DESY 15-110
July 6, 2015

# Renormalization Group Constraints on New Top Interactions from Electroweak Precision Data

**J. de Blas**[a†], **M. Chala**[b‡] and **J. Santiago**[c§]

[a]*INFN, Sezione di Roma, Piazzale A. Moro 2, I-00185 Rome, Italy*

[b]*DESY, Notkestrasse 85, 22607 Hamburg, Germany*

[c]*Departamento de Física Teórica y del Cosmos and CAFPE, Universidad de Granada, Campus de Fuentenueva, E-18071 Granada, Spain*



**Abstract**

Anomalous interactions involving the top quark contribute to some of the most difficult observables to directly access experimentally. They can give however a sizeable correction to very precisely measured observables at the loop level. Using a model-independent effective Lagrangian approach, we present the leading indirect constraints on dimension-six effective operators involving the top quark from electroweak precision data. They represent the most stringent constraints on these interactions, some of which may be directly testable in future colliders.

## 1 Introduction

Once the presence of the Higgs boson has been firmly established at the Large Hadron Collider (LHC) the focus has turned towards the discovery of new physics (NP). Already at Run 1 the absence of significant deviations from the Standard Model (SM) predictions has put stringent bounds on the mass scale of NP. Indirect constraints on new particles, when they are beyond the kinematic reach of the LHC, are becoming competitive and in some cases complementary to the indirect constraints from electroweak precision data (EWPD) [1]. It is expected that, with the increased energy available during Run 2, any new particle within the kinematic reach will be discovered, or very stringent constraints will be placed in case they cannot be directly produced. Still, there are certain interactions that, even with the increased energy, will be very

†E-mail: Jorge.DeBlasMateo@roma1.infn.it
‡E-mail: mikael.chala@desy.de
§E-mail: jsantiago@ugr.es

difficult to directly probe at the LHC with a significant precision. A notable example is the interactions involving the top quark that we consider in this work.

Many well-motivated models of NP predict the largest deviations from the SM results in processes involving the top quark. Using an effective Lagrangian description, several groups have studied the potential of the LHC to constrain higher-dimensional operators involving the top quark in single and pair top production [2], including in some cases next-to-leading order predictions (see [3] and references there in). Some of these interactions can be directly constrained to a reasonable accuracy for the first time at the LHC. However, the complexity of the $t(\bar{t})$ system limits the precision that one can achieve with these direct probes. In some cases, certain higher-dimensional operators are just inaccessible at the LHC. Many of these interactions, however, contribute at the loop level to EWPD. The very stringent constraints that can be derived from EWPD, together with the fact that the relevant coupling is usually the top Yukawa coupling, can compensate for the loop suppression, thus producing the most stringent constraints on many higher-dimensional operators involving the top quark. In particular, this includes those that cannot be directly probed at the LHC.

In this article we use EWPD to place bounds on NP interactions. We use a model-independent effective Lagrangian approach going beyond the usual analysis of dimension-six operators correcting precision observables at the tree level. Analyses including one-loop contributions from higher-dimensional operators have been done for a small subset of purely bosonic operators in the past [4]. Here we use the calculation of the renormalization group equations (RGE) for the entire dimension-six effective Lagrangian [5, 6, 7] (see also [8]) to determine, without restricting to any particular set of operators, which interactions can be constrained by EWPD at the $O(0.1)$ or better. This precision can be achieved by higher-dimensional operators that give a sizeable loop contribution to EWPD. If we further neglect operators that can be directly probed in current or past experiments, we are basically left with dimension-six operators involving top quarks. We will show that indirect constraints from EWPD can be quite stringent for these interactions, some of which could be tested in future lepton colliders.

We discuss the global fit to EWPD, including the leading, currently unconstrained, loop effects in section 2. The corresponding constraints on the coefficients of the dimension-six operators involving the top quark are presented in section 3 and we conclude in section 4.

# 2 The global electroweak fit for new physics to dimension six

Assuming that NP is heavier than the energies currently probed by experiments, its effects can be described by an effective Lagrangian,

$$\mathcal{L}_{\mathrm{Eff}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda^2}\mathcal{L}_6 + \ldots, \qquad (1)$$

where $\mathcal{L}_d = \sum c_i \mathcal{O}_i$, with $\mathcal{O}_i$ Lorentz and SM gauge invariant operators of mass dimension $d$ built from the SM fields. $\Lambda$ stands for the cut-off scale of the effective theory, and we have neglected lepton number violating effects. There is a total of 59 operators (up to flavor indices) at dimension six [9]. However, only a few of these directly contribute at leading order to EWPD. We consider in our analysis the following set of EWPD:

$$\Big\{ M_H,\, m_t,\, \alpha_S(M_Z^2),\, \Delta\alpha_{\rm had}^{(5)}(M_Z^2),\, M_W,\, \Gamma_W,\, {\rm Br}_W^{e\nu,\mu\nu,\tau\nu},\, M_Z,\, \Gamma_Z,\, \sigma_{\rm had},\, R_{e,\mu,\tau}, \\ A_{FB}^{e,\mu,\tau},\, A_{e,\mu,\tau}({\rm SLD}),\, A_{e,\tau}(P_\tau),\, R_{b,c},\, A_{\rm FB}^{b,c},\, A_{b,c},\, A_{\rm FB}^{s},\, A_s,\, R_u/R_{u+d+s},\, Q_{\rm FB}^{\rm had} \Big\}, \tag{2}$$

whose definition, experimental values, errors and correlation matrices are taken from [10]. In our fits to NP the SM inputs are also taken as floating parameters. Due to the precision of their direct measurements, however, the effect of the variation of the coefficients of the higher-dimensional operators in the value of these input parameters is minimal.

The analysis of bounds on dimension-six interactions from EWPD has been considered extensively in the literature (see for instance [11]). In order to include dimension-six effects in a consistent way, we limit the contributions to EWPD to order $1/\Lambda^2$. At the tree level, only the following coefficients of dimension-six operators modify the observables in (2):

$$c_{\rm EWPD}^{\rm tree} = \Big\{ c_{\phi l}^{(1)},\, c_{\phi q}^{(1)},\, c_{\phi e}^{(1)},\, c_{\phi u}^{(1)},\, c_{\phi d}^{(1)},\, c_{\phi l}^{(3)},\, c_{\phi q}^{(3)},\, c_{\phi D},\, c_{WB},\, c_{ll}^{(1)} \Big\}. \tag{3}$$

The definition of the operator basis used in this work follows closely the original one in [9] except for the four-fermion sector for which we use the one in Appendix A of [12].[1] At the loop level, however, many other dimension-six operators contribute to EWPD, including some to which we have little or no direct experimental access. In this latter case, the high precision of the EWPD can compensate the loop suppression and give the most stringent constraints on these operators. The complete one-loop calculation of the EWPD including dimension-six operators is beyond the scope of this article but the logarithmically enhanced contributions can be computed by means of the RGE recently computed in [5, 6, 7]. An analysis of the loop-improved electroweak constraints on the dimension-six interactions in (3) will be presented elsewhere [13]. Here we want to focus on those operators that, as explained above, have not been directly probed by any experiment yet, and to determine which ones can be constrained to a reasonable accuracy with current data.

The largest RGE effects on EWPD are those proportional to the top Yukawa coupling, $y_t$. (The strong coupling $g_3$ does not enter in any of the anomalous dimensions for the interactions in (3).) Inspecting the RGE for the operators in (3) and looking for $y_t$ effects then allows us to identify which ones could be significantly constrained from

[1]For completeness, the dimension-six operators in (3) are presented here: $\mathcal{O}_{\phi\psi}^{(1)}=(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\overline{\psi}\gamma^\mu\psi)$, $\mathcal{O}_{\phi\psi}^{(3)}=(\phi^\dagger i\overleftrightarrow{D}_\mu^a\phi)(\overline{\psi_L}\gamma^\mu\sigma_a\psi_L)$ (with $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$ and $\overleftrightarrow{D}_\mu^a = \sigma_a\overrightarrow{D}_\mu - \overleftarrow{D}_\mu\sigma_a$), $\mathcal{O}_{\phi D}=(\phi^\dagger D_\mu\phi)((D^\mu\phi)^\dagger\phi)$, $\mathcal{O}_{WB}=(\phi^\dagger\sigma_a\phi)\, W_{\mu\nu}^a B^{\mu\nu}$ and $\mathcal{O}_{ll}^{(1)}=\frac{1}{2}\left(\overline{l_L}\gamma_\mu l_L\right)\left(\overline{l_L}\gamma^\mu l_L\right)$.

the low-energy bounds. Restricting to operators that have not been directly tested in experiments so far leaves us with the following set, containing always the top quark:[2]

$$\begin{aligned}
\mathcal{O}_{\phi q}^{(1)} &= (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\overline{q_L}\gamma^\mu q_L), & \mathcal{O}_{\phi q}^{(3)} &= (\phi^\dagger i \overleftrightarrow{D}^a_\mu \phi)(\overline{q_L}\gamma^\mu \sigma_a q_L), \\
\mathcal{O}_{\phi u}^{(1)} &= (\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\overline{u_R}\gamma^\mu u_R), & & \\
\mathcal{O}_{lq}^{(1)} &= (\overline{l_L}\gamma_\mu l_L)(\overline{q_L}\gamma^\mu q_L), & \mathcal{O}_{lq}^{(3)} &= (\overline{l_L}\gamma_\mu \sigma_a l_L)(\overline{q_L}\gamma^\mu \sigma_a q_L), \\
\mathcal{O}_{eu} &= (\overline{e_R}\gamma_\mu e_R)(\overline{u_R}\gamma^\mu u_R), & \mathcal{O}_{lu} &= (\overline{l_L}\gamma_\mu l_L)(\overline{u_R}\gamma^\mu u_R), \\
\mathcal{O}_{qe} &= (\overline{q_L}\gamma_\mu q_L)(\overline{e_R}\gamma^\mu e_R), & & \\
\mathcal{O}_{qq}^{(1)} &= \frac{1}{2}(\overline{q_L}\gamma_\mu q_L)(\overline{q_L}\gamma^\mu q_L), & \mathcal{O}_{qq}^{(8)} &= \frac{1}{2}(\overline{q_L}\gamma_\mu T_A q_L)(\overline{q_L}\gamma^\mu T_A q_L) \\
\mathcal{O}_{uu}^{(1)} &= \frac{1}{2}(\overline{u_R}\gamma_\mu u_R)(\overline{u_R}\gamma^\mu u_R), & \mathcal{O}_{ud}^{(1)} &= (\overline{u_R}\gamma_\mu u_R)(\overline{d_R}\gamma^\mu d_R), \\
\mathcal{O}_{qu}^{(1)} &= (\overline{q_L}\gamma_\mu q_L)(\overline{u_R}\gamma^\mu u_R), & \mathcal{O}_{qd}^{(1)} &= (\overline{q_L}\gamma_\mu q_L)(\overline{d_R}\gamma^\mu d_R), \\
\mathcal{O}_{uB} &= (\overline{q_L}\sigma^{\mu\nu} u_R)\tilde{\phi} B_{\mu\nu}, & \mathcal{O}_{uW} &= (\overline{q_L}\sigma^{\mu\nu}\sigma_a u_R)\tilde{\phi} W^a_{\mu\nu}.
\end{aligned} \tag{4}$$

An analysis of EWPD constraints on a small subset of these operators was performed in [15]. However, at one loop only non-logarithmic finite contributions were included and the corresponding bounds are much weaker. Regarding the operators $\mathcal{O}_{\phi q}^{(1),(3)}$, we will only consider the combination $\mathcal{O}_{\phi q}^{(t)} \equiv (\mathcal{O}_{\phi q}^{(1)} - \mathcal{O}_{\phi q}^{(3)})_{tt}$, which (up to corrections suppressed by products of $V_{td}$ and $V_{ts}$ with $V$ the Cabibbo-Kobayashi-Maskawa matrix) modifies the neutral current top couplings, without inducing any tree-level correction in the bottom ones. As we will also see, because EWPD is only sensitive to $\mathcal{O}_{\phi q}^{(b)} \equiv (\mathcal{O}_{\phi q}^{(1)} + \mathcal{O}_{\phi q}^{(3)})_{bb}$ at the tree level, NP effects coming from $\mathcal{O}_{qq}^{(8)}$ cannot be constrained by the data. Similarly, EWPD is not sensitive to $(\mathcal{O}_{\phi u}^{(1)})_{tt}$, so $\mathcal{O}_{uu}^{(1)}$ cannot be constrained if it only involves the third generation.

As we mentioned above, we compute the predictions for physical observables consistently with the approximation of a dimension-six effective Lagrangian, including only the interference between the SM amplitudes and the NP effects (i.e. terms linear in $1/\Lambda^2$). We also use the leading logarithmic approximation for the solution of the RGE

$$\frac{dC_i}{d\log\mu} = \frac{1}{16\pi^2}\gamma_i^j C_j \Longrightarrow C_i(\mu) \approx \left(\delta_i^j + \frac{1}{16\pi^2}\gamma_i^j(\Lambda)\log\frac{\mu}{\Lambda}\right) C_j(\Lambda), \tag{5}$$

where we have defined the dimension-six coefficients $C_i \equiv c_i/\Lambda^2$. We include in the anomalous dimensions $\gamma_i^j$ the full dependence on the SM gauge couplings [7] and the leading contributions from the Yukawa interactions [6], i.e. $Y_e, Y_d \approx 0$, and $Y_u \approx \mathrm{diag}(0,0,y_t)$. The dependence on the Higgs self-coupling [5] is irrelevant for

[2] Apart from these, we will also comment on another operator (see footnote 3): the pure scalar interaction $\mathcal{O}_{\phi\Box} = (\phi^\dagger\phi)\Box(\phi^\dagger\phi)$, which does not contribute to EWPD proportionally to $y_t$, but enters in the anomalous dimension of $c_{\phi D}$ with a large coefficient $\sim 20 g_1^2/3$ [7]. Although this operator can be in principle tested in Higgs physics, its constraints are still very weak [14].

our analysis. Finite one-loop contributions beyond the logarithmically enhanced terms included in the RGE can in some cases be relevant [16]. We include these finite terms in our analysis below whenever they are available. Note that, for any interaction that has no direct tree-level contribution to the EWPD, the physical predictions depend always on $C_i \log \mu/\Lambda$.

# 3 Loop constraints on new top interactions

In this section we present the constraints that EWPD impose on the operators involving the top quark in (4) due to RGE effects. We report these bounds in Table 1 assuming that only one operator is generated at the ultraviolet scale, $\Lambda$, at a time.[3] We present the results from two different types of fits. The limits on $C_i \log M_Z/\Lambda$ (and $c_i$, for $\Lambda = 1$ TeV) are obtained assuming the dimension-six coefficients can have either sign. On the other hand, the bounds on the NP scale $\Lambda$ are derived from a fit with the extra assumption that the $c_i$ have a definite sign, and then setting this to some illustrative values, $c_i = \pm 1$. As we mentioned above, the precision of EWPD overcomes the loop suppression of the RGE effects and allows to constrain most of the interactions at the few percent level for $\Lambda = 1$ TeV, or alternatively pushes the scale of NP in the top sector to a few TeV for $c_i = \pm 1$, thus fully justifying the use of an effective Lagrangian description. (Note that, even for the weakest bounds, the NP scale is always pushed significantly above the $Z$ mass, where the EWPD are measured.) The leading-log approximation used is also justified provided the value of $\Lambda$ is not too large, so that $\left|\frac{\alpha_i}{4\pi} \log \frac{M_Z}{\Lambda}\right| \ll 1$, with $\alpha_i$ the relevant SM parameter. In the rest of this section we discuss the origin of the constraints on the different operators.

The relatively strong constraints on $\ell^+\ell^- t\bar{t}$ interactions can be understood from the fact that all those interactions contribute to the running of $c_{\phi l}^{(1)}$, $c_{\phi l}^{(3)}$ or $c_{\phi e}^{(1)}$. The corresponding operators provide direct corrections to the neutral current couplings of the charged leptons, and are bounded at the per mile level (see [17] for an earlier partial analysis). Note that the bounds for some of these interactions, e.g. $\mathcal{O}_{lq}^{(1)}$ and $\mathcal{O}_{lu}$, are almost identical (up to a sign). This correlation follows directly from the RGE for the leptonic interactions in (3). In particular,

$$
\begin{aligned}
\frac{d(C_{\phi l}^{(1)})_{ij}}{d\log\mu} &= \frac{N_c}{8\pi^2}\left\{\left(Y_u^\dagger Y_u\right)_{lk}\left(C_{lq}^{(1)}\right)_{ijkl} - \left(Y_u Y_u^\dagger\right)_{lk}\left(C_{lu}\right)_{ijkl}\right\} + \ldots, \\
\frac{d(C_{\phi e}^{(1)})_{ij}}{d\log\mu} &= \frac{N_c}{8\pi^2}\left\{\left(Y_u^\dagger Y_u\right)_{lk}\left(C_{qe}\right)_{klij} - \left(Y_u Y_u^\dagger\right)_{lk}\left(C_{eu}\right)_{ijkl}\right\} + \ldots. \qquad (7)
\end{aligned}
$$

[3]For completeness, we include here the 95% probability interval for the operator coefficient $c_{\phi\Box}$:

$$
\frac{c_{\phi\Box}}{\Lambda^2}\log\frac{M_Z}{\Lambda} \in [-4.63, 0.65]\ \text{TeV}^{-2} \quad (c_{\phi\Box} \in [-0.27, 1.93]\ \text{for}\ \Lambda = 1\ \text{TeV})\,. \qquad (6)
$$

For negative values of the coefficients, the corresponding bound is then somewhat better than the one obtained from Higgs observables [14].

| Operator | 95% prob. interval | | 95% prob. lower bound $\Lambda$ [TeV] | |
|---|---|---|---|---|
| | $\frac{c_i}{\Lambda^2}\log\frac{M_Z}{\Lambda}$ $\left[\mathrm{TeV}^{-2}\right]$ | $c_i$ $(\Lambda = 1$ TeV$)$ | $(c_i = +1)$ | $(c_i = -1)$ |
| $(\mathcal{O}_{lq}^{(1)})_{eett}$ | $[-0.15, 0.38]$ | $[-0.16, 0.06]$ | 4.4 | 3.2 |
| $(\mathcal{O}_{lq}^{(3)})_{eett}$ | $[-0.26, 0.36]$ | $[-0.15, 0.11]$ | 3.7 | 3.3 |
| $(\mathcal{O}_{eu})_{eett}$ | $[-0.21, 0.44]$ | $[-0.18, 0.09]$ | 3.8 | 2.9 |
| $(\mathcal{O}_{lu})_{eett}$ | $[-0.40, 0.16]$ | $[-0.07, 0.17]$ | 3.1 | 4.3 |
| $(\mathcal{O}_{qe})_{ttee}$ | $[-0.42, 0.20]$ | $[-0.08, 0.18]$ | 3 | 3.9 |
| $(\mathcal{O}_{lq}^{(1)})_{\mu\mu tt}$ | $[-0.91, 0.25]$ | $[-0.11, 0.38]$ | 1.9 | 2.9 |
| $(\mathcal{O}_{lq}^{(3)})_{\mu\mu tt}$ | $[-0.04, 0.54]$ | $[-0.22, 0.02]$ | 4.8 | 2.6 |
| $(\mathcal{O}_{eu})_{\mu\mu tt}$ | $[-1.29, 0.22]$ | $[-0.09, 0.54]$ | 1.5 | 2.6 |
| $(\mathcal{O}_{lu})_{\mu\mu tt}$ | $[-0.26, 0.95]$ | $[-0.40, 0.11]$ | 2.8 | 1.9 |
| $(\mathcal{O}_{qe})_{tt\mu\mu}$ | $[-0.22, 1.24]$ | $[-0.52, 0.09]$ | 2.7 | 1.6 |
| $(\mathcal{O}_{lq}^{(1)})_{\tau\tau tt}$ | $[-0.52, 0.96]$ | $[-0.40, 0.22]$ | 2.3 | 1.8 |
| $(\mathcal{O}_{lq}^{(3)})_{\tau\tau tt}$ | $[-0.86, 0.69]$ | $[-0.29, 0.36]$ | 1.9 | 2.1 |
| $(\mathcal{O}_{eu})_{\tau\tau tt}$ | $[-0.58, 1.18]$ | $[-0.49, 0.24]$ | 2.1 | 1.6 |
| $(\mathcal{O}_{lu})_{\tau\tau tt}$ | $[-1.01, 0.54]$ | $[-0.23, 0.42]$ | 1.8 | 2.2 |
| $(\mathcal{O}_{qe})_{tt\tau\tau}$ | $[-1.14, 0.56]$ | $[-0.23, 0.48]$ | 1.7 | 2.2 |
| $(\mathcal{O}_{lq}^{(1)})_{\ell\ell tt}$ | $[-0.16, 0.26]$ | $[-0.11, 0.07]$ | 4.7 | 3.9 |
| $(\mathcal{O}_{lq}^{(3)})_{\ell\ell tt}$ | $[-0.07, 0.29]$ | $[-0.12, 0.03]$ | 5.9 | 3.8 |
| $(\mathcal{O}_{eu})_{\ell\ell tt}$ | $[-0.24, 0.33]$ | $[-0.14, 0.10]$ | 3.8 | 3.4 |
| $(\mathcal{O}_{lu})_{\ell\ell tt}$ | $[-0.27, 0.17]$ | $[-0.07, 0.11]$ | 3.8 | 4.6 |
| $(\mathcal{O}_{qe})_{tt\ell\ell}$ | $[-0.32, 0.23]$ | $[-0.10, 0.13]$ | 3.4 | 3.9 |
| $(\mathcal{O}_{qq}^{(1)})_{tttt}$ | $[-0.55, 1.38]$ | $[-0.58, 0.23]$ | 2.1 | 1.5 |
| $(\mathcal{O}_{ud}^{(1)})_{ttbb}$ | $[0.25, 10.9]$ | $[-4.6, -0.10]$ | 0.89 | 0.37 |
| $(\mathcal{O}_{qu}^{(1)})_{tttt}$ | $[-1.47, 0.59]$ | $[-0.25, 0.62]$ | 1.4 | 2 |
| $(\mathcal{O}_{qd}^{(1)})_{ttbb}$ | $[-9.7, -0.07]$ | $[0.03, 4.06]$ | 0.41 | 0.95 |
| $(\mathcal{O}_{uB})_{tt}$ | $[-0.35, 0.10]$ | $[-0.04, 0.15]$ | 3.4 | 5.1 |
| $(\mathcal{O}_{uW})_{tt}$ | $[-0.39, 0.11]$ | $[-0.05, 0.17]$ | 3.2 | 4.7 |

Table 1: EWPD bounds on top interactions, assuming one operator at a time at the scale $\Lambda$. The bounds on the NP scale $\Lambda$ are obtained from two independent types of fits, assuming a definite sign for the coefficients $c_i$. The results for the operators $(\mathcal{O}_i)_{\ell\ell tt,tt\ell\ell}$ are obtained assuming lepton universality in the interactions. The bounds for $(\mathcal{O}_{qq}^{(8)})_{tttt}$ are too weak and have been omitted, while the operator coefficient for $(\mathcal{O}_{uu}^{(1)})_{tttt}$ cannot be constrained within our approximations (see text for details).

Thus, only the combinations of operators appearing in Eq. (7) can be constrained by EWPD, up to corrections in the RGE induced by the gauge interactions.

Constraints on four-quark interactions involving only the third family are dominated by the contributions they generate to the $Zb\bar{b}$ couplings, via the operators $\mathcal{O}_{\phi q}^{(1),(3)}$ and $\mathcal{O}_{\phi d}^{(1)}$, and are therefore somewhat weaker than the leptonic ones. Limits on $c_{qq}^{(1)}$ and $c_{qu}^{(1)}$ arise from the bounds on the left-handed bottom couplings, and are significantly stronger than those of $c_{ud}^{(1)}$ and $c_{qd}^{(1)}$, which contribute to the $Zb_R\overline{b_R}$ interactions. In particular, the strong preference for a positive (negative) value of $(c_{ud}^{(1)})_{ttbb}$ $((c_{qd}^{(1)})_{ttbb})$ follows from the corresponding preference for a large correction to the right-handed bottom coupling, $\delta g_R^b = -\frac{1}{2}(c_{\phi d}^{(1)})_{bb}\frac{v^2}{\Lambda^2}$, with $v \approx 246$ GeV, to alleviate the $-2.5$-$\sigma$ deviation in the bottom forward-backward asymmetry at the $Z$-pole. Again, some of the bounds on these four-quark operators can be easily correlated from the corresponding contributions to the running for the quark interactions in (3),

$$\frac{d(C_{\phi q}^{(1)}+C_{\phi q}^{(3)})_{ij}}{d\log\mu} = \frac{N_c}{16\pi^2}\Big\{\left(Y_u^\dagger Y_u\right)_{lk}\Big(\left(C_{qq}^{(1)}\right)_{ijkl}+\left(C_{qq}^{(1)}\right)_{klij}\Big) - 2\left(Y_u Y_u^\dagger\right)_{lk}\left(C_{qu}^{(1)}\right)_{ijkl}\Big\}+\ldots,$$
$$\frac{d(C_{\phi d}^{(1)})_{ij}}{d\log\mu} = \frac{N_c}{8\pi^2}\left(Y_u^\dagger Y_u\right)_{lk}\left(\left(C_{qd}^{(1)}\right)_{klij} - \left(C_{ud}^{(1)}\right)_{klij}\right)+\ldots\ , \quad (8)$$

that determine which combinations of operators can be constrained by EWPD. In the first line of Eq. (8) there is no contribution from $\mathcal{O}_{qq}^{(8)}$, because the corresponding corrections to the running of $C_{\phi q}^{(1)}$ and $C_{\phi q}^{(3)}$ cancel each other. There is a suppressed contribution to the running of $C_{\phi q}^{(1)} + C_{\phi q}^{(3)}$ from $C_{qq}^{(8)}$ proportional to the electroweak gauge couplings, which results in much weaker constraints. This explains the absence of a bound on $C_{qq}^{(8)}$ in Table 1. Finally, the coefficient $(C_{uu}^{(1)})_{tttt}$ only contributes to the $Zt_R\overline{t_R}$ couplings through the RGE for $(C_{\phi u}^{(1)})_{tt}$, and therefore cannot be bounded by EWPD at the order we are working.

Four-quark operators involving two quarks of the third generation and two of either the first or second generations contribute through RGE to operators that modify the electroweak couplings of the quarks in the first two generations, measured with worse precision than those of the charged leptons or bottom quark and the bounds are therefore much weaker and not reported here. If one assumes universality among the three families then the bounds are still mostly dominated by the operators involving only third generation quarks. The exception is the case of the operators $\mathcal{O}_{qd,ud}^{(1)}$, for which there is a tension between the required contribution to $\delta g_R^b$ and the corresponding one for the first two generations. This tension results in significantly improved bounds, reducing the size of the corresponding 95% probability intervals by a factor of two, e.g. $(C_{qd}^{(1)})_{ttqq}\log\frac{M_Z}{\Lambda} \in [-4.09, 0.65]$ TeV$^{-2}$.

The limits on the electroweak top dipole interactions, $\mathcal{O}_{uB}$ and $\mathcal{O}_{uW}$, come exclusively from their contributions to the running of $c_{WB}$ ($c_{WB}/\Lambda^2 \in [-0.009, 0.003]$ TeV$^{-2}$ at 95% probability), which is related to the $S$ parameter [18]. Hence, only the approximate combination $g_2(C_{uB})_{tt} + 2g_1(\frac{1}{6}+\frac{2}{3})(C_{uW})_{tt}$ (where the 1/6 and 2/3 factors are the $q_L$ and $u_R$ hypercharges, respectively), which enters in the RGE for $c_{WB}$, can be constrained.

Finally, we have not included in Table 1 the constraints on the operators that induce direct corrections to the top electroweak couplings,

$$\delta g_L^t = -\frac{1}{2}\left(V\left(c_{\phi q}^{(1)} - c_{\phi q}^{(3)}\right)V^\dagger\right)_{tt}\frac{v^2}{\Lambda^2} = -c_{\phi q}^{(t)}\frac{v^2}{\Lambda^2}, \qquad \delta g_R^t = -\frac{1}{2}\left(c_{\phi u}^{(1)}\right)_{tt}\frac{v^2}{\Lambda^2}. \quad (9)$$

Note that, to dimension six, the effects on the left-handed sector are also correlated with the direct corrections of the charged current couplings, $\delta V_{tb} = (V c_{\phi q}^{(3)})_{tb} v^2/\Lambda^2$. (We work in a flavor basis in which the SM Yukawa couplings for the down sector are diagonal.) The constraints on the combinations in Eq. (9) follow from the one-loop contributions to the $T$ parameter and corrections to the $Zb\bar{b}$ vertices. These corrections contain logarithmically enhanced terms that can be read from the RGE of the operators $\mathcal{O}_{\phi D}$ (equivalent to the $T$ parameter in our basis), $\mathcal{O}_{\phi q}^{(b)} \equiv (\mathcal{O}_{\phi q}^{(1)} + \mathcal{O}_{\phi q}^{(3)})_{bb}$ and $(\mathcal{O}_{\phi d}^{(1)})_{bb}$. We have also included finite (not proportional to logarithms) one-loop effects by integrating out the top quark with the anomalous couplings defined in Eq. (9) [19, 20]. In particular, the finite contribution to the $T$ parameter is given by

$$\alpha \Delta T = \frac{N_c}{16\pi^2} y_t^2 \mathrm{Re}\left\{(V\alpha_{\phi q}^{(3)})_{tb} V_{tb}^*\right\}\frac{v^2}{\Lambda^2}. \quad (10)$$

Because of these finite terms, the $\chi^2$ depends on both $C_i$ and $C_i \log\frac{M_Z}{\Lambda}$, so we vary $C_i$ and $\Lambda$ independently in our fits. We impose $\Lambda \geq 1$ TeV, to avoid regions where the effective Lagrangian description may break down. In the bounds below, when no mention to $\Lambda$ is made, we take the most conservative bound that is reached for $\Lambda = 1$ TeV.

Considering only one of the combinations in Eq. (9) at a time we obtain the following 95% probability interval for $\delta g_L^t / g_L^{t\,\mathrm{SM}}$,

$$\frac{\delta g_L^t}{g_L^{t\,\mathrm{SM}}} \in [-0.016, 0.002] \qquad \left(c_{\phi q}^{(t)} \in [-0.01, 0.09]\right), \quad (11)$$

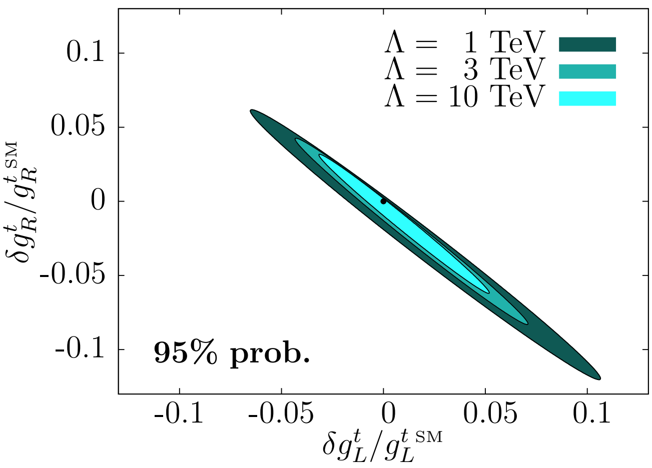
while for $\delta g_R^t / g_R^{t\,\mathrm{SM}}$ we get

$$\frac{\delta g_R^t}{g_R^{t\,\mathrm{SM}}} \in [-0.017, 0.002] \qquad \left((c_{\phi u}^{(1)})_{tt} \in [-0.08, 0.01]\right), \quad (12)$$

where $g_L^{t\,\mathrm{SM}} = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W$ and $g_R^{t\,\mathrm{SM}} = -\frac{2}{3}\sin^2\theta_W$, with $\theta_W$ the weak angle. When we consider dimension-six effects correcting both electroweak top couplings at the same time, the 95% probability bounds change to:

$$\begin{array}{cc} \dfrac{\delta g_L^t}{g_L^{t\,\mathrm{SM}}} \in [-0.048, 0.089], & \dfrac{\delta g_R^t}{g_R^{t\,\mathrm{SM}}} \in [-0.102, 0.044]. \\ \Big(c_{\phi q}^{(t)} \in [-0.52, 0.28], & (c_{\phi u}^{(1)})_{tt} \in [-0.50, 0.21].\Big) \end{array} \quad (13)$$

The weaker bounds compared with Eqs. (11) and (12) follow from a strong correlation



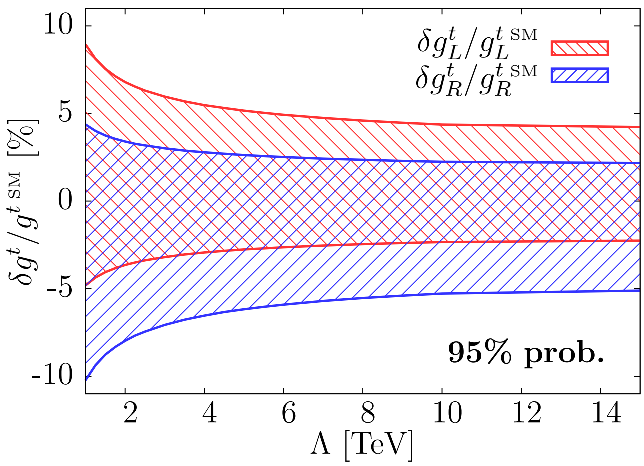




Figure 1: *Left)* 95% probability regions in the $\frac{\delta g_L^t}{g_L^{t\,\mathrm{SM}}} - \frac{\delta g_R^t}{g_R^{t\,\mathrm{SM}}}$ plane for $\Lambda$ =1, 3 and 10 TeV. *Right)* Boundaries of the 95% probability intervals for $\frac{\delta g_{L,R}^t}{g_{L,R}^{t\,\mathrm{SM}}}$ as a function of $\Lambda$.

($\approx -99\%$) between these two couplings, as can be seen in Figure 1 left, which can be understood from the leading logarithmic contributions. Indeed, neglecting the finite contributions, there is a large correlation between the corresponding dimension-six operators, which comes from the RGE for $C_{\phi D}$ (whose limit dominates the constraints in Eqs. (11) and (12) via the $T$ parameter),

$$\frac{dC_{\phi D}}{d\log\mu} = \frac{N_c}{2\pi^2}\left\{\left(C_{\phi q}^{(1)}\right)_{ij}\left(Y_u^\dagger Y_u\right)_{ji} - \left(Y_u^\dagger Y_u\right)_{ij}\left(C_{\phi u}^{(1)}\right)_{ji}\right\} + \ldots \,. \tag{14}$$

At this level there is an approximate flat direction for $\delta g_L^t/g_L^{t\,\mathrm{SM}} = -\delta g_R^t/g_R^{t\,\mathrm{SM}}$ (we have used $g_L^{t\,\mathrm{SM}} \approx -2 g_R^{t\,\mathrm{SM}}$ and $c_{\phi q}^{(3)} = -c_{\phi q}^{(1)}$), which is also reflected in the equality of the bounds in Eqs. (11) and (12). This flat direction is lifted however by the logarithmic contributions to the $Zb_L\overline{b_L}$ vertex and, to a less extent, the contributions in the running from gauge interactions. The global factors in front of the finite terms turn out to be smaller than the ones in the logarithmic terms, so the effects from the former are not very important for values of $\Lambda$ consistent with the effective Lagrangian description. In Figure 1 right, we show how the bounds obtained in Eq. (13) for $\Lambda = 1$ TeV evolve as we increase the value of the scale of NP.

The results in Eq. (13) imply quite strong bounds on deviations with the respect to the SM electroweak theory, at the few percent level. One may wonder though about the robustness of these bounds when interpreted within particular models. For instance, as explained above, the effects in the $T$ parameter have a strong impact in these bounds. Now, the $T$ parameter is known to have a strong correlation with the $S$ parameter (we obtain an 89% correlation), and many SM extensions in which the NP interacts mainly with the third generation come also usually accompanied by relatively large positive contributions to $S$. Still, considering a positive value for $S = 0.2$ (about the 95% probability limit obtained from the $S$-$T$ fit) does not have a dramatic impact on the bounds in Eq. (13), due to the constraints on the logarithmic contributions to $Zb_L\overline{b_L}$. We thus obtain only a moderate shift in the bounds for the right-handed couplings,

$$\frac{\delta g_L^t}{g_L^{t\,\mathrm{SM}}} \in [-0.050, 0.088]\,, \qquad \frac{\delta g_R^t}{g_R^{t\,\mathrm{SM}}} \in [-0.123, 0.023] \quad (S = 0.2), \tag{15}$$

but we can still conclude that NP contributions to $Zt\bar{t}$ couplings beyond 10% are disfavored by the data.

## 4 Conclusions

Among the possible NP deformations of the SM there are several on which no direct experimental information can be extracted from present or past experiments. Indirect constraints can however be obtained in some cases from their loop contribution to the very precisely measured EWPD. Using a model-independent effective Lagrangian approach, we have explored in this article the potential that EWPD have to put constraints on dimension-six effective operators on which no precise direct information is currently available. We have shown that, despite the loop suppression, the leading contributions proportional to the top Yukawa coupling are large enough to place significant bounds on several interactions involving the top quark.

Our results, reported in Table 1, show that, for NP in the TeV region, EWPD can constrain the dimension-six operator coefficients for a large set of interactions involving the top quark to $O(0.1)$ values. These results are obtained using the logarithmically enhanced one-loop contributions as given by the RGE together with the finite terms in the most significant cases. Barring accidental cancellations, similar bounds will apply for all the interactions we have considered, even if the missing finite terms give a comparable effect to the logarithmically enhanced ones.

The bounds presented here have been computed assuming only one operator at a time at the scale of NP. There are several scenarios with new scalars, quarks or vector bosons for which such operators can be generated alone, or whose effects are not correlated with the contributions to other dimension-six interactions [12, 21, 22]. Nevertheless, we have discussed the origin of the leading constraints and described the most relevant approximate flat directions so that bounds on more complicated models can be estimated. As an example, let us discuss the case of lepton-top four-fermion interactions. For NP at the TeV scale we are able to place $O(0.1)$ bounds on the coefficients of contact interactions resulting from the products of vector currents involving two electron and two top fields, with all possible chiralities. As we have mentioned, the effects of some of these operators are correlated in the RGE that enter in the EWPD so, in practice, we can only constrain three combinations of these $e^+e^-t\bar{t}$ operators. Removing the two redundant interactions from a simultaneous global fit to all the operators we obtain the following limits (for $\Lambda = 1$ TeV):

$$\begin{aligned} (c_{lq}^{(1)} - c_{lu})_{eett} &\in [-0.21, 0.08]\,, \\ (c_{lq}^{(3)})_{eett} &\in [-0.31, 0.06]\,, \\ (c_{qe} - c_{eu})_{eett} &\in [-0.10, 0.29]\,. \end{aligned} \tag{16}$$

In particular, these $e^+e^-t\bar{t}$ contact interactions can be tested in future colliders [23], like the ILC [24] or the FCC-ee [25], and are therefore relevant to guide NP searches in these facilities. The same applies for our analysis of the $Zt\bar{t}$ couplings, which could be directly measured at this kind of experiments with great precision (around 1% for the ILC with 500 fb$^{-1}$ [26]). Finally, while less precise, our limits on four-top interactions are comparable (and in some cases significantly better) than the latest ones obtained by the LHC [27].

Higher-dimensional operators involving the top quark are not only among the least constrained operators that contribute the most to EWPD, they are also among the best motivated ones in models that attempt to solve the hierarchy problem. In many of these models, a sizeable contribution to the $S$ parameter is generated at the NP scale. We have considered that possibility, see Eq. (15), and shown that while inducing a shift in the allowed values of the coefficients of the higher-dimensional operators involving the top, these can still be constrained to a similar level of accuracy. This also illustrates the fact that, the limits we have computed, being indirect limits, are sensitive to assumptions about operators that can give tree-level contribution to EWPD. In the presence of such operators the quantitative results may change but the qualitative fact that one-loop contributions can place the most stringent bounds on currently untested operators still holds. These bounds are therefore a crucial piece of information, both for model building purposes and as a guide for future experimental searches.

**Note added:** While this manuscript was being prepared for submission, ref. [28] appeared in the arXiv. In that work, a global fit to dimension-six effective operators involving the top quark is performed using top production data from the LHC and the Tevatron. Our approach is complementary to theirs, in the sense that our analysis is sensitive to a different class of operators. In the few cases in which there is overlap, our results provide more stringent bounds on the coefficients of the operators.

# Acknowledgements

We thank L. Silvestrini for useful discussions. The work of JB has been supported by the European Research Council under the European Union's Seventh Framework Programme (FP / 2007-2013) / ERC Grant Agreement n. 279972. The work of JS has been partially supported by the European Commission (PITN-GA-2012-316704 HIGGSTOOLS), by MINECO under grants (FPA2010-17915 and FPA2013-47836-C3-2-P) and by Junta de Andalucía grants FQM 101 and FQM 6552. JS thanks the Pauli Center Visitor Program for financial support.

# References


[1] J. de Blas, M. Chala and J. Santiago, Phys. Rev. D **88** (2013) 095011 [arXiv:1307.5068 [hep-ph]].

[2] J. A. Aguilar-Saavedra, Nucl. Phys. B **821** (2009) 215 [arXiv:0904.2387 [hep-ph]]; Nucl. Phys. B **821** (2009) 215 [arXiv:0904.2387 [hep-ph]]; Nucl. Phys. B **843** (2011) 638 [Nucl. Phys. B **851** (2011) 443] [arXiv:1008.3562 [hep-ph]]; C. Zhang and S. Willenbrock, Phys. Rev. D **83** (2011) 034006 [arXiv:1008.3869 [hep-ph]]; C. Degrande, J. M. Gerard, C. Grojean, F. Maltoni and G. Servant, JHEP **1103** (2011) 125 [arXiv:1010.6304 [hep-ph]]; J. A. Aguilar-Saavedra and M. Perez-Victoria, JHEP **1105** (2011) 034 [arXiv:1103.2765 [hep-ph]]; C. Degrande, J. M. Gerard, C. Grojean, F. Maltoni and G. Servant, Phys. Lett. B **703** (2011) 306 [arXiv:1104.1798 [hep-ph]]; JHEP **1207** (2012) 036 [JHEP **1303** (2013) 032] [arXiv:1205.1065 [hep-ph]].

[3] C. Zhang, Phys. Rev. D **90** (2014) 1, 014008 [arXiv:1404.1264 [hep-ph]]; C. Zhang, J. Phys. Conf. Ser. **556** (2014) 1, 012030 [arXiv:1410.2825 [hep-ph]].

[4] A. De Rujula, M. B. Gavela, P. Hernandez and E. Masso, Nucl. Phys. B **384** (1992) 3; K. Hagiwara, S. Ishihara, R. Szalapski and D. Zeppenfeld, Phys. Lett. B **283** (1992) 353; Phys. Rev. D **48** (1993) 2182; S. Alam, S. Dawson and R. Szalapski, Phys. Rev. D **57** (1998) 1577 [hep-ph/9706542]; H. Mebane, N. Greiner, C. Zhang and S. Willenbrock, Phys. Rev. D **88** (2013) 1, 015028 [arXiv:1306.3380 [hep-ph]].

[5] E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1310** (2013) 087 [arXiv:1308.2627 [hep-ph]].

[6] E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1401** (2014) 035 [arXiv:1310.4838 [hep-ph]].

[7] R. Alonso, E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1404** (2014) 159 [arXiv:1312.2014 [hep-ph]].

[8] J. Elias-Miró, J. R. Espinosa, E. Masso and A. Pomarol, JHEP **1308** (2013) 033 [arXiv:1302.5661 [hep-ph]]; JHEP **1311** (2013) 066 [arXiv:1308.1879 [hep-ph]]; J. Elias-Miró, C. Grojean, R. S. Gupta and D. Marzocca, JHEP **1405** (2014) 019 [arXiv:1312.2928 [hep-ph]].

[9] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010** (2010) 085 [arXiv:1008.4884 [hep-ph]].

[10] G. Aad *et al.* [ATLAS and CMS Collaborations], Phys. Rev. Lett. **114** (2015) 191803 [arXiv:1503.07589 [hep-ex]]; [ATLAS and CDF and CMS and D0 Collaborations], arXiv:1403.4427 [hep-ex]; K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38** (2014) 090001; M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C **71** (2011) 1515 [Eur. Phys. J. C **72** (2012) 1874] [arXiv:1010.4180 [hep-ph]]; T. E. W. Group [CDF and D0 Collaborations],

arXiv:1204.0042 [hep-ex]; LEP Electroweak Working Group [ALEPH and CDF and D0 and DELPHI and L3 and OPAL and SLD and LEP Electroweak Working Group and Tevatron Electroweak Working Group and SLD Electroweak and Heavy Flavour Groups Collaborations], arXiv:1012.2367 [hep-ex]; S. Schael *et al.* [ALEPH and DELPHI and L3 and OPAL and SLD and LEP Electroweak Working Group and SLD Electroweak Group and SLD Heavy Flavour Group Collaborations], Phys. Rept. **427** (2006) 257 [hep-ex/0509008].

[11] Z. Han and W. Skiba, Phys. Rev. D **71** (2005) 075009 [hep-ph/0412166]; F. del Aguila and J. de Blas, Fortsch. Phys. **59** (2011) 1036 [arXiv:1105.6103 [hep-ph]]; M. Ciuchini, E. Franco, S. Mishima and L. Silvestrini, JHEP **1308** (2013) 106 [arXiv:1306.4644 [hep-ph]]; J. de Blas, EPJ Web Conf. **60** (2013) 19008 [arXiv:1307.6173 [hep-ph]]; A. Pomarol and F. Riva, JHEP **1401** (2014) 151 [arXiv:1308.2803 [hep-ph]]; J. Ellis, V. Sanz and T. You, JHEP **1503** (2015) 157 [arXiv:1410.7703 [hep-ph]]; A. Efrati, A. Falkowski and Y. Soreq, arXiv:1503.07872 [hep-ph].

[12] J. de Blas, M. Chala, M. Perez-Victoria and J. Santiago, JHEP **1504** (2015) 078 [arXiv:1412.8480 [hep-ph]].

[13] J. de Blas et al. In preparation.

[14] J. de Blas, M. Ciuchini, E. Franco, D. Ghosh, S. Mishima, M. Pierini, L. Reina and L. Silvestrini, arXiv:1410.4204 [hep-ph].

[15] C. Zhang, N. Greiner and S. Willenbrock, Phys. Rev. D **86** (2012) 014024 [arXiv:1201.6670 [hep-ph]].

[16] C. Hartmann and M. Trott, arXiv:1505.02646 [hep-ph]; M. Ghezzi, R. Gomez-Ambrosio, G. Passarino and S. Uccirati, arXiv:1505.03706 [hep-ph].

[17] M. Carpentier and S. Davidson, Eur. Phys. J. C **70** (2010) 1071 [arXiv:1008.0280 [hep-ph]].

[18] M. E. Peskin and T. Takeuchi, Phys. Rev. D **46** (1992) 381.

[19] A. Pomarol and J. Serra, Phys. Rev. D **78** (2008) 074026 [arXiv:0806.3247 [hep-ph]].

[20] C. Anastasiou, E. Furlan and J. Santiago, Phys. Rev. D **79** (2009) 075003 [arXiv:0901.2117 [hep-ph]].

[21] F. del Aguila, M. Perez-Victoria and J. Santiago, JHEP **0009** (2000) 011 [hep-ph/0007316].

[22] F. del Aguila, J. de Blas and M. Perez-Victoria, JHEP **1009** (2010) 033 [arXiv:1005.3998 [hep-ph]].

[23] B. Grzadkowski, Z. Hioki and M. Szafranski, Phys. Rev. D **58** (1998) 035002 [hep-ph/9712357].

[24] H. Baer, T. Barklow, K. Fujii, Y. Gao, A. Hoang, S. Kanemura, J. List and H. E. Logan *et al.*, arXiv:1306.6352 [hep-ph].

[25] M. Bicer *et al.* [TLEP Design Study Working Group Collaboration], JHEP **1401** (2014) 164 [arXiv:1308.6176 [hep-ex]].

[26] E. Devetak, A. Nomerotski and M. Peskin, Phys. Rev. D **84** (2011) 034029 [arXiv:1005.1756 [hep-ex]].

[27] G. Aad *et al.* [ATLAS Collaboration], arXiv:1505.04306 [hep-ex].

[28] A. Buckley, C. Englert, J. Ferrando, D. J. Miller, L. Moore, M. Russell and C. D. White, arXiv:1506.08845 [hep-ph].