

# Electroweak corrections to $Z$ -boson hadroproduction at finite transverse momentum

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## Abstract

We calculate the full one-loop electroweak radiative corrections, of  $\mathcal{O}(\alpha^2\alpha_s)$ , to the cross section of single  $Z$ -boson inclusive hadroproduction at finite transverse momentum ( $p_T$ ). This includes the  $\mathcal{O}(\alpha)$  corrections to  $Z+j$  production, the  $\mathcal{O}(\alpha_s)$  corrections to  $Z+\gamma$  production, and certain QCD-electroweak interference contributions involving a single quark trace. We present  $p_T$  and rapidity distributions for the experimental conditions at the Fermilab Tevatron and the CERN LHC, and compare the electromagnetic and purely weak corrections with the QCD ones at next-to-leading order.

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# 1 Introduction

The study of single electroweak-gauge-boson hadroproduction, via the so-called Drell–Yan process, has a long history, starting from the discovery of the  $W$  [1] and  $Z$  [2] bosons at the CERN Super Proton Synchrotron more than three decades ago, which marked a breakthrough for the Standard Model (SM). These processes remain to be of paramount importance also at modern hadron colliders, such as the Fermilab Tevatron and the CERN LHC. On the one hand, they have large cross sections and clean decay signatures in the detectors. This renders them particularly useful for calibrating and monitoring the luminosities at hadron colliders, which affects all other cross section measurements performed there as well. By the same token, they are sensitive probes of the parton density functions (PDFs), in particular those of the quarks and antiquarks. On the other hand, singly produced  $W$  and  $Z$  bosons form important backgrounds for searches of new physics beyond the SM, such as anomalous couplings, extra vector bosons, etc.

To achieve an adequate theoretical description, radiative corrections, both of QCD and electroweak type, must be taken into account. As for the total cross sections of single  $W$ - and  $Z$ -boson hadroproduction, the next-to-leading-order (NLO) [3] and next-to-next-to-leading-order (NNLO) [4] QCD corrections were calculated a long time ago, and also partial results at next-to-next-to-next-to-leading-order are available [5]. These corrections are of relative orders  $\mathcal{O}(\alpha_s^n)$  with  $n = 1, 2, 3$ , respectively, in the strong-coupling constant  $\alpha_s$ . The one-loop electroweak corrections, of relative order  $\mathcal{O}(\alpha)$  in Sommerfeld’s fine-structure constant  $\alpha$ , were studied for the  $W$  boson in Ref. [6] and for the  $Z$  boson in Ref. [7]. At the mixed two-loop order  $\mathcal{O}(\alpha\alpha_s)$ , the corrections to the  $q\bar{q}Z$  form factor were evaluated for light quarks  $q \neq b, t$  in Ref. [8] using the techniques developed in Ref. [9], and the treatment of the nonfactorizable corrections in the resonance region was discussed in Ref. [10].

In order for the  $W$  and  $Z$  bosons to acquire finite transverse momenta ( $p_T$ ), they must be produced in association with additional particles or hadron jets ( $j$ ). The QCD corrections to the  $p_T$  distributions of  $W$ - and  $Z$ -boson inclusive hadroproduction were computed at NLO [11,12] and NNLO [13]. The  $\mathcal{O}(\alpha)$  corrections were investigated for the  $W$  boson in Refs. [14,15] and for the  $Z$  boson in Refs. [16,17].

Specifically, in Ref. [14], the electroweak  $\mathcal{O}(\alpha)$  corrections to the  $\mathcal{O}(\alpha\alpha_s)$  partonic subprocesses of  $W$ -boson production were calculated imposing a minimum-transverse-momentum cut on outgoing gluons to prevent soft-gluon singularities. However, this cut was not applied to outgoing quarks and antiquarks as well, which renders it impractical at the hadron level, where gluon and light-quark jets are hard to distinguish on an event-by-event basis. Such a cut is also problematic from the conceptual point of view because, as a matter of principle, a collinear gluon-photon system cannot be distinguished from a single gluon with the same momentum. In Ref. [15], the results of Ref. [14] were confirmed, but the soft-gluon singularities were properly eliminated by including also the  $\mathcal{O}(\alpha_s)$  corrections to the  $\mathcal{O}(\alpha^2)$  partonic subprocesses. Furthermore, the  $\mathcal{O}(\alpha^3)$  contributions due to direct and resolved photoproduction by elastic and inelastic scattering off the incoming (anti)proton were taken into account in Ref. [15].

In contrast to the charged-current case, the separation of the electroweak  $\mathcal{O}(\alpha)$  corrections to the neutral-current Drell–Yan process into an electromagnetic and a weak part is meaningful with regard to infrared (IR) and ultraviolet (UV) finiteness and gauge independence. In Refs. [16,17], the purely weak  $\mathcal{O}(\alpha)$  corrections to the  $\mathcal{O}(\alpha\alpha_s)$  partonic subprocess  $q\bar{q} \rightarrow Zg$  and its crossed versions were computed. They make up an important subset of the contributions of absolute order  $\mathcal{O}(\alpha^2\alpha_s)$  to the inclusive hadroproduction of finite- $p_T$   $Z$  bosons. It is the purpose of this work to complete our knowledge of these contributions, which have several sources, and to check the results presented in Ref. [17]. To start with, we need to complement the purely weak  $\mathcal{O}(\alpha)$  corrections to the  $\mathcal{O}(\alpha\alpha_s)$  partonic subprocess  $q\bar{q} \rightarrow Zg$  and its crossed versions by the QED ones, which have virtual and real parts. The  $\mathcal{O}(\alpha^2\alpha_s)$  partonic subprocesses that we are then led to consider include  $q\bar{q} \rightarrow Zg\gamma$ . As in the charged-current case [15], we thus inevitably encounter a soft-gluon singularity. To cancel it, we need to also include the  $\mathcal{O}(\alpha_s)$  QCD corrections to the  $\mathcal{O}(\alpha^2)$  partonic subprocess  $q\bar{q} \rightarrow Z\gamma$ . Furthermore,  $\mathcal{O}(\alpha^2\alpha_s)$  contributions may also arise from interferences of  $\mathcal{O}(\alpha^{1/2}\alpha_s)$  and  $\mathcal{O}(\alpha^{3/2})$  Feynman diagrams yielding a single Dirac spinor trace with nonvanishing color factor  $\text{tr} T^a T^a = N_c C_F$ . This happens for the partonic subprocess  $q\bar{q} \rightarrow Zq\bar{q}$  when a diagram involving a virtual gluon in the  $s$  ( $t$ ) channel is connected with a diagram involving a photon or a  $Z$  boson in the  $t$  ( $s$ ) channel, and for the subprocesses  $qq \rightarrow Zqq$  and  $\bar{q}\bar{q} \rightarrow Z\bar{q}\bar{q}$  when a gluon-exchange diagram and a photon/ $Z$ -boson-exchange diagram are connected with twisted quark lines. For completeness, we also recalculate the  $\mathcal{O}(\alpha_s)$  QCD corrections to the inclusive hadroproduction of finite- $p_T$   $Z$  bosons [11,12] and thus recover the analytic results specified in Ref. [12] apart from a few misprints that we correct. Finally, we also study the LO photon-induced subprocesses, of order  $\mathcal{O}(\alpha^2)$ , which, after convolution with the photon PDFs of the (anti)proton, yield contributions of absolute order  $\mathcal{O}(\alpha^3)$ . As in Refs. [12,17], we list full analytic results in a compact form.

The  $\mathcal{O}(\alpha)$  corrections to  $l^+\nu+j$  and  $l^+l^-+j$  inclusive hadroproduction were considered in Refs. [18,19], respectively. The numerical results presented in Refs. [18,19] do not allow one to extract the  $p_T$  distributions of the  $W$  and  $Z$  bosons, respectively, and thus cannot be compared with the results obtained in Refs. [14,15,16,17] and here. The distributions in the lepton-pair invariant mass  $M_{ll}$  shown in Figs. 7 and 8 of Ref. [19] exhibit a rapid fall-off at the shoulders of the peak at  $M_{ll} = M_Z$  indicating that the narrow-width approximation adopted in Refs. [14,15,16,17] and here is quite appropriate for the Tevatron and the LHC. The  $\mathcal{O}(\alpha^2\alpha_s)$  interference contributions mentioned above were neglected in Ref. [19].

This paper is organized as follows. In Sec. 2, we explain our analytic calculations. In Sec. 3, we present our numerical results. Our conclusions are contained in Sec. 4. Our analytic results are listed in Appendices A–C.

## 2 Analytic results

We consider the inclusive production of a  $Z$  boson in the collision of two hadrons  $h_1$  and  $h_2$ ,

$$h_1(P_1) + h_2(P_2) \rightarrow Z(q) + X, \quad (1)$$

where the four-momenta are indicated in parentheses and  $X$  collectively denotes the residual particles in the final state. We take the  $Z$  boson to be on mass shell,  $q^2 = M_Z^2$ , neglect the hadron masses,  $P_1^2 = P_2^2 = 0$ , and define the hadronic Mandelstam variables as

$$S = (P_1 + P_2)^2, \quad T = (P_1 - q)^2, \quad U = (P_2 - q)^2, \quad Q^2 = q^2. \quad (2)$$

In the center-of-mass frame, we write  $q^\mu = (q^0, \mathbf{q}_T, q^3)$ , where  $\mathbf{q}_T$  is the transverse momentum, and define  $q_T = |\mathbf{q}_T|$  and the rapidity  $y = (1/2) \ln[(q^0 + q^3)/(q^0 - q^3)]$ . Using Eq. (2), we have

$$q_T^2 = \frac{TU - Q^2(S + T + U - Q^2)}{S}, \quad y = \frac{1}{2} \ln \frac{U - Q^2}{T - Q^2}. \quad (3)$$

We work in the collinear parton model of QCD with  $n_f = 5$  massless quark flavors. We write the partonic subprocesses that contribute to the hadronic reaction in Eq. (1) generically as

$$i(p_1) + j(p_2) \rightarrow Z(q) + X(p_X), \quad (4)$$

where  $p_i = x_i P_i$  with  $i = 1, 2$ . The partonic Mandelstam variables are defined as

$$s = (p_1 + p_2)^2, \quad t = (p_1 - q)^2, \quad u = (p_2 - q)^2, \quad Q^2 = q^2, \quad s_2 = p_X^2, \quad (5)$$

and satisfy

$$s + t + u = Q^2 + s_2. \quad (6)$$

The hadronic and partonic Mandelstam variables are related as follows:

$$\begin{aligned} s &= x_1 x_2 S, & t &= x_1(T - Q^2) + Q^2, & u &= x_2(U - Q^2) + Q^2, \\ s_2 &= x_1 x_2 S + x_1(T - Q^2) + x_2(U - Q^2) + Q^2. \end{aligned} \quad (7)$$

The differential cross section for reaction (1) may be evaluated according to

$$\frac{d\sigma}{dq_T^2 dy} = \sum_{ij} \int dx_1 dx_2 f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2) \frac{s d\sigma_{ij}}{dt du}(x_1 P_1, x_2 P_2, \mu_F^2), \quad (8)$$

where the sum runs over all the partons  $i$  and  $j$ ;  $f_{i/h}(x, \mu_F^2)$  is the PDF of parton  $i$  in hadron  $h$ ; and  $\mu_F$  is the factorization scale. The partonic cross sections  $d\sigma_{ij}/(dt du)$  may be computed in perturbation theory as double series in  $\alpha_s$  and  $\alpha$ . Apart from the Feynman rules of QCD, we also need those for the couplings of a quark  $q$  to the photon  $\gamma$  and the

$Z$  boson. They are given by the vertices  $ieQ_q\gamma_\mu$  and  $ie\gamma_\mu(v_q - \gamma_5 a_q)$ , respectively, where  $Q_q$  is the electric charge of  $q$  in units of the positron charge  $e = \sqrt{4\pi\alpha}$ ,

$$v_q = \frac{I_3 - 2Q_q \sin^2 \theta_w}{2 \sin \theta_w \cos \theta_w}, \quad a_q = \frac{I_3}{2 \sin \theta_w \cos \theta_w} \quad (9)$$

are its vector and axial vector couplings to the  $Z$  boson,  $I_3$  is its third component of weak isospin, and  $\theta_w$  is the weak mixing angle.

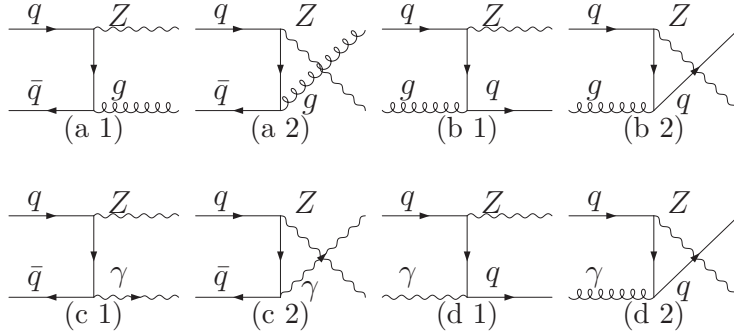


Figure 1: Tree-level diagrams contributing to the partonic subprocesses  $q + \bar{q} \rightarrow Z + g$  and  $q + g \rightarrow Z + q$  at  $\mathcal{O}(\alpha\alpha_s)$  and to the partonic subprocesses  $q + \bar{q} \rightarrow Z + \gamma$  and  $q + \gamma \rightarrow Z + q$  at  $\mathcal{O}(\alpha^2)$ .

In the remainder of this section, we list the relevant partonic subprocesses with the contributing Feynman diagrams and outline the computation and its organization. At LO, we consider the  $2 \rightarrow 2$  subprocesses

$$q + \bar{q} \rightarrow Z + g, \quad (10)$$

$$q + \bar{q} \rightarrow Z + \gamma, \quad (11)$$

$$q + g \rightarrow Z + q, \quad (12)$$

$$q + \gamma \rightarrow Z + q, \quad (13)$$

where it is understood that  $q$  may also be an antiquark, in which case  $\bar{q}$  is a quark. They are mediated by the Feynman diagrams depicted in Fig. 1. At LO, subprocesses (10) and (12) are of  $\mathcal{O}(\alpha\alpha_s)$ , and subprocesses (11) and (13) are of  $\mathcal{O}(\alpha^2)$ .

We include the  $\mathcal{O}(\alpha_s)$  QCD corrections to subprocesses (10)–(12) and the  $\mathcal{O}(\alpha)$  electroweak corrections to subprocesses (10) and (12), while we treat subprocess (13) only at LO because of the additional  $\mathcal{O}(\alpha)$  suppression due to the photon emission by the incoming hadrons. The virtual QCD corrections to subprocesses (10) and (11) and the virtual QED corrections to subprocess (10) arise from the Feynman diagrams in Fig. 2. The virtual QCD and QED corrections to subprocess (12) are obtained by appropriately crossing external legs. The virtual weak corrections to subprocess (10) arise from the Feynman diagrams in Fig. 3, and those to subprocess (12) emerge by crossing.

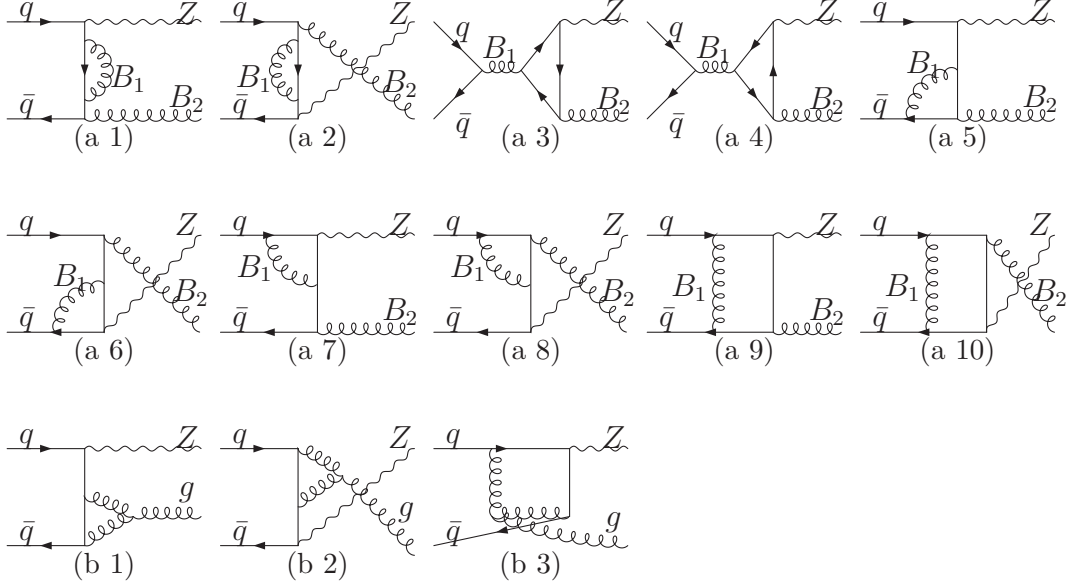


Figure 2: QCD and QED one-loop diagrams contributing to the partonic subprocess  $q + \bar{q} \rightarrow Z + g$  at  $\mathcal{O}(\alpha\alpha_s^2)$  ( $B_1 = B_2 = g$ ) and  $\mathcal{O}(\alpha^2\alpha_s)$  (diagrams (a 1)–(a 10) with  $B_1 = \gamma$  and  $B_2 = g$ ) and to the partonic subprocess  $q + \bar{q} \rightarrow Z + \gamma$  at  $\mathcal{O}(\alpha^2\alpha_s)$  (diagrams (a 1)–(a 10) with  $B_1 = g$  and  $B_2 = \gamma$ ). The QCD and QED one-loop diagrams contributing to the partonic subprocess  $q + g \rightarrow Z + q$  at  $\mathcal{O}(\alpha\alpha_s^2)$  and  $\mathcal{O}(\alpha^2\alpha_s)$  are obtained by appropriately crossing external legs.

The real QCD and QED corrections arise from the  $2 \rightarrow 3$  subprocesses

$$q + \bar{q} \rightarrow Z + q + \bar{q}, \quad (14)$$

$$q + \bar{q} \rightarrow Z + q' + \bar{q}', \quad (15)$$

$$q + \bar{q} \rightarrow Z + g + g, \quad (16)$$

$$q + \bar{q} \rightarrow Z + g + \gamma, \quad (17)$$

$$q + q \rightarrow Z + q + q, \quad (18)$$

$$q + q' \rightarrow Z + q + q', \quad (19)$$

$$q + g \rightarrow Z + q + g, \quad (20)$$

$$q + g \rightarrow Z + q + \gamma, \quad (21)$$

$$g + g \rightarrow Z + q + \bar{q}. \quad (22)$$

Subprocesses (14) and (18) contribute both at  $\mathcal{O}(\alpha\alpha_s^2)$  and  $\mathcal{O}(\alpha^2\alpha_s)$ , subprocesses (15), (16), (19), (20), and (22) contribute at  $\mathcal{O}(\alpha\alpha_s^2)$ , and subprocesses (17) and (21) contribute at  $\mathcal{O}(\alpha^2\alpha_s)$ . The tree-level diagrams contributing to subprocess (14) are shown in Fig. 4, those contributing to subprocesses (16) and (17) in Fig. 5, and those contributing to subprocesses (18) and (19) in Fig. 6.

As already mentioned in Sec. 1, the  $\mathcal{O}(\alpha^2\alpha_s)$  contributions to subprocesses (14) and (18) are generated by interferences of  $2 \rightarrow 3$  tree-level diagrams with a virtual gluon in one factor and a virtual photon or  $Z$  boson in the other one in such a way that one closed quark line is formed yielding a nonvanishing color factor,  $\text{tr} T^a T^a = N_c C_F$ , as indicated in Fig. 7. In the case of subprocess (14), this is achieved when the gluon is in the  $s$  channel and the photon or  $Z$  boson is in the  $t$  channel or vice versa. In the case of subprocess (18), this is achieved by twisting the quark lines in one of the interfering diagrams. These interference contributions exhaust the  $\mathcal{O}(\alpha^2\alpha_s)$  corrections to subprocesses (14) and (18) and are thus finite and gauge-independent by themselves. On the other hand, interferences leading to two closed quark lines are nullified by  $(\text{tr} T^a)^2 = 0$ . This explains, why subprocesses (15) and (19) do not receive  $\mathcal{O}(\alpha^2\alpha_s)$  contributions. Obviously, these types of mixed QCD-QED corrections may not be obtained by merely manipulating coupling constants and color factors as is often the case for pure QED corrections.

We compute the full  $\mathcal{O}(\alpha\alpha_s^2)$  and  $\mathcal{O}(\alpha^2\alpha_s)$  corrections to the cross section of the hadronic process (1) according to Eq. (8) by including all the partonic subprocesses (10)–(22). We regularize both the UV and IR divergences using dimensional regularization in  $D = 4 - 2\varepsilon$  space-time dimensions. The UV divergences arise from the  $2 \rightarrow 2$  one-loop diagrams and are removed by renormalizing the coupling constants, masses, and wave functions in the respective  $2 \rightarrow 2$  tree-level diagrams. We renormalize  $\alpha_s$  and  $\alpha$  according to the modified minimal-subtraction ( $\overline{\text{MS}}$ ) scheme and employ the electroweak on-shell renormalization scheme otherwise. In particular, we define the weak mixing angle  $\theta_w$  in terms of the pole masses as  $\cos \theta_w = M_W/M_Z$ . The IR divergences, both of soft and collinear types, are generated by  $2 \rightarrow 2$  one-loop and  $2 \rightarrow 3$  tree-level diagrams. The soft and collinear divergences related to the final states are canceled by integrating over the kinematic degrees of freedom of the three-particle phase space that are related to the systems  $X$  and combining the outcome with the virtual corrections. Specifically, the three-particle kinematics turns into the two-particle one by taking the limit  $s_2 \rightarrow 0$ . This is implemented in dimensional regularization using the relationship

$$\frac{1}{s_2^{1+\varepsilon}} = \delta(s_2) \left( -\frac{1}{\varepsilon} + \ln s_2^{\text{max}} - \frac{\varepsilon}{2} \ln^2 s_2^{\text{max}} + \dots \right) + \left( \frac{1}{s_2} \right)_+ + \left( \frac{\ln s_2}{s_2} \right)_+ + \dots, \quad (23)$$

where  $s_2^{\text{max}}$  is the maximum value of  $s_2$  allowed for given values of  $p_T$  and  $y$  and the plus distributions are defined for smooth test functions  $f(s_2)$  as

$$\begin{aligned} \int_0^{s_2^{\text{max}}} ds_2 \left( \frac{1}{s_2} \right)_+ f(s_2) &= \int_0^{s_2^{\text{max}}} ds_2 \frac{1}{s_2} [f(s_2) - f(0)], \\ \int_0^{s_2^{\text{max}}} ds_2 \left( \frac{\ln s_2}{s_2} \right)_+ f(s_2) &= \int_0^{s_2^{\text{max}}} ds_2 \frac{\ln s_2}{s_2} [f(s_2) - f(0)]. \end{aligned} \quad (24)$$

There remain collinear divergences related to the initial states, which are universal and are absorbed into the bare PDFs so to render them finite. In the  $\overline{\text{MS}}$  factorization scheme,

this PDF renormalization is implemented as

$$f_{i/h}(x, \mu_F^2) = \sum_j \int_x^1 \frac{dy}{y} f_{j/h}^{\text{bare}}\left(\frac{x}{y}\right) \left[ \delta_{ij} \delta(1-y) - \frac{\mu_F^{-2\varepsilon}}{\varepsilon} \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} P_{ij}(y) + \dots \right], \quad (25)$$

where  $P_{ij}(y)$  are the  $j \rightarrow i$  splitting functions. In the one-loop approximation of QCD, the latter read [20]

$$\begin{aligned} P_{qq}(y) &= C_F \left[ \frac{3}{2} \delta(1-y) + 2 \left( \frac{1}{1-y} \right)_+ - 1 - y \right], \\ P_{gq}(y) &= C_F \frac{1 + (1-y)^2}{y}, \\ P_{gg}(y) &= \left( \frac{11}{6} C_A - \frac{2}{3} T n_f \right) \delta(1-y) + 2C_A \left[ \left( \frac{1}{1-y} \right)_+ + \frac{1}{y} - 2 + y(1-y) \right], \\ P_{qg}(y) &= T [y^2 + (1-y)^2], \end{aligned} \quad (26)$$

where  $C_F = (N_c^2 - 1)/(2N_c) = 4/3$ ,  $C_A = N_c = 3$ ,  $T = 1/2$ , and  $n_f = 5$  is the number of active quark flavors. The QED counterparts are obtained from Eq. (26) by substituting  $\alpha_s \rightarrow \alpha Q_q^2$ ,  $C_F \rightarrow 1$ , and  $C_A \rightarrow 0$ .

To exploit Eq. (23), it is useful to introduce  $s_2$  as an integration variable in Eq. (8), in lieu of  $x_2$ , say. This leads to

$$\frac{d\sigma}{dq_T^2 dy} = \sum_{i,j} \int_{x_1^{\min}}^1 dx_1 \int_0^{s_2^{\max}} \frac{ds_2}{x_1 S + U - Q^2} f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2) \frac{s}{dt} \frac{d\sigma_{ij}}{du}(x_1 P_1, x_2 P_2, \mu_F^2), \quad (27)$$

where

$$\begin{aligned} x_1^{\min} &= \frac{-U}{S + T - Q^2}, & s_2^{\max} &= U + x_1(S + T - Q^2), & x_2 &= \frac{s_2 - Q^2 - x_1(T - Q^2)}{x_1 S + U - Q^2}, \\ T &= Q^2 - e^{-y} \sqrt{S(Q^2 + q_T^2)}, & U &= Q^2 - e^y \sqrt{S(Q^2 + q_T^2)}. \end{aligned} \quad (28)$$

At this point, we compare our analytic results with the literature [12,17]. The NLO QCD corrections, of relative order  $\mathcal{O}(\alpha_s)$ , due to the virtual contributions from subprocesses (10) and (12), and the real contributions from subprocesses (14)–(16), (18)–(20), and (22) are listed in Ref. [12]. Apart from some misprints,<sup>1</sup> we find agreement with Ref. [12]. The weak  $\mathcal{O}(\alpha)$  corrections to subprocess (10) are listed in Ref. [17]. In Ref. [17], collinear divergences are regularized by introducing an infinitesimal quark mass  $\lambda$ , while

<sup>1</sup>In Eq. (2.12) of Ref. [12],  $B_2^{qG}(s, t, u, Q^2)$  should be replaced with  $[B_2^{qG}(s, t, u, Q^2) + C_2^{qG}(s, t, u, Q^2)]$  and  $C_2^{qG}(s, t, u, Q^2)$  with  $C_3^{qG}(s, t, u, Q^2)$ .



we employ dimensional regularization. The  $\lambda$ -dependent one-loop scalar integrals  $J_{12}$ ,  $J_{13}$ , and  $J_{14}$  in Eq. (39) of Ref. [17] may be conveniently converted to dimensional regularization using the results of Ref. [21]. In Ref. [17], the renormalization is performed both in the  $\overline{\text{MS}}$  scheme and in the on-shell scheme implemented with a running fine-structure constant as explained in Eqs. (49) and (50) of Ref. [17]. While the latter construction is well defined at one loop, it becomes ambiguous at higher orders because of the required separation of fermionic and bosonic contributions. By contrast, we work in a hybrid renormalization scheme, which uses the  $\overline{\text{MS}}$  definition of  $\alpha$ , but the electroweak on-shell scheme otherwise. Taking these conceptual differences into account, we fully agree with Ref. [17].

In this paper, we only list those analytic results that may not be found in the previous literature. Specifically, we consider  $\mathcal{O}(\alpha^2\alpha_s)$  contributions to  $q+\bar{q} \rightarrow Z+X$ ,  $q+g \rightarrow Z+X$ , and  $q+q \rightarrow Z+X$  in Appendices A, B, and C, respectively. In the case of  $q+\bar{q} \rightarrow Z+X$ , this includes the virtual QED corrections to subprocess (10), the virtual QCD corrections to subprocess (11), the real corrections from subprocess (17), and the above-mentioned interference contributions from subprocess (14). In the case of  $q+g \rightarrow Z+X$ , this includes the virtual QED corrections to subprocess (12) and the real corrections from subprocess (21). In the case of  $q+q \rightarrow Z+X$ , this includes the above-mentioned interference contributions from subprocess (18).

As already mentioned in Sec. 1, we also include the LO contributions from photoproduction. Incoming photons can participate in the hard scattering either directly or indirectly, i.e. through their quark and gluon content. The contributions from direct and resolved photoproduction are formally of the same order in the perturbative expansion. This may be understood by observing that the PDFs of the photon have a leading behavior proportional to  $\alpha \ln(\mu_F^2/\Lambda_{\text{QCD}}^2) \propto \alpha/\alpha_s(\mu_F^2)$ , where  $\Lambda_{\text{QCD}}$  is the asymptotic scale parameter of QCD. At LO, direct photoproduction proceeds via subprocess (13) and resolved photoproduction via subprocesses (10) and (12). The cross section of subprocess (13) reads [22]

$$\frac{d\sigma_{q\gamma}}{dt} = -\frac{2\pi\alpha^2 Q_q^2 (v_q^2 + a_q^2)}{N_c s^2} \left( \frac{t}{s} + \frac{s}{t} + \frac{2uQ^2}{st} \right), \quad (29)$$

where the Mandelstam variables and gauge coupling constants are defined in Eqs. (5) and (9), respectively. The ones of subprocesses (10) and (12) may be read off from Eqs. (30) and (39), respectively. The emission of photons off the (anti)proton can happen either elastically or inelastically, i.e. the (anti)proton stays intact or is destroyed, respectively. In both cases, an appropriate PDF can be evaluated in the Weizsäcker-Williams approximation [23,24,25,26]. Since these PDFs are of  $\mathcal{O}(\alpha)$ , the LO photoproduction contributions are of  $\mathcal{O}(\alpha^3)$ . Although photoproduction contributions are parametrically suppressed by a factor of  $\alpha/\alpha_s$  relative to the  $\mathcal{O}(\alpha^2\alpha_s)$  corrections discussed above, we include them in our analysis because they may turn out to be sizable in certain regions of phase space.

### 3 Numerical analysis

We are now in a position to present our numerical analysis. As input we use the pole masses  $M_W = 80.385$  GeV,  $M_Z = 91.1876$  GeV,  $M_H = 125$  GeV,  $m_b = 4.89$  GeV,  $m_t = 173.07$  GeV, and the  $\overline{\text{MS}}$  coupling constants  $\bar{\alpha}(M_Z^2) = 1/127.944$  [27] and  $\alpha_s^{(5)}(M_Z^2) = 0.1180$  [26] to gauge  $\bar{\alpha}(\mu_R^2)$  and  $\alpha_s^{(5)}(\mu_R^2)$ . We employ the NNPDF2.3QED NLO set of proton PDFs [26], which also include QED evolution and provide a photon distribution function. This allows us to consistently treat direct and resolved photoproduction via inelastic scattering off the (anti)proton along with ordinary hadroproduction. As for photoproduction via elastic scattering off the (anti)proton, we adopt the photon flux function from Ref. [23] and the resolved-photon PDFs from Ref. [28]. For definiteness, we identify the renormalization and factorization scales with the  $Z$ -boson transverse mass,  $\mu_R = \mu_F = \sqrt{p_T^2 + M_Z^2}$ .

In Fig. 8, we study the cross section of  $p\bar{p} \rightarrow Z + X$  at center-of-mass energy  $\sqrt{S} = 1.96$  TeV appropriate for Tevatron run II (a) differential in  $p_T$  integrated over  $y$  and (b) differential in  $y$  imposing the acceptance cut  $p_T > 10$  GeV. In Fig. 9, we do the same for  $pp \rightarrow Z + X$  at  $\sqrt{S} = 14$  TeV appropriate for the LHC. In each plot, we show (i) the NLO QCD result considered in Ref. [12], i.e. the sum of the  $\mathcal{O}(\alpha\alpha_s)$  and  $\mathcal{O}(\alpha\alpha_s^2)$  results (thin solid lines); (ii) the  $\mathcal{O}(\alpha^2)$  Born result (thin dot-dashed lines); (iii) the purely weak  $\mathcal{O}(\alpha^2\alpha_s)$  corrections to subprocesses (10) and (12) considered in Ref. [17] (thick dashed lines); (iv) the residual electroweak  $\mathcal{O}(\alpha^2\alpha_s)$  corrections (thin dashed lines); (v) the  $\mathcal{O}(\alpha^3)$  photoproduction contributions (dotted lines); and (vi) the total sum (thick solid lines). The  $p_T$  distributions in Figs. 8(a) and 9(a) are plotted on logarithmic scales. Since those of contribution (iii) are negative in the considered  $p_T$  ranges, their moduli are shown. The  $y$  distributions in Figs. 8(b) and 9(b) are plotted on linear scales. For better visibility, contributions (ii), (iv), and (v) are amplified by a factor of 100. In Figs. 8(b) and 9(b), we do not consider negative  $y$  values because the  $y$  distributions are symmetric by charge conjugation invariance.

From Figs. 8 and 9, we observe that the combined effect of the electroweak contributions (ii)–(v) is to reduce the NLO QCD predictions (i). The reduction ranges from a few percent at low  $p_T$  values to a few tens of percent in the large- $p_T$  domain, where large Sudakov logarithms dominate. The bulk of the electroweak contributions (ii)–(v) is made up by the purely weak  $\mathcal{O}(\alpha^2\alpha_s)$  corrections to subprocesses (10) and (12) [contribution (iii)], which are negative throughout the kinematic ranges considered here. The residual types of electroweak effects taken into account here, namely the  $\mathcal{O}(\alpha^2)$  Born result (ii), the residual electroweak  $\mathcal{O}(\alpha^2\alpha_s)$  corrections (iv), and the  $\mathcal{O}(\alpha^3)$  photoproduction contributions (v), are all positive, but numerically suppressed by typically one order of magnitude or more relative to contribution (iii), except for contribution (ii) in the small- $p_T$  range.

## 4 Conclusions

We studied the inclusive hadroproduction of large- $p_T$   $Z$  bosons including both the QCD and electroweak NLO corrections and presented  $p_T$  and  $y$  distributions under Tevatron and LHC kinematic conditions. Our analytic results agree with the literature as far as the latter goes. Specifically, we recovered the well-known NLO QCD corrections [12], of absolute order  $\mathcal{O}(\alpha\alpha_s^2)$ , and the purely weak one-loop corrections, of absolute order  $\mathcal{O}(\alpha^2\alpha_s)$ , to the partonic subprocess  $q\bar{q} \rightarrow Zg$  and its crossed versions [17]. We completed our knowledge of the  $\mathcal{O}(\alpha^2\alpha_s)$  corrections by providing also the QED corrections and some missing weak contributions due to interferences of tree-level  $2 \rightarrow 3$  scattering amplitudes in analytic form. While the new  $\mathcal{O}(\alpha^2\alpha_s)$  contributions turned out to be numerically small, their knowledge helps us to reduce the theoretical uncertainty on this important benchmark cross section. We also considered direct and resolved photoproduction in elastic and inelastic scattering.

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## A Cross section of $\bar{q} + q \rightarrow Z + X$ through $\mathcal{O}(\alpha^2\alpha_s)$

In this appendix, we consider  $\mathcal{O}(\alpha^2\alpha_s)$  contributions to  $\bar{q} + q \rightarrow Z + X$ . This includes the virtual QED corrections to subprocess (10), the virtual QCD corrections to subprocess (11), the real corrections from subprocess (17), and the interference contributions from subprocess (14) involving a virtual photon or  $Z$  boson. We have

$$\begin{aligned} \frac{s d\sigma^{\bar{q}q}}{dt du} = & \frac{2\pi\alpha^2 Q_q^2 (v_q^2 + a_q^2)}{N_c s} \left[ \delta(s_2) A_0 \right. \\ & \left. + C_F \frac{\alpha_s}{\pi} \left( \delta(s_2) A_1 + A_2 + A_3 + \frac{v_q^4 + 6v_q^2 a_q^2 + a_q^4}{v_q^2 + a_q^2} A_4 \right) \right], \end{aligned} \quad (30)$$

where

$$\begin{aligned} A_0 = & \frac{t}{u} + \frac{u}{t} + 2 \left( \frac{s}{t} + \frac{s}{u} + \frac{s^2}{tu} \right), \\ A_1 = & A_0 \left[ L_{\mu_F} (2L_t + 2L_u - 4L_A - 3) + 4L_A (L_A + L_s - L_t - L_u - 1) + (L_t + L_u)^2 \right] \\ & + L_s^2 \left( -\frac{2s^2}{tu} - A_0 \right) + L_s \left( \frac{8s}{t+u} + \frac{4s^2}{(t+u)^2} \right) \end{aligned} \quad (31)$$

$$\begin{aligned}
& + \left( 2A_0 + \frac{4s^2}{tu} \right) \left( L_s \ln \frac{s-Q^2}{Q^2} - \text{Li}_2 \left( \frac{Q^2}{s} \right) \right) + \zeta(2) \left( \frac{4s^2}{tu} + 8A_0 \right) \\
& + \left\{ L_t \left( \frac{t+4s}{s+u} + \frac{st}{(s+u)^2} \right) + L_s L_t \left( \frac{2(2s+t)}{u} - 4A_0 \right) \right. \\
& + \frac{2(2s^2+2su+u^2)}{tu} \left( L_t \ln \frac{s+u}{Q^2} + \text{Li}_2 \left( \frac{t}{Q^2} \right) \right) \\
& \left. - \frac{9t+17s}{u} - \frac{8s^2}{tu} + \frac{2s}{t+u} + \frac{s}{s+t} \right\} + \left\{ u \leftrightarrow t \right\}, \tag{32}
\end{aligned}$$

$$\begin{aligned}
A_2 = & \left[ 4A_0 \left( -\ln \frac{(s_2-t)(s_2-u)}{s^2} - L_{\mu_F} + 2 \ln \frac{s_2}{Q^2} - L_s \right) \frac{1}{s_2} \right]_{A+} \\
& + \ln \frac{tu - Q^2 s_2}{(s_2-t)(s_2-u)} \left( \frac{4s^2(s-Q^2)}{tu(s_2-t)(s_2-u)} + \frac{2(2s+u-t)}{t(s_2-t)} + \frac{2(2s+t-u)}{u(s_2-u)} \right. \\
& \left. + \frac{4Q^2 A_0 - 2u - 2t - 2Q^2 - 6s}{tu - Q^2 s_2} + \frac{2(t+u+s+Q^2)}{tu} \right) \\
& + \ln \frac{(s_2-t)(s_2-u)}{s s_2} \left( \frac{4s^2(s-Q^2)}{tu(s_2-t)(s_2-u)} + \frac{2(2s+u-t)}{t(s_2-t)} + \frac{2(2s+t-u)}{u(s_2-u)} \right) \\
& + \ln \frac{\mu_F^2}{s_2} \left( \frac{s}{(s_2-t)^2} + \frac{s}{(s_2-u)^2} - \frac{2s+u+t}{t(s_2-t)} - \frac{2s+t+u}{u(s_2-u)} + \frac{Q^2}{t^2} + \frac{Q^2}{u^2} \right. \\
& \left. - \frac{4Q^2 A_0 - 2u - 2t - 2Q^2 - 6s}{tu - Q^2 s_2} \right) \\
& + \frac{s}{(s_2-t)^2} + \frac{s}{(s_2-u)^2} + \frac{2(u/t + s/t - s/u)}{s_2-t} + \frac{2(t/u + s/u - s/t)}{s_2-u} \\
& + \frac{2(s-Q^2 + Q^2 u/t + Q^2 t/u)}{tu - Q^2 s_2} + \frac{2(t+u-s)}{tu}, \tag{33}
\end{aligned}$$

$$\begin{aligned}
A_3 = & \frac{9}{4\lambda} L_\lambda \left( \frac{3s(t-u)^2(t+u)(4Q^2 s + (t+u)^2)}{2tu\lambda^4} \right. \\
& - \frac{2s(t^3+u^3) + Q^2(-4s(t-u)^2 + 2s^2(t+u) - (t-u)^2(t+u))}{tu\lambda^2} \\
& + \frac{(t+u)(4s^2+t^2+u^2+2s(t+u))}{s_2 tu} + \frac{6(s-Q^2)(t^2+u^2)}{stu} + \frac{2(6s+t+u)}{s+Q^2-s^2} \\
& \left. + \frac{(9s-18Q^2+8Q^4/s)(t+u)}{2tu} + \frac{3(t^3+t^2u+tu^2+u^3)}{stu} - \frac{4Q^2}{s} + 2 \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{9}{8st} L_{\lambda t} \left( \frac{4s^2 + t^2 + 4su + u^2}{s + Q^2 - s_2} + 2Q^2 - 4s - t - 3u \right) \\
& + \frac{9}{8su} L_{\lambda u} \left( \frac{4s^2 + u^2 + 4st + t^2}{s + Q^2 - s_2} + 2Q^2 - 4s - u - 3t \right) \\
& + \frac{9}{8t} L_{st} \left( \frac{-2s_2 + 3t + u}{s} + \frac{2(2s^2 + 2su + u^2)}{s_2 u} \right. \\
& \quad \left. - \frac{2(2s - 2s_2 + t + 2u)}{t} + \frac{4s^2 + t^2 + 4su + u^2}{s(s + Q^2 - s_2)} \right) \\
& + \frac{9}{8u} L_{su} \left( \frac{-2s_2 + 3u + t}{s} + \frac{2(2s^2 + 2st + t^2)}{s_2 t} \right. \\
& \quad \left. - \frac{2(2s - 2s_2 + u + 2t)}{u} + \frac{4s^2 + u^2 + 4st + t^2}{s(s + Q^2 - s_2)} \right) \\
& + \frac{27(t - u)^2(t + u)(-2Q^2(2s + t + u) + (t + u)(4s + t + u))}{8tu\lambda^4} \\
& - \frac{9(t^3 + t^2u + tu^2 + u^3 - Q^2(t + u)(2s + t + u) + s(6t^2 - 4tu + 6u^2))}{4tu\lambda^2} \\
& + \frac{9s^2}{4u(s_2 - t)^2} + \frac{9s^2}{4t(s_2 - u)^2} + \frac{9}{4} \left( 2 + \frac{3s^2}{tu} \right) \left( \frac{1}{s_2 - t} + \frac{1}{s_2 - u} \right) \\
& + \frac{9}{4s} \left( \frac{1}{t^2} + \frac{1}{u^2} \right) \left( 4Q^4 + 2tu - Q^2(7s + 4(t + u)) \right) + \frac{9(3s(t + u) - 4tu)}{8stu}, \tag{34}
\end{aligned}$$

$$\begin{aligned}
A_4 = & \frac{L_\lambda}{\lambda^5} \left( \frac{3}{4} su^3 - \frac{3}{2} stu^2 - \frac{3}{2} st^2 u + \frac{3}{4} st^3 + 3s^2 us_2 + 3s^2 ts_2 - 6s^2 tu - 3s^3 u - 3s^3 t \right. \\
& + \frac{3su^4}{4t} + \frac{3st^4}{4u} - 3 \frac{s^2 u^2 s_2}{t} + 3 \frac{s^2 u^3}{t} + \frac{3s^2 u^3}{4(s_2 - Q^2)} + \frac{3s^2 u^3}{4(s - Q^2)} + \frac{3s^2 u^4}{4t(s - Q^2)} \\
& + \frac{3s^2 u^4}{4t(s_2 - Q^2)} - \frac{3s^2 tu^2}{2(s_2 - Q^2)} - \frac{3s^2 tu^2}{2(s - Q^2)} - 3 \frac{s^2 t^2 s_2}{u} - \frac{3s^2 t^2 u}{2(s_2 - Q^2)} - \frac{3s^2 t^2 u}{2(s - Q^2)} \\
& + 3 \frac{s^2 t^3}{u} + \frac{3s^2 t^3}{4(s_2 - Q^2)} + \frac{3s^2 t^3}{4(s - Q^2)} + \frac{3s^2 t^4}{4u(s - Q^2)} + \frac{3s^2 t^4}{4u(s_2 - Q^2)} + 3 \frac{s^3 u^2}{t} \\
& \left. + 3 \frac{s^3 t^2}{u} \right) + \frac{L_\lambda}{\lambda^3} \left( \frac{1}{2} us_2 + \frac{1}{2} ts_2 - tu + 4ss_2 - \frac{5}{2} su - \frac{5}{2} st - 6s^2 - \frac{1}{2} \frac{u^2 s_2}{t} + \frac{1}{2} \frac{u^3}{t} \right. \\
& - \frac{1}{2} \frac{t^2 s_2}{u} + \frac{1}{2} \frac{t^3}{u} - 2 \frac{sus_2}{t} + \frac{1}{2} \frac{sus_2}{(s - Q^2)} + \frac{3su^2}{2t} - \frac{1}{2} \frac{su^2 s_2}{t(s - Q^2)} + \frac{1}{2} \frac{su^3}{t(s - Q^2)} \\
& \left. + \frac{1}{2} \frac{su^3}{t(s_2 - Q^2)} - 2 \frac{sts_2}{u} + \frac{1}{2} \frac{sts_2}{(s - Q^2)} - \frac{stu}{(s_2 - Q^2)} - \frac{stu}{(s - Q^2)} + \frac{3st^2}{2u} \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} \frac{st^2s_2}{u(s-Q^2)} + \frac{1}{2} \frac{st^3}{u(s-Q^2)} + \frac{1}{2} \frac{st^3}{u(s_2-Q^2)} + \frac{s^2s_2}{t} + \frac{s^2s_2}{u} + \frac{s^2u}{t} - \frac{1}{2} \frac{s^2u}{(s-Q^2)} \\
& -\frac{1}{2t} \frac{s^2u^2}{(s-Q^2)} - \frac{s^2u^2}{t(s_2-Q^2)} + \frac{s^2t}{u} - \frac{1}{2} \frac{s^2t}{(s-Q^2)} - \frac{1}{2} \frac{s^2t^2}{u(s-Q^2)} - \frac{s^2t^2}{u(s_2-Q^2)} \\
& -\frac{s^3}{t} - \frac{s^3}{u} \Big) + \frac{L_\lambda}{\lambda} \left( -2 + 2\frac{s_2^2}{u(s-Q^2)} + \frac{s_2^2}{u(s_2-Q^2)} + 2\frac{s_2^2}{t(s-Q^2)} + \frac{s_2^2}{t(s_2-Q^2)} \right. \\
& + 2\frac{s_2^2}{su} + 2\frac{s_2^2}{st} - \frac{us_2}{t(s-Q^2)} - \frac{us_2}{t(s_2-Q^2)} - 4\frac{us_2}{st} + \frac{1}{2} \frac{u^2}{t(s-Q^2)} + \frac{1}{2} \frac{u^2}{t(s_2-Q^2)} \\
& + \frac{7u^2}{2ts_2} - \frac{u^2}{s(s+Q^2-s_2)} + \frac{7u^2}{2st} - \frac{ts_2}{u(s-Q^2)} - \frac{ts_2}{u(s_2-Q^2)} - 4\frac{ts_2}{su} \\
& - 2\frac{tu}{s(s+Q^2-s_2)} + \frac{1}{2} \frac{t^2}{u(s-Q^2)} + \frac{1}{2} \frac{t^2}{u(s_2-Q^2)} + \frac{7t^2}{2us_2} - \frac{t^2}{s(s+Q^2-s_2)} + \frac{7t^2}{2su} \\
& - \frac{1}{2} \frac{ss_2}{u(s-Q^2)} - \frac{ss_2}{u(s_2-Q^2)} - \frac{1}{2} \frac{ss_2}{t(s-Q^2)} - \frac{ss_2}{t(s_2-Q^2)} + \frac{su}{(s+Q^2-s_2)(s-Q^2)} \\
& + \frac{1}{2} \frac{su}{t(s-Q^2)} + \frac{1}{2} \frac{su}{t(s_2-Q^2)} + 10\frac{su}{ts_2} + \frac{st}{(s+Q^2-s_2)(s-Q^2)} + \frac{1}{2} \frac{st}{u(s-Q^2)} \\
& + \frac{1}{2} \frac{st}{u(s_2-Q^2)} + 10\frac{st}{us_2} + 6\frac{s^2}{(s+Q^2-s_2)(s-Q^2)} + \frac{3}{4} \frac{s^2}{u(s-Q^2)} + \frac{9}{4} \frac{s^2}{u(s_2-Q^2)} \\
& + 8\frac{s^2}{us_2} + \frac{3}{4} \frac{s^2}{t(s-Q^2)} + \frac{9}{4} \frac{s^2}{t(s_2-Q^2)} + 8\frac{s^2}{ts_2} - \frac{17s}{4t} - \frac{17s}{4u} + 20\frac{s}{s_2} \\
& + 6\frac{s}{(s+Q^2-s_2)} + 3\frac{s}{(s_2-Q^2)} + \frac{9t}{2s} + \frac{3t}{2u} + \frac{19t}{2s_2} + 3\frac{t}{(s+Q^2-s_2)} + \frac{1}{2} \frac{t}{(s_2-Q^2)} \\
& + \frac{5}{2} \frac{t}{(s-Q^2)} + \frac{9u}{2s} + \frac{3u}{2t} + \frac{19u}{2s_2} + 3\frac{u}{(s+Q^2-s_2)} + \frac{1}{2} \frac{u}{(s_2-Q^2)} + \frac{5}{2} \frac{u}{(s-Q^2)} \\
& - 6\frac{s_2}{s} - \frac{3s_2}{2t} - \frac{3s_2}{2u} - 2\frac{s_2}{(s_2-Q^2)} - 6\frac{s_2}{(s-Q^2)} \Big) + \frac{1}{\lambda^4} \left( -\frac{3}{4}u^3 - 3tus_2 + \frac{3}{2}tu^2 \right. \\
& + \frac{3}{2}t^2u - \frac{3}{4}t^3 - 3sus_2 - 3sts_2 + 3stu + 3s^2u + 3s^2t + \frac{3u^3s_2}{2t} - \frac{3u^4}{4t} + \frac{3t^3s_2}{2u} \\
& - \frac{3t^4}{4u} + 3\frac{su^2s_2}{t} - \frac{3su^3}{2t} - \frac{3}{4} \frac{su^3}{(s_2-Q^2)} - \frac{3}{4} \frac{su^3}{(s-Q^2)} + \frac{3}{2} \frac{su^3s_2}{t(s-Q^2)} \\
& + \frac{3}{2} \frac{su^3s_2}{t(s_2-Q^2)} - \frac{3}{4} \frac{su^4}{t(s-Q^2)} - \frac{3}{4} \frac{su^4}{t(s_2-Q^2)} - 3\frac{stus_2}{(s_2-Q^2)} - 3\frac{stus_2}{(s-Q^2)} \\
& + \frac{3}{2} \frac{stu^2}{(s_2-Q^2)} + \frac{3}{2} \frac{stu^2}{(s-Q^2)} + 3\frac{st^2s_2}{u} + \frac{3}{2} \frac{st^2u}{(s_2-Q^2)} + \frac{3}{2} \frac{st^2u}{(s-Q^2)} - \frac{3st^3}{2u} \\
& - \frac{3}{4} \frac{st^3}{(s_2-Q^2)} - \frac{3}{4} \frac{st^3}{(s-Q^2)} + \frac{3}{2} \frac{st^3s_2}{u(s-Q^2)} + \frac{3}{2} \frac{st^3s_2}{u(s_2-Q^2)} - \frac{3}{4} \frac{st^4}{u(s-Q^2)} \\
& \left. - \frac{3}{4} \frac{st^4}{u(s_2-Q^2)} - 3\frac{s^2u^2}{t} - 3\frac{s^2t^2}{u} \right) + \frac{1}{\lambda^2} \left( -s_2 + u + t + 5s - \frac{1}{2} \frac{us_2}{t} \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} \frac{us_2}{(s_2 - Q^2)} + \frac{1}{2} \frac{u^2 s_2}{t(s_2 - Q^2)} - \frac{1}{4} \frac{u^3}{t(s_2 - Q^2)} - \frac{1}{2} \frac{ts_2}{u} - \frac{1}{2} \frac{ts_2}{(s_2 - Q^2)} \\
& + \frac{1}{2} \frac{tu}{(s_2 - Q^2)} + \frac{1}{2} \frac{t^2 s_2}{u(s_2 - Q^2)} - \frac{1}{4} \frac{t^3}{u(s_2 - Q^2)} - \frac{ss_2}{t} - \frac{ss_2}{u} + \frac{ss_2}{(s_2 - Q^2)} \\
& + \frac{ss_2}{(s - Q^2)} - \frac{3}{2} \frac{su}{t} + \frac{1}{2} \frac{su}{(s - Q^2)} - \frac{3}{2} \frac{sus_2}{t(s - Q^2)} - \frac{3}{2} \frac{sus_2}{t(s_2 - Q^2)} + \frac{1}{2} \frac{su^2}{t(s - Q^2)} \\
& + \frac{su^2}{t(s_2 - Q^2)} - \frac{3}{2} \frac{st}{u} + \frac{1}{2} \frac{st}{(s - Q^2)} - \frac{3}{2} \frac{sts_2}{u(s - Q^2)} - \frac{3}{2} \frac{sts_2}{u(s_2 - Q^2)} + \frac{1}{2} \frac{st^2}{u(s - Q^2)} \\
& + \frac{st^2}{u(s_2 - Q^2)} + \frac{s^2}{t} + \frac{s^2}{u} \Big) + 2 \frac{s_2^2}{u^2(s - Q^2)} + \frac{3}{2} \frac{s_2^2}{u^2(s_2 - Q^2)} + 2 \frac{s_2^2}{t^2(s - Q^2)} \\
& + \frac{3}{2} \frac{s_2^2}{t^2(s_2 - Q^2)} - 2 \frac{us_2}{t^2(s - Q^2)} - \frac{3}{2} \frac{us_2}{t^2(s_2 - Q^2)} - \frac{u^2}{t^2(s_2 - t)} - 2 \frac{ts_2}{u^2(s - Q^2)} \\
& - \frac{3}{2} \frac{ts_2}{u^2(s_2 - Q^2)} - \frac{t^2}{u^2(s_2 - u)} - 2 \frac{ss_2}{u^2(s - Q^2)} - \frac{3}{2} \frac{ss_2}{u^2(s_2 - Q^2)} - 2 \frac{ss_2}{t^2(s - Q^2)} \\
& - \frac{3}{2} \frac{ss_2}{t^2(s_2 - Q^2)} - 2 \frac{su}{t^2(s_2 - t)} - 2 \frac{st}{u^2(s_2 - u)} + 3 \frac{s^2}{(s_2 - u)^3} + 3 \frac{s^2}{(s_2 - t)^3} \\
& + \frac{s^2}{u(s_2 - u)^2} + \frac{3}{2} \frac{s^2}{u(s_2 - t)(s_2 - Q^2)} - \frac{13}{2} \frac{s^2}{u(s_2 - t)^2} - \frac{s^2}{u^2(s_2 - u)} \\
& + \frac{3}{2} \frac{s^2}{t(s_2 - u)(s_2 - Q^2)} - \frac{13}{2} \frac{s^2}{t(s_2 - u)^2} + \frac{s^2}{t(s_2 - t)^2} + 3 \frac{s^2}{tu(s_2 - Q^2)} \\
& + \frac{1}{2} \frac{s^2}{tu(s_2 - u)} + \frac{1}{2} \frac{s^2}{tu(s_2 - t)} - \frac{s^2}{t^2(s_2 - t)} + \frac{1}{2} \frac{s^2 s_2}{u(s_2 - t)^2(s_2 - Q^2)} \\
& + \frac{1}{2} \frac{s^2 s_2}{t(s_2 - u)^2(s_2 - Q^2)} + 3 \frac{s^3}{u(s_2 - t)^3} + 3 \frac{s^3}{t(s_2 - u)^3} + \frac{s^3}{tu(s_2 - u)^2} \\
& + \frac{s^3}{tu(s_2 - t)^2} + \frac{s}{(s_2 - u)(s - Q^2)} - 4 \frac{s}{(s_2 - u)^2} + \frac{s}{(s_2 - t)(s - Q^2)} \\
& - 4 \frac{s}{(s_2 - t)^2} + \frac{1}{4} \frac{s}{u(s - Q^2)} - \frac{1}{4} \frac{s}{u(s_2 - Q^2)} + 5 \frac{s}{u(s_2 - t)} + \frac{1}{2} \frac{s}{u^2} \\
& + \frac{1}{4} \frac{s}{t(s - Q^2)} - \frac{1}{4} \frac{s}{t(s_2 - Q^2)} + 5 \frac{s}{t(s_2 - u)} + \frac{1}{2} \frac{s}{t^2} + \frac{t}{u(s - Q^2)} + \frac{1}{4} \frac{t}{u(s_2 - Q^2)} \\
& + \frac{1}{2} \frac{t}{u^2} - 2 \frac{t}{s^2} + \frac{u}{t(s - Q^2)} + \frac{1}{4} \frac{u}{t(s_2 - Q^2)} + \frac{1}{2} \frac{u}{t^2} - 2 \frac{u}{s^2} - 2 \frac{s_2}{u(s - Q^2)} \\
& - \frac{s_2}{u(s_2 - Q^2)} + \frac{1}{2} \frac{s_2}{u^2} - 2 \frac{s_2}{t(s - Q^2)} - \frac{s_2}{t(s_2 - Q^2)} + \frac{1}{2} \frac{s_2}{t^2} + 2 \frac{s_2}{s^2} - 5 \frac{1}{s} - \frac{3}{4} \frac{1}{t} \\
& - \frac{3}{4} \frac{1}{u} + 2 \frac{1}{(s_2 - t)} + 2 \frac{1}{(s_2 - u)} - \frac{1}{2} \frac{1}{(s_2 - Q^2)} - \frac{1}{(s - Q^2)} \\
& + \left\{ L_t \left( \frac{t^2}{u^2(s_2 - u)} - \frac{t^3}{u^2(s_2 - u)^2} - 4 \frac{st}{(s_2 - u)^3} + 2 \frac{st}{u^2(s_2 - u)} - 3 \frac{st^2}{u^2(s_2 - u)^2} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -12 \frac{s^2}{(s_2 - u)^3} + \frac{s^2}{u^2(s_2 - u)} + 8 \frac{s^2}{t(s_2 - u)^2} - 4 \frac{s^2}{tu(s_2 - u)} + 4 \frac{s^2}{tus_2} + 3 \frac{s^2 t}{(s_2 - u)^4} \\
& + \frac{s^2 t}{u(s_2 - u)^3} - 3 \frac{s^2 t}{u^2(s_2 - u)^2} + 6 \frac{s^3}{(s_2 - u)^4} + 2 \frac{s^3}{u(s_2 - u)^3} - \frac{s^3}{u^2(s_2 - u)^2} \\
& - 8 \frac{s^3}{t(s_2 - u)^3} + 3 \frac{s^4}{t(s_2 - u)^4} + \frac{s^4}{tu(s_2 - u)^3} + 9 \frac{s}{(s_2 - u)^2} - 4 \frac{s}{u(s_2 - u)} + 6 \frac{s}{us_2} \\
& - 4 \frac{s}{t(s_2 - u)} + 4 \frac{s}{ts_2} + 2 \frac{t}{(s_2 - u)^2} - \frac{t}{u(s_2 - u)} + \frac{5}{2} \frac{t}{us_2} + \frac{3}{2} \frac{t}{su} + \frac{u}{ts_2} - \frac{s_2}{su} \\
& + \left. \frac{31}{2s} + \frac{1}{u} + 3 \frac{1}{s_2} - 3 \frac{1}{(s_2 - u)} \right) \\
& + L_{\lambda t} \left( \frac{u^2/4 + su + s^2}{t(s + Q^2 - s_2)(s - Q^2)} - \frac{1}{2} \frac{s}{t(s - Q^2)} + \frac{1}{4} \frac{t}{(s + Q^2 - s_2)(s - Q^2)} \right. \\
& \left. - \frac{1}{4} \frac{u}{t(s - Q^2)} - \frac{1}{2} \frac{s_2}{t(s - Q^2)} + \frac{1}{4} \frac{1}{(s - Q^2)} \right) + L_{st} \left( \frac{s_2^2 - us_2}{t^2(s_2 - Q^2)} \right. \\
& + \frac{1}{4} \frac{u^2}{t(s + Q^2 - s_2)(s - Q^2)} - \frac{ss_2}{t^2(s_2 - Q^2)} + \frac{su}{t(s + Q^2 - s_2)(s - Q^2)} \\
& + \frac{s^2}{t(s + Q^2 - s_2)(s - Q^2)} + \frac{s^2}{tu(s_2 - Q^2)} + \frac{1}{2} \frac{s}{t(s - Q^2)} + \frac{s}{t(s_2 - Q^2)} \\
& + \frac{1}{4} \frac{t}{(s + Q^2 - s_2)(s - Q^2)} + \frac{1}{4} \frac{u}{t(s - Q^2)} + \frac{1}{2} \frac{u}{t(s_2 - Q^2)} - \frac{1}{2} \frac{s_2}{t(s - Q^2)} \\
& \left. - \frac{s_2}{t(s_2 - Q^2)} + \frac{3}{4} \frac{1}{(s - Q^2)} \right) + I_4(1, 1, t, -1) \left( \frac{15}{2} s_2 - \frac{3}{4} u + \frac{13}{4} t + \frac{35}{2} s - 6 \frac{s_2^2}{s} \right. \\
& - 6 \frac{s_2^2}{t} + \frac{s_2^2}{u} + 2 \frac{s_2^3}{st} + \frac{23}{2} \frac{us_2}{s} + 10 \frac{us_2}{t} - 6 \frac{us_2^2}{st} - \frac{23}{4} \frac{u^2}{s} - 2 \frac{u^2}{t} - 8 \frac{u^2}{s_2} \\
& - \frac{3}{4} \frac{u^2}{(s + Q^2 - s_2)} + \frac{13}{2} \frac{u^2 s_2}{st} - \frac{5}{2} \frac{u^3}{ts_2} + \frac{1}{4} \frac{u^3}{s(s + Q^2 - s_2)} - \frac{5}{2} \frac{u^3}{st} + 6 \frac{ts_2}{s} - 3 \frac{ts_2}{u} \\
& - \frac{21}{4} \frac{tu}{s} - \frac{19}{2} \frac{tu}{s_2} - \frac{1}{2} \frac{tu}{(s + Q^2 - s_2)} + \frac{1}{2} \frac{tu^2}{s(s + Q^2 - s_2)} - 2 \frac{t^2}{s} + 3 \frac{t^2}{u} - 5 \frac{t^2}{s_2} \\
& - \frac{1}{4} \frac{t^2}{(s + Q^2 - s_2)} + \frac{1}{4} \frac{t^2 u}{s(s + Q^2 - s_2)} - \frac{t^3}{us_2} + 2 \frac{ss_2}{t} - 6 \frac{ss_2}{u} + 6 \frac{su}{t} \\
& - 24 \frac{su}{s_2} - 2 \frac{su}{(s + Q^2 - s_2)} - \frac{17}{2} \frac{su^2}{ts_2} + 12 \frac{st}{u} - 21 \frac{st}{s_2} - \frac{st}{(s + Q^2 - s_2)} - 6 \frac{st^2}{us_2} \\
& \left. + 6 \frac{s^2}{t} + 12 \frac{s^2}{u} - 24 \frac{s^2}{s_2} - \frac{s^2}{(s + Q^2 - s_2)} - 10 \frac{s^2 u}{ts_2} - 12 \frac{s^2 t}{us_2} - 8 \frac{s^3}{us_2} - 4 \frac{s^3}{ts_2} \right) \\
& + I_4(1, 1, t, 1) \left( -\frac{1}{2} s_2 - \frac{3}{4} u + \frac{1}{4} t - \frac{1}{2} s - \frac{1}{2} \frac{us_2}{s} - \frac{1}{4} \frac{u^2}{s} - 3 \frac{u^2}{t} \right)
\end{aligned}$$



$$\begin{aligned}
& -\frac{3}{4} \frac{u^2}{(s+Q^2-s_2)} + \frac{1}{4} \frac{u^3}{s(s+Q^2-s_2)} - \frac{u^3}{st} + \frac{1}{4} \frac{tu}{s} - \frac{1}{2} \frac{tu}{(s+Q^2-s_2)} \\
& + \frac{1}{2} \frac{tu^2}{s(s+Q^2-s_2)} - \frac{1}{4} \frac{t^2}{(s+Q^2-s_2)} + \frac{1}{4} \frac{t^2u}{s(s+Q^2-s_2)} - 3 \frac{su}{t} \\
& - 2 \left( \frac{su}{(s+Q^2-s_2)} - \frac{st}{(s+Q^2-s_2)} - \frac{s^2}{t} - \frac{s^2}{(s+Q^2-s_2)} \right) \\
& + I_4(1, 2, t, -1) \left( 10s_2^2 - 20us_2 + 10u^2 - 20ts_2 + 20tu + 10t^2 - 16ss_2 + 16su \right. \\
& + 16st + 8s^2 - 2\frac{s_2^3}{s} + 6\frac{us_2^2}{s} - 6\frac{u^2s_2}{s} + 2\frac{u^3}{s} + 6\frac{ts_2^2}{s} - 12\frac{tus_2}{s} + 6\frac{tu^2}{s} - 6\frac{t^2s_2}{s} \\
& \left. + 6\frac{t^2u}{s} + 2\frac{t^3}{s} \right) + \left\{ t \longleftrightarrow u \right\}. \tag{35}
\end{aligned}$$

Here,  $A = \frac{s_2^{\max}}{\lambda(x, y, z)}$  as defined in Eq. (28),  $\lambda = \lambda(s, Q^2, s_2)$  with  $\lambda(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2(xy + yz + zx)}$  being Källén's function, we have introduced the short-hand notations

$$\begin{aligned}
L_s &= \ln \frac{s}{Q^2}, \quad L_t = \ln \frac{-t}{Q^2}, \quad L_u = \ln \frac{-u}{Q^2}, \quad L_A = \ln \frac{A}{Q^2}, \quad L_{\mu_F} = \ln \frac{\mu_F^2}{Q^2}, \\
L_{st} &= \ln \frac{st^2}{Q^2(s_2-t)}, \quad L_{su} = \ln \frac{su^2}{Q^2(s_2-u)}, \quad L_\lambda = \ln \frac{s+Q^2-s_2+\lambda}{s+Q^2-s_2-\lambda}, \\
L_{\lambda t} &= \ln \frac{sQ^2(s_2-t)^2}{[s_2(2Q^2-u) - Q^2t]^2}, \quad L_{\lambda u} = \ln \frac{sQ^2(s_2-u)^2}{[s_2(2Q^2-t) - Q^2u]^2}, \tag{36}
\end{aligned}$$

and we have adopted the following phase-space integrals from Appendix C in Ref. [29]:

$$\begin{aligned}
I_4(1, 1, t, 1) &= \frac{1}{\sqrt{X_+}} \ln \frac{u(Q^2-s_2) + 2Q^2s + \sqrt{X_+}}{u(Q^2-s_2) + 2Q^2s - \sqrt{X_+}}, \\
I_4(1, 1, t, -1) &= \frac{1}{\sqrt{X_-}} \ln \frac{2Q^4 - Q^2(t+u) - st + \sqrt{X_-}}{2Q^4 - Q^2(t+u) - st - \sqrt{X_-}}, \\
I_4(1, 2, t, -1) &= \frac{4Q^6 + st(t+u) - 2Q^4(s+2(t+u)) + Q^2(-2s^2 - 2s(t+u) + (t+u)^2)}{2Q^2sX_-} \\
& + \frac{2Q^6 - s(s+u)t + Q^2u(s+t+u) - Q^4(2s+3u+t)}{2X_-} I_4(1, 1, t, -1), \tag{37}
\end{aligned}$$

where

$$\begin{aligned}
X_+ &= [u(Q^2-s_2) + 2Q^2s]^2 - 4Q^4s(s+u), \\
X_- &= [2Q^4 - Q^2(t+u) - st]^2 - 4Q^4s(s+u). \tag{38}
\end{aligned}$$

## B Cross section of $q + g \rightarrow Z + X$ through $\mathcal{O}(\alpha^2\alpha_s)$

In this appendix, we consider  $\mathcal{O}(\alpha^2\alpha_s)$  contributions to  $q + g \rightarrow Z + X$ . This includes the virtual QED corrections to subprocess (12) and the real corrections from subprocess (21). We have

$$\frac{s d\sigma^{qg}}{dt du} = \frac{2\pi\alpha\alpha_s Q_q^2 (v_q^2 + a_q^2) C_F}{(N_c^2 - 1)s} \left[ \delta(s_2) B_0 + \frac{\alpha}{\pi} (\delta(s_2) B_1 + B_2) \right], \quad (39)$$

where

$$B_0 = - \left[ \frac{s}{t} + \frac{t}{s} + 2 \left( \frac{u}{t} + \frac{u}{s} + \frac{u^2}{st} \right) \right], \quad (40)$$

$$\begin{aligned} B_1 = & B_0 \left[ L_{\mu_F} (L_u - L_A - \frac{3}{4}) + \frac{1}{2} L_A^2 - \frac{3}{4} L_A \right] \\ & + \frac{(t+u)^2 + u^2}{2st} \left( 2\text{Li}_2\left(\frac{Q^2}{s}\right) + L_s^2 + 2L_s L_u - 2L_s \ln \frac{s-Q^2}{Q^2} \right) \\ & - \frac{(s+u)^2 + u^2}{st} \left( \text{Li}_2\left(\frac{t}{Q^2}\right) - L_t L_u + L_t \ln \frac{s+u}{Q^2} \right) \\ & - \frac{(s+u)^2 + (t+u)^2 + 2u^2}{st} \left( \text{Li}_2\left(\frac{u}{Q^2}\right) + L_u \ln \frac{s+t}{Q^2} \right) \\ & + \frac{(s+u)^2 + (t+u)^2}{2st} L_u^2 - \frac{2u(2s+2t+u)}{(s+t)^2} L_u + \left( -\frac{2u+t}{s+u} + \frac{st}{2(s+u)^2} \right) L_t \\ & - \left( \frac{s+4u}{2(t+u)} + \frac{su}{2(t+u)^2} \right) L_s - \frac{2(2s^2+4su+5u^2)}{st} \zeta(2) - \frac{u}{2(t+u)} \\ & - \frac{2u}{s+t} + \frac{s}{2(s+u)} + \frac{11(s^2+t^2) - 2st + 20(su+tu) + 18u^2}{4st}, \end{aligned} \quad (41)$$

$$\begin{aligned} B_2 = & \frac{1}{\lambda} L_\lambda \left[ \frac{3s(t-u)^2(t+u)(-2Q^2+t+u)}{8t\lambda^4} \right. \\ & + \frac{1}{\lambda^2} \left( \frac{(t-u)^2(t+u) - s(t^2+u^2)}{4t} + \frac{(t-u)(t+u)^2}{8s} \right. \\ & \left. \left. + \frac{Q^2(-t^3 + s^2(3t-u) + tu^2 - 2s(3t^2 - 4tu + u^2))}{4st} \right) \right. \\ & \left. + \frac{64Q^4 + 7s^2 + 7t^2 + 21tu + 16u^2 + 2s(t+u) - 2Q^2(30s + 15t + 16u)}{8st} \right] \end{aligned}$$

$$\begin{aligned}
& - \frac{t^3 + 3t^2u + 4tu^2 + 2u^3}{2st(s_2)_{A+}} \Big] + L_{\mu_F} \frac{s^2 + t^2 + 2su + 2tu + 2u^2}{st(s_2)_{A+}} \\
& + L_{su} \left[ - \frac{t^2 + 2tu + 2u^2}{2st(s_2)_{A+}} + \frac{1}{2s} - \frac{1}{2t} + \frac{Q^2}{st} + \frac{Q^2(Q^2 - t - s)}{2su^2} \right. \\
& \quad \left. - \frac{1}{u} + \frac{Q^2}{su} - \frac{t}{2su} + \frac{u}{st} - \frac{2Q^4 - 2Q^2(s+t) + (s+t)^2}{2st(Q^2 - u)} \right] \\
& + \ln \frac{tu - s_2Q^2}{(s_2 - t)(s_2 - u)} \left[ - \frac{s^2 + 2su + 2u^2}{st(s_2)_{A+}} + \left( Q^2 + t - \frac{2Q^2}{st} \left( (s-u)^2 + (t-u)^2 \right) \right. \right. \\
& \quad \left. \left. + \frac{t^2 + (t-2u)^2}{s} \right) \frac{1}{tu - s_2Q^2} + \frac{2Q^2(u - s_2) + (s+t)^2}{st(Q^2 - u)} + \frac{-2Q^2 + 3t - 2u}{st} \right] \\
& + L_{\lambda t} \left( \frac{2Q^2(u - s_2) + (s+t)^2}{2st(Q^2 - u)} - \frac{s_2 - 2Q^2}{st} \right) \\
& - \left[ \frac{(s+u)^2 + (t+u)^2}{sts_2} \ln \frac{s_2}{Q^2} \right]_{A+} - \ln \frac{\mu_F^2}{s_2} \left( - \frac{st}{(s_2 - t)^3} + \frac{u+t}{(s_2 - t)^2} \right. \\
& \quad - \frac{3s^2 - 4st + 2t^2 + 6su - 4tu + 4u^2}{2st(s_2 - t)} - \frac{1}{s_2 - u} + \frac{Q^2(u - s_2)}{2u^2s} \\
& \quad \left. + \frac{t((s+t)^2 + su + (t-4u)^2) - Q^2(2(s^2 + t^2) + st + 4(su - tu + u^2))}{st(tu - s_2Q^2)} \right. \\
& \quad \left. + \frac{5}{2s} - \frac{Q^2}{2t^2} + \frac{3}{2t} - \frac{1}{u} + \frac{Q^2}{su} - \frac{t}{2su} \right) \\
& + \frac{3(s^2 + t^2 + 2su + 2tu + 2u^2)}{4st(s_2)_{A+}} + \frac{3s(t-u)^2(t+u)}{4t\lambda^4} \\
& + \frac{-2t^3 - 4Q^2(s-2t)(t-u) + 2tu^2 - 2s^2(t+u) + s(7t^2 - 10tu + 3u^2)}{8st\lambda^2} \\
& + \frac{t(s^2 + su + 4(t-u)u) - Q^2(s(t-4u) + 4u(t-u))}{2st(tu - s_2Q^2)} + \frac{4st}{(s_2 - t)^3} \\
& - \frac{4u + 8t - 3s}{2(s_2 - t)^2} + \frac{-s^2 - 4st + 3t^2 + 2su - 2tu + u^2}{2st(s_2 - t)} - \frac{s}{t(s_2 - u)} \\
& - \frac{3}{2s} - \frac{Q^2}{4t^2} + \frac{3}{8t} - \frac{Q^2(u - s_2)}{2u^2s} - \frac{1}{u} + \frac{Q^2}{su} - \frac{t}{su}. \tag{42}
\end{aligned}$$

## C Cross section of $q + q \rightarrow Z + X$ through $\mathcal{O}(\alpha^2\alpha_s)$

In this appendix, we consider  $\mathcal{O}(\alpha^2\alpha_s)$  contributions to  $q + q \rightarrow Z + X$ . This includes the interference contributions from subprocess (18) involving a virtual photon or  $Z$  boson.

We have

$$\frac{s d\sigma^{qq}}{dt du} = \frac{2\pi\alpha_s\alpha^2 C_F}{N_c s} [Q_q^2(v_q^2 + a_q^2)C_1 + (v_q^4 + 6v_q^2 a_q^2 + a_q^4)C_2], \quad (43)$$

where

$$\begin{aligned} C_1 = & \frac{9(2Q^4 + 2s^2 + 2s(t+u) + (t+u)^2 - 2Q^2(2s+t+u))}{4tu} \ln \frac{(s_2-t)(s_2-u)}{ss_2} \\ & + \frac{9(2s_2Q^2(t^2+u^2) - tu(t+u)^2)}{4t^2u^2} \\ & + s \left\{ L_{\lambda t} \frac{9(2s^2 + (t+u-2s_2)(s+Q^2-s_2))}{8(s+Q^2-s_2)(s_2-t)t} \right. \\ & + \frac{9}{4} L_{st} \left( \frac{u-t}{2t(s_2-t)} + \frac{1}{s} + \frac{s-s_2+u}{t^2} + \frac{s_2^2}{stu} + \frac{t-2s_2}{2su} + \frac{u-2s_2}{2st} \right. \\ & \left. \left. + \frac{s^2}{t(s_2-t)(s+Q^2-s_2)} \right) \right\} + s \left\{ u \leftrightarrow t \right\}, \quad (44) \end{aligned}$$

$$\begin{aligned} C_2 = & \left\{ -\frac{1}{4(Q^2+s)^2tu} \ln \frac{(Q^2t - s_2(s+Q^2))^2}{Q^2(s+t)(s_2-t)^2} \right. \\ & \left( Q^2(s_2^2 + (s_2-t)^2 - u(2s+u)) + s(2s_2(s-Q^2) + (t+u)^2) \right) \\ & + L_{st} \left[ \frac{-s^2}{2(Q^2+s)^2t} + \frac{s^2}{2(Q^2+s-s_2)t(s+u)} - \frac{s+t}{4t(s+u)} - \frac{1}{4t} \right. \\ & \left. + \frac{t^2 - u^2 + 2s_2^2 - 2s_2t}{4(Q^2+s)tu} + \frac{s_2Q^2}{2t^2(s+u)} \right] + \frac{L_{\lambda t}(s^2 + (Q^2 - s_2)^2)}{4(Q^2+s-s_2)t(s+u)} \\ & + \ln \frac{Q^2}{s+t} \frac{Q^2(s+t-Q^2)}{2u^2(s+t)} + \frac{1}{4} I_4(1, 1, t, 1) \left[ \frac{s^2(Q^2 - s - s_2)}{(Q^2+s-s_2)(s+u)} \right. \\ & \left. + \frac{(2s_2^2 - 2ts_2 + t(Q^2+s_2))s}{t(s+u)} - \frac{(-2s_2+t+u)^2}{t} \right] \\ & + I_4(1, 1, t, -1) \left[ -\frac{s^3}{2(Q^2+s-s_2)(s+u)} - \frac{s^2Q^2}{2(Q^2+s)^2} + \frac{s(Q^2-u)}{Q^2+s} \right. \\ & + \frac{s_2s(u-Q^2)}{2(Q^2+s)u} - \frac{s(u-t)^2}{4t(Q^2+s)} + \frac{s_2s(u-s_2)}{2(Q^2+s)t} + \frac{s^2(2t-s_2)}{4t(s+u)} \\ & - \frac{5s_2t}{4(s+u)} + \frac{s_2^2(2t-s_2)}{t(s+u)} + \frac{t^2}{4(s+u)} + \frac{s(2s_2-t)}{4(s+u)} + \frac{u^2}{2t} - \frac{5s_2}{2} + \frac{t}{2} + u \\ & \left. - \frac{9s_2u}{4t} + \frac{3s_2^2}{t} - \frac{s_2^2}{u} - \frac{t^2}{4u} + \frac{s_2t}{u} - \frac{5s_2s}{4t} + \frac{s_2s}{2u} \right] - I_4(1, 2, t, -1) \left[ \frac{Q^2s^2}{Q^2+s} \right. \\ & \left. + \frac{s_2(Q^2-u)}{t^2(Q^2+s)} + \frac{2s_2t - u(t+u)}{2(Q^2+s)u^2} \right] \left. \right\} + \left\{ t \leftrightarrow u \right\}. \quad (45) \end{aligned}$$

Here, we have again used the phase-space integrals given in Eq. (37).

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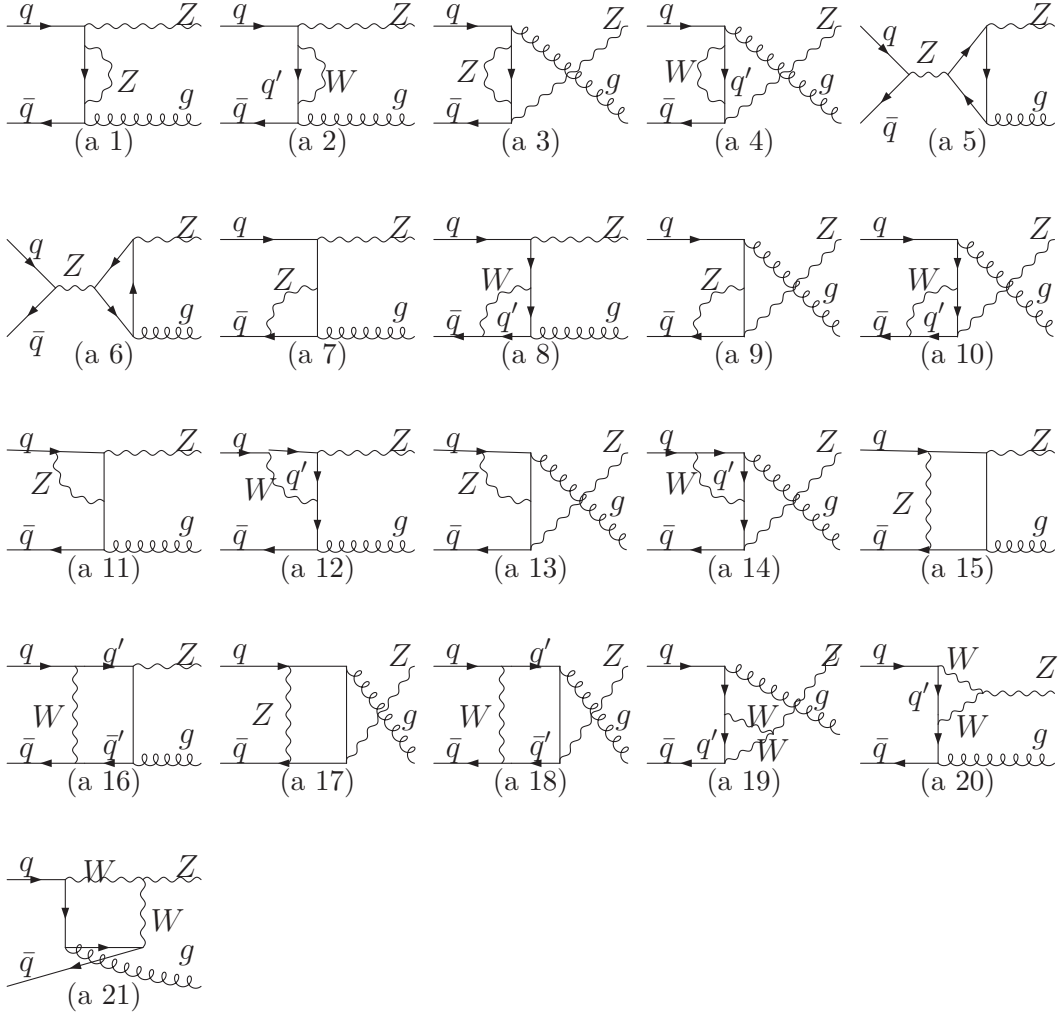


Figure 3: Weak-interaction one-loop diagrams contributing to the partonic subprocess  $q + \bar{q} \rightarrow Z + g$  at  $\mathcal{O}(\alpha^2\alpha_s)$ . The weak-interaction one-loop diagrams contributing to the partonic subprocess  $q + g \rightarrow Z + q$  at  $\mathcal{O}(\alpha^2\alpha_s)$  are obtained by appropriately crossing external legs.



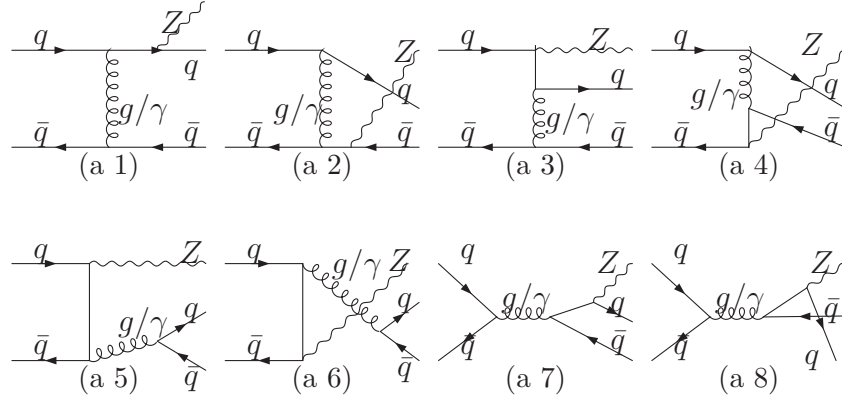


Figure 4: Tree-level diagrams contributing to the partonic subprocess  $q + \bar{q} \rightarrow Z + q + \bar{q}$  at  $\mathcal{O}(\alpha_s^2)$  (with a virtual gluon). Interferences of diagrams (a 5)–(a 8) with a virtual photon or Z boson (gluon with diagrams (a 1)–(a 4) with a virtual gluon (photon or Z boson) contribute at  $\mathcal{O}(\alpha^2\alpha_s)$ .

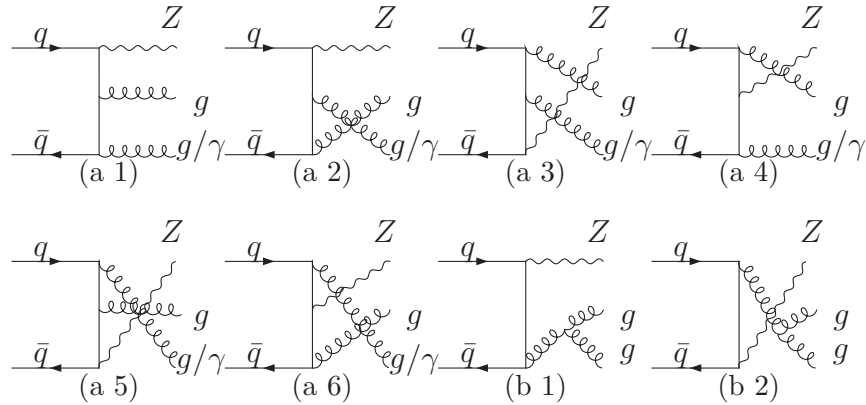


Figure 5: Tree-level diagrams contributing to the partonic subprocess  $q + \bar{q} \rightarrow Z + g + g$  at  $\mathcal{O}(\alpha_s^2)$  and to the partonic subprocess  $q + \bar{q} \rightarrow Z + g + \gamma$  at  $\mathcal{O}(\alpha^2\alpha_s)$ .

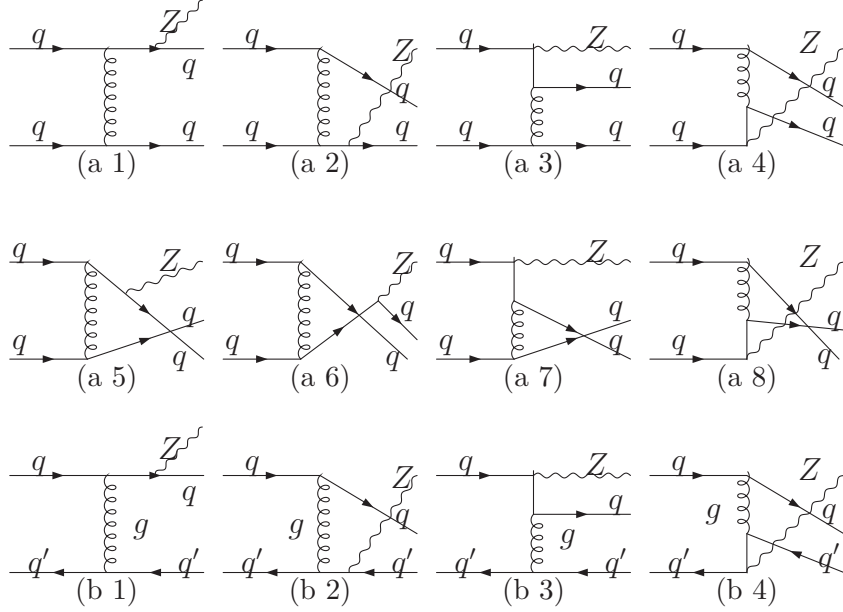


Figure 6: Tree-level diagrams contributing to the partonic subprocesses  $q + q \rightarrow Z + q + q$  and  $q + q' \rightarrow Z + q + q'$  at  $\mathcal{O}(\alpha_s^2)$ . Interferences of diagrams (a 5)–(a 8) with the gluon replaced by a photon or  $Z$  boson with diagrams (a 1)–(a 4) contribute at  $\mathcal{O}(\alpha^2\alpha_s)$ .

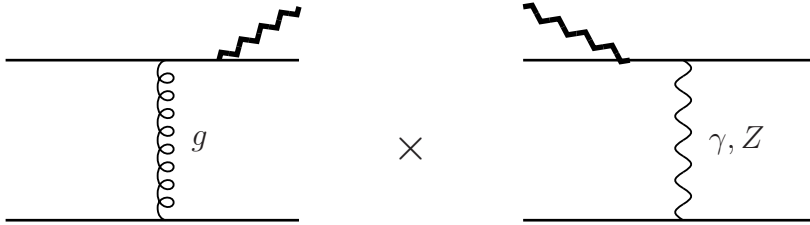
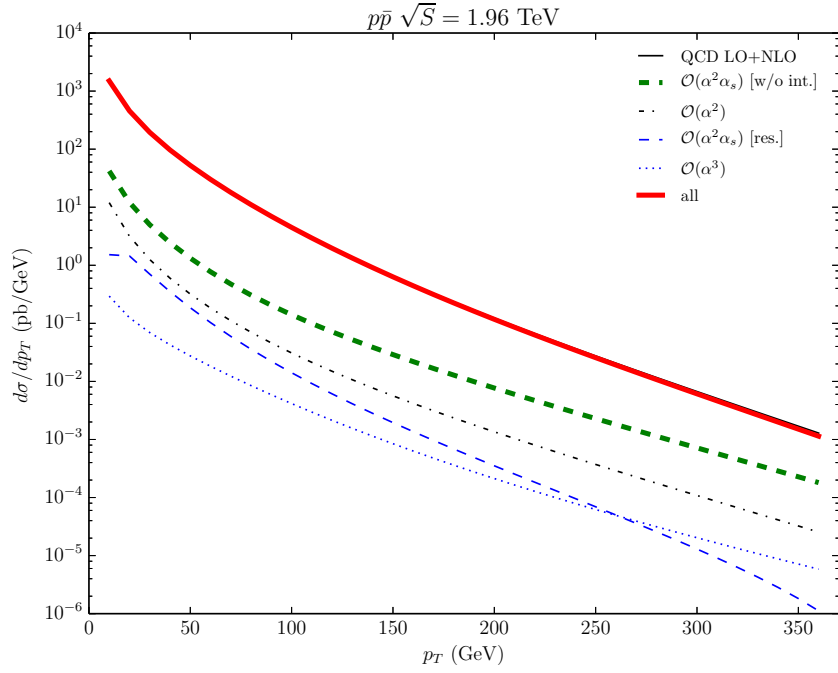
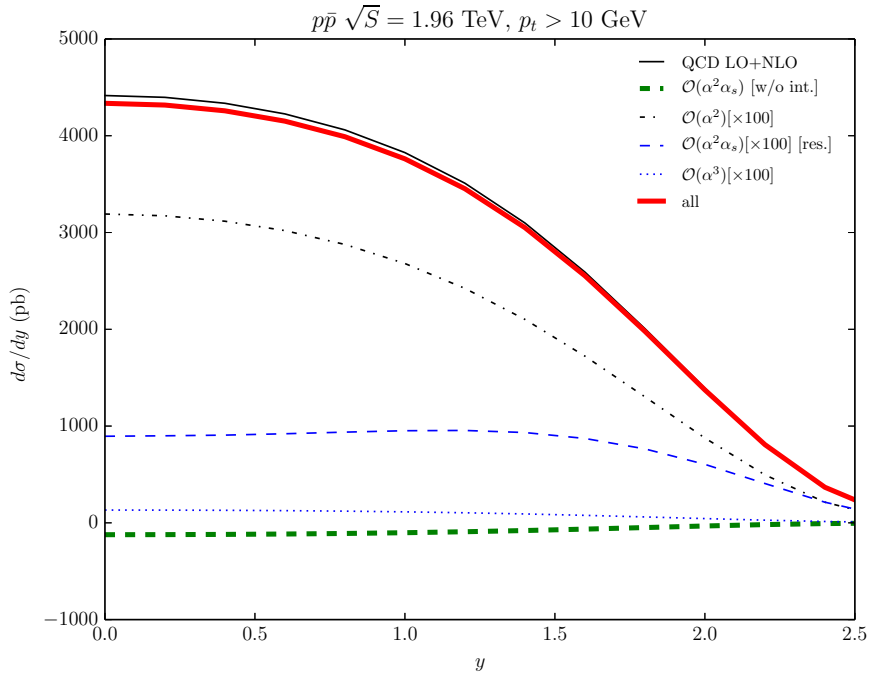


Figure 7:  $\mathcal{O}(\alpha^{1/2}\alpha_s)$  and  $\mathcal{O}(\alpha^{3/2})$  tree-level diagrams interfering to yield  $\mathcal{O}(\alpha^2\alpha_s)$  contributions to subprocesses  $q + \bar{q} \rightarrow Z + q + \bar{q}$  and  $q + q \rightarrow Z + q + q$ .

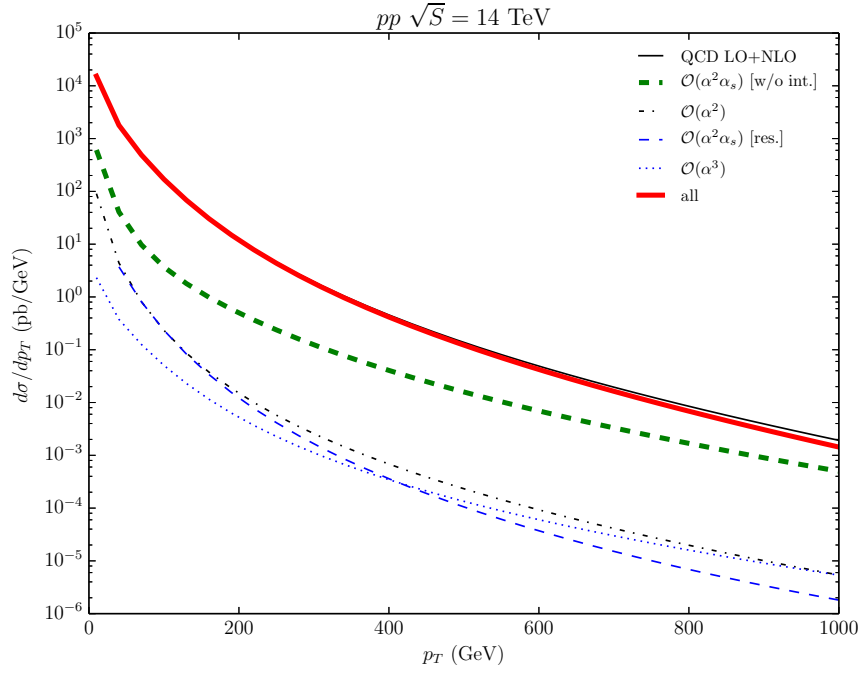


(a)

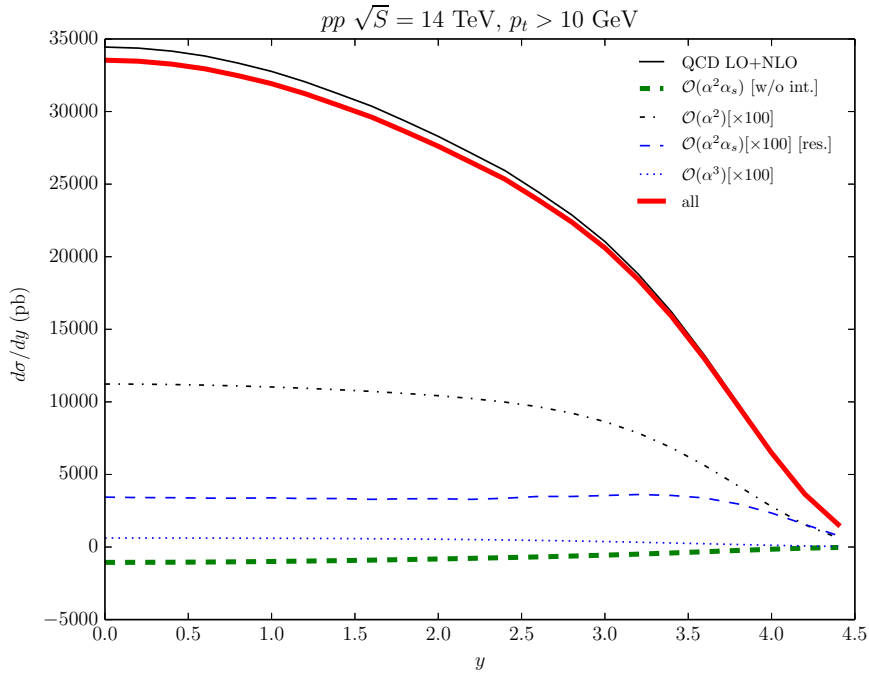


(b)

Figure 8: Cross section distributions in (a)  $p_T$  and (b)  $y$  for  $p_T > 10 \text{ GeV}$  of  $p\bar{p} \rightarrow Z + X$  at  $\sqrt{S} = 1.96 \text{ TeV}$  (Tevatron run II).



(a)



(b)

Figure 9: Cross section distributions in (a)  $p_T$  and (b)  $y$  for  $p_T > 10 \text{ GeV}$  of  $pp \rightarrow Z + X$  at  $\sqrt{S} = 14 \text{ TeV}$  (LHC).