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N-Relaxion

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Relaxion models are an interesting new avenue to explain the radiative stability of the Standard Model scalar sector. They require very large field excursions, which are difficult to generate in a consistent UV completion and to reconcile with the compact field space of the relaxion. We propose an N-site model which naturally generates the large decay constant needed to address these issues. Our model offers distinct advantages with respect to previous proposals: the construction involves non-abelian fields, allowing for controlled high energy behaviour and more model building possibilities, both in particle physics and inflationary models, and also admits a continuum limit when the number of sites is large, which may be interpreted as a warped extra dimension.

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I. INTRODUCTION

Large field excursions have long been known to be an ingredient of slow roll theories of inflation [1, 2], and have recently become a requirement for relaxation solutions to the hierarchy problem of the Standard Model (SM), through the cosmological relaxation of the Electroweak scale [3]. In these scenarios we have a scalar field starting at some large initial value and slowly going to smaller values during the inflationary epoch. To illustrate how such a situation is reached, consider the relaxion model [3, 4]:

$$V(\phi, H) = \Lambda^3 g\phi - \frac{1}{2}\Lambda^2\left(1-\frac{g\phi}{\Lambda}\right)H^2 + \Lambda_c^4(H)\cos(\phi/f) + \cdots, \quad (1)$$

where H is the Higgs field, Λ is the cutoff of the model, ϕ is the relaxion field (assumed to the be a pseudo-Nambu-Goldstone-Boson (pNGB) of a symmetry breaking at scale f and thus protected by a discrete shift symmetry), the spurion g quantifies the explicit breaking of the discrete shift symmetry and $\Lambda_c(H)$ is a scale depending on the Higgs vev so that $\Lambda_c(H) \neq 0 \leftrightarrow \langle H\rangle \neq 0$.

The idea is straightforward: it is technically natural to set g to very small values, so the first term in Eq. (1) is responsible for the slow roll of ϕ to smaller field values. Once ϕ is such that the coefficient of H^2 on the second term becomes negative, this has two effects: (i) the H field adquires a vev and one can easily show that the Higgs mass is much smaller than Λ for small g; (ii) once $\langle H\rangle \neq 0$ the amplitude of the last term becomes different from zero and the potential of ϕ becomes oscillatory, trapping the field value close to this phase transition (which in turn fixes the Higgs vev). If this is to work in a natural way, one cannot set the initial value of ϕ very close to the phase transition, instead, we must assume it scanned the typical range of field values $\Delta\phi \sim \Lambda/g \gg \Lambda$, which for small g means the field has an excursion much greater than the cutoff of the model.

There are relevant concerns regarding this idea that we summarize below:

- While having field excursions larger than the cutoff of the effective theory is not a problem in itself, it might be problematic to construct a theory that could consistently generate these large excursions, specially if the UV theory includes quantum gravity [5–8].
- Another crucial feature of Eq. (1) is the presence of a linear term that explicit breaks a gauge symmetry (the axion shift symmetry), which is inconsistent with the assumption that the relaxion is a pNGB [9].

This second point can be avoided if all operators involving ϕ are periodic, but with very different periods, and the linear term is nothing but a small region in an oscillation of longer period. Effectively, a simple way to consistently generate such hierarchical oscillations is to produce a large hierarchy between the decay constants [10–13]:

$$V(\phi, H) \sim \Lambda^4 \cos\left(\frac{\phi}{F}\right) + \Lambda_c^4(H)\cos\left(\frac{\phi}{f}\right), \quad (2)$$

where $F \gg f$. If F is also larger than the cutoff scale Λ then the first point is also addressed, because ϕ will have a compact field space of size $2\pi F$ (this does not address gravity related problems, we will comment on those below).

In [10] they propose an explicit example to generate an effective super-Planckian field range. In this model they consider $N+1$ complex scalars with the same decay constant $f < M_{\rm Pl}$. The key point is that by adding a conveniently chosen breaking term, the global $U(1)^{N+1}$ is explicitly broken to $U(1)$ and the remaining pNGB effectively has a super-Planckian decay constant, which exponentially depends on the number of fields as $F \gg e^{cN} f$,

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where $c \sim \mathcal{O}(1)$. It is emphasized in [10] that this construction cannot be interpreted as a deconstructed extra dimension, i.e. there is no continuum limit for this model in theory space. Other approaches achieving similar results are employed in [14–16].

In the following we present a different approach that can deal with the issues discussed previously and at the same time indicates a different strategy to search for UV completions for the relaxation mechanism. The two main advantages of our approach are that: (i) the model does have a continuum limit that could be interpreted as an extra dimension; and (ii) we show that the desired features can be obtained from non-abelian groups, allowing for controlled (asymptotically free) UV behaviour.

A relevant concern arising when attempting to include gravity in the UV theory is the so-called Weak Gravity Conjecture (WGC) [5], which limits how small the coupling constants in gauge theories may be. In a non-abelian setup, the conjecture is not yet sufficiently explored, however, since embedded abelian Yang-Mills black hole solutions exist [17–20] it is expected that the usual arguments will also apply to the non-abelian case. We leave this matter for future work.

This paper is organized as follows: in Section II, we present a minimal model that generates large field excursions and may be interpreted as a discretized extra dimension. In Section III, we analyse the relaxation mechanism in our setup and conclude in Section IV. We discuss possible UV scenarios related to new strongly coupled models in Appendix A and warped extra dimensions in Appendix B.

II. MINIMAL MODEL

We consider a $2N$-site model schematically represented in Fig. 1, where on each site lies a global symmetry group, $SU(2)$ (the construction is trivially generalized for other groups).

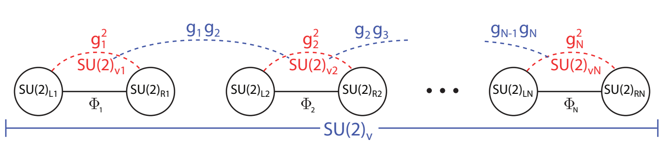

FIG. 1. Diagram for a $2N$-site model. The initial symmetry groups and link fields are in black. In red we show the effect of the g_j^2 explicit breakings, and the resulting preserved groups. The same is shown in blue for the g_jg_{j+1} breakings.

The Lagrangian for the link fields reads:

$$\begin{aligned}\mathcal{L}_\Phi = \sum_{j=1}^{N} \Big\{ &\mathrm{Tr}\left[(\partial_\mu \Phi_j)^\dagger \partial^\mu \Phi_j\right] + \\ &+ \frac{f^3}{2}(2-\delta_{j,1}-\delta_{j,N}) g_j^2 \mathrm{Tr}\left[\Phi_j + \Phi_j^\dagger\right]\Big\} \\ &- \frac{f^2}{2}\sum_{j=1}^{N-1} g_j g_{j+1} \mathrm{Tr}\left[(\Phi_j - \Phi_j^\dagger)(\Phi_{j+1}-\Phi_{j+1}^\dagger)\right], \quad (3)\end{aligned}$$

where the Φ_j are scalars transforming as bifundamentals, $\Phi_j \to L_j \Phi_j R_j^\dagger$, under adjacent $SU(2)$ groups. We assume the Φ_j acquire a vev $\langle \Phi_j \rangle \equiv f/2$, spontaneously breaking $SU(2)_{L_j} \times SU(2)_{R_j} \to SU(2)_{V_j}$. In the low energy limit, these fields are non-linearly realized as:

$$\Phi_j \to \frac{f}{2} e^{i\vec{\pi}_j\cdot\vec{\sigma}/f} = \frac{f}{2}\cos\left(\frac{\pi_j}{f}\right) + i\frac{f}{2}\frac{\vec{\pi}_j\cdot\vec{\sigma}}{\pi_j}\sin\left(\frac{\pi_j}{f}\right), \quad (4)$$

where $\vec{\sigma}$ are the Pauli matrices, $\vec{\pi}_j$ are the NGB multiplets in each link field and $\pi_j \equiv \sqrt{\vec{\pi}_j\cdot\vec{\pi}_j}$.

In addition to the kinetic terms, the Lagrangian contains terms that explicitly break some global symmetries. These parameters are assumed to be small spurions generated at a higher scale and may be chosen such that they give a mass to all but one linear combination of the $\vec{\pi}_j$. More concretely, the terms with g_j explicitly break the chiral symmetries to the vector combination, $SU(2)_{L_j} \times SU(2)_{R_j} \to SU(2)_{V_j}$, while the terms with g_jg_{j+1} break $SU(2)_{V_j} \times SU(2)_{V_{j+1}} \to SU(2)_{V_{j,j+1}}$. Taken together, in the end these terms break explicitly all symmetries down to a final diagonal $SU(2)_V$. However, due to the peculiar structure of the breaking parameters, one combination of the $\vec{\pi}_j$ remains accidentally lighter, gaining a small mass only at higher order. Additional breaking terms (involving three or more powers of the Φ_j fields) could be present, but we will assume that they are suppressed in relation to those in Eq. (3) and we discuss a UV scenario that can implement that suppression in the Appendix A .

The Lagrangian may be written in terms of the Goldstone fields as:

$$\begin{aligned}\mathcal{L}_\pi = \sum_{j=1}^{N}\Big[&\frac{1}{2}\partial_\mu \vec{\pi}_j \cdot \partial^\mu \vec{\pi}_j + \\ &+ f^4(2-\delta_{j,1}-\delta_{j,N}) g_j^2 \cos\left(\frac{\pi_j}{f}\right) + \cdots\Big] \\ &+ f^4 \sum_{j=1}^{N-1} g_j g_{j+1} \frac{\vec{\pi}_j\cdot\vec{\pi}_{j+1}}{\pi_j \pi_{j+1}} \sin\left(\frac{\pi_j}{f}\right)\sin\left(\frac{\pi_{j+1}}{f}\right), \quad (5)\end{aligned}$$

where the omitted terms correspond to interaction operators with two derivatives. Expanding to quadratic order, we obtain the mass matrix for the $\vec{\pi}_j$, which is independent of the $SU(2)$ index:

$$\vec{\pi}^T \cdot M_\pi^2 \cdot \vec{\pi} \equiv \sum_{j=1}^{N-1} f^2 (g_j \vec{\pi}_j - g_{j+1}\vec{\pi}_{j+1})^2, \qquad (6)$$

where we defined the theory space vector $\vec{\pi}^T \equiv \{\vec{\pi}_1, \cdots, \vec{\pi}_N\}$.

We now parametrize the mass matrix as $g_j \to q^j$, with $0 < q < 1$, obtaining a matrix for the pNGBs that is identical to the one obtained for a pNGB Wilson line (zero mode) in the deconstruction of AdS_5 [21, 22] (see Appendix (B)):

$$M_\pi^2 = f^2 \begin{pmatrix} q^2 & -q^3 & 0 & \dots & 0 & 0 \\ -q^3 & 2q^4 & -q^5 & \dots & 0 & 0 \\ 0 & -q^5 & 2q^6 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2q^{2(N-1)} & -q^{2N-1} \\ 0 & 0 & 0 & \dots & -q^{2N-1} & q^{2N} \end{pmatrix}. \qquad (7)$$

Note that since $\mathrm{Det}[M_\pi^2] = 0$, this matrix has a zero mode (at tree level), as advertised. In this case, its profile is given by:

$$\vec{\eta}_0 = \sum_{j=1}^{N} \frac{q^{N-j}}{\sqrt{\sum_{k=1}^N q^{2(k-1)}}} \vec{\pi}_j, \qquad (8)$$

which is similar to the result found in [10]. One sees that for $0 < q < 1$, the zero mode is exponentially localized at the last site. It is important to note that, in contrast with [10], since $q < 1$ our matrix does admit a continuum limit, which should correspond to some bulk scalar in AdS_5.

Since η_0 has a mass much smaller than the other states[1], one is justified to consider it as the relaxion field that slow rolls during inflation, since the other modes rapidly lose coherence on scales larger than their Compton wavelength and may thus be assumed to be constant on the scale $m_{\eta_0}^{-1}$. They correspond to immaterial phase shifts in the potential of η_0. In terms of η_0, then, one obtains the following Lagrangian after integrating out the other pNGBs:

$$\mathcal{L}_\eta = \sum_{j=1}^{N} \left[\frac{1}{2} \partial_\mu \vec{\eta}_0 \cdot \partial^\mu \vec{\eta}_0 + f^4 (2 - \delta_{j,1} - \delta_{j,N}) q^{2j} \cos \frac{\eta_0}{f_j} \right] + \sum_{j=1}^{N-1} f^4 q^{2j+1} \sin \frac{\eta_0}{f_j} \sin \frac{\eta_0}{f_{j+1}}, \qquad (9)$$

[1] At tree level, for $q \ll 1$, the spectrum is approximately given by $m_j^2 \approx f^2 q^{2(j-1)}$ for $1 < j \leq N$ plus an exact zero mode. Expanding Eq. (9), a quartic term is generated of order $q^{2N}\eta_0^4$. Closing the loop, one obtains a mass for η_0 of order $m_{\eta_0} = f^2 q^{2N}$, which is a factor of q^2 smaller than the lightest tree level mass, hence the approximation scheme is consistent.

where $\eta_0 \equiv \sqrt{\vec{\eta}_0 \cdot \vec{\eta}_0}$ and the effective decay constants are given by:

$$f_j \equiv f \frac{\sqrt{\sum_{k=1}^N q^{2(k-1)}}}{q^{N-j}} \equiv f q^{j-N} \mathcal{C}_N, \qquad (10)$$

where $\mathcal{C}_N = \sqrt{\frac{q^{2N}-1}{q^2-1}}$ is a normalization constant. One sees that a large hierarchy of decay constants is generated, from the largest $f_{\max} = f_1 \approx f/q^{N-1}$ to the smallest $f_{\min} = f_N \approx f$, as we wanted.

Regarding the radiative stability of the potential, we find that interactions with m external η_0 legs scale as $c_m \sim q^{2N} f^{4-m}$ and renormalize multiplicatively (as expected, since all the couplings in the Lagrangian Eq. (9) are spurions), so the whole potential is radiatively stable up to small corrections.

III. HIGGS-AXION INTERPLAY

If the lightest pNGB field is to function as a relaxion, its potential must be such that no local minima stops the relaxion field when the Higgs vev is zero, as the relaxion must keep rolling down. The general behavior of the potential in Eq. (9) is dominated by the oscillation with the largest amplitude and period: $-f^4 q^2 \cos \frac{\eta_0}{f_1}$, which grows monotonically in $0 < \eta_0 < \pi f_1$ (which will be our region of interest from now on). To check that the other oscillations do not get the field stuck we need to consider the derivative of the potential:

$$\frac{\partial V_\eta}{\partial \eta_0} = \frac{f^3 q^N}{\mathcal{C}_N} \sum_{j=1}^{N} q^j \sin\left(\frac{\eta_0}{f_j}\right) \Big\{ (2 - \delta_{j,1} - \delta_{j,N}) + -(1-\delta_{j,1}) \cos\left(\frac{\eta_0}{f_{j-1}}\right) - (1-\delta_{j,N}) \cos\left(\frac{\eta_0}{f_{j+1}}\right) \Big\}. \qquad (11)$$

The constant $\frac{f^3 q^N}{\mathcal{C}_N}$ is positive for any $q < 1$ and $N > 1$, and the term between braces is bounded between 0 and 4 (for $j = 1$ and $j = N$ it is bounded between 0 and 2). The leading term for small q is given by:

$$\frac{f^3 q^N}{\mathcal{C}_N} q \sin\left(\frac{\eta_0}{f_1}\right) \Big\{ 1 - \cos\left(\frac{\eta_0}{f_2}\right) \Big\}, \qquad (12)$$

which is never negative for $0 < \eta_0 < \pi f_1$ and is only zero at particular points, given by $\eta_0^m \equiv 2\pi m q f_1$, with $m = \{0, 1, 2 \dots\}$. Close to these points the sign of the derivative will come from the sines accompanying the higher powers of q. The one multiplying q^{N+2} is:

$$\sin\left(\frac{\eta_0^m}{f_2}\right) \approx \frac{\eta_0}{q f_1} - 2\pi m. \qquad (13)$$

This sine will push the derivative to negative values near η_0^m, generating shallow minima (similar arguments apply to the next terms in the q-expansion). The derivative only remains negative while the term in Eq. (12) is smaller than the $\mathcal{O}(q^{N+2})$ term, so these minima become less and less important as q gets smaller. In fact, for small q the height of the barrier between two adjacent minima decreases as q^4 and the width decreases as $q^{2-N}\mathcal{C}_N$ and we expect the field to be able to proceed rolling down for the typical values of q considered below. The shape of the potential with decreasing q can be seen in Figure 2. One can see that, despite the use of quite large values of q and a scaling factor α to exacerbate the features of the potential, the slope gets quickly smooth with decreasing q.

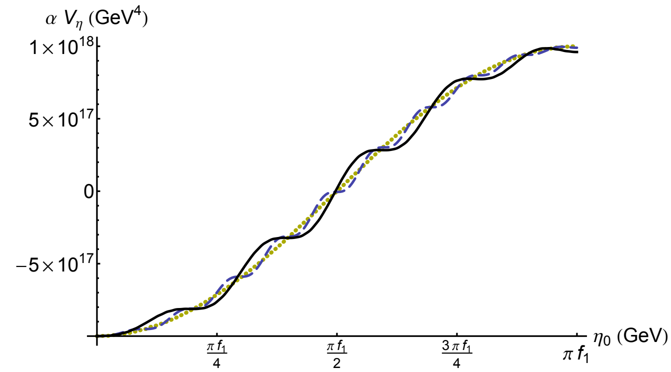

FIG. 2. Potential $V_\eta(\eta_0)$ for different values of q and $N = 3$. A re-scaling factor α was introduced to allow easy comparison between the curves (the potential gets flatter with decreasing q). The black, blue (dashed) and yellow (dotted) curves have respectively ($q = 0.1, \alpha = 1$), ($q = 0.05, \alpha = 10^3$) and ($q = 0.01, \alpha = 10^{10}$). Note that these values of q are much larger than the realistic ones, in order to exacerbate the features in the potential.

We now turn on the Higgs field by multiplying the Lagrangian by the electroweak singlet $1 + c|H|^2/\Lambda^2$, where H is the Higgs doublet, c is an order one coupling constant which we take equal to one, for simplicity, and the cutoff Λ is of order $4\pi f$. Adding in the Higgs potential and kinetic term, the full Lagrangian is now given by:

$$\mathcal{L}_{\eta,H} = \left(1 + \frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta + |D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4. \tag{14}$$

Once the Higgs is set to its vev, $\langle h\rangle = v > 0$, the slope equation is the same as Eq. (11), multiplied by $(1 + v^2/(2\Lambda^2))$. The field ϕ should stop rolling when this expression is approximately zero. However, this clearly has no solutions apart from the trivial one $v^2 = -2\Lambda^2$, which is clearly undesirable.

One sees that with the current Lagrangian, having $v \ll f$ is untenable. In order to fix this, we consider adding the following breaking term at the last site which, in the continuum limit, will be equivalent to a deformation of the metric (as discussed in Appendix B) in the infrared (IR):

$$\mathcal{L}_{\eta,H} \to \mathcal{L}_{\eta,H} + \epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N + \Phi_N^\dagger]|H|^2 \tag{15}$$

where ϵ is a small parameter, and Λ_c is a new scale, which we could assume is generated at lower energies, of order Λq^N, in order to avoid spoiling the results of the previous section (see Eq. (2)). However, as pointed out in [4], a small Λ_c scale leads to a coincidence problem (i.e., $\Lambda \gg \Lambda_c \sim$ TeV) for the model. We will then take $\Lambda_c \approx \Lambda \approx 4\pi f$ and discuss below how to avoid the problems generated by this choice.

Once this term is generated, the relaxion potential acquires the term

$$\epsilon f^2|H|^2\cos\frac{\eta_0}{f_N}, \tag{16}$$

giving the relaxion a small mass. By closing the loop of H, one sees that a term $\epsilon f^4\cos(\eta_0/f_N)$ is generated, which can spoil the relaxation mechanism. One possible solution is to adopt the double scanner mechanism of [4], that is, we may add a scalar singlet to control the amplitude of the additional term. As emphasized in [4], the new singlet field needs an even larger field excursion than the relaxion. This can be easily accommodated in our framework by replicating this scalar on the N-sites, provided we choose a smaller value of the q parameter (or, equivalently, of the couplings g_j) for this scalar. The details of this construction and the continuum limit thereof are beyond the scope of our paper and left for future work.

With the inclusion of this term, the new slope equation is given by:

$$\frac{\partial V_{\eta,H}}{\partial\eta_0} = \frac{f^3q^{N+1}}{\mathcal{C}_N}\left\{\left(1 + \frac{v^2}{2\Lambda^2}\right)\sin\left(\frac{\eta_0}{f_1}\right)\left[1 - \cos\left(\frac{\eta_0}{f_2}\right)\right.\right.$$
$$\left.\left. + \mathcal{O}(q)\right] - \epsilon\frac{v^2}{2f^2q^{N+1}}\sin\left(\frac{\eta_0}{f_N}\right)\right\} + \cdots . \tag{17}$$

This slope should be zero when the vev of H is of the order of the electroweak scale. Solving for this yields

$$v^2 \sim \frac{f^2}{\epsilon}q^{N+1}. \tag{18}$$

One sees that for small q and $q^{N+1} < \epsilon < 1$, a natural electroweak scale is obtainable and that q^{N+1} should be identified with the relaxion coupling g of [4], as in Eq. (1).

The cutoff for our model can be estimated along the lines of [4] by considering additional constraints besides Eq. (18). The main bounds come from requiring that η_0 does not drive inflation, i.e. $\Lambda^2 \lesssim H_I M_{\rm Pl}$, where H_I is the inflation scale and $M_{\rm Pl}$ is the reduced Planck scale, and the requirement that quantum fluctuations of η_0 are less important than its classical rolling. This yields the condition that $H_I^3 \lesssim q^{N+1}f^3$. Finally, suppressing higher order terms like $\epsilon^2 f^4\cos(\eta_0/f)^2$ requires $\epsilon \lesssim v^2/f^2 \sim 10^{-12}$, for $f = 10^8$ GeV [4]. Combining these with Eq. (18), we obtain:

$$\frac{\Lambda^6}{f^3M_{\rm Pl}^3} \lesssim q^{N+1} \lesssim \frac{v^4}{f^4}. \tag{19}$$

From this, we find the upper bound of $f \lesssim 10^8$ GeV and also that $q \lesssim 10^{-23/(N+1)}$.

Finally, using all these constraints, we find that for $q \approx 10^{-24/(N+1)}$ and $\epsilon \approx 10^{-12}$, we obtain $v \sim 10^{-6} f$ which is of the order of the electroweak scale for $f \approx 10^8$ GeV. Note that for these parameter choices, Eq. (18) does not depend on N. Of course, having a large value for N allows for a larger value of q.

IV. DISCUSSION

We have constructed a simple $2N$-site model capable of addressing two problematic points of the relaxation mechanism, namely the necessity for (i) large field excursions and (ii) a linear term that explicit breaks the axion shift symmetry. Our model generates a potential composed of many oscillatory terms with very different periods (see Eq. (9)), the term with the larger period plays the role of the linear term in Eq. (1). From N fields acquiring expectation values of order f, an effective scale $f_1 = \mathcal{C}_N f/q^{N-1} \gg f$ (see Eq. (10)) is generated and the pNGBs have a compact field space of $2\pi f_1$, which allows for large field excursions.

The present model has some distinctive features when compared with previous many-field models that also address the points above [10, 11]:

- The N fields are bi-fundamentals of $2N$ non-abelian $SU(2)$ groups and the formalism employed can be trivially generalized to any non-abelian group. This allows for a controlled UV behavior and opens up many possibilities of model building in particle physics and inflation.

- The model has a well defined continuum limit $N \to \infty$, $q \to 1$, with q^{N+1} kept fixed, and the mass matrix for the pNGBs in Eq. (7) is exactly the one obtained from a pNGB Wilson line in the deconstruction of AdS_5 [21, 22]. Even the desired relation between v and f (in Eq. (18)) is maintained in the continuum limit, as $f^2 q^{N+1} \to M/g_5^2 \, e^{-kL}$, where L is the size of the extra dimension, k is the curvature, g_5 is the $5d$ gauge coupling, and M is the cutoff of the UV theory (see Appendix B, in particular, Eq. (B5)). In addition, we find that (up to suppressed terms) in the continuum limit (see Eq. (10)), $f_1 = \mathcal{C}_N f q^{1-N} \to M/(g_5\sqrt{2k}) e^{kL}$ and $f_N = \mathcal{C}_N f \to M/(g_5\sqrt{2k})$, that is $f_1/f_N \to e^{kL}$, i.e. they are related by the AdS_5 warp factor. These expressions are in agreement with those obtained by [23] in AdS_5.

While the potential of Eq. (9) has shallow minima that do not affect the slow roll of the relaxion, adding the Higgs requires the introduction of a new term that generates large barriers for $\langle H \rangle \neq 0$. This extra breaking is proportional to a new spurion ϵ and ultimately controls the magnitude of the Higgs vev via Eq. (18). In the continuum limit, this breaking should correspond to an IR deformation of the extra dimensional metric. This term may also spoil the relaxation mechanism via higher order corrections, but we expect these can be amended by adopting the double scanner scenario of [4].

In the viable region of parameter space, we find that the cutoff of the model can be pushed up to $\Lambda \approx 4\pi f \sim 10^9$ GeV.

The breaking term of Eq. (15) is not unique, and it may be possible to avoid introducing it by considering different terms in Eq. (3) that automatically generate the large barriers needed to stop the rolling of η_0. Alternatively, one might be able to achieve the same result through changing the parametrization of the g_j couplings in the Lagrangian in order to mimic a metric that is slightly deformed from AdS_5.

It will also be interesting to investigate the continuum limit of this model (i.e. a warped extra dimension), which is a possible direction to achieve an UV completion that is compatible with the WGC [24].

Additionally, the framework established here could find application in model building of the inflation sector, which also requires large field excursions, for instance, in models with observable primordial gravitational waves [25].

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Appendix A: Fermionic UV Model

A simple UV model generating the Lagrangian of eq.(3) is given by adding $2N$ multiplets of Dirac fermions, transforming as $SU(2)$ doublets, at a high energy scale, with the following lagrangian:

$$\begin{aligned}\mathcal{L}_{UV} = &\sum_{j=1}^{N} \left\{ \bar{\psi}_j \not{p} \psi_j + \bar{\chi}_j \not{p} \chi_j \right\} \\ &+ \sum_{j=1}^{N-1} \Big\{ \bar{\psi}_{Lj} \left[\lambda_j \phi_j + \lambda_{j+1}\phi_{j+1} - \lambda'_j f \right] \psi_{Rj} \\ &+ \bar{\chi}_{Lj} \left[\tilde{\lambda}_j \phi_j - \tilde{\lambda}_{j+1} \phi^\dagger_{j+1} - \tilde{\lambda}'_j f \right] \chi_{Rj} + \text{h.c.} \Big\} ,\end{aligned} \tag{A1}$$

where L, R denote chirality projections and the couplings λ_j, λ'_j, $\tilde{\lambda}_j$, $\tilde{\lambda}'_j$ are assumed small. Upon integrating out these fermions and matching the couplings, one obtains the Lagrangian of Eq. (3), plus terms suppressed by higher orders of the couplings.

The additional term of Eq. (15) can be similarly generated by

$$\mathcal{L}'_{UV} = \xi^\dagger \not{p}\xi + \zeta\not{p}\zeta^\dagger + \xi(\epsilon\phi_N - m)\zeta + \text{h.c.}, \qquad \text{(A2)}$$

where $\xi,\ \zeta$ are a set of chiral fermions located at the last site. The Higgs may then be added trivially by multiplying the entire Lagrangian by the EW singlet $1+HH^\dagger/\Lambda^2$.

Appendix B: pNGB Wilson line in deconstructed AdS$_5$

Consider the action for the gauge field of a group $\mathcal{G}$ in a slice of AdS$_5$ in proper coordinates [26], $ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$:

$$\begin{aligned} S_5^A &= \int d^4x \int_0^{\pi R} dy\sqrt{-g}\left\{-\frac{1}{2g_5^2}\text{Tr}\left[F_{MN}^2\right]\right\} \\ &= \int d^4x \int_0^{\pi R} dy \left\{-\frac{1}{2g_5^2}\ \text{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right]\right. \\ &\left.+\frac{1}{g_5^2}e^{-2ky}\ \text{Tr}\left[(\partial_5 A_\mu - \partial_\mu A_5)^2\right]\right\}. \end{aligned} \qquad \text{(B1)}$$

We discretize the extra dimension by substituting

$$\begin{aligned} \textstyle\int_0^{\pi R} dy &\to \sum_{j=0}^{N} a, \\ \partial_5 A_\mu &\to \frac{A_{\mu,j} - A_{\mu,j-1}}{a}, \end{aligned} \qquad \text{(B2)}$$

where a is the lattice spacing (inverse cutoff). We obtain:

$$\begin{aligned} S_5^A = \frac{a}{g_5^2}\int d^4x &\left\{-\frac{1}{2}\sum_{j=0}^{N}\text{Tr}\left[F_{\mu\nu,j}F_j^{\mu\nu}\right]\right. \\ &\left.+\sum_{j=1}^{N}\frac{e^{-2kaj}}{a^2}\text{Tr}\left[(A_{\mu,j} - A_{\mu,j-1} - a\partial_\mu A_{5,j})^2\right]\right\}. \end{aligned} \qquad \text{(B3)}$$

Consider now a theory of $N+1$ gauged non-linear sigma model fields, U_j. The scalar fields act like linking fields in a lattice, transforming under adjacent gauge groups (assumed to be all equal to $\mathcal{G}$) as $U_j \to L_j U_j R_j^\dagger$, where $L_j,\ R_j$ are the gauge symmetries on sites $j,\ j+1$, respectively. The U_j spontaneously break $L_j \times R_j \to V_j$ at an scale f_j, yielding $N+1$ multiplets of NGB fields, π_j. We can match the discretized action above to this gauged non-linear sigma model action, by expanding it at the quadratic level in the Nambu-Goldstone fields:

$$\begin{aligned} S_4^A = \frac{1}{g^2}\int d^4x &\left\{-\frac{1}{2}\sum_{j=0}^{N}\text{Tr}\left[F_{\mu\nu,j}\ F_j^{\mu\nu}\right]+\right. \\ &\left.\sum_{j=1}^{N} f^2 g^2 q^{2j}\,\text{Tr}\left[\left(A_{\mu,j} - A_{\mu,j-1} - \partial_\mu\frac{\pi_j}{f_j}\right)^2\right]\right\}, \end{aligned} \qquad \text{(B4)}$$

where π_j is a Goldstone mode transforming in the adjoint of the vector symmetry V_j, and we take $f_j \equiv fq^j$, by making the identifications [27–30]:

$$\begin{aligned} \frac{g_5^2}{a} &\leftrightarrow g^2, \\ f &\leftrightarrow \frac{1}{\sqrt{a}g_5} = \frac{1}{ag}, \\ q &\leftrightarrow e^{-ka}, \end{aligned} \qquad \text{(B5)}$$

we see the Goldstone mode is identified with the scalar component of the gauge field. Or, equivalently, the non-linear linking field $U_j = e^{i\pi_j/f_j}$ is identified with the Wilson line $\exp\left[i\int_{aj}^{a(j+1)} dy\, A_5 e^{-2ky}\right]$.

Now, consider the breaking $\mathcal{G} \to \mathcal{H}$ by boundary conditions in theory space, that is, we assume that the first and last sites, the symmetry group is reduced to $\mathcal{H}$ [2]. Denoting the broken generators by hatted indexes, it is straightforward to see that we can remove the mixing between Goldstone modes and gauge fields by adding the gauge fixing term:

$$\mathcal{L}_G = -\sum_{j=1}^{N-1}\frac{1}{2\xi}\left[\partial_\mu A_j^{\mu,\hat{a}} + \xi\left(f_j\pi_j^{\hat{a}} - f_{j+1}\pi_{j+1}^{\hat{a}}\right)\right]^2. \qquad \text{(B6)}$$

One may then verify that the mass matrix obtained for the NGB fields parametrizing $\mathcal{G}/\mathcal{H}$ is given by: [21, 22]

$$M_\pi^2 = f^2\xi\begin{pmatrix} q^2 & -q^3 & 0 & \cdots & 0 & 0 \\ -q^3 & 2q^4 & -q^5 & \cdots & 0 & 0 \\ 0 & -q^5 & 2q^6 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2q^{2(N-1)} & -q^{2N-1} \\ 0 & 0 & 0 & \cdots & -q^{2N-1} & q^{2N} \end{pmatrix}. \qquad \text{(B7)}$$

reproducing Eq. (7). Note that while the massive modes have gauge dependent masses, the zero mode is physical.

[2] Alternatively, we can implement this breaking by localized scalar fields, then take their vev to infinity, decoupling the massive gauge modes.

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