = \lim_{g\to\infty} h_g \in \tfrac{1}{2}\mathbb{Z}\ . \tag{13}$$

The multiplicities of $\frac{1}{2}$BPS states at any total conformal weight $\Delta = h + \bar{h}$ remain finite as the coupling g tends to infinity, as can be seen with the help of eq. (10) together with eq. (3). For $j_1, j_2 \in \mathbb{Z}$ we find

$$\Delta_\infty^{\min} = h_\infty^{\min}\big([j_1, l_1]\big) + \bar{h}_\infty^{\min}\big([j_2, l_2]\big) = j_1^2 + 2j_1 + j_2^2 + 2j_2 + l_1 + l_2 + 2. \tag{14}$$

When either j_1 or j_2 are half-integer, $\frac{1}{4}$ gets added to the above formula. If they are both half-integer, we must add $\frac{1}{2}$. Since all the labels are non-negetive, the total energy grows strictly monotonically in them. Therefore, multiplicities of $\frac{1}{2}$BPS states remain finite for any given value of Δ_∞. Moreover, Δ_∞ remains non-negative and the only state that goes to $\Delta_\infty = 0$ is the ground state of the WZNW model.

We will now describe the spectrum at $g = \infty$ up to $\Delta_\infty = 5$. The analysis is organized according to the right moving conformal weight $\bar{h}_\infty$, i.e. we shall start by listing all the $\frac{1}{2}$BPS states that possess $\bar{h}_\infty = 0$, i.e. the chiral states of the Gross-Neveu model at strong coupling $g = \infty$. Obviously, all chiral $\frac{1}{2}$BPS states of the WZNW model, that is those that are built with the right moving vacuum state and hence have weights $(h_0, 0)$, do not acquire an anomalous contribution to their conformal weights. Hence, chiral states of the WZNW model give states with $(h_\infty = h_0, \bar{h}_\infty = 0)$. That does not mean, however, that the chiral $\frac{1}{2}$BPS spectrum at $g = \infty$ is the same as it is at $g = 0$. Indeed, starting from $h_\infty = 3$ we see new chiral states appearing.

The first ones originate from an operator multiplet at $\big(h_0, \bar{h}_0\big) = (4,1)$ that transforms in the representation $[0,1]^L \otimes [\frac{1}{2}]^R$ in the WZNW model. Under the diagonal action D, this product decomposes into

$$[0,1] \otimes [\tfrac{1}{2}] = 6[0] + 6[\tfrac{1}{2}] + 4[1] + [\tfrac{3}{2}] + \text{typicals}. \tag{15}$$

Hence, this multiplet of the WZNW model contributes plenty of chiral fields at strong coupling. At $h_\infty = 4$ we only need to account for the holomorphic derivative of this operator. For $h_\infty = 5$, finally, there exist three multiplets in the representation $[0,1]^L \otimes [\frac{1}{2}]^R$. Additionally, we obtain a contribution from a multiplet that transforms in $[0,2]^L \otimes [0,1]^R$. Its $\frac{1}{2}$BPS content in the decomposition with respect to the diagonal action is the same as for the previous operator. Summing everything up, the chiral spectrum to this level is given by

$$\begin{array}{ll} h_\infty = 0 & [0] \\ h_\infty = 1 & [\tfrac{1}{2}] \\ h_\infty = 2 & [0] + [\tfrac{1}{2}] \\ h_\infty = 3 & 10[0] + 9[\tfrac{1}{2}] + 6[1] + [\tfrac{3}{2}] \\ h_\infty = 4 & 11[0] + 10[\tfrac{1}{2}] + 6[1] + [\tfrac{3}{2}] \\ h_\infty = 5 & 38[0] + 37[\tfrac{1}{2}] + 24[1] + 5[\tfrac{3}{2}] \ . \end{array} \tag{16}$$

The analysis for the next cases with $\bar{h}_\infty > 0$ proceeds along the same lines. For $\bar{h}_\infty = 1$ one finds,

$$\begin{array}{ll} h_\infty = 1 & 4[0] + 2[\tfrac{1}{2}] + 2[1] \\ h_\infty = 2 & 4[0] + 3[\tfrac{1}{2}] + 2[1] \\ h_\infty = 3 & 18[0] + 20[\tfrac{1}{2}] + 14[1] + 5[\tfrac{3}{2}] \\ h_\infty = 4 & 22[0] + 23[\tfrac{1}{2}] + 16[1] + 5[\tfrac{3}{2}] \ . \end{array} \tag{17}$$

Similarly, the results for $\bar{h}_\infty = 2$ read

$$\begin{array}{ll} h_\infty = 2 & 19[0] + 16[\tfrac{1}{2}] + 8[1] + [\tfrac{3}{2}] \\ h_\infty = 3 & 58[0] + 61[\tfrac{1}{2}] + 46[1] + 17[\tfrac{3}{2}] + 2[2]. \end{array} \tag{18}$$

For higher values of $\bar{h}_\infty \leq 5$ the multiplicities of $\frac{1}{2}$BPS multiplicities in the $g = \infty$ Gross-Neveu model can be inferred from the list we provided, exploiting that the spectrum is certainly symmetric under the exchange of left- and right movers.

There exist actually a few more states at $\Delta_\infty = 5$ that we have not listed yet. In fact, $\Delta_\infty = 5$ marks the first level at which states with negative left moving weight $h_\infty < 0$ appear in the spectrum. At the same time, $\Delta_\infty = 5$ is also the lowest value of the scaling weight at which half-integer conformal weights $(h_\infty, \bar{h}_\infty)$ are actually observed. The additional states are generated by two WZNW operators that transform in the representation $[\frac{1}{2}]^L \otimes [\frac{1}{2}, \frac{3}{2}]^R$. In this case, the decomposition of the diagonal action can be worked out to give

$$2[\tfrac{1}{2}] \otimes [\tfrac{1}{2}, \tfrac{3}{2}] = 4[0] + 8[\tfrac{1}{2}] + 12[1] + 8[\tfrac{3}{2}] + 2[2] + \text{typicals}. \tag{19}$$

Note that we multiplied the left hand side by a factor 2 so that the left hand side accounts for all operators that possess weights $\big(h_\infty, \bar{h}_\infty\big) = \big(-\frac{1}{2}, \frac{11}{2}\big)$ at $g = \infty$. Of course, the spectrum is symmetric under the exchange of the holomorphic and anti-holomorphic sectors so that the same content appears with $\big(h_\infty, \bar{h}_\infty\big) = \big(\frac{11}{2}, -\frac{1}{2}\big)$.

6 Conclusions and open problems

Using exact results on the anomalous dimensions of operators in perturbed WZNW models, we were able to show analytically that the $\mathfrak{psl}(2|2)$ WZNW model does not contain RG-relevant high-gradient operators, confirming the findings of [2] and extending them to all orders in the coupling and all invariant operators of the model. This shows that the $\mathfrak{psl}(N|N)$ WZNW models, at least for $N = 2$, take a special role among models with target-space supergroup symmetry. There is only one other case, namely that of the boundary $\mathfrak{osp}(4|2)$ Gross-Neveu model, in which similar stability statements have been established [13], at least against boundary perturbations. Bulk perturbations, on the other hand, were recently seen to produce strongly RG-relevant operators, much in the same way as for other sigma models [14].

We also observed that, as the coupling g tends to plus infinity, the spectrum becomes half-integer valued, indicating that the theory could possess a free-field description. For the boundary $\mathfrak{osp}(4|2)$ Gross-Neveu model, a similar study has been performed and the resulting spectrum has been identified with a boundary spectrum of the free sigma model on the supersphere $S^{3|2}$, see [13]. It has been argued several times before that the $\mathfrak{psl}(2|2)$ Gross-Neveu model should be dual to the sigma model on $\mathbb{CP}^{1|2}$ [5]. At zero sigma model coupling (infinite radius), the spectrum of boundary operators in that model has been worked out in [15]. It is not difficult to extend that analysis to the bulk, but unfortunately, the resulting spectrum bears no resemblance with what we saw in the previous section. In any case, it would be very interesting to identify a free field theory that gives rise to the spectrum of the $\mathfrak{psl}(2|2)$ model at $g = \infty$ and possibly to understand its precise relation to the $\mathbb{CP}^{1|2}$ model.

Acknowledgments

The authors wish to thank Constantin Candu, Vladimir Mitev, Andreas Ludwig, Christopher Mudry, Thomas Quella and Hubert Saleur for comments and interesting discussions. The research leading to these results has received funding from the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme FP7/2007-2013/ under REA Grant Agreement No 317089 (GATIS).

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