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Single-Superfield Helical-Phase Inflation

Sergei V. Ketov [a,b,c] and Takahiro Terada [d,e]

[a] *Department of Physics, Tokyo Metropolitan University*
Minami-ohsawa 1-1, Hachioji-shi, Tokyo 192-0397, Japan
[b] *Kavli Institute for the Physics and Mathematics of the Universe (IPMU)*
The University of Tokyo, Chiba 277-8568, Japan
[c] *Institute of Physics and Technology, Tomsk Polytechnic University*
30 Lenin Ave., Tomsk 634050, Russian Federation
[d] *Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan*
[e] *Deutsches Elektronen-Synchrotron (DESY), 22607 Hamburg, Germany*

ketov@tmu.ac.jp, takahiro@hep-th.phys.s.u-tokyo.ac.jp

Abstract

Large field inflation in supergravity requires an approximate global symmetry to ensure flatness of the scalar potential. In helical phase inflation, a U(1) symmetry of the Kähler potential is used, the phase part of the complex scalar of a chiral superfield plays the role of inflaton, and the radial part is strongly stabilized. The original model of helical phase inflation, proposed by Li, Li and Nanopoulos (LLN), employs an extra (stabilizer) superfield. We propose a more economical and new class of the helical phase inflationary models without the stabilizer superfield. As the examples, the quadratic, the natural, and the Starobinsky-type inflationary models are studied in our approach.

1 Introduction

Inflation well explains the origin of the primordial density fluctuations as well as flatness and homogeneity of the universe. The general idea is so far quite successful, and inflationary models are confronted with precise observational data [1, 2]. Because inflation is a high-energy phenomenon, it is important to study inflation in a more fundamental framework such as supergravity [3] which is motivated by particle (or string) physics. In particular, if tensor perturbations are detected in a near future, it would imply large field excursions of the inflaton [4]. In that case, the Planck suppressed corrections cannot be neglected. Even if supersymmetry is broken at a higher scale than that of inflation, supergravity corrections have substantial impact on the scalar potential.

As is well known, a generic scalar potential in supergravity tends to be very steep in the large field region, because of the exponential factor of the Kähler potential. Accordingly, it is hard to tune the flatness of the scalar potential along the whole inflaton trajectory in the case of large field models. Therefore, some symmetries are imposed in the inflationary model building in supergravity. A good example is the axion-like shift symmetry in the non-SUSY model [5] and in the supergravity-based ones [6, 7].

Another example is given by the U(1) symmetry that is equally ubiquitous as the shift symmetry in particle physics. The inflation with U(1) symmetry and the monodromy structure of the superpotential is known under the name of helical phase inflation, because its inflaton is the phase component of a complex scalar field that rolls down a helicoid potential [8–11]. Similarly to the standard approach in the shift symmetric case, a stabilizer superfield is used in the known helical phase inflationary models. As a matter of fact, the models based on U(1) and the shift symmetric models with a stabilizer superfield are equivalent, since they are related by field redefinition. However, a minimal U(1) symmetric Kähler potential corresponds to the infinite shift symmetric series with specific coefficients. Therefore, certain formulae become simple in a particular formalism, and it is worth studying the helical phase inflation separately.

In the context of the shift symmetric approach, the stabilizer superfield is needed to ensure the positivity of the potential. In our previous work [12, 13] (see also Ref. [14]), we proposed the alternative framework to ensure the positivity by stabilizing the scalar superpartner of the inflaton. In our approach, a shift symmetric quartic term is added to the Kähler potential.

In this paper, we study a helical phase inflation without a stabilizer superfield. The radial part is stabilized in Section 2 by employing a higher order term in the Kähler potential, similarly to Refs. [12,13]. Particular models are studied in Section 3. We conclude in Section 4. Throughout the paper, we take the natural (reduced) Planck units, $c = \hbar = M_{\rm P}/\sqrt{8\pi} = 1$.

2 Stabilization of the radial component of complex inflaton

Since the inflationary trajectory is supposed to be in the phase direction in the helical phase inflation, the radial direction has to be constant during inflation. For example, in the LLN model of helical phase inflation, it is achieved by taking the superpotential proportional to a negative power of the inflaton. Balancing the superpotential contribution diverging at the origin with the exponentially rising contribution due to the Kähler potential results in stabilization of the radial part at a value of the order of the Planck scale.

In our case without a stabilizer superfield, the inflaton potential includes both the superpotential itself and its derivative, and the formulae become rather complicated. Therefore, instead of dealing with a numerical minimization of the radial part, we employ a strong stabilization mechanism by using a higher order term in the Kähler potential,

$$K = \left(\bar{\Phi}\Phi - \Phi_0^2\right) - \frac{\zeta}{4}\left(\bar{\Phi}\Phi - \Phi_0^2\right)^4. \tag{1}$$

The first term is the usual minimal Kähler potential. The constant term is added so that the expectation value of the Kähler potential approximately vanishes. The second term is introduced for the purpose of stabilization. Thanks to that term, the radial part is stabilized at $|\Phi| \simeq \Phi_0$. More general Kähler potentials with similar features may exist, but we find the above example to be simple and efficient. Similar stabilization mechanisms are used in the literature [12–17].

The stabilization parameter ζ in eq. (1) is a real positive parameter. Some comments on its magnitude are in order. For too large ζ with a fixed Φ_0, the Kähler metric (the coefficient at the kinetic term) may change its sign before reaching the symmetric phase, $\langle\Phi\rangle = 0$. In such a case, the above Kähler potential should be regarded as the effective description of the Higgsed phase, $\langle\Phi\rangle \neq 0$. It is enough for our purpose, since the radial part is stabilized throughout the process of inflation and we do not have to consider its dynamics. Conversely, if ζ becomes small for a fixed Φ_0, the stabilized position of the radial part shifts inward, $|\Phi| < \Phi_0$ and eventually moves to the origin for $\zeta \to 0$. Depending on the value of ζ, the inflaton may move inward to the origin during inflation by the classical dynamics, quantum fluctuation, or quantum tunneling. To avoid this situation, we take values of ζ at least slightly smaller than the critical value for $K_{\Phi\bar{\Phi}} \leq 0$.[1]

As regards the strength of the stabilization, we obtain an expression for the mass of the radial part. We assume the potential can be approximated as $(K^{\Phi}K_{\Phi} - 3)|W|^2 = (\Phi_0^2 -$

[1] For example, in the cases of $\Phi_0 = 1.8$ (*cf.* Figs. 1 and 2) and $\Phi_0 = 5$ (*cf.* Fig. 3), the field region with $K_{\Phi\bar{\Phi}} < 0$ appears when $\zeta \gtrsim 0.12$ and $\zeta \gtrsim 2.5 \times 10^{-4}$, and we take $\zeta = 0.11$ and $\zeta = 0.03$ in the Figures, respectively. The situation with the radial part stabilized at $|\Phi| \simeq 5$ without the region of $K_{\Phi\bar{\Phi}} < 0$ inside the disk ($|\Phi| \leq 5$) can be realized *e.g.* with $\Phi_0 = 7$ and $\zeta = 3.399 \times 10^{-5}$.

$3)|W|^2$, and neglect derivatives of the superpotential since they are proportional to the slow-roll parameters. At $|\Phi| = \Phi_0$, the canonically normalized squared mass of the radial part is

$$m_{\text{radial}}^2 \simeq \frac{3(12\zeta\Phi_0^6 + 2\Phi_0^4 - \Phi_0^2 - 2)}{\Phi_0^2 - 3} H^2 \gtrsim 20H^2. \tag{2}$$

In the last inequality, we assume $\zeta \geq 0$ and $\Phi_0^2 > 3$. Thus, it is not difficult to strongly stabilize the radial part. As long as the radial component is stabilized with its mass much larger than the Hubble scale, the following discussion is independent of the detailed mechanism of the stabilization.

3 Helical phase inflationary models in our approach

Having stabilized the radial mode at $|\Phi| = \Phi_0$, let us consider typical inflationary models, without introducing a stabilizer superfield. Let us parametrize the inflaton field as $\Phi = \Phi_0 e^{i\theta/\sqrt{2}\Phi_0}$. The phase is scaled so that it is canonically normalized. The superpotential breaks the U(1) symmetry in the Kähler potential, and generates the inflaton (scalar) potential. We study chaotic inflation with the quadratic potential, the Starobinsky-like plateau potential, and a sinusoidal potential in this Section.

3.1 Quadratic helical-phase inflation

The logarithmic singularity in the superpotential is the heart of the helical phase inflation, which is needed to realize a nontrivial spiral shape. Let us take the simplest Ansatz

$$W = m \log \frac{\Phi}{f} \quad , \tag{3}$$

where m sets the scale of inflation, and $f \equiv f_0 e^{i\theta_0/\sqrt{2}\Phi_0}$ (with f_0 and θ_0 real) is the dimensional parameter controlling the cosmological constant.

After stabilization, the inflaton potential becomes

$$V = \frac{1}{2} m_{\text{inf}}^2 (\theta - \theta_0)^2 + \Lambda, \tag{4}$$

with

$$m_{\text{inf}} = \frac{|m|\sqrt{\Phi_0^2 - 3}}{\Phi_0} \quad , \tag{5}$$

$$\Lambda = |m|^2 \left(\frac{1}{\Phi_0^2} + 2\log\frac{\Phi_0}{f_0} + \left(\Phi_0^2 - 3\right) \left|\log\frac{\Phi_0}{f_0}\right|^2 \right). \tag{6}$$

Thus, the quadratic scalar potential is obtained under the condition $\Phi_0 > \sqrt{3}$. The cosmological constant can be eliminated by choosing

$$f_0 = \Phi_0 e^{\frac{1}{\Phi_0(\Phi_0 \pm \sqrt{3})}}. \tag{7}$$

The full potential is shown in Fig. 1 for a limited field range. As is clear from the Figure, the radial part is strongly stabilized, while its mass increases with the potential. This is also implied by Eq. (2).

In this model, the gravitino mass at the vacuum is

$$m_{3/2} = \frac{|m|}{\Phi_0(\Phi_0 \pm \sqrt{3})} \quad . \tag{8}$$

On the one hand, in the case of $(\Phi_0 - \sqrt{3}) \ll 1$, the inflaton becomes much lighter than the gravitino, $m_{\rm inf} \ll m_{3/2}$. On the other hand, in the large Φ_0 limit, the inequality is reversed, $m_{3/2} \ll m_{\rm inf}$, and one gets the cosmological gravitino problem.

Though the quadratic potential is already excluded by Planck observations, some modifications or coupling to other sectors may make the quadratic model consistent with the data (see *e.g.*, Ref. [18]). Instead of studying such possibilities, we directly construct some viable inflationary models in the next subsections.

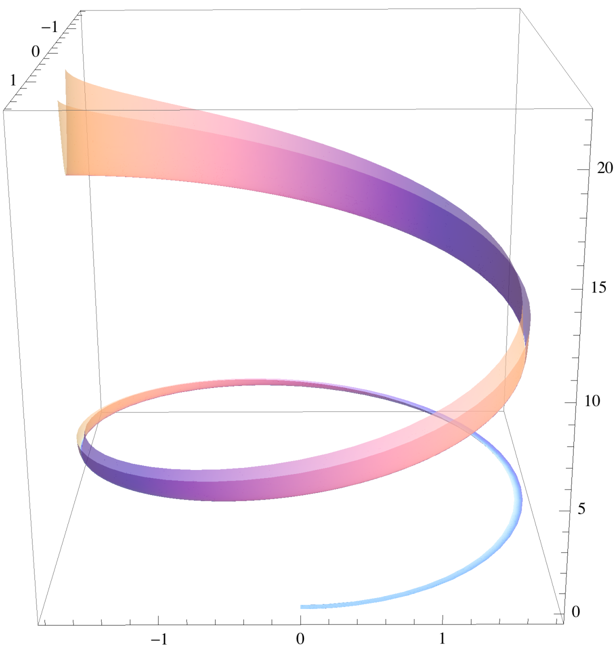

Figure 1: The quadratic potential for helical phase inflation. The parameters are chosen as $\Phi_0 = 1.8$ and $\zeta = 0.11$.

3.2 Starobinsky-like helical-phase inflation

In the previous Subsection, the logarithm $\log\Phi = \log\Phi_0 + i\theta/(\sqrt{2}\Phi_0)$ in the superpotential leads to the quadratic potential. A plateau-type potential consists of the exponential factors like $e^{-\theta}$, so let us consider the exponential of the logarithm, $e^{i\log\Phi} = \Phi^i$ which is equivalent to the imaginary power of the superfield. In other words, let us take the following superpotential:

$$W = m\left(c + \Phi^i\right), \tag{9}$$

where m and c are the constant parameters that determine the scale of inflation and the cosmological constant (see below).

After stabilization, the inflaton potential becomes

$$V = |m|^2\left(A + Be^{-\theta/\sqrt{2}\Phi_0} + Ce^{-2\theta/\sqrt{2}\Phi_0}\right), \tag{10}$$

with the coefficients

$$A = |c|^2\left(\Phi_0^2 - 3\right), \tag{11}$$

$$B = 2|c|\left[\left(\Phi_0^2 - 3\right)\cos\left(\log\Phi_0 - \varphi\right) - \sin\left(\log\Phi_0 - \varphi\right)\right], \tag{12}$$

$$C = \Phi_0^2 - 3 + \frac{1}{\Phi_0^2}, \tag{13}$$

where the phase φ is defined by $c = |c|e^{i\varphi}$.

For any Φ_0 larger than $\sqrt{3}$, A and C are positive definite, and the sign of B depends on φ. There exists a solution of φ such that B is negative and, moreover, the cosmological constant vanishes. The potential is a generalization of the Starobinsky potential. Such potentials are often called “Starobinsky-like” in the literature. Our Starobinsky-like scalar potential is shown in Fig. 2.

The masses of inflaton and gravitino are given by

$$m_{\rm inf} = \frac{|mc|}{\Phi_0}\sqrt{\Phi_0^2 - 3}, \tag{14}$$

$$m_{3/2} = |mc|\left|e^{i\varphi} - e^{i\log\Phi_0}\sqrt{\frac{\Phi_0^2(\Phi_0^2-3)}{\Phi_0^2(\Phi_0^2-3)+1}}\right|. \tag{15}$$

Similarly to the previous case, $m_{\rm inf} \ll m_{3/2}$ for small $(\Phi_0^2 - 3)$, whereas the opposite relation takes place for large Φ_0^2.

The spectral index is the same as that of the Starobinsky model, but the tensor-to-scalar ratio is different:

$$1 - n_{\rm s} = \frac{2}{N} \qquad \text{and} \qquad r = \frac{16\Phi_0^2}{N^2}, \tag{16}$$

in the leading order of N^{-1}. With $\Phi_0^2 > 3$, the tensor-to-scalar ratio is enhanced, when being compared to the Starobinsky model ($r = 12/N^2$). With an arbitrary imaginary power Φ^{bi}

instead of Φ^i in Eq. (9), where b is a real parameter, the tensor-to-scalar ratio is divided by $|b|$ as $r = 16\Phi_0^2/|b|N^2$.

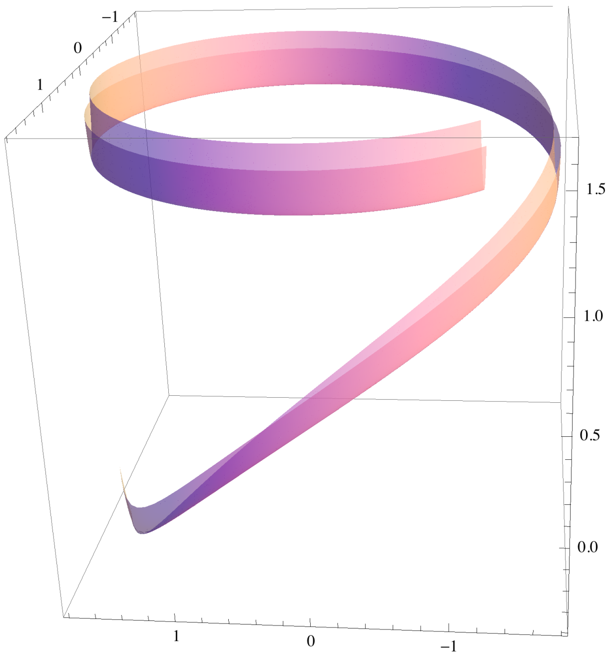

Figure 2: The Starobinsky-like potential for helical phase inflation. The parameters are chosen as $\Phi_0 = 1.8$, $\zeta = 0.11$ and $\varphi = 3.85$.

3.3 Natural helical-phase inflation

The previous examples are based on the superpotentials having the singularity at the origin. However, it is not the necessary feature of our mechanism because of the super-Planckian value of the radial component. Let us take the superpotential of the previous Subsection and replace its imaginary power by a real power as

$$W = m\,(c + \Phi)\,. \tag{17}$$

This is simply a linear function without the monodromy structure. In this case, a large value of $|\Phi|$ is required not only by the positivity of the stabilized potential but also by the observational status of natural inflation.

After stabilization, the inflaton potential becomes

$$V = |m|^2 \left[D + E \cos\left(\frac{\theta}{\sqrt{2}\Phi_0} - \varphi \right) \right] \,, \tag{18}$$

with the coefficients

$$D = |c|^2(\Phi_0^2 - 3) + \Phi_0^4 - \Phi_0^2 + 1 \quad , \tag{19}$$

$$E = 2|c|\Phi_0(\Phi_0^2 - 2) \quad , \tag{20}$$

and φ is again the argument of c, $c = |c|e^{i\varphi}$. The cosmological constant vanishes when

$$|c| = \frac{\Phi_0(\Phi_0^2 - 2) \pm \sqrt{3}}{\Phi_0^2 - 3} \quad . \tag{21}$$

In this case, the potential is positive when $\Phi_0^2 > 3$ (2) for the upper (lower) sign, and the sinusoidal scalar potential of natural inflation is obtained. The potential is shown in Fig. 3.

The masses of inflaton and gravitino are given by

$$m_{\text{inf}} = |m|\sqrt{\frac{\Phi_0^2 - 2}{\Phi_0}} \quad , \tag{22}$$

$$m_{3/2} = \frac{|m|}{\Phi_0 \mp \sqrt{3}} \quad . \tag{23}$$

Again, if the absolute value of the field is barely larger than the critical value $\sqrt{2}$ (this is for the lower sign), the inflaton is much lighter than the gravitino. In the large VEV case, gravitino becomes much lighter than the inflaton.

The parameter of the natural inflation is tightly constrained by the CMB observations. The decay constant (in our case $\sqrt{2}\Phi_0$) must be larger than 6.9 at 95% confidence level [2], so that the lower bound on the absolute value is obtained as $\Phi_0 \gtrsim 4.9$.

4 Conclusion

In this paper we studied helical phase inflation with a single chiral superfield in supergravity, *i.e.* without the stabilizer superfield used in the known versions of helical phase inflation in the literature.

In order to ensure positivity of the scalar potential and avoid computational complexity, we introduced a stabilization term to the Kähler potential that fixes the radial component of the inflaton complex scalar at a sufficiently large value. It results in technical simplification also. After the stabilization, a slow-roll inflation occurs in the direction of the phase component.

We exemplified our findings on the three simple models of the single-superfield helical-phase inflation. It implies that there should be many more possibilities to obtain viable inflationary potentials in our approach. One such noticeable generalization is a hybrid version of the models in Subsections 3.2 and 3.3. Let us take an arbitrary complex power of the inflaton superfield, $W = m(c + \Phi^{a+ib})$ with a and b real. This model interpolates between the natural inflation and

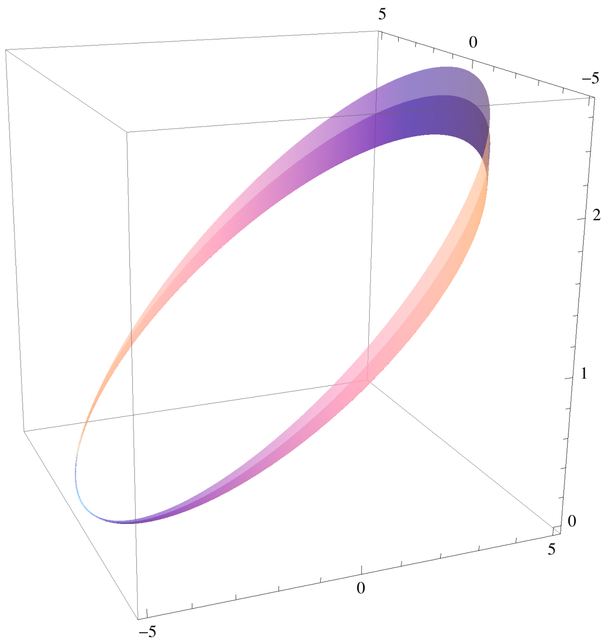

Figure 3: The sinusoidal potential for helical phase inflation. The parameters are chosen as $\Phi_0 = 5$ and $\zeta = 0.03$, and the upper sign is taken in Eq. (21).

the Starobinsky-like inflation. A similar model was studied in the presence of the stabilizer superfield in Ref. [11].

In summary, we proposed the new type of inflationary mechanism in supergravity, combining the ideas of helical phase inflation [8–11] and single-superfield inflation with the higher dimensional stabilization term in the Kähler potential [12, 13]. Our models are simple: the kinetic term is approximately canonical, the superpotential is very economical, and no stabilizer superfield (or extra d.o.f.) is present.

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