Looking for dark matter on the light side DESY 15-003

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Abstract. Among the prominent low-mass dark matter candidates is the QCD axion but also other light and weakly interacting particles beyond the Standard Model. We review briefly the case for such dark matter and give an overview on most recent experimental efforts within laboratory searches, where we focus on experiments exploiting a potential electromagnetic coupling of such particles.

1. Three ultra-light dark matter candidates

It would be a huge break-through to find out what dark matter (DM) is made of. Whilst its constituents could be rather heavy and, e.g., leave an 'imprint' at the LHC, show up in astrophysics signatures or reveal themselves through recoil energy at direct detection setups [1], there are well-motivated candidates also on the 'light side'. Our main concern in this article are henceforth DM candidates below the eV mass-scale.

Of such ultra-light particles, the QCD axion is the most prominent dark matter candidate, see, e.g., [2] for a review. On the theoretical side, the axion is a pseudo-scalar pseudo-Goldstone boson that is a consequence of the Peccei-Quinn solution [3] to the strong CP problem. The strong CP problem amounts to the question why CP violation in QCD is unmeasurably small (or even absent). The effective parameter for CP violation receives contributions from the θ -angle of QCD, being essentially unconstrained a priori, and the quark mass matrix. As these parameters are unrelated from the outset, it arises the question for a natural explanation on why the CP-violating parameter is so close to zero. In essence, the axion solution to the strong CP-problem makes the parameter a dynamical variable which naturally relaxes to zero.

From an experimental viewpoint, it is most interesting that axions - in certain parameter regimes - constitute a perfect candidate to make up the cold dark matter (CDM) in our universe. Although the axion is very light, it can be non-thermally produced in the early universe [4, 5].

A particularly attractive feature of axion dark matter is that its viable parameter range is comparatively small and it is thus a realistic aim to confirm or exclude axions as main dark matter component with current and near-future technology. For axions as cold DM, two natural cosmological windows exist, see, e.g., [6]: In the post-inflation scenario, the spontaneous breaking of the Peccei-Quinn symmetry at a scale f, which gives rise to the axion as its pseudo Goldstone boson, takes place only after inflation. In the other scenario, the Peccei-Quinn phase transition happens before inflation. The former scenario is typically related to axions with higher masses than the latter because the decay of axionic topological defects (absent in the latter) also produces DM axions and generically the DM abundance grows with the decay constant (i.e. decreases with the axion mass).

The situation for the axion is sketched in Fig. 1 where performed experiments and the foreseen reach of some experiments that are planned for current and near future are shown. In addition, some astrophysical bounds are shown, for more insight on this matter see, e.g. [7, 8]. In Fig. 1, for the black line labeled 'axion' we have implemented $g = \frac{\alpha}{2\pi f}$ as relation of the axion-to-photon coupling and the symmetry breaking scale. For specific axion models, a $\mathcal{O}(1)$ factor enters in the relation between g and f, cf. [8]. Axions are a good cold dark matter candidate roughly between masses of $10^{-6} \mathrm{eV}$ and $10^{-3} \mathrm{eV}$ [9, 10]. Below $10^{-6} \mathrm{eV}$, axions tend to produce too much DM and they are disfavored, although some models can justify the selection of suitable initial conditions. Note that at very low masses, there might be also effects on black hole dynamics [11]. In laboratory searches, to probe the very tiny couplings that correspond to high values of the axion symmetry breaking scale, resonant search strategies are mostly employed and will be briefly sketched in Sect. 2.1

The axion is very light and very weakly interacting. Thus it exhibits the properties of a more general class of particles dubbed 'Weakly Interacting Slim Particles' (WISPs) [12, 13, 14]. Among the WISPs that could be cold dark matter [15] are also axion-like particles (ALPs). Such general ultralight pseudo-scalars could be pseudo-Nambu Goldstone bosons (pNGB) associated with a symmetry breaking scale different from the Peccei-Quinn scale [16, 17]. Effectively they can have a coupling to photons similarly to the QCD axion, but a 'relaxed' mass-coupling relation (i.e., they are not confined to the parameter region around the axion line in Fig. 1). Their existence is motivated in Standard Model extensions [12], and they have been also evoked to explain some astrophysical puzzles such as the observed transparency of the universe to high-energetic photons, see, e.g., [18].

Similar to axions, ALP CDM [15] can be realized through a misalignment mechanism: The Axion mass m is strongly temperature dependent (same might apply to ALPs as pNGBs). For axions above the symmetry breaking scale (in the early universe), the axion is essentially massless. Thus at very high energies, the initial angle is not fixed (for the axion it needs not to be at its CP conserving value), i.e., it can be mis-aligned from its minimum. The equation of motion for the field in the expanding universe is that of a damped harmonic oscillator (where 3H quantifies the damping term and H is the Hubble expansion parameter) and initially the field is frozen when $3H \gg m$. At later times t_1 , when the particle mass $m_1 = 3H$, the field starts to oscillate and behaves as cold dark matter fluid. The situation for ALP DM is reviewed in part through the dotted black lines in Fig. 1: The upper line, labeled $m_1 > 3H(T_{eq})$ is an upper bound for any such ALP DM model: The mass m_1 at the which the oscillations start should be attained latest at matter-radiation equality. The lower black dotted line labeled $m_1 = m_0$ denotes 'Standard ALP DM' and is the simplest ALP DM model in which m_1 is the same as the ALP mass today (m_0) . Models in which $m_1 \ll m_0$ can in principle create a sufficient DM abundance at slightly higher coupling values [15], but most model-building efforts favor ALP DM within roughly an order of magnitude above this line.

Beyond ALPs, hidden photons (HPs, reviewed, e.g., in [19]), which are hidden sector U(1)s coupled kinetically to the photons of the visible sector, are WISPy cold dark matter candidates [20, 15]. Such particles would - with a hidden Higgs or Stückelberg generated mass term - be manifest in photon hidden-photon oscillations similar to what is observed with neutrinos. Fig. 2 shows the viable parameter space for HP DM according to [15], in which the orange region labeled 'Xenon' denotes limits inferred from the XENON10 experiment [21] (see also [22] for novel bounds on the longitudinal HP component).

In summary: For both ALPs and hidden photons the parameter regime in which they can constitute DM is much larger than for QCD axions. Thus, in laboratory searches, also non-resonant techniques have become attractive as they allow faster scanning (albeit at reduced overall sensitivity). This is discussed in Sect 2.2. In the following we give a brief overview on laboratory searches for axion and WISP DM. We focus on searches exploiting coupling to

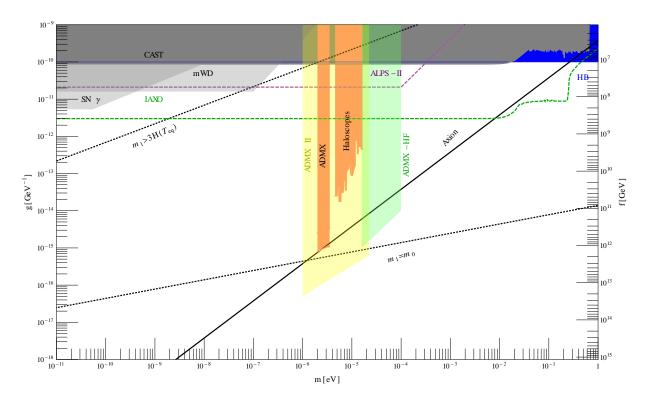


Figure 1. Parameter space of axion and axion-like particle DM (photon-coupling) in the sub-eV range. See text for details.

light. Recent reviews on ultra-light particle dark matter with a more theory-based focus include [23, 17].

2. Laboratory setups looking for ultra-light dark matter

It might be helpful to first categorize experiments that are sensitive to light & weakly interacting particles of the above described types in the following manner:

Firstly, there are setups which could find such particles outside the range in which they likely constitute dark matter in the most immediate models. This is because these setups cannot probe small enough couplings. Two such examples are first generation light-shining-through-a-wall (LSW) experiments [24] or polarimetric setups [25] (the latter search for WISPs as by-product on their way to measure birefringence of the QED vacuum).

Secondly, there are experiments which could find these particles inside the range in which they could be cold dark matter, however independent of the fact whether they actually are dark matter (indirect searches). An example is second-generation LSW like ALPS-II [26]. Another example are Helioscopes, that aim to convert axions or the like emitted from our sun into a detectable X-ray signal, such as with the Cern Axion Solar Telescope (CAST) ([27] for recent results, grey in Fig. 1) or the future International Axion Observatory (IAXO) [28]. In Fig. 1 the foreseen reach of ALPS-II and IAXO is shown in dashed pink and green in the upper left, respectively. The grayish regions labeled 'SN γ ' and 'mWD' show astrophysical exclusions according to [29] and [30], respectively.

Thirdly, there are experiments which are conceived such that they (probably) find only something if the above described particles are dark matter. In the following, we mention only such recent and planned setups/ideas. As motivated above, we group the searches into 'resonant' and 'non-resonant' although some setups can do both sort of scans. The former technique has

an advantage of highest sensitivity, the latter of an increased scanning speed.

2.1. Resonant searches

A resonant Haloscope is the most prominent and thus far most sensitive experiment searching for ultra-light dark matter. Based on the concept suggested in [31], several such experiments have been performed, see [32, 33] for recent reviews: A narrow band microwave resonator is placed in a (typically solenoidal) strong magnetic field. If axions constitute dark matter, they should then stream in large numbers through this experimental apparatus. In the magnetic field, axions would then be converted into photons and be visible at the resonant frequency of the cavity (related to the DM particle mass). Here, the conversion probability of the axions into photons is increased by the power built-up factor of the resonator. Hence, the sensitivity of such experiments can be increased by several orders of magnitudes compared to non-resonant techniques. Due to the pseudo-scalar nature of the axion, the electric fields of the cavity-mode and the external magnetic field should have a sizable overlap. In Fig. 1, obtained Haloscope limits are shown in orange (labeled 'Haloscopes' and 'ADMX') [34, 35, 36, 37, 38, 39, 40]. Currently two further, more sensitive Haloscope measurements are prepared at Seattle (ADMX II) and Yale (ADMX-HF). Prospects are adapted from [41] in the yellow and green shaded areas of Fig. 1, respectively.

For Haloscope searches in the solenoid configuration, it eventually can become difficult at the high mass range to keep the cavity resonant whilst retaining a sizable cavity volume (high mass \rightarrow smaller length scales) ¹. Thus, for higher-mass DM axion searches, additional techniques have been suggested and corresponding studies are underway. An example is the ORPHEUS setup [43] based on wire-planes in a confocal resonator. The wire-planes allow to alternate the B-field direction and keep a sizable overlap of external magnetic field and electric photon fields and thus probe higher frequencies [44]. Besides these efforts, let us also point to the emerging activities at the 'Center for Axion and Precision Physics Research' (CAPP) in Korea [45] from where novel contributions to DM axion detection are emerging currently.

A possibility for either resonant (or broadband, see below) direct dark matter searches might open also up with IAXO [28]. IAXO, although its primary mission is to act as a Helioscope like described above, could host, e.g., a cavity set-up in one of its bores. IAXO will be based on a toroidal magnet. Thus, cylindrical cavities do not suggest themselves. However, e.g., a rectangular cavity structure [46, 47] is conceivable. In principle, rectangular cavities could be feasible to explore a mass range below ADMX (making use of the huge magnetic volume of IAXO). If long thin cavities are realized, this opens up possibilities also for the higher massrange [48]. For testing such rectangular cavities before IAXO exists, e.g., the CAST magnet as well as a straightened² HERA dipole magnet could be an appropriate test-bench.

Another Haloscope experiment focusing (so far) on HP DM also exists at DESY, called WISPDMX [49]. It employs a former HERA proton cavity and is most sensitive in the 200-600 MHz range and can also run in 'broadband mode', see [50]. The 'broadband mode' is possible since for HPs no mode-overlap with an external field is needed.

A recent suggestion that could complement cavity searches at *very low masses* is the resonant axion DM detection through cryogenic LC-circuits in a strong external magnetic field [51]: axion CDM could induce a current in the part of an LC circuit that is immersed in an external magnetic field. Nicely, such setups are also apt to scan a large part of the HP cold DM parameter space, see [52, 53], although a careful design is likely needed that accounts for the proper shielding of

¹ & at the same time detection techniques need careful study at higher masses [42].

² For HERA magnets, the cold-mass (including the beam pipe) was originally fabricated straight and then bent by its outer shell. It has been shown by the ALPS-II collaboration that the beam-pipe can be straightened to almost the full aperture [26] and thus fit appropriate test cavities (the CAST magnet has a straight beam pipe to start with).

the setup: As HP DM does not need the magnetic field for its conversion in a photon signal, the HP DM stream will also excite electrons in the shield, causing a parasitic contribution that needs to be accounted for in the experiment's design [53].

So far we have have described setups that exploit coupling of the light dark matter particles to photons only. Of course there are alternative techniques, focusing on the coupling of axions to other particles. Examples of recent works point out that axion DM could be found through time-varying CP-odd nuclear moments [54, 55], see also [56], atomic transitions [57] or NMR techniques [58]. As mentioned, we focus on DM-to-photon couplings in the following and turn to the non-resonant techniques now.

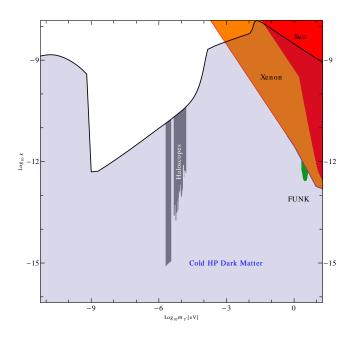


Figure 2. Parameter space of hidden photon dark matter in the sub-eV range. The blue shaded region is the viable HP DM parameter space as discussed in [15]. Other regions see text.

2.2. Broadband search

As has become obvious from the above considerations, the resonant enhancement is a crucial factor in becoming sensitive to very small couplings and thus eventually to QCD axions. On the other hand, it can be favorable to abstain from tuning to resonance, thereby losing sensitivity in coupling, but gaining in the width of the mass-range that can be covered within a fixed amount of time³.

Such a concept was proposed in [59, 60] as DM search with a dish antenna: axions and ALP DM (in combination with a magnetic field) as well as HP DM can excite electrons at the surface of a conductor, owing to their (albeit very weak) electromagnetic coupling to them. If this conductor is a spherical mirror, a consequence of this process would a photonic signal at the center of the mirror sphere (whereas most background photons are scattered to the focal point). Such a setup has a directional sensitivity [61] as the exact position of a signal spot will depend on the velocity distribution of the DM with respect to the dish. A global off-set of the signal can be expected due to the movement of the sun in the galactic rest frame as well as a daily modulation on the same order of magnitude (the yearly modulation is negligible due to the small velocity of the earth around the sun). Besides the signal-spot movement, a velocity distribution of the DM leads to a broadening of the signal spot. Such considerations can ultimately help to

³ This assumes that appropriate low-noise broad-band detectors are available which can be difficult over the entire spectral range of interest.

verify the dark matter nature of a signal. A dish dark matter search (Finding U(1)'s of a Novel Kind - FUNK) is currently being set up [62] at Karlsruhe. It uses a prototype mirror built for the fluorescence detector for the Pierre Auger observatory. Fig. 2 shows a plausible parameter reach for a first measurement in the optical in green.

3. Summary

In this talk, we have briefly reviewed the case for dark matter candidates below the eV-scale, notably axions, axion-like-particles and hidden photons. We have reviewed recent experiments thriving to find them through their effective coupling to electromagnetic waves. As of today, dark matter could be light, medium mass or heavy, a single species or a mixture of particles. Thus it seems well advised to have a well-balanced variety of experimental searches until we learn more about it. For dark matter candidates in the sub-eV regime, in view of the upcoming experiments described above, we are entering very interesting times.

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References

- [1] See the contributions of Lars Bergstrom, Roberto Battiston and Gernot Maier to these proceedings
- [2] P. Sikivie, In *Bertone, G. (ed.): Particle dark matter* 204-227
- [3] R. D. Peccei and H. R. Quinn, Phys. Rev. D 16, 1791 (1977). R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977).
- [4] J. Preskill, M. B. Wise and F. Wilczek, Phys. Lett. B 120 (1983) 127; L. F. Abbott and P. Sikivie, Phys. Lett. B 120 (1983) 133; M. Dine and W. Fischler, Phys. Lett. B 120 (1983) 137.
- P. Sikivie, Lect. Notes Phys. 741, 19 (2008) [astro-ph/0610440].
- [6] M. P. Hertzberg, M. Tegmark and F. Wilczek, Phys. Rev. D 78, 083507 (2008) [arXiv:0807.1726 [astro-ph]].
- [7] G. G. Raffelt, Lect. Notes Phys. **741**, 51 (2008) [hep-ph/0611350].
- [8] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).
- [9] M. Archidiacono, S. Hannestad, A. Mirizzi, G. Raffelt and Y. Y. Y. Wong, JCAP 1310, 020 (2013) [arXiv:1307.0615 [astro-ph.CO]].
- [10] J. E. Kim, Phys. Rev. Lett. 43, 103 (1979);
 - M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 166, 493 (1980).
- [11] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper and J. March-Russell, Phys. Rev. D 81, 123530 (2010) [arXiv:0905.4720 [hep-th]].
- [12] J. Jaeckel and A. Ringwald, Ann. Rev. Nucl. Part. Sci. 60, 405 (2010) [arXiv:1002.0329 [hep-ph]].
- [13] K. Baker et al. Ann. Phys. (Berlin) 525, No. 6, A93-A99 (2013) [arXiv:1306.2841 [hep-ph]].
- $[14]\,$ R. Essig, et al., arXiv:1311.0029 [hep-ph].
- [15] P. Arias, D. Cadamuro, M. Goodsell, J. Jaeckel, J. Redondo and A. Ringwald, JCAP 1206, 013 (2012) [arXiv:1201.5902 [hep-ph]].
- [16] A. Ringwald, Phys. Dark Univ. 1, 116 (2012) [arXiv:1210.5081 [hep-ph]].
- [17] A. Ringwald, arXiv:1310.1256 [hep-ph].
- [18] D. Horns and M. Meyer, JCAP **1202**, 033 (2012) [arXiv:1201.4711 [astro-ph.CO]].
- [19] J. Jaeckel, Frascati Phys. Ser. **56**, 172 (2012) [arXiv:1303.1821 [hep-ph]].
- [20] A. E. Nelson and J. Scholtz, Phys. Rev. D 84, 103501 (2011) [arXiv:1105.2812 [hep-ph]].
- [21] H. An, M. Pospelov and J. Pradler, Phys. Rev. Lett. 111, 041302 (2013) [arXiv:1304.3461 [hep-ph]].
- [22] N. Vinyoles, A. Serenelli, F. L. Villante, S. Basu, J. Redondo and J. Isern, arXiv:1501.01639 [astro-ph.SR].
 [23] M. Kawasaki and K. Nakayama, Ann. Rev. Nucl. Part. Sci. 63, 69 (2013) [arXiv:1301.1123 [hep-ph]].
- [24] J. Redondo and A. Ringwald, Contemp. Phys. 52, 211 (2011) [arXiv:1011.3741 [hep-ph]].
- [25] F. Della Valle, E. Milotti, A. Ejlli, G. Messineo, L. Piemontese, G. Zavattini, U. Gastaldi and R. Pengo et al., Phys. Rev. D 90, no. 9, 092003 (2014).

- [26] R. Bähre, et al. JINST 8, T09001 (2013) [arXiv:1302.5647 [physics.ins-det]].
- [27] M. Arik et al. [CAST Collaboration], Phys. Rev. Lett. 112, no. 9, 091302 (2014) [arXiv:1307.1985 [hep-ex]].
 S. Aune et al. [CAST Collaboration], Phys. Rev. Lett. 107, 261302 (2011) [arXiv:1106.3919 [hep-ex]].
- [28] E. Armengaud et al., JINST 9, T05002 (2014) [arXiv:1401.3233 [physics.ins-det]].
- [29] A. Payez, C. Evoli, T. Fischer, M. Giannotti, A. Mirizzi and A. Ringwald, arXiv:1410.3747 [astro-ph.HE].
- [30] R. Gill and J. S. Heyl, Phys. Rev. D 84, 085001 (2011) [arXiv:1105.2083 [astro-ph.HE]].
- [31] P. Sikivie Phys. Rev. Lett. **51**, 1415 (1983) [Erratum-ibid. **52**, 695 (1984)].
- [32] G. Rybka, Phys. Dark Univ. 4, 14 (2014).
- [33] I. P. Stern, AIP Conf. Proc. 1604, 456 (2014) [arXiv:1403.5332 [physics.ins-det]].
- [34] C. Hagmann, P. Sikivie, N. S. Sullivan and D. B. Tanner, Phys. Rev. D 42, 1297 (1990).
- [35] W. Wuensch, S. De Panfilis-Wuensch, Y. K. Semertzidis, J. T. Rogers, A. C. Melissinos, H. J. Halama, B. E. Moskowitz and A. G. Prodell et al., Phys. Rev. D 40, 3153 (1989).
- [36] S. De Panfilis, A. C. Melissinos, B. E. Moskowitz, J. T. Rogers, Y. K. Semertzidis, W. Wuensch, H. J. Halama and A. G. Prodell et al., Phys. Rev. Lett. 59, 839 (1987).
- [37] S. J. Asztalos et al. [ADMX Collaboration], Phys. Rev. D 69, 011101 (2004) [astro-ph/0310042].
- [38] S. J. Asztalos et al. [ADMX Collaboration], Astrophys. J. 571, L27 (2002) [astro-ph/0104200].
- [39] S. J. Asztalos et al. [ADMX Collaboration], Phys. Rev. D 64, 092003 (2001).
- [40] S. J. Asztalos et al. [ADMX Collaboration], Phys. Rev. Lett. 104, 041301 (2010) [arXiv:0910.5914 [astro-ph.CO]].
- [41] T. M. Shokair, J. Root, K. A. Van Bibber, B. Brubaker, Y. V. Gurevich, S. B. Cahn, S. K. Lamoreaux and M. A. Anil et al., Int. J. Mod. Phys. A 29, 1443004 (2014) [arXiv:1405.3685 [physics.ins-det]].
- [42] S. K. Lamoreaux, K. A. van Bibber, K. W. Lehnert and G. Carosi, Phys. Rev. D 88, no. 3, 035020 (2013) [arXiv:1306.3591 [physics.ins-det]].
- [43] G. Rybka and A. Wagner, arXiv:1403.3121 [physics.ins-det].
- [44] P. Sikivie, D. B. Tanner and Y. Wang, Phys. Rev. D 50, 4744 (1994) [hep-ph/9305264].
- [45] http://capp.ibs.re.kr/html/capp_en/
- [46] O. K. Baker et al. Phys. Rev. D 85, 035018 (2012) [arXiv:1110.2180 [physics.ins-det]]; M. Betz, F. Caspers and K. Zioutas, ATS/Note/2011/066 (2011).
- [47] I. G. Irastorza and J. A. Garcia, JCAP 1210, 022 (2012) [arXiv:1207.6129 [physics.ins-det]].
- [48] J. Redondo, I. G. Irastorza, B. Gimeno, private communication
- [49] D. Horns, A. Lindner, A. Lobanov and A. Ringwald, arXiv:1309.4170 [physics.ins-det].
- [50] D. Horns, A. Lindner, A. Lobanov and A. Ringwald, arXiv:1410.6302 [hep-ex].
- [51] P. Sikivie, N. Sullivan and D. B. Tanner, Phys. Rev. Lett. 112, 131301 (2014) [arXiv:1310.8545 [hep-ph]].
- [52] P. Arias, A. Arza, B. Döbrich, J. Gamboa and F. Mendez, arXiv:1411.4986 [hep-ph].
- [53] S. Chaudhuri, P. W. Graham, K. Irwin, J. Mardon, S. Rajendran and Y. Zhao, arXiv:1411.7382 [hep-ph].
- [54] P. W. Graham and S. Rajendran, Phys. Rev. D 88, 035023 (2013) [arXiv:1306.6088 [hep-ph]].
- [55] D. Budker et al. Phys. Rev. X 4, 021030 (2014) [arXiv:1306.6089 [hep-ph]].
- [56] B. M. Roberts, Y. V. Stadnik, V. A. Dzuba, V. V. Flambaum, N. Leefer and D. Budker, Phys. Rev. Lett. 113, 081601 (2014) [arXiv:1404.2723 [hep-ph]].
- [57] P. Sikivie, Phys. Rev. Lett. 113, no. 20, 201301 (2014) [arXiv:1409.2806 [hep-ph]].
- [58] A. Arvanitaki and A. A. Geraci, Phys. Rev. Lett. 113, 161801 (2014) [arXiv:1403.1290 [hep-ph]].
- [59] D. Horns et al., JCAP **1304**, 016 (2013) [arXiv:1212.2970].
- [60] J. Jaeckel and J. Redondo, Phys. Rev. D 88, 115002 (2013) [arXiv:1308.1103 [hep-ph]].
- [61] J. Jaeckel and J. Redondo, JCAP 1311, 016 (2013) [arXiv:1307.7181].
- [62] B. Döbrich, K. Daumiller, R. Engel et al., arXiv:1410.0200 [physics.ins-det]; J. Redondo and B. Döbrich, arXiv:1311.5341 [hep-ph].