Form factors for $B_s \to K\ell\nu$ decays in Lattice QCD

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We present the current status of the computation of the form factor $f_+(q^2)$ for the semi-leptonic decay $B_s \to K\ell\nu$ by the ALPHA collaboration. We use gauge configurations which were generated as part of the Coordinated Lattice Simulations (CLS) effort. They have $N_f = 2$ non-perturbatively $O(a)$ improved Wilson fermions, and pion masses down to $\approx 250\,\mathrm{MeV}$ with $m_\pi L \geq 4$. The heavy quark is treated in non-perturbative Heavy Quark Effective Theory (HQET).

We discuss how to extract the form factors from the correlation functions and present first results for the form factor at $q^2 = 21.23\,\mathrm{GeV}^2$ extrapolated to the continuum. Next-to-leading order terms in HQET and the chiral extrapolation still need to be included in the analysis.

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1 Introduction

Determinations of the CKM matrix element $|V_{ub}|$ from different exclusive (and inclusive) decays tend to disagree at the $\sim 2 - 3\sigma$ level [1]. Both theoretical and experimental improvements are needed to clarify the situation.

In this work, we report on our ongoing effort to non-perturbatively determine the form factors for $B_s \to K\ell\nu$ decays from Lattice Quantum Chromodynamics (LQCD) with $N_f = 2$ sea quarks. Although no experimental data is available yet for this decay, the heavier spectator s-quark renders the LQCD computations technically simpler than for $B \to \pi\ell\nu$, and thus provides a good starting point to gain solid control on all systematic errors (and to make an LQCD prediction).

The decay amplitude for $B_s \to K\ell\nu$ is proportional to $|V_{ub}|$ times the hadronic matrix element $\langle K(p_K)|V_\mu|B_s(p_{B_s})\rangle$ of the vector current $V_\mu(x) = \overline{\psi}_u(x)\gamma_\mu\psi_b(x)$. The matrix element is parametrised by two form factors, $f_0(q^2)$ and $f_+(q^2)$, which depend on $q^2 = (p_{B_s} - p_K)^2$, the invariant mass of the lepton pair. In the limit of vanishing lepton masses only $f_+(q^2)$ contributes to the decay rate.

$|V_{ub}|$ can then be determined by combining the differential decay rate from experiment with $f_+(q^2)$ from theory. In principle, it is sufficient to do this at a single value of q^2. In practice, experimental data is provided over a range (of bins) of q^2, and one can use the BCL paramterisation [2] to express the form factor $f_+(q^2)$ as a continuous function of q^2. Then, a theoretical prediction of $f_+(q^2)$, e.g. from LQCD, for at least a single q^2 allows to extract $|V_{ub}|$.

Here we report on preliminary work to study the feasibility of a precise determination of the form factor in the continuum limit and at a fixed q^2.

2 HQET on the lattice

On the lattice (with spatial extent L and lattice spacing a) the large mass of the b quark gives rise to a hierarchy of scales

$$L^{-1} \ll m_\pi \approx 140\,\text{MeV} \ll m_B \approx 5\,\text{GeV} \ll a^{-1}, \tag{1}$$

which cannot be directly simulated with present computing resources. Instead, we follow the strategy devised by the ALPHA collaboration [3] to treat the heavy quark within the framework of non-perturbative Lattice HQET. It is an expansion in inverse powers of the heavy quark mass m_h and valid for kaon momenta $p_K \ll m_h$. In practice, we require $p_K \lesssim 1\,\text{GeV}$. A key feature of Lattice HQET is that it is (believed to be) non-perturbatively renormalisable order by order in $1/m_h$, and thus the computations have a well-defined continuum limit. The expectation value of a product of local fields, $\mathcal{O}$, up to and including $O(1/m_h)$ in HQET on the lattice is

$$\langle\mathcal{O}\rangle = \langle\mathcal{O}\rangle_{\text{stat}} + \omega_{\text{kin}}a^4\sum_x \langle\mathcal{O}\mathcal{O}_{\text{kin}}(x)\rangle_{\text{stat}} + \omega_{\text{spin}}a^4\sum_x \langle\mathcal{O}\mathcal{O}_{\text{spin}}(x)\rangle_{\text{stat}}, \tag{2}$$

where $\langle\ldots\rangle_{\text{stat}}$ is the expectation value in the static approximation. On the right hand side, also $\mathcal{O}$ needs to be expanded in $1/m_{\text{h}}$, for instance,

$$\text{V}_k^{\text{HQET}}(x) = Z_{V_k}^{\text{HQET}}\left[\text{V}_k^{\text{stat}}(x) + \sum_{j=1}^{4} c_{\text{V}_{k,j}}\text{V}_{k,j}(x)\right], \tag{3}$$

for the spatial components of the vector current, and analogous for V_0. The HQET parameters ω_{kin}, ω_{spin}, and $c_{\text{V}_{\mu,j}}$ are of order $1/m_{\text{h}}$, while $Z_{V_0}^{\text{HQET}}$ and $Z_{V_k}^{\text{HQET}}$ are of order 1. They can be determined fully non-perturbatively by matching HQET and QCD [4]. Thus, perturbation theory can be avoided at any stage of the computation.

Since the non-perturbative matching is still in progress, we present in this exploratory work only results in the static approximation, i.e. setting $\omega_{\text{kin}} = \omega_{\text{spin}} = c_{\text{V}} = 0$. For the renormalisation constants we follow the lines of [5, 6] and write Z^{HQET} as a product of matching factors, C_{PS} or C_{V}, which are known at three loops in perturbation theory [7], and $Z_{\text{A,RGI}}^{\text{stat}}$ which is known non-perturbatively [8]. Truncation of the lattice theory at static order is expected to be a 10-20% effect.

To perform the continuum extrapolation of the form factors at a fixed value of q^2, we employ flavour twisted boundary conditions [9] for the s quark, $\psi(x + L\hat{k}) = e^{i\theta_k}\psi(x)$. In this way, the quark momentum is altered from $\vec{p} = 2\pi\vec{n}/L$ to $\vec{p}^\theta = (2\pi\vec{n} + \vec{\theta})/L$, with $\vec{n} \in \mathbb{N}^3$. Choosing the twist angle θ_k, one can freely tune the momentum of the s quark, and thus of the kaon. The heavy quark is twisted by the same angle to remain in the rest frame of the B_{s} meson.

Our computations are performed on gauge field ensembles generated with $N_{\text{f}} = 2$ dynamical sea quarks within the CLS effort. They use non-perturbatively $O(a)$-improved Wilson fermions and the scale is set via f_{K} [10]. All ensembles have $m_\pi L \gtrsim 4$. In this work, we present results from measurements on the three CLS ensembles A5, F6 and N6. Their properties are listed in ref. [10]. Error estimates take into account correlations and autocorrelations [11].

3 Computation of the form factor

On the gauge configurations we measure the two- and three-point correlation functions

$$\mathcal{C}^{\text{K}}(x^0 - y^0; \vec{p}) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}\langle P^{\text{su}}(x)P^{\text{us}}(y)\rangle, \quad \mathcal{C}_{ij}^{\text{B}}(x^0 - y^0; \vec{0}) = \sum_{\vec{x},\vec{y}}\langle P_i^{\text{sb}}(x)P_j^{\text{bs}}(y)\rangle,$$

$$\mathcal{C}_{\mu,j}^3(t_{\text{K}}, t_{\text{B}}; \vec{p}) = \sum_{\vec{x}_{\text{K}},\vec{x}_V,\vec{x}_{\text{B}}} e^{-i\vec{p}\cdot(\vec{x}_{\text{K}}-\vec{x}_V)}\langle P^{\text{su}}(x_{\text{K}})\text{V}_\mu(x_V)P_j^{\text{bs}}(x_{\text{B}})\rangle, \tag{4}$$

where $P_i^{\text{q}_1\text{q}_2}(x)$ are interpolating fields, like $\overline{\psi}_{\text{q}_1}(x)\gamma_5\psi_{\text{q}_2}(x)$, for the mesons. The subscripts i or j indicate different levels of Gaussian smearing [12] of the s quark

in the heavy-light meson, i.e. different trial wave functions. In the limit of large Euclidean times, $t_B \equiv x_B^0 - x_V^0$ and $t_K \equiv x_V^0 - x_K^0$, the ratio

$$f_{\mu,i}^{\mathrm{ratio}}(t_B, t_K; q^2) \equiv \frac{\mathcal{C}_{\mu,i}^3(t_K, t_B)}{\sqrt{\mathcal{C}^K(t_K)\mathcal{C}_{ii}^B(t_B)}} \, e^{E_K t_K/2} e^{E_B t_B/2} \tag{5}$$

will then give the desired matrix element (for any suitable smearing i)

$$\langle K(p_K^\theta)|V_\mu|B_s(0)\rangle = \lim_{T, t_B, t_K \to \infty} f_{\mu,i}^{\mathrm{ratio}}(t_B, t_K; q^2) \tag{6}$$

Alternatively, we can parameterise the correlation functions as

$$\mathcal{C}^K(t_K) = \sum_m (\kappa^{(m)})^2 e^{-E_K^{(m)} t_K}, \quad \mathcal{C}_{ij}^B(t_B) = \sum_n \beta_i^{(n)} \beta_j^{(n)} e^{-E_B^{(n)} t_B}, \tag{7}$$

$$\mathcal{C}_{\mu,i}^3(t_B, t_K) = \sum_{m,n} \kappa^{(m)} \varphi_\mu^{(m,n)} \beta_i^{(n)} e^{-E_B^{(n)} t_B} e^{-E_K^{(m)} t_K}, \tag{8}$$

and determine $\{\kappa^{(m)}, E_K^{(m)}\}$ from a fit to $\mathcal{C}^K$, and $\{\beta_i^{(n)}, \varphi_\mu^{(n,m)}, E_B^{(n)}\}$ from a combined fit to $\mathcal{C}_{\mu i}^3$ and $\mathcal{C}_{ij}^B$. Then, $\varphi_\mu^{(1,1)}$ is equal to the matrix element of eq. (6). We take $m = 1$ and $n = 1, 2$, i.e. we include the first excited B_s state, but only the kaon ground state.

In fig. 1 we show the ratio f_μ^{ratio} of eq. (5) at fixed $t_K = 20$ as a function of t_B. For comparison, we also indicate the value of $\varphi_\mu^{(1,1)}$ resulting from the fit.
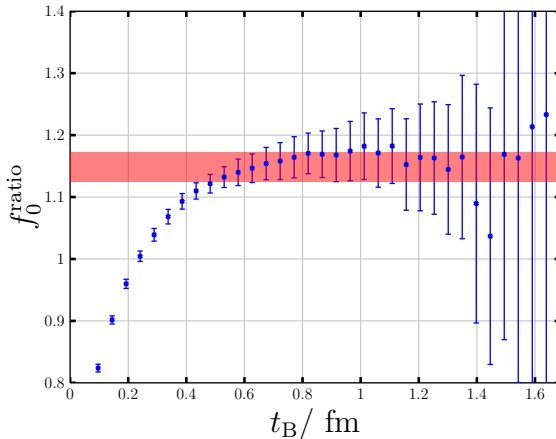
In the rest frame of the B_s meson, the matrix elements have the form

$$\langle K|V_0|B_s\rangle = \sqrt{2m_{B_s}} \, f_\|(q^2),$$
$$\langle K|V_i|B_s\rangle = \sqrt{2m_{B_s}} \, p_K^i \, f_\perp(q^2),$$

where the form factors $(f_\|, f_\perp)$ are related to (f_+, f_0). In particular, we have

$$f_+ = \frac{1}{\sqrt{2m_{B_s}}} f_\| + \frac{1}{\sqrt{2m_{B_s}}}(m_{B_s} - E_K) f_\perp. \tag{9}$$

Figure 1: The ratio f_μ^{ratio} (blue points) and the fit result $\varphi_\mu^{(1,1)}$ (red band) for lattice N6, $\mu = 0$ and fixed $t_K = 20$.

Fig. 2 shows f_+, as extracted from the fitted $\varphi_\mu^{(1,1)}$, for different lattice spacings. Working in the static approximation of HQET, we are free to keep or drop terms of order $1/m_h$ in eq. (9) for computing f_+. To illustrate this $O(1/m_h)$ ambiguity, we

show in fig. 2 (and 3) two sets of data points: the upper one corresponds to using all terms in eq. (9), the lower one to dropping the term proportional to $f_{\parallel}$. Once we include all $O(1/m_{\mathrm{h}})$ terms of HQET, this ambiguity will disappear. For both sets we show a constant continuum extrapolation and one linear in a^2. The latter has by far the larger error and within this error is consistent with the result of the constant extrapolation.

In fig. 3, we compare our results from the linear continuum extrapolation of $f_+(q^2)$ to recent results of HPQCD [13] (at their smallest $a = 0.09\,\mathrm{fm}$ and $m_\pi = 320\,\mathrm{MeV}$).
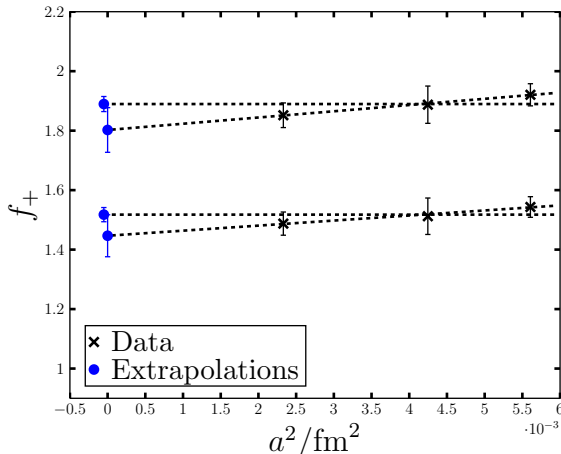

Figure 2: Continuum extrapolation of our data for f_+ at $q^2 = 21.23\,\mathrm{GeV}^2$.
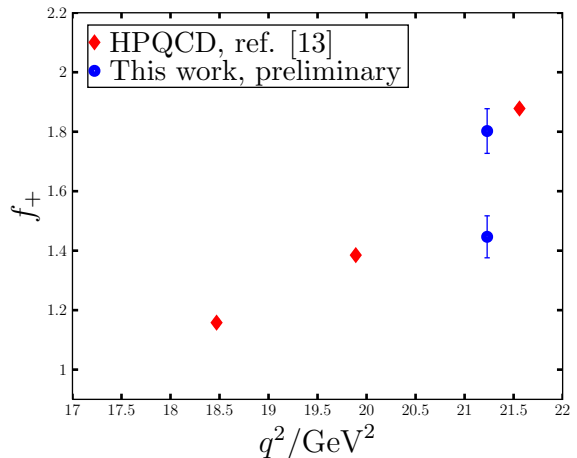
Figure 3: Comparison of LQCD results at various values of q^2.

4   Conclusion

We presented the current status of our computation of the form factor $f_+(q^2)$ for the semi-leptonic decay $\mathrm{B_s} \to \mathrm{K}\ell\nu$ at a fixed value of $q^2 = 21.23\,\mathrm{GeV}^2$ using HQET on the lattice. We compare two different methods to extract the form factors, either from the plateau value of a suitable ratio of correlators, or from a simultaneous fit to the functional form of the correlators.

We also have performed a continuum extrapolation of our lattice data and find small $O(a^2)$ effects. The preliminary results reported here are still computed in the static approximation and an extrapolation to the physical pion mass has yet to be performed. Our preliminary value of f_+ at this stage is in rough agreement with the results from other collaborations.

All $O(1/m_{\mathrm{h}})$ effects of HQET will be included in the analysis once the HQET parameters are known non-perturbatively. We also plan to extend the computation to $\mathrm{B} \to \pi\ell\nu$ decays, several values of q^2, and $N_{\mathrm{f}} = 2 + 1$ flavours of sea quarks.

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References

[1] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).

[2] C. Bourrely, I. Caprini and L. Lellouch, Phys. Rev. D **79**, 013008 (2009) [Erratum-ibid. D **82**, 099902 (2010)] [arXiv:0807.2722 [hep-ph]].

[3] J. Heitger and R. Sommer [ALPHA Collaboration], JHEP **0402**, 022 (2004) [hep-lat/0310035].

[4] B. Blossier *et al.* [ALPHA Collaboration], JHEP **1209** (2012) 132 [arXiv:1203.6516 [hep-lat]].

[5] R. Sommer, *Introduction to Non-perturbative Heavy Quark Effective Theory*, published in *Les Houches Summer School in Theoretical Physics, Modern perspectives in lattice QCD, Les Houches, France, 3 – 28 August 2009, Oxford Univ. Pr.* (2010) 517.

[6] J. Heitger *et al.* [ALPHA Collaboration], JHEP **0411**, 048 (2004) [hep-ph/0407227].

[7] K. G. Chetyrkin and A. G. Grozin, Nucl. Phys. B **666**, 289 (2003) [hep-ph/0303113].

[8] M. Della Morte, P. Fritzsch and J. Heitger, JHEP **0702**, 079 (2007) [hep-lat/0611036].

[9] P. F. Bedaque, Phys. Lett. B **593**, 82 (2004) [nucl-th/0402051].

[10] P. Fritzsch *et al.*, Nucl. Phys. B **865**, 397 (2012) [arXiv:1205.5380 [hep-lat]].

[11] S. Schaefer *et al.* [ALPHA Collaboration], Nucl. Phys. B **845**, 93 (2011) [arXiv:1009.5228 [hep-lat]].

[12] S. Güsken, U. Löw, K. H. Mütter, R. Sommer, A. Patel and K. Schilling, Phys. Lett. B **227**, 266 (1989).

[13] C. M. Bouchard, G. P. Lepage, C. Monahan, H. Na and J. Shigemitsu, arXiv:1406.2279 [hep-lat].