

# Prompt double $J/\psi$ production in proton-proton collisions at the LHC

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We provide a detailed study of prompt double  $J/\psi$  production within the non-relativistic QCD (NRQCD) framework in proton-proton collisions at the LHC. We confront the recent LHC data with the results obtained at leading-order (LO) in the NRQCD framework within two approaches of the collinear factorization and the  $k_T$ -factorization. We show that the LHCb data can be fairly described within the  $k_T$ -factorized LO NRQCD, while the collinearly factorized LO NRQCD significantly overshoots the LHCb data at low  $J/\psi$ -pair invariant mass. We show that the LO NRQCD formalism cannot describe the recent CMS data, with about one order of magnitude discrepancy. If the CMS data are confirmed, this indicates rather large higher-order corrections for prompt double  $J/\psi$  production. We provide various predictions which can further test the NRQCD-based approach at the LHC in a kinematic region that LO contributions dominate. We also investigate long-range in rapidity double  $J/\psi$  correlations. We found no evidence of a ridge-like structure for double  $J/\psi$  production in proton-proton collisions at the LHC up to subleading  $\alpha_s^6$  accuracy.

## I. INTRODUCTION

The nonrelativistic QCD (NRQCD) factorization [1] is an effective theory for description of the heavy quarkonium production [2]. In the NRQCD, the production and decay of heavy quarkonium factorizes into two stages, a heavy quark-antiquark pair is first created perturbatively at short distances, calculated by expansion in the strong-coupling constant  $\alpha_s$ , and then nonperturbatively evolves into quarkonium at long distance. The last stage is described via a universal long-distance matrix elements which are strongly ordered in size by relative velocity  $v$  (of the heavy quark) scaling rules [3]. In the NRQCD framework, the heavy quark-antiquark pairs may appear both as color singlet (CS) and color octet (CO) states. The latter gives rise to the CO mechanism. It is still an open question to understand to what degree the CO mechanism plays a role in quarkonium production. In this paper, we investigate if the current LHC data for prompt double  $J/\Psi$  can provide any extra information to address this question.

During last decade there has been tremendous progress in systematic studies of heavy quarkonium production in the NRQCD framework, for a recent review see Ref. [2]. In particular, it was recently shown that the next-to-leading-order (NLO) calculation for single heavy quarkonium production provide a good description of the experimental data at the LHC and Tevatron [4]. In contrast, the prompt double heavy quarkonium production is a more complicated process, and at the moment full NLO calculation for this process in the NRQCD framework is not yet available. Recently He and Kniesl in a nice paper [5] showed that the full LO up to  $\mathcal{O}(\alpha_s^4)$  accuracy for the prompt double  $J/\psi$  production, calculated in the collinearly factorized NRQCD formalism has sizable discrepancies with the recent LHC data. In this paper, we analyse the recent LHC data measured by the LHCb [6] and the CMS [7] collaborations for the prompt double  $J/\psi$  within the LO NRQCD framework but in the  $k_T$ -factorization approach. The main advantage of the  $k_T$ -factorization [8–10] over the collinear factorization [11] approach is that one can effectively incorporate some of small- $x$  dynamics via initial-state gluon radiations. This is important given the fact that at the LHC, Bjorken  $x$  can be small (approximately  $x \leq 10^{-2}$  in all experimental data considered here). Here we also extend the previous studies by quantifying various theoretical uncertainties including uncertainties associated to the unintegrated gluon density and our freedom to choose different values for the factorization/renormalization scale. Having quantified all theoretical uncertainties, we examine the importance of CS and CO prescription of the LHC data for double  $J/\psi$  production. We show that within the theoretical errors, the LHCb data are consistent with the LO NRQCD results obtained in the  $k_T$ -factorization approach by including only CS contribution, while there are more than one order of magnitude discrepancy between LO NRQCD results and the CMS data both in the collinear factorization and the  $k_T$ -factorization approaches. Note that the LHCb and the CMS kinematic coverage for the measurements of prompt double  $J/\psi$  production are complementary to each other. Therefore, if the CMS data are confirmed, this indicates rather large higher-order corrections for prompt double  $J/\psi$  production, see also Refs. [12–15]. We provide various predictions which can further test the NRQCD-based approach at the LHC in a kinematic region that LO contributions dominate.

The discovery of the so-called ridge phenomenon, namely the near-side di-hadron correlations which extend to a large pseudorapidity separation, in high-multiplicity events selection in both proton-proton and proton(nucleon)-nucleus collisions at the LHC and RHIC [16–22] triggered an on-going debate about the underlying dynamics of

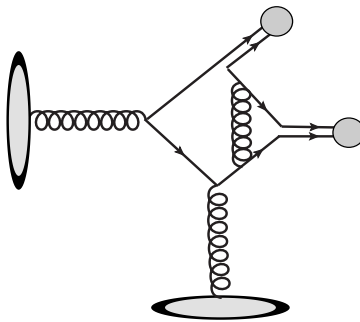


FIG. 1: Typical non-fragmentation diagram contributing to the prompt double  $J/\Psi$  production at leading-order  $\mathcal{O}(\alpha_s^4)$  in proton-proton collisions. The remaining 30 Feynman diagrams due to gluon-gluon fusion subprocesses are not shown here.

such correlations, see for example Refs. [23–28]. A similar ridge-type structure has also been observed in heavy-ion collisions at RHIC and the LHC and is understood as a phenomenon related to hydrodynamical behavior of the quark-gluon-plasma [29, 30]. Nevertheless, it is unknown how/why a small system like the one produced in proton-proton collisions can/should exhibit a hydro type behavior. It is therefore still an open question to understand the true nature of the observed ridge phenomenon, whether it is mainly due to initial-state or final-state physics. Another interesting question is whether the observed ridge phenomenon is universal for all two-particle correlations [31] like for example  $J/\psi$ -pair correlations. In this paper, we also study long-range in rapidity near-side and away-side double  $J/\psi$  correlations. We investigate if within the NRQCD framework upto LO, there is any ridge-like structure for  $J/\psi$  pair production in proton-proton collisions at the LHC.

This paper is organized as follows: In Sec. II we introduce the main formalism and setup for calculating the cross-section of inclusive prompt double  $J/\Psi$  production in proton-proton collisions within the NRQCD framework. We then present our main results and compare with the recent LHC data in Sec. III. We summarize our main results in Sec. IV.

## II. THEORETICAL FORMALISM

In the NRQCD approach, the cross-section of prompt double  $J/\psi$  at high-energy can be written in the following general factorization form,

$$d\sigma(p + p \rightarrow J/\psi J/\psi + X) = \sum_{m,n} \int \mathcal{F}_g(x_1, k_{1T}) \otimes \mathcal{F}_g(x_2, k_{2T}) \otimes d\sigma(g + g \rightarrow c\bar{c}(n) + c\bar{c}(m)) \langle O_n^{J/\Psi} \rangle \langle O_m^{J/\Psi} \rangle, \quad (1)$$

where  $\mathcal{F}_g$  is unintegrated gluon density of the projectile (or target) proton with the longitudinal momentum fraction  $x_{1,2}$  (carried by a parton with respect to the beam proton) and transverse momentum  $k_{1T}$  (or  $k_{2T}$ ). The convolution symbol stands for integral phase factors.  $d\sigma(g + g \rightarrow c\bar{c}(n) + c\bar{c}(m))$  is the short-distance coefficients of double charm-pair production in the gluon-gluon (gg) fusion, calculable in powers of  $\alpha_s$ . The quark-pair may appear in any Fock state  $n, m = 2S+1 L_j$ , both as color singlet (CS) and color octet (CO) states. The heavy-quark pair then evolve nonperturbatively into  $J/\Psi$ . In the case of CO states, extra soft gluon emissions are needed in order to bring the  $c\bar{c}$  quantum numbers to the  $J/\psi$  state. The long-distance nature of the heavy quarkonium is factored out into the NRQCD universal long-distance matrix elements (LDMEs) denoted by  $\langle O_n^{J/\Psi} \rangle$ , and are assumed to obey certain hierarchy in powers of the relative quark velocity  $v$  [3] in the bound state (this is in a full analog to the classical multipole radiation theory). The leading contribution is still due to states with exact  $J/\psi$  quantum numbers (color-singlet) which require no further soft emission. In order to match with the quantum numbers of the final state mesons, one has to project the production amplitude to a proper spin and color state. Note that the CS contribution dominates for small values of transverse momentum of  $J/\Psi$  while the CO contribution are expected to take over at large transverse momentum of  $J/\Psi$  [5, 32–34].

For heavy quarkonium production, the light  $q\bar{q}$  contributions are negligible. Moreover, at the LHC energies, one probes the small-x region of parton distribution where gluons dominate. At leading-order  $\mathcal{O}(\alpha_s^4)$ , the gluon-gluon fusion subprocess leads to 31 Feynman diagrams, the so-called box diagrams [35–39]. A typical diagram of such processes is shown in Fig. 1. We mainly concentrate here in a kinematic region with low transverse momenta of  $J/\Psi$  pairs. At low transverse momentum of  $J/\Psi$  the gluon-fragmentation contributions to the double  $J/\Psi$  production is less important, and will not be considered here.

Here for the first time we confront the  $k_T$ -factorization based NRQCD results with the LHC data. The main advantage of the  $k_T$ -factorization is that one can incorporate the small- $x$  dynamics in a minimal way via the unintegrated gluon density  $\mathcal{F}_g$  and its evolution. The unintegrated gluon densities absorb the effects of soft gluon resummation; this regularizes infrared divergences and makes our theory applicable to even small transverse momentum region. At the LHC kinematics that we consider here, the Bjorken  $x$  is quite small and therefore small- $x$  physics is important. An approximate relation between the mass  $M$  of the produced  $J/\psi$ -pair and the center-of-mass-energy squared  $s$  of the colliding protons is  $x_1 x_2 = M^2/s$ . Therefore for both the CMS and the LHCb kinematic coverage shown in Figs. 2,3, we have approximately  $x_{1,2} \leq 10^{-2}$ . Note that in the  $k_T$ -factorization approach, the evaluation of Feynman diagrams is straightforward and follows standard QCD rules, with one reservation: in accordance with the  $k_T$  prescription [8], the initial gluon spin density matrix is taken in the form  $\overline{\epsilon_g^\mu \epsilon_g^{*\nu}} = k_T^\mu k_T^\nu / |k_T|^2$ , where  $k_T$  is the component of the gluon momentum perpendicular to the beam axis. In the collinear limit, when  $k_T \rightarrow 0$ , this expression converges to the ordinary  $\overline{\epsilon_g^\mu \epsilon_g^{*\nu}} = -g^{\mu\nu}/2$ , while in the case of off-shell gluons it contains an admixture of longitudinal polarization. We have checked that in the collinear limit we reproduce the results of Refs. [5, 37–39]. We also compare our results with those results coming from the collinear factorization which is based on a similar factorization given in Eq. (1) by replacing the  $\mathcal{F}_g$  convolution to a convolution with parton distribution functions (PDFs).

In the NRQCD factorization, in principle,  $J/\Psi$  can also be produced by a branching processes of intermediate hadrons such as  $\chi_{cJ}$  and  $\psi'$ . The quantum numbers of  $\psi'$  are identical to those of  $J/\psi$ , and therefore the production mechanism and all observables are also identical. Given that the long-distance matrix elements for  $J/\psi$  and  $\psi'$  are proportional to the respective leptonic decay widths, we have,  $\langle O_m^{\psi'} \rangle \simeq (1/2) \langle O_m^{J/\psi} \rangle$ . Therefore, we find that  $\psi'$  is produced with twice as small probability. Moreover, there is a 50% branching fraction for  $\psi' \rightarrow J/\psi + X$ . Therefore, the feed-down from  $\psi'$  reduces to a multiplicative factor for the cross-section which can be incorporated as a pre-factor. Production of  $\chi_c$  in combination with  $J/\psi$  is forbidden by charge parity conservation (at LO). Production of  $\chi_c$  pairs is suppressed by their wave functions by two orders of magnitude compared to  $J/\psi$  pairs ( $P$ -waves versus  $S$ -waves). Therefore, this feed-down can safely be neglected. The production of  $\chi_c$  pairs in the collinear factorization was considered in [40].

### III. NUMERICAL RESULTS AND DISCUSSION

The parameter settings used in our numerical study is as follows. Similar to the previous studies, we perform our computation at the LO in the fixed-flavor number scheme with three massless quark flavors and a charm quark with a mass approximately taken to be half of  $J/\Psi$  mass  $m_c \approx m_{J/\Psi}/2$ . Note that there is large uncertainties on the extracted value of charm mass from the current data for the charm structure functions and diffractive processes at HERA at the small- $x$  region [41]. Following the standard pQCD practice at LO, we assume the factorization and the renormalization scale to be equal. We use the LO running coupling  $\alpha_s(\mu_R^2)$  with a renormalization scale  $\mu_R^2 = \hat{s}/4$  where  $\hat{s}$  is the partonic subprocess invariant energy squared. Note that this choice of  $\mu_R$  is Lorentz invariant and symmetric with respect to  $J/\Psi$  pair. In the theoretical bands shown in Figs. 2,3 we incorporate the uncertainties associated to our freedom to choose different factorization scale by varying  $\mu_R$  in a range of  $\hat{s}/8 < \mu_R^2 < \hat{s}/2$ . We employed the unintegrated gluon density (UGD)  $\mathcal{F}_g$  obtained from the CCFM evolution [42] constrained by data on the structure function. The CCFM evolution equation matches to the (LO) DGLAP evolution equation at moderate  $x$  and it embodies BFKL dynamics at low  $x$  [42]. The free parameters of the UGD was obtained from a fit to the HERA structure functions data with  $x < 5 \cdot 10^{-3}$  and  $Q^2 > 4.5 \text{ GeV}^2$  [43]. Note that the current HERA data alone cannot uniquely determine the free parameters of the UGDs. We incorporate the uncertainties associated to our freedom to choose different UGDs by incorporating in the theoretical bands shown in Figs. 2,3 the effect of employing different UGD sets ( $A^0$ ,  $A^+$  and  $A^-$  sets given in Ref. [43]). The only free parameter of the long-distance matrix element  $\langle O_1^{J/\Psi}(^3S_1^{[1]}) \rangle$  in Eq. (1) (in the CS channel) is the  $J/\Psi$  radial wavefunction at the origin of coordinate space  $\mathcal{R}_{J/\Psi}(0)$ . The parameter  $\mathcal{R}_{J/\Psi}(0)$  is determined via a fit to the leptonic decay width [44] with an extracted value of  $|\mathcal{R}_{J/\Psi}^2(0)| = 0.8 \text{ GeV}^3$ .

At the LHCb, the  $J/\Psi$  pair production are sampled at  $\sqrt{s} = 7 \text{ TeV}$  in the following kinematic phase space for the individual  $J/\Psi$  with transverse momentum  $p_T^{J/\psi}$  and rapidity  $y^{J/\psi}$  [6]:

$$0 < p_T^{J/\psi} [\text{GeV}] < 10 \quad \text{for} \quad 2 < y^{J/\psi} < 4.5. \quad (2)$$

Note that in this paper, the transverse momentum of a  $J/\Psi$ -pair and an individual  $J/\Psi$  are denoted by  $p_T^{J/\Psi J/\Psi}$  and  $p_T^{J/\psi}$ , respectively. The CMS experiment samples the  $J/\Psi$  pair production at the same energy  $\sqrt{s} = 7 \text{ TeV}$  but complementary to the LHCb kinematics, with coverage to higher transverse momentum of a pair at more central

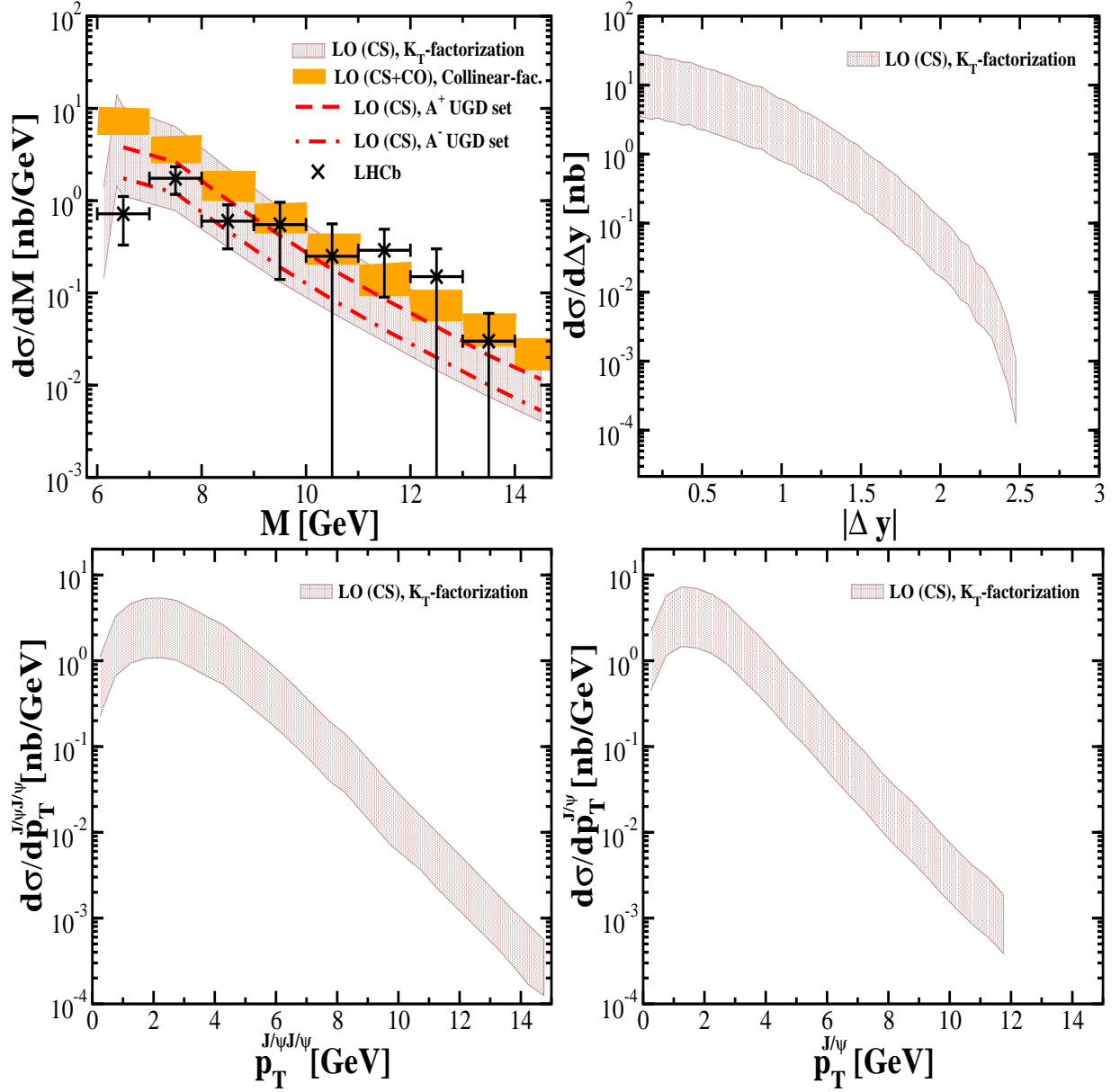


FIG. 2: Upper left: The  $M$ -distribution of prompt double  $J/\Psi$  hadroproduction measured by the LHCb collaboration [6] is compared to LO NRQCD predictions calculated in two different approaches: the  $k_T$ -factorization by including only CS contribution labeled by "LO (CS),  $k_T$ -factorization" and the collinear factorization by including CS and CO contributions labeled by "LO (CS+CO), collinear fac.". In upper left panel, we show the results obtained in the LO NRQCD  $k_T$ -factorization approach by taking two different unintegrated gluon density distributions (UGD) sets labeled by  $A^+$  and  $A^-$ . We also show predictions for  $|\Delta y|$ -distribution (top right panel),  $p_T^{J/\Psi J/\Psi}$ -distribution (lower left) and  $p_T^{J/\Psi}$ -spectra (lower right) of prompt double  $J/\Psi$  hadroproduction calculated in the NRQCD framework by including only CS contribution in the  $k_T$ -factorization approach. The band incorporates various theoretical uncertainties, see the text. The orange band (in upper left panel) labeled by "LO (CS+CO), collinear fac." is taken from [5].

rapidity region. The CMS experiment provides access to  $p_T^{J/\Psi J/\Psi}$  measurements above 15 GeV. The CMS measured the cross-section in a phase space defined by the individual  $J/\Psi$  [7]:

$$\begin{aligned}
 p_T^{J/\psi}[\text{GeV}] > 6.5 & \quad \text{for } |y^{J/\psi}| < 1.2 \\
 p_T^{J/\psi}[\text{GeV}] > 6.5 \rightarrow 4.5 & \quad \text{for } 1.2 < |y^{J/\psi}| < 1.43 \\
 p_T^{J/\psi}[\text{GeV}] > 4.5 & \quad \text{for } 1.43 < |y^{J/\psi}| < 2.2.
 \end{aligned} \tag{3}$$

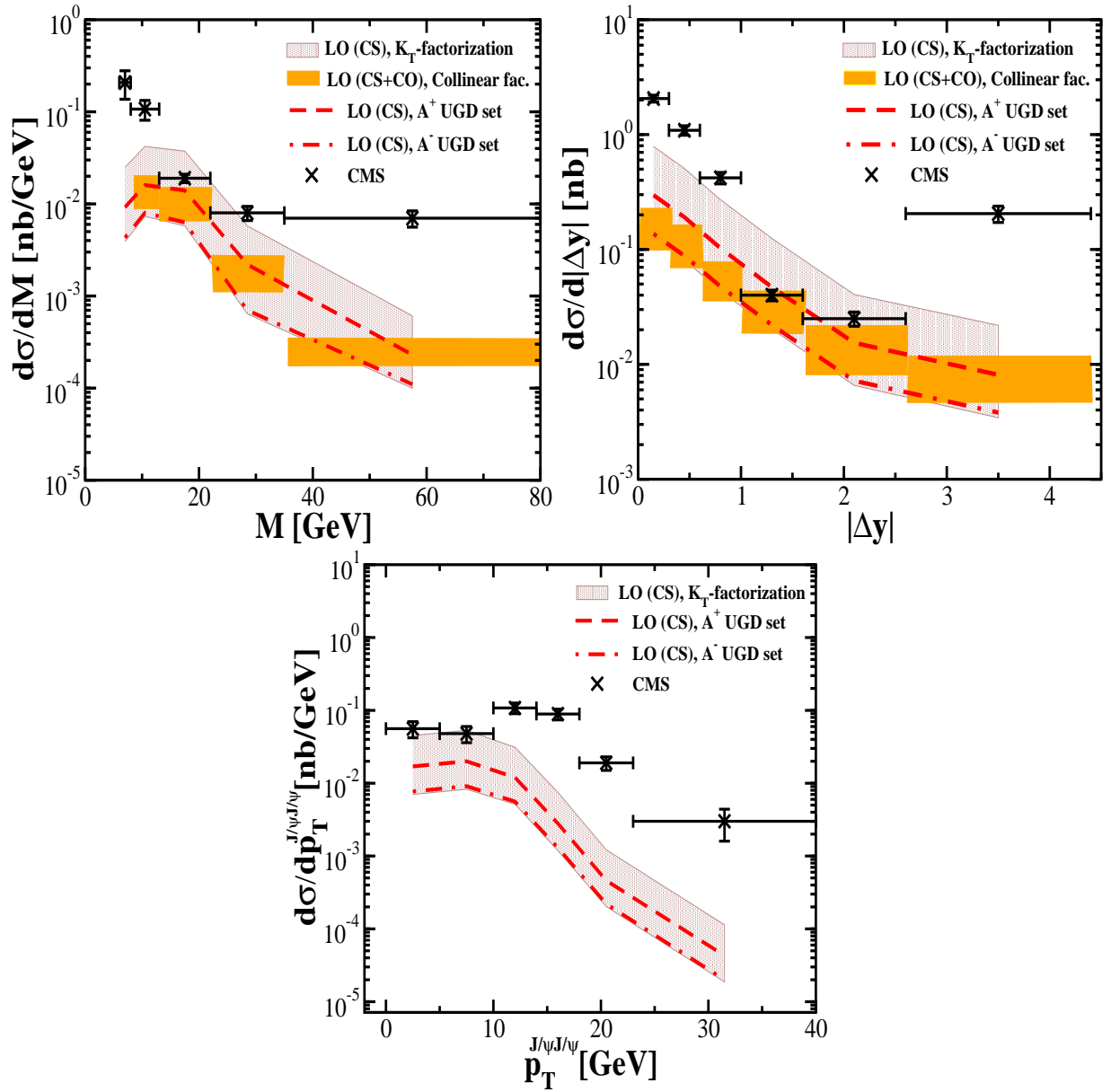


FIG. 3: The  $M$ -distribution (top left panel),  $|\Delta y|$ -distribution (top right panel) and  $p_T^{J/\Psi J/\Psi}$ -distribution (lower panel) of prompt double  $J/\Psi$  hadroproduction measured by the CMS collaboration [7] are compared to LO NRQCD predictions calculated in two different approaches of the collinear factorization and the  $k_T$ -factorization. The notation of curves and bands are the same as in Fig. 2.

For a comparison with the LHCb and CMS data, in our numerical calculation we impose the same kinematic constrains as employed by the LHCb and the CMS experiments, given in Eqs. (2,3).

The theoretical uncertainties are shown by a band in Figs. 2,3. In the  $k_T$ -factorization approach, main uncertainties come from our freedom to choose different factorization/renormalization scale and different UGDs. To highlight the effect of different UGDs, we also show in Figs. 2,3 the results obtained by two different sets of the UGD [43] labeled by  $A^+$  and  $A^-$  UGD sets. It is seen that the uncertainties associated to the UGDs are rather large. Note that both the collinear factorization and the  $k_T$ -factorization bands shown in Figs. 2,3 include all feed-down channels at LO.

In Fig. 2, we first confront the LHCb data for the differential cross-section for  $J/\Psi$  pairs as a function of the invariant mass  $M$  of the  $J/\Psi$ -pair system with LO NRQCD results calculated in two different formalisms of the collinear factorization and the  $k_T$ -factorization. It is seen in Fig. 2 (left panel) that at low  $M$ , the  $k_T$ -factorization approach provides a better description of the LHCb data compared to the results based on the collinear factorization. This is due to the fact that soft gluon radiations (or initial-state radiation) incorporated in the  $k_T$ -factorization

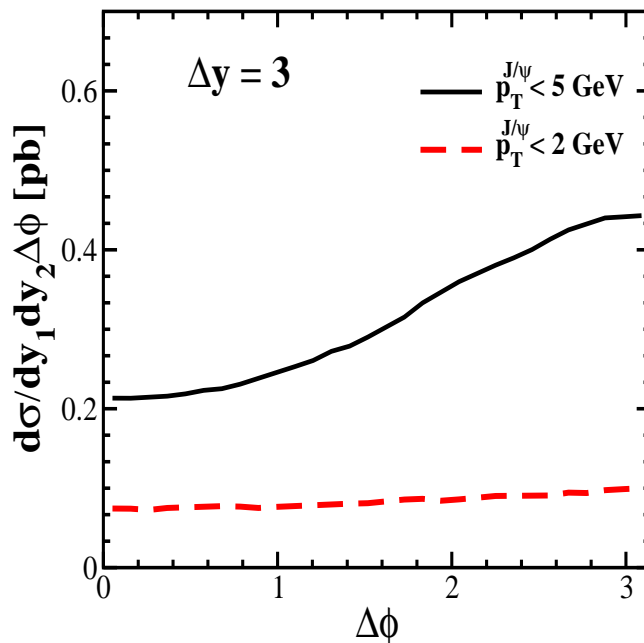


FIG. 4: The prompt double  $J/\Psi$  cross-section as a function of angle between two  $J/\Psi$  at the LHC energy 8 TeV. The rapidities of two  $J/\Psi$  are taken at  $y_1^{J/\Psi} = 1$  and  $y_2^{J/\Psi} = 4$  (with  $\Delta y = 3$ ). The solid and dashed curves are obtained by integrating over the individual transverse momentum of  $J/\Psi$  for  $p_T^{J/\Psi} < 5$  GeV and  $p_T^{J/\Psi} < 2$  GeV, respectively.

formalism are important and drastically modifies the shape of  $p_T^{J/\Psi J/\Psi}$ ,  $p_T^{J/\Psi}$  and  $M$  distribution at low transverse momentum. It is generally seen that within the error band, the LO NRQCD results obtained in the  $k_T$ -factorization are in a good agreement with the LHCb data over entire range of  $M$  shown in Fig. 2 while the collinear factorization results overestimate the LHCb data at low  $M < 8$  GeV by about an order of magnitude. At high  $M$ , both the collinear and the  $k_T$ -factorization results are consistent with each other. In order to understand the importance of the CS contribution at the LHCb kinematics, we compare our results obtained by only taking CS contribution (using the  $k_T$ -factorization) with those results which contain both CS and CO contributions (using the collinear factorization). It is seen that within theoretical uncertainties, the LHCb data can be fairly described by only taking into account the CS contribution in the  $k_T$ -factorization.

In order to further test the NRQCD framework at the LHC, in Fig. 2, we show our predictions for the rapidity distribution of double  $J/\Psi$  production as a function of rapidity difference between two  $J/\Psi$  (top right panel),  $p_T^{J/\Psi J/\Psi}$ -distribution (lower left) and  $p_T^{J/\Psi}$ -spectra (lower right) of prompt double  $J/\Psi$  hadroproduction obtained by only taking into account the CS contribution at LO in the  $k_T$ -factorization approach employing the same kinematic phase space as taken at the LHCb (given in Eq. (2)). Again similar to upper-left panel in Fig. 2, the band labeled by "LO(CS),  $k_T$ -factorization" includes various theoretical uncertainties outlined above.

In Fig. 3, we compare our results with the CMS data for differential cross-section of prompt  $J/\Psi$  pair production as a function of the  $J/\Psi$ -pair invariant mass  $M$ , the absolute rapidity difference  $|\Delta y|$  between two  $J/\Psi$ , and the  $J/\Psi$  pair transverse momentum  $p_T^{J/\Psi J/\Psi}$ . We recall that the CMS data shown in Fig. 3 are taken in the kinematic phase space given in Eq. (3). The notation and description of theoretical bands are similar to Fig. 2. Similar to Fig. 2 within the theoretical uncertainties (detailed above), the results obtained in the  $k_T$ -factorization and the collinear factorization approaches are consistent at large  $p_T^{J/\Psi J/\Psi}$  (and large  $M$ ). Given rather large theoretical uncertainties at LO, it is not possible to make any firm conclusion if the CO contribution is essential at the CMS kinematics. It is seen that LO NRQCD results based on both the  $k_T$ -factorization and the collinear factorization formalisms, significantly underestimate the CMS data in most of  $M$ ,  $p_T^{J/\Psi J/\Psi}$  and  $|\Delta y|$  bins. Note that  $M$ ,  $p_T^{J/\Psi J/\Psi}$  and  $|\Delta y|$  are related, and a large  $p_T^{J/\Psi J/\Psi}$  corresponds to a large  $M$ . It is generally seen in Fig. 3 that the discrepancy between theoretical results at LO and the CMS data (within error bars) can be as large as about two orders of magnitude at large  $M$  (and/or large  $p_T^{J/\Psi J/\Psi}$ ). Taking the CMS data at face value, the flatness of the data at large value of  $M$  and  $|\Delta y|$ , and raise of the  $|\Delta y|$  distribution at large  $|\Delta y|$  indicate a large missing anomalous contribution beyond the full LO approximation, see also Ref. [15].

In Fig. 4, we show the prompt double  $J/\Psi$  cross-section as a function of angle  $\Delta\phi$  between two  $J/\Psi$  at the LHC energy 8 TeV for a case that the rapidity interval between two  $J/\Psi$  is taken  $\Delta y = 3$  (with  $y_1^{J/\Psi} = 1$  and  $y_2^{J/\Psi} = 4$ ) and transverse momenta of each of  $J/\Psi$  was integrated over  $p_T^{J/\Psi} < 5$  (solid line) and  $p_T^{J/\Psi} < 2$  (dashed line). The result shown in Fig. 4 was obtained by employing  $A^0$  UGD set and a fixed factorization/renormalization scale  $\mu_F^2 = \mu_R^2 = \hat{s}/4$ . It is seen in Fig. 4 that in both cases of different transverse momentum bins, there is no enhancement at near-side  $\Delta\phi \approx 0$  while the way-side  $\Delta\phi \approx \pi$  correlations are enhanced at larger  $p_T^{J/\Psi}$  bins due to standard back-to-back kinematics.

Although the standard double parton interactions contributions [45] (not considered here) are important at large rapidity interval, and may change the normalization of the total cross-section, but such contributions will not modify the flatness of the correlations in angle between two  $J/\Psi$  at near-side. We have also checked that the inclusion of subleading  $\alpha_s^6$  corrections (due to gluon exchanges to the box-diagrams) which dominate at very large rapidity interval [45], will not change the flatness of the near-side correlation for  $J/\Psi$ -pairs. Note that such subleading  $\alpha_s^6$  corrections are negligible at  $\Delta y = 3$  employed in Fig. 4. We recall that at the same rapidity and transverse momentum coverage considered here, di-hadron correlations exhibit a ridge-like structure in proton-proton collisions at the LHC in high-multiplicity event selections. Note that the experimental data for  $J/\Psi$ -pairs correlations in proton-proton and proton-nucleus collisions are not yet available. Our results show that at LO in NRQCD framework, there is no a ridge-like behaviour (a second local maximum at the near-side) for prompt double  $J/\Psi$  production in proton-proton collisions at the LHC.

#### IV. CONCLUSION

In this paper we confronted the LO NRQCD results obtained in two approaches of the collinear factorization and the  $k_T$ -factorization with the recent LHC data for prompt double  $J/\Psi$  production in proton-proton collisions. We showed that the LO  $k_T$ -factorized NRQCD results provide a good description of the LHCb data. Here we provided various predictions at the LHCb kinematic phase space which can further test the NRQCD formalism, see Fig. 2. We quantified various theoretical uncertainties in the NRQCD formalism and showed that the CMS data cannot be described at LO with about a factor of 10 discrepancy, see Fig. 3. It remains to be seen if higher order corrections, in particular next-to-leading-order (NLO) corrections can fill such a large gap between LO results and the CMS data. Although the relative importance of the NLO corrections is process-dependent, we recall that the NLO corrections enhance the LO NRQCD cross-section for single inclusive  $J/\Psi$  production by a factor about 10 [4, 46]. Note that in this paper, we only focused on the single parton scattering (SPS) contributions. The contribution of double parton scattering (DPS) [45, 47] can be important for double  $J/\Psi$  production at small-x region at forward rapidities. The D0 collaboration [48], for the first time, separated the double  $J/\Psi$  production cross section at  $\sqrt{s} = 1.96$  TeV into contributions due to single and double parton scatterings, and found that  $(46 \div 22)\%$  of cross-section is due to DPS. Although, the relative importance of the DPS cannot be unambiguously quantified till full NLO correlations are known, it is hard to believe that one order of magnitude discrepancy seen between theory and CMS data at about midrapidity can be accounted for by the DPS.

Within the  $k_T$ -factorized NRQCD approach for prompt double  $J/\Psi$  production, we quantified various theoretical uncertainties including the uncertainties associated to the unintegrated gluon density which was previously neglected. Within these theoretical uncertainties and rather large experimental error bars, with only the current LHC data for prompt double  $J/\Psi$  production, one cannot conclusively pin down the importance of the CO contribution. In particular, we showed that the LHCb data can be described by only taking into account the CS channel. Nevertheless, in order to further substantiate the importance of the CO contribution, it is indispensable to complete the full NLO analysis of prompt double  $J/\Psi$  production by including all CO channels.

We also investigated the long-range in rapidity azimuthal angle correlations between  $J/\Psi$  pairs in the NRQCD formalism. We found that there is no near-side ridge-like structure up to subleading  $\alpha_s^6$  accuracy in this formalism. It is of great interest to see whether the inclusion of higher order corrections, the DPS effect and in particular density effect at high multiplicity events, and in general small-x dynamics due to gluon saturation or color-glass-condensate physics, can change this outcome. Note that there are growing evidence that the gluon saturation physics (which was not considered here) is important in small-x region in proton-proton collisions at the LHC, see e.g. Refs. [23–27, 49]. These are important issues which are beyond the scope of this paper and certainly deserve separate studies.

## Acknowledgments

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