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# Evolution equation for the higher-twist B-meson distribution amplitude

V.M. Braun,[1] A.N. Manashov,[2,1,3] and N. Offen[1]

[1]*Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
[2]*Institut für Theoretische Physik, Universität Hamburg, D-22761 Hamburg, Germany*
[3]*Department of Theoretical Physics, St.-Petersburg State University, 199034, St.-Petersburg, Russia*

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We find that the evolution equation for the three-particle quark-gluon B-meson light-cone distribution amplitude (DA) of subleading twist is completely integrable in the large $N_c$ limit and can be solved exactly. The lowest anomalous dimension is separated from the rest, continuous, spectrum by a finite gap. The corresponding eigenfunction coincides with the contribution of quark-gluon states to the two-particle DA $\phi_-(\omega)$ so that the evolution equation for the latter is the same as for the leading-twist DA $\phi_+(\omega)$ up to a constant shift in the anomalous dimension. Thus "genuine" three-particle states that belong to the continuous spectrum effectively decouple from typical observables to the leading-order accuracy. Our results suggest that the study of $1/m_b$ corrections to heavy-meson decays in the framework of QCD factorization or light-cone sum rules requires a much simpler nonperturbative input than it is usually assumed.

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B-meson light-cone distribution amplitudes (DAs) are the main nonperturbative input to the QCD description of weak decays involving light hadrons in the final state [1, 2]. In particular the leading-twist DA gives a dominant contribution in the heavy quark expansion and it received considerable attention already [3–9]. Utility of the QCD factorization techniques depends, however, on the possibility to control, or at least estimate, the corrections suppressed by powers of the $b$-quark mass that involve higher-twist DAs. This task is attracting increasing attention and in the last years there have been several efforts to combine light-cone sum rules with the expansion in terms of B-meson DAs [10–13]. This technique allows one to tame infrared divergences which appear in higher-twist contributions in the purely perturbative framework. The problem is that higher-twist B-meson DAs involve contributions of multiparton states and are very poorly known.

In this letter we point out that the structure of subleading twist DAs is much simpler as compared to what one may assume from their general partonic decomposition [14, 15]. This structure is revealed by considering the scale dependence of the DAs in the limit of large number of colors, $N_c \to \infty$, i.e. neglecting the $1/N_c^2$ corrections to the renormalization group equations. It turns out that the evolution equation for the three-particle DA in this approximation is completely integrable and can be solved exactly. The lowest anomalous dimension is separated from the rest, continuous, spectrum by a finite gap. The corresponding eigenfunction defines what can be called the "asymptotic" three-particle B-meson DA and has a relatively simple form. Most remarkably, it turns out that the higher-twist contribution to the two-particle B-meson DA $\phi_-(\omega)$ that is related to the three-particle DA by QCD equations of motion (EOM), is expressed entirely in terms of this "asymptotic" state, the states that belong to the continuous spectrum do not contribute. As the result the DA $\phi_-(\omega)$ evolves autonomously and does not mix with "genuine" three-particle contributions. The evolution equation for $\phi_-(\omega)$ is the same as for the leading-twist DA $\phi_+(\omega)$ up to a constant shift in the anomalous dimension. We suggest a simple model for $\phi_-(\omega)$ that can be used in phenomenology.

Following established conventions [3] we define the B-meson DAs as matrix elements of the renormalized nonlocal operators built of an effective heavy quark field $h_v(0)$, a light (anti)quark and gluons at a light-like separation:

$$
\begin{aligned}
iF(\mu)\Phi_+(z,\mu) &= \langle 0|\bar q(nz)\not{n}\gamma_5 h_v(0)|\bar B(v)\rangle, \\
iF(\mu)\Phi_-(z,\mu) &= \langle 0|\bar q(nz)\not{\bar n}\gamma_5 h_v(0)|\bar B(v)\rangle, \qquad (1)
\end{aligned}
$$

and

$$
\begin{aligned}
&-2iF(\mu)\Phi_3(z_1,z_2,\mu) = \\
&= \langle 0|\bar q(nz_1) gG_{\mu\nu}(nz_2) n^\nu \sigma^{\mu\rho} n_\rho \gamma_5 h_v(0)|\bar B(v)\rangle. (2)
\end{aligned}
$$

Here $v_\mu$ is the heavy quark velocity, $n_\mu$ is the light-like vector, $n^2 = 0$, such that $n\cdot v = 1$, $\Gamma$ stands for an arbitrary Dirac structure, $|\bar B(v)\rangle$ is the $\bar B$-meson state, $\mu$ is the factorization scale and $F(\mu)$ is the B-meson decay constant in the heavy quark effective theory (HQET). Wilson lines connecting the fields are not shown for brevity; they are always implied.

The functions $\Phi_+$ and $\Phi_-$ are the leading- and subleading-twist two-particle B-meson DAs [2], and

$\Phi_3$ is the (lowest twist) three-particle DA that is the only one relevant for the present study. In notations of [14] $\Phi_3 = \Psi_A - \Psi_V$. These three DAs are related by an EOM [2, 14]

$$\partial_z z\, \Phi_-(z) = \Phi_+(z) + 2\int_0^z wdw\, \Phi_3(z,w) \qquad (3)$$

that can be solved to obtain $\Phi_-$ as a sum of the so-called Wandzura-Wilczek (WW) term expressed in terms of $\Phi_+$ [2], and a certain integral of the quark-gluon DA $\Phi_3$. The latter contribution is nontrivial because it involves a function of two variables. We will demonstrate, however, that this complication is to a large extent illusory as the integral appearing in the EOM essentially decouples from "genuine" quark-gluon correlations. This simplification is exactly analogous to what has been observed before [16–19] for the structure function $g_2(x,Q^2)$ in polarized deep-inelastic lepton-proton scattering.

The following discussion is based on properties of the renormalization group equations for heavy-light operators under collinear conformal transformations. The corresponding generators read

$$S_+ = z^2\partial_z + 2jz\,, \quad S_0 = z\partial_z + j\,, \quad S_- = -\partial_z\,, \quad (4)$$

where $j=1$ is the conformal spin, $j_q=1$ for the light quark and $j_g = 3/2$ for the gluon. The generators satisfy the standard $SL(2)$ commutation relations $[S_+,S_-] = 2S_0\,,\quad [S_0,S_\pm] = \pm S_\pm\,.$ We distinguish the generators acting on quark and gluon coordinates by the subscript $S_q$ and $S_g$, respectively.

The starting observation is that both the one-loop renormalization group equations (RGE) for the DAs and the EOM relations are invariant under special conformal transformations [8, 20]. It is therefore natural to expand the DAs in terms of the eigenfunctions of the corresponding generator [8]

$$Q_s^{(j)}(z) = \frac{e^{-i\pi j}}{z^{2j}} e^{is/z}\,, \qquad iS_+^{(j)} Q_s^{(j)} = sQ_s^{(j)}\,. \qquad (5)$$

They form a complete orthonormal set

$$\langle Q_s^{(j)} | Q_{s'}^{(j)}\rangle_j = \Gamma(2j) s^{1-2j}\,\delta(s-s')\,, \qquad (6)$$

with respect to the $SL(2)$ invariant scalar product [21]

$$\langle \Phi_1|\Phi_2\rangle_j = \int_{\mathbb{C}_-} \mathcal{D}_j z\, \overline{\Phi_1(z)}\, \Phi_2(z)\,, \qquad (7)$$

where $\mathcal{D}_j z = \frac{2j-1}{\pi} d^2z\,[i(z-\bar z)]^{2j-2}$.

Thus we write the two-particle DAs as

$$\begin{aligned}\Phi_+(z) &= -\frac{1}{z^2}\int_0^\infty ds\, s\, e^{is/z}\widetilde{\phi}_+(s)\,,\\ \Phi_-(z) &= -\frac{i}{z}\int_0^\infty ds\, e^{is/z}\widetilde{\phi}_-(s)\,, \end{aligned} \qquad (8)$$

and the three-particle DA

$$\Phi_3(z_1,z_2) = \frac{-i}{z_1^2 z_2^3}\int_0^\infty ds\, s^4 \int_0^1 du\, u\bar u^2\, e^{is(\frac{u}{z_1}+\frac{\bar u}{z_2})}\widetilde{\phi}_3(s,u). \qquad (9)$$

Here and below $\bar u = 1-u$. Inserting these expressions in the EOM relation (3) one derives for the expansion coefficients

$$\widetilde{\phi}_-(s,\mu) = \widetilde{\phi}_+(s,\mu) - 2s^2\int_0^1 du\, u\bar u\, \widetilde{\phi}_3(s,u,\mu)\,. \quad (10)$$

Invariance under special conformal transformations means that terms with different values of $s$ cannot get mixed by the RGE. The leading twist contributions $\widetilde{\phi}_+(s,\mu)$ have autonomous scale dependence:

$$\Big(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \frac{\alpha_s}{2\pi}\mathcal{E}_+(s,\mu)\Big) F(\mu)\,\widetilde{\phi}_+(s,\mu) = 0\,,$$

where [7, 8]

$$\mathcal{E}_+(s,\mu) = 2C_F\left[\ln(\mu s) - \psi(1) - 5/4\right]\,. \qquad (11)$$

The RGE for the three-particle DA $\Phi_3$ is more complicated,

$$\Big(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \frac{\alpha_s}{2\pi}\mathcal{H}\Big) F(\mu)\,\Phi_3(z_1,z_2,\mu) = 0\,,$$

where the "Hamiltonian" $\mathcal{H}$ to the one-loop accuracy is given by a sum of two-particle kernels

$$\mathcal{H} = N_c\,\mathbb{H} + N_c^{-1}\,\delta\mathbb{H} = H_{qg} + H_{gh} + H_{qh}\,. \qquad (12)$$

The kernels take the following form [8, 20, 22, 23]:

$$\begin{aligned} H_{qg} &= N_c\Big[\psi\left(J_{qg}{+}3/2\right) + \psi\left(J_{qg}{-}3/2\right) - 2\psi(1) - 3/4\Big]\\ &\quad + \frac{2}{N_c}(-1)^{J_{qg}-3/2}\frac{\Gamma\left(J_{qg}-3/2\right)}{\Gamma\left(J_{qg}+3/2\right)}\,,\\ H_{gh} &= N_c\left[\ln\left(i\mu S_g^+\right) - \psi(1) - 1/2\right]\,,\\ H_{qh} &= -\frac{1}{N_c}\left[\ln\left(i\mu S_q^+\right) - \psi(1) - 5/4\right]\,, \end{aligned} \qquad (13)$$

where $J_{qg}$ is defined in terms of the corresponding quadratic Casimir operator $J_{qg}(J_{qg}-1) = (\vec S_q + \vec S_g)^2$. Note that in difference to [8, 20, 23] we include the QCD coupling in the definition of the quark-antiquark-gluon operator: $G_{\mu\nu} \mapsto gG_{\mu\nu}$.

The Hamiltonian $\mathcal{H}$ commutes with the generator of special conformal transformations

$$\mathbb{Q}_1 \;=\; i\big(S_q^+ + S_g^+\big)\,. \qquad (14)$$

This implies that the RGE is "diagonal" in $s$ but this symmetry alone is not sufficient to find the solution.

It turns out, however, that the large-$N_c$ Hamiltonian, $\mathbb{H}$, has an additional "hidden" symmetry. Namely, it is possible to show that the operator

$$\mathbb{Q}_2 = \frac{9}{4} i S_q^+ - i S_g^+ \big( S_g^+ S_q^- + S_g^0 S_q^0 \big) - i S_g^0 \big( S_q^0 S_g^+ - S_g^0 S_q^+ \big) \tag{15}$$

commutes with $\mathbb{Q}_1$ and the large-$N_c$ kernel $\mathbb{H}$:

$$[\mathbb{Q}_1, \mathbb{Q}_2] = [\mathbb{Q}_1, \mathbb{H}] = [\mathbb{Q}_2, \mathbb{H}] = 0 . \tag{16}$$

This property is known as complete integrability. In the formalism of the quantum inverse scattering method (QISM) [24] the charges $\mathbb{Q}_1, \mathbb{Q}_2$ appear in the expansion of the element $C(u)$ of the monodromy matrix for an open spin chain, $C(u) \propto u^2 \mathbb{Q}_1 + \mathbb{Q}_2$. The commutation relation $[C(u), \mathbb{H}] = 0$ can be verified by a direct calculation, or with help of the QISM techniques. The derivation will be given elsewhere [25].

The "conserved charges" $\mathbb{Q}_1$, $\mathbb{Q}_2$ and the "Hamiltonian" $\mathbb{H}$ are self-adjoint operators with respect to the $SL(2)$ scalar product (7):

$$\langle \Psi | \Phi \rangle = \int_{\mathbb{C}_-} \mathcal{D}_1 z_1 \int_{\mathbb{C}_-} \mathcal{D}_{\frac{3}{2}} z_2 \, \overline{\Psi(z_1, z_2)} \Phi(z_1 . z_2) \,,$$

and can be diagonalized simultaneously:

$$\begin{aligned} \mathbb{Q}_1 Y_{s,x}(z_1, z_2) &= s \, Y_{s,x}(z_1, z_2) \,, & s > 0 \\ \mathbb{Q}_2 Y_{s,x}(z_1, z_2) &= -s x^2 \, Y_{s,x}(z_1, z_2) \,, & x^2 \in \mathbb{R} \\ \mathbb{H} \, Y_{s,x}(z_1, z_2) &= \mathbb{E}(s,x) \, Y_{s,x}(z_1, z_2) \,. & \end{aligned} \tag{17}$$

The eigenfunctions $Y_{s,x}$ are labeled by two "quantum numbers" and provide the basis of the Sklyanin's representation of Separated Variables. They can be found with help of the method developed in [26],

$$Y_{s,x}(z_1, z_2) = \frac{i s^2}{z_1^2 z_2^3} \int_0^1 du \, u \bar{u} \, e^{i s (u/z_1 + \bar{u}/z_2)} \times {}_2F_1 \left( \left. {-\tfrac{1}{2} - i x, -\tfrac{1}{2} + i x \atop 2} \right| - \frac{u}{\bar{u}} \right) . \tag{18}$$

The functions $Y_{s,x}$ are symmetric under reflection $x \to -x$. Since the eigenvalue $x^2$ has to be real, $x$ can take real or imaginary values. There exists only one normalizable solution corresponding to imaginary $x$, $x = i/2$, in which case the hypergeometric function disappears:

$$Y_s^{(0)}(z_1, z_2) = \frac{i s^2}{z_1^2 z_2^3} \int_0^1 du \, u \bar{u} \, e^{i s (u/z_1 + \bar{u}/z_2)} . \tag{19}$$

This solution (ground state) has the minimal energy

$$\mathbb{E}_0 \equiv \mathbb{E}(s, x = i/2) = \ln(\mu s) - \psi(1) - 1/4 \tag{20}$$

and describes the "asymptotic" quark-gluon DA with the lowest anomalous dimension. The state is normalized as

$$\langle Y_s^{(0)} | Y_{s'}^{(0)} \rangle = \delta(s - s') \,. \tag{21}$$

The eigenfunctions corresponding to real values of $x$ belong to the continuous spectrum. They are orhogonal to the ground state and normalized as

$$\langle Y_{s,x} | Y_{s',x'} \rangle = \delta(s - s') \delta(x - x') \frac{\coth \pi x}{x (x^2 + 9/4)} \,, \tag{22}$$

The corresponding eigenvalue (energy) is

$$\mathbb{E}(s,x) = \ln(\mu s) + \psi\big(3/2 + i x\big) + \psi\big(3/2 - i x\big) - 3\psi(1) - 5/4 \,. \tag{23}$$

The gap between the ground state and the continuous spectrum $\Delta\mathbb{E} = \mathbb{E}(s,0) - \mathbb{E}_0 = 2\psi(3/2) - \psi(2) - \psi(1)$ coincides with the gap in the spectrum of anomalous dimensions of twist-three quark-gluon operators with large number of derivatives, see Ref. [27].

The $1/N_c^2$ corrections to the ground state energy $\mathcal{E}_0(s) = N_c \mathbb{E}_0 + 1/N_c \delta\mathbb{E}_0$ can be calculated in a standard quantum-mechanical perturbation theory. The answer can be written as

$$\mathcal{E}_0(s) = \mathcal{E}_+(s) + \Delta + O(1/N_c^{-3}) \,, \tag{24}$$

where $\mathcal{E}_+(s)$ is the lowest anomalous dimension for subleading twist operators, Eq. (11), and the difference does not depend on $s$:

$$\Delta = N_c + N_c^{-1} \left( \pi^2/3 - 3 \right) . \tag{25}$$

It coincides exactly with the gap between the spectrum of anomalous dimensions of twist-three quark-antiquark-gluon operators and the leading-twist quark-antiquark operators at $N \to \infty$ (here $N$ is number of derivatives), cf. [18].

The three-particle DA $\Phi_3(z_1, z_2, \mu)$ can be expanded in eigenfunctions of the large-$N_c$ Hamiltonian

$$\Phi_3(z_1, z_2, \mu) = \int_0^\infty ds \Big[ \eta_0(s, \mu) \, Y_s^{(0)}(z_1, z_2) + \int_{-\infty}^{\infty} dx \, \eta(s, x, \mu) \, Y_{s,x}(z_1, z_2) \Big], \tag{26}$$

where the coefficient functions $\eta_0(s,\mu)$ and $\eta(s,x,\mu)$ are multiplicatively renormalized up to $1/N_c^2$ corrections. Remarkably enough, only the ground state contributes to the integral in the EOM relation (3); contributions from the continuum vanish identically (for arbitrary $\eta(s,x,\mu)$). One finds

$$\int_0^z w dw \, \Phi_3(z, w, \mu) = -\frac{1}{2 z^2} \int_0^\infty s ds \, e^{i s/z} \, \eta_0(s, \mu) \,,$$

leading to the following very simple relation

$$\widetilde{\phi}_-(s,\mu) = \widetilde{\phi}_+(s,\mu) + \eta_0(s,\mu)\,. \qquad (27)$$

Going over to the momentum space

$$\Phi_\pm(z) = \int_0^\infty d\omega\, e^{-i\omega z}\phi_\pm(\omega)\,, \qquad (28)$$
$$\Phi_3(z_1,z_2) = \int_0^\infty d\omega_1\, d\omega_2\, e^{-i(\omega_1 z_1+\omega_2 z_2)}\phi_3(\omega_1,\omega_2)$$

one obtains the following representation for the two-particle DAs [8, 9]

$$\phi_+(\omega,\mu) = \int_0^\infty ds\, \widetilde{\phi}_+(s,\mu)\,\sqrt{\omega s}\, J_1(2\sqrt{\omega s})\,,$$
$$\phi_-(\omega,\mu) = \int_0^\infty ds\, \widetilde{\phi}_-(s,\mu) J_0(2\sqrt{\omega s})\,, \qquad (29)$$

and the asymptotic quark-gluon DA

$$\phi_3^{\rm as}(\omega_1,\omega_2,\mu) = -\omega_2\sqrt{\omega_1}\int_0^\infty ds\sqrt{s}\eta^{(0)}(s,\mu)\int_0^1 du\sqrt{u}J_1\Big(2\sqrt{s\omega_1 u}\Big)J_2\Big(2\sqrt{s\omega_2\bar u}\Big)$$
$$= \frac{\omega_1\omega_2}{\omega_1+\omega_2}\Big[f_1(\omega_1+\omega_2) - f_0(\omega_1+\omega_2)\Big] + \omega_1\Big[f_1(\omega_1+\omega_2) - f_1(\omega_1)\Big], \qquad (30)$$

where

$$f_k(\omega) = \int_0^\infty ds\, \eta^{(0)}(s,\mu)\big(\sqrt{\omega s}\big)^{-k} J_k\big(2\sqrt{\omega s}\big)\,. \quad (31)$$

For small momenta $\phi_3^{\rm as}(\omega_1,\omega_2,\mu) \sim \mathcal{O}(\omega_1)$ and $\sim \mathcal{O}(\omega_2^2)$ in the limits $\omega_1\to 0$ and $\omega_2 \to 0$, respectively. This behavior is in agreement with arguments based on quark-gluon duality [10]. If both quark and gluon momenta are small one obtains

$$\phi_3^{\rm as}(\omega_1,\omega_2,\mu) \overset{\omega_1,\omega_2\to 0}{=} -\frac{\omega_1\omega_2^2}{12}\int_0^\infty ds\, s^2\,\eta^{(0)}(s,\mu)\,.$$

An interesting property of the asymptotic DA (30) is that it does not decrease for large gluon momenta $\omega_2 \to \infty$ (because of the last term that is $\omega_2$-independent). As a consequence, existence of the limit $z_1,z_2 \to 0$ and the relation of the normalization of the asymptotic DA to matrix elements of local operators even at a single scale requires specific cancellations that amount to a fine-tuning, see below.

The scale-dependence of the coefficients $\widetilde{\phi}_+(s,\mu)$ and $\eta_0(s,\mu)$ differs by a simple factor

$$F(\mu)\,\widetilde{\phi}_+(s,\mu) = E(s;\mu,\mu_0)\,F(\mu_0)\,\widetilde{\phi}_+(s,\mu_0)\,, \qquad (32)$$
$$F(\mu)\,\eta_0(s,\mu) = L^{\Delta/\beta_0}E(s;\mu,\mu_0)\,F(\mu_0)\,\eta_0(s,\mu_0)\,,$$

where $\Delta$ is defined in Eq. (24) and

$$E(s;\mu,\mu_0) = \exp\left[-\int_{\mu_0}^{\mu}\frac{d\tau}{\tau}\,\Gamma_{cusp}(\alpha_s(\tau))\,\ln(\tau s/s_0)\right]$$
$$= \left(\frac{\mu}{\mu_0}\right)^{-\frac{2C_F}{\beta_0}}\left(\frac{\mu_0 s}{s_0}\right)^{\frac{2C_F}{\beta_0}\ln L} L^{-\frac{4C_F\pi}{\beta_0^2\,\alpha_s(\mu_0)}}$$

Here $L=\alpha_s(\mu)/\alpha_s(\mu_0)$, $s_0=e^{5/4-\gamma_E}$, $\beta_0=\frac{11}{3}N_c-\frac{2}{3}n_f$ and $\Gamma_{cusp}(\alpha_s)=\frac{\alpha_s}{\pi}C_F+\ldots$ is the cusp anomalous dimension [28, 29]. These equations present our main result.

For the simplest phenomenologically acceptable model of the leading-twist B-meson DA at a low scale $\mu=\mu_0$ one usually takes [3]

$$\phi_+(\omega) = \frac{\omega}{\lambda_B^2}e^{-\omega/\lambda_B} \quad \mapsto \quad \widetilde{\phi}_+(s) = e^{-s\lambda_B}\,, \quad (33)$$

where $\lambda_B$ is defined as

$$\frac{1}{\lambda_B} = \int_0^\infty \frac{d\omega}{\omega}\,\phi_+(\omega)\,. \qquad (34)$$

This is the most important nonperturbative parameter in the QCD factorization approach [1, 2], $\lambda_B \simeq 300-600$ MeV [5, 15].

We suggest a similar simple model for the subleading twist DA

$$\eta_0(s,\mu_0) = \phi_3\, s^2\, e^{-s\lambda_3}\,, \quad \phi_3 = \frac{1}{6}\Big[\lambda_E^2-\lambda_H^2\Big]\,, \quad (35)$$

where $\lambda_E^2,\lambda_H^2$ are the local matrix elements of quark-gluon operators [3, 15] and $\lambda_3$ is a parameter (similar to $\lambda_B$) that characterizes the spread of the DA in $s$-space. One can take $\lambda_3 \simeq \lambda_B$ as the simplest assumption.

For this model one obtains

$$\phi_-(\omega) = \frac{e^{-\omega/\lambda_B}}{\lambda_B} + \frac{\phi_3}{\lambda_3^3}e^{-\omega/\lambda_3}\Big(2-4\frac{\omega}{\lambda_3}+\frac{\omega^2}{\lambda_3^2}\Big)\,,$$
$$\phi_3^{\rm as}(\omega_1,\omega_2) = -\frac{\phi_3\omega_1}{\lambda_3^4}e^{-\frac{\omega_1+\omega_2}{\lambda_3}}\Big[\Big(\omega_1-2\lambda_3\Big)\Big(1-e^{\frac{\omega_2}{\lambda_3}}\Big) + \frac{\omega_2}{\lambda_3}\big(\omega_2+\omega_1-2\lambda_3\big)\Big]. \qquad (36)$$

The recent QCD sum rule calculation [15] gives $\lambda_E^2 - \lambda_H^2 = -0.03 \pm 0.03\,\mathrm{GeV}^2$. This is a rather small number so that $\phi_3 \ll \lambda_B^2$ and the corresponding contribution is further suppressed at large scales by the factor $(\alpha_s(\mu)/\alpha_s(\mu_0))^{\Delta/\beta_0}$ (33). Hence the DA $\phi_-(\omega,\mu)$ is likely to be dominated by the WW contribution.

Note that despite the fact that $\eta_0(s,\mu_0)$ is exponentially suppressed at large $s$, the quark-gluon DA $\phi_3^{\rm as}(\omega_1,\omega_2)$ does not decrease at large gluon momenta $\omega_2 \to \infty$; the prefactor $s^2$ in (35) is chosen for the existence of the $z_1, z_2 \to 0$ limit. This fine-tuning is lifted at higher scales so that the relation of the moments of DAs with matrix elements of local operators is lost [5]. It would be interesting to study the arising large-momentum contributions using the expansion of the type suggested in [6] (see also [9]).

To summarize, we have shown that "genuine" three-particle contributions of quark-gluon states essentially decouple from the subleading-twist two-particle DA $\phi_-(\omega)$ [2] so that its properties are similar to the leading-twist DA. We expect that such "genuine" three-particle contributions do not contribute directly to many physical observables at the tree level because three-particle and two-particle twist-three contributions to the products of currents are typically related by Ward identities; hence they cannot have a different scale dependence. We also expect that similar simplification holds for twist-four distributions. This study is in progress [25]. The goal is to find important degrees of freedom in multiparticle quark-gluon distributions in heavy mesons that can be parametrized by a minimum number of nonperturbative parameters. This would allow one to increase significantly the accuracy of QCD predictions for heavy meson (and baryon) decays based on the heavy-quark expansion.

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