

Misconception Regarding Conventional Coupling of Fields and Particles in XFEL Codes

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Abstract

Maxwell theory is usually treated in the laboratory frame under the standard time order, that is the usual light-signal clock synchronization. In contrast, particle tracking in the laboratory frame usually treats time as an independent variable. As a result, here we argue that the evolution of electron beams is usually treated according to the absolute time convention i.e. using a different time order defined by a non-standard clock synchronization procedure. This essential point has never received attention in the accelerator community. There are two possible ways of coupling fields and particles in this situation. The first, Lorentz's prerelativistic way, consists in a 'translation' of Maxwell's electrodynamics to the absolute time world-picture. The second, Einstein's way, consists in a 'translation' of particle tracking results to the electromagnetic world-picture, obeying the standard time order. Conventional particle tracking shows that the electron beam direction changes after a transverse kick, while the orientation of the microbunching phase front stays unvaried. Here we show that in the ultrarelativistic asymptotic $v \rightarrow c$, the orientation of the planes of simultaneity, i.e. the orientation of the microbunching fronts, is always perpendicular to the electron beam velocity when the evolution of the modulated electron beam is treated under Einstein's time order. This effect allows for the production of coherent undulator radiation from a modulated electron beam in the kicked direction without suppression. We hold a recent FEL study at the LCLS as a direct experimental evidence that the microbunching wavefront indeed readjusts its direction after the electron beam is kicked by a large angle, limited only by the beamline aperture. In a previous paper we quantitatively described this result invoking the aberration of light effect, which corresponds to Lorentz's way of coupling fields and particles. The purpose of this paper is to provide the details of Lorentz's prerelativistic approach used in our previous paper. Here we also demonstrate that an 'inverse translation' of particle tracking results to the standard time order resolves all puzzles related with the strong qualitative disagreement between simulations and experiments. We conclude that previous simulation results in ultrarelativistic electron beam physics should be reexamined in the light of the pointed difference between particle tracking and electromagnetic world-pictures.

1 Introduction

A recent ‘beam splitting’ experiment at the LCLS [1] apparently demonstrated that after a microbunched electron beam is kicked on a large angle compared to the divergence of FEL radiation pulses, the microbunching wave front is readjusted along the new direction of motion of the kicked beam. Therefore, coherent radiation from an undulator placed after the kicker is emitted along the kicked direction without suppression. Computers allow for simulations of the beam splitting setup, but the picture of coherent undulator radiation in the kicked direction cannot be explained in the framework of existing beam physics codes: the experimental result is in strong qualitative disagreement with simulations. Therefore, the question arises about how to explain coherent undulator radiation emission, which occurs in nature, in the framework of classical mechanics and electrodynamics.

In our previous paper [2] we demonstrated that the effect of aberration of light supplies the basis for a quantitative description of this phenomenon. Stellar aberration was discovered by Bradley in 1728. The aberration of light can be easily explained on the basis of the corpuscular theory of light and the rule for computing its effect is simple. One takes the velocity of light with respect to the source and adds it vectorially to the velocity of the source with respect to the observer. The direction of the resulting vector is the apparent direction of the light source as measured at the observer position. The angle of aberration is the difference between true and apparent angular positions of the source. Application of this rule for the case when the angle of aberration is at its maximum, i.e. when the direction of the observer’s motion is perpendicular to the direction of the source radiation, results in an angle of aberration of v_{\perp}/c radians. For this result to hold, it is important that the transverse speed of the observer v_{\perp} is very much smaller than speed of light c . Since all the velocities mentioned in the rule are relative velocities, the rule conforms to the principle of relativity. This is not so surprising as long as we work within the corpuscular theory of light. It is just what car drivers experience if they drive through a vertically falling shower of rain [3, 4]. The apparent readjusting of the microbunching front orientation observed at the LCLS beam splitting experiment can be quantitatively explained with the help of the rule described above for calculating the angle of aberration.

The above rule for source aberration conforms to the principle of relativity if we think of light as corpuscles. However, aberration of light can also be explained on the basis of the wave theory of light. This is plausible if one keeps in mind that a light signal represents a certain amount of electromagnetic energy. Energy, like mass, is a quantity that is conserved, so that a light signal resembles, in many aspects, a material particle. Therefore we expect

that group velocities of light signals obey the same addition theorem for particle velocities. A closer treatment based on Maxwell's electrodynamics confirm this expectation. Lorentz (1895) showed that aberration arises in Maxwell's theory in such a way that the principle of Galilean relativity holds. He argued that Maxwell's theory depends on a single preferred ether state of rest. However, nothing in what is observable about aberration can reveal that state of rest. Lorentz's explanation of this fact involved the demonstration of the existence of a sort of 'nature conspiracy', such that it is impossible to exhibit any effect of the relative motion of the observer relative to ether by 'first order optical experiments'. This accuracy is more than sufficient in our case of interest, because the maximal kick angle in the LCLS beam splitting experiment was very small (smaller than 0.1 mrad), so that a first approximation over the parameter v_{\perp}/c yields a correct quantitative description. We thus conclude that in our case of interest we find ourselves in a very unusual situation where we can quantitatively explain the result of the beam spitting experiment at the LCLS with the help of 18th and 19th century physics, while numerical simulations of the 21st century demonstrate a strong qualitative disagreement between theory and experiment. If this correct, then we are inclined to say that there must be something wrong with the coupling between equations of motion and field equations in conventional accelerator physics codes.

Before continuing our analysis it is necessary to mention that in the case of the beam splitting setup at the LCLS we deal with an ultra relativistic electron beam, $\gamma^2 \sim 10^8$, and with fine intra beam microbunching structures. In the theory of particle accelerators and XFELs it is generally accepted that there is no need to use the laws of relativistic kinematics. The only relativistic equation necessary for description of the relativistic electron beam is $d\vec{p}/dt = \vec{F}$, where $\vec{p} = m\gamma\vec{v}$. In other words, in order to describe the dynamical processes in a relativistic electron beam, it is sufficient to take into account the relativistic dependence of the electron momentum on the velocity. Such an approach always gives a reliable way to describe single particle dynamics in given electromagnetic fields. However, this method should be used with caution in the case of a modulated electron beam where one needs to consider the evolution of the microbunching wavefront.

The microbunching wavefront can be considered as a plane of simultaneous events. Accelerator physicists never considered the role of the concept of simultaneity within the laboratory system, nor the need to attach some physical meaning to it. *What is important is to know exactly what we are doing and what we are measuring, when we discuss about simultaneous events.* In fact, we cannot give any experimental method by which simultaneity between two events in different places can be ascertained. Determining simultaneous events is only a matter of convention. More precisely, it is a matter of convention which events should be considered simultaneous for a given

observer. Einstein's procedure to synchronize clocks at different points in space is only one out of many possible alternatives. In particular, another convention about clock synchronization can be chosen, which maintains the concept of absolute simultaneity between inertial frames.

Particle tracking in the laboratory frame treats time as an independent variable. This fact is equivalent, indeed, to a non-standard choice of synchronization where the electron beam evolution is treated in terms of absolute time (or simultaneity). Such a non-standard convention of absolute time or simultaneity would have been the logical consequence of the development along the line of reasoning of 19th century physics.

One might be inclined to think that the concept of absolute simultaneity necessarily violates relativity, but this is not right. Minkowski space-time can be described equally well from the point of view of any spread time through space: the standard and non-standard clock synchronization reveals no deep facts about Minkowski space-time and special relativity; it is, rather, a trivial consequence of general covariance [5]. We may use language as we want and define a meaning for the names 'synchronization', 'simultaneity' and 'speed' in many different ways. Reality is independent of the names we use to describe it. What really matters is that one makes sure that *none of these words is used in some context and interpreted in a different one*. In contrast, conventional coupling of fields and particles in accelerator physics codes is based on the use of results of particle tracking treated according to the absolute time convention and Maxwell's equations treated according to the standard synchronization convention. Here lies the misconception that led to the strong qualitative disagreement between theory and experiment in the LCLS beam splitting experiment.

One might think that particle tracking under the convention of absolute simultaneity is in contradiction with Maxwell's equations. This is only true if Maxwell's equations are written in their standard form in an inertial frame with standard synchronization. However, they can be written in general covariant form with the help of the metric tensor and 'translated', in particular, to the language of absolute time synchronization. Such way of describing microbunching beam physics is a consequence of the development along the line of Lorentz's reasoning in the 19th century. Lorentz actually first performed such prerelativistic 'translation' for the case of 'first order optical experiments'. In our previous paper [2] we followed this way of reasoning. We concluded that if there is a microbunch front tilt with respect to the direction of motion, then, due to aberration of light effect, the undulator radiation wavefront is tilted to be orthogonal to the direction of the ultrarelativistic electron beam motion. As a result, one cannot tell the difference with the case when the microbunch front tilt is absent. From Lorentz's pre-relativistic analysis we conclude that there exists a sort of 'conspiracy of nature'.

Our fruitless attempt to find any influence of the microbunch front tilt on the emission of coherent undulator radiation convinced us that this result has general validity and the front tilt of the ultrarelativistic electron beam has no definite objective meaning. All physical phenomena take the same course of development independently of the microbunch front tilt and one can never decide with an experiment if an ultrarelativistic electron beam has a microbunch front tilt.

Here we try to remove some of barriers to understand our paper [2], by providing the details of Lorentz's 'translation' of Maxwell's electrodynamics to the absolute time order. The purpose of this paper is also to demonstrate that an 'inverse translation' of particle tracking results to the standard time order explains the meaning of the 'conspiracy of nature', by which the microbunch front tilt has no influence on the emission of coherent radiation. We show that in the ultrarelativistic asymptotic $v \rightarrow c$, the orientation of the planes of simultaneity, i.e. the orientation of the microbunching fronts, is always perpendicular to the electron beam velocity when the evolution of the modulated electron beam is treated under Einstein's time order. In particular, this effect of microbunching phase front readjustment allows for the production of coherent undulator radiation from a modulated electron beam in the kicked direction without suppression.

In [2] we examined an XFEL under the steady state assumption, that is a harmonic time dependence. Recently, we extended our analysis of the influence of microbunching front tilt by taking into account time-dependent effects [6]. The structure of Minkowski space-time dictates that a modulated electron bunch in the ultrarelativistic asymptote has the same form as a laser pulse. In the time domain it is convenient to introduce a carrier frequency and to write the modulation as a product of a slowly varying modulation envelope and a phase term of a plane wave. In analogy with a laser pulse, the carrier microbunch phase front is always orthogonal to the propagation direction. Whatever we know about the kinematics of a laser pulse can immediately be applied to an ultrarelativistic modulated electron bunch. As is well-known in laser beam optics, a kick introduces a tilt of the envelope with respect to the phase front of the laser pulse i.e. a kick introduces a spatiotemporal coupling. The phase front of a laser pulse after a kick is readjusted and is always orthogonal to the propagation direction. As we just pointed out, while learning about pulse front tilt phenomena in laser beam optics, we also learn about ultrarelativistic electron bunch tilt phenomena. In [6] we demonstrated that by a direct translation of the solution for the kinematics of a laser pulse it is possible to solve the kinematical problem for a kicked modulated ultrarelativistic electron bunch with the same ease ¹.

¹ Or with the same difficulty, because usually researchers are specifying that spatiotemporal coupling phenomena are the most complex part of laser beam optics.

2 Conventinality in distant simultaneity

In this section we discuss the essence of the conventionality of simultaneity. A convention is something which has no physical relevance, and yet is necessary for communicating physical ideas. In particular, in order to give a meaning to the concept of simultaneity, we must first synchronize distant clocks in a single frame, which we assume to be inertial. According to the thesis of conventionality of simultaneity [5, 7, 8, 9, 10], simultaneity of distant events is a conventional matter, as it can be legitimately fixed in different manners in any given inertial reference frame.

In order to measure the one-way speed of light one has first to synchronize the clocks. A clear deadlock appears if one synchronizes the clocks by assuming that the one-way speed of light is c . The one-way speed of light measured with these clocks, that is the Einstein speed of light, will always be c : this is because the clocks have been set assuming that particular one-way speed in advance, and it cannot be otherwise. Therefore, it can be said that the value of the one-way speed of light is just a matter of convention. While the one-way speed of light can only be determined by human convention, the directly measurable two-way (round-trip) speed of light has physical meaning. Round-trip experiments rely upon the observation of simultaneity or non simultaneity of events at a *single point* for determining their outcomes.

Let us illustrate a particular special example of the simultaneity convention. Let the synchronization of clocks in different spatial points be provided by light signals having, respectively, velocity c_1 in the direction parallel to the positive axis x , and velocity c_2 in the opposite direction. Then, a light signal sent from point A at time t_A will arrive at point B at time $t_B = t_A + x_{AB}/c_1$. The reflected signal will arrive back at point A at time $t'_A = t_B + x_{AB}/c_2$. From these two expressions we get $t'_A - t_A = x_{AB}(c_2 + c_1)/(c_1c_2)$. Summing up we have $t_B = t_A + c_2(t'_A - t_A)/(c_1 + c_2)$. So we come to the synchronization first proposed in [7]: $t_B = t_A + \epsilon(t'_A - t_A)$, where $0 < \epsilon < 1$. Einstein's choice for ϵ is $\epsilon = 1/2$. If $\epsilon \neq 1/2$, then speed of light from A to B differs from the speed of light from B to A . In the following we shall call the choice $\epsilon = 1/2$ standard 'light-signal' synchronization. This time order, as fixed by the standard synchronization, is frame dependent. This is the well-known thesis of relativity of simultaneity. The conventional nature of distant simultaneity in special relativity is not to be confused with the relativity of simultaneity. Clearly, the conventionality of simultaneity within a single inertial frame is quite distinct from the relativity of simultaneity.

The idea of simultaneity changed first when the theory of relativity showed that simultaneity was not absolute, and that it is meaningful to discuss about

the simultaneity of events only relatively to a given reference frame. A second change of view on simultaneity is connected with the elucidation of the fact that even within a single reference frame the definition of simultaneity of events is not absolute, but is matter of convention.

There are several a priori independent ways of fixing the time order of events. Laws of physics are simplified in the Einstein convention whereas, in any other synchronization convention with $\epsilon \neq 1/2$, they lose their symmetry. This does not mean at all that non-standard synchronization choices are a-priori inconvenient. As a matter of fact, the most suitable formalism is usually suggested by the problem itself. The fact that the standard formalism of the theory of relativity is only one of the possible legitimate choices, allows for a freedom that may prove useful in many instances.

Let us consider a special example of non standard simultaneity convention. The Galilean transformation of coordinates between rest and moving frames is given by $x' = x - vt$, $t' = t$. Let us suppose, for simplicity, that we consider only the relative speed v , which satisfies the condition $v/c \ll 1$, so that $\gamma \simeq 1$. The convention on the synchronization of clocks corresponds, in this case, to the choice $\epsilon = 1/2$ in the rest frame and $\epsilon = (1 - v/c)/2$ in the moving frame [10]. Galilean time appears to be an absolute time, since it is the same in all inertial systems. In the moving frame, the one-way speed of light is not the same in different directions ($c_{1,2} = c \pm v$) and c_1 is always bigger than c . And yet, the two-way speed of light is always equal to c (here we neglect terms of second and higher orders in v/c).

It should not be surprising that a Galilean transformation is mathematically equivalent to the standard synchronized (Lorentz) transformation and vice versa. Hence, it does not matter which transformation is used to describe the same reality. If we know the Galilean coordinates of a certain event, then we can immediately know its standard synchronized coordinates [11]. Once the clocks in both frames have been non-standardly synchronized as described above, a Lorentz transformation can be easily obtained by correcting in a particular way the time reading of the moving clocks. In our accepted approximation, the rhythm of the clocks will not change, but they will start from a different condition. The moving clocks will be delayed by a factor that is proportional to the distance x' to the reference position $x' = 0$, which is given by vx'/c^2 (if x' is negative, this corresponds to advancing the clock) [11]. We thus have $t'_L = t' - vx'/c^2$, $x'_L = x'$ where t'_L and x'_L are the Lorentzian time and coordinate in the moving frame, respectively. Note that this temporal transformation incorporates discernible relativity of simultaneity as long as distances are big enough. We conclude that generally (in particular, when we discuss Maxwell's electrodynamics) we have two non-relativistic limits [8]. Condition $\gamma \rightarrow 1$ is necessary but not sufficient for form-invariance of physical laws with respect to Galilean transformations.

3 Translation of electrodynamics to absolute time world picture and vice versa

Conventional particle tracking states that after an electron beam is kicked there is a change in the trajectory of the electron beam, while the orientation of the microbunching phase front remains as before. In other words, the kick results in a difference between the directions of the electron motion and the normal to the microbunching phase front. In sum, after a kick along the x axis, Cartesian coordinates transform as $x' = x - v_x t$. This transformation is completed with the invariance in simultaneity $\Delta t = \Delta t'$. The absolute character of the temporal coincidence of two events is a consequence of the as well absolute concept of time $t' = t$. As a result of the kick, the transformation of the time and coordinates t, x of any event has the form of a Galilean transformation. Particle tracking under the convention of absolute simultaneity is in contradiction with Maxwell's equations. In fact, the d'Alembertian, which enters in the basic equation of electromagnetism, is not a Galilean invariant. It is a partial differential operator whose change of form under Galilean transformation can be verified by replacing $\partial/\partial t = \partial/\partial t' - v_x \partial/\partial x'$, $\partial/\partial x = \partial/\partial x'$.

There are two satisfying ways of coupling dynamics and electrodynamics in the situation considered when the solution of the dynamical problem (i.e. finding the motion of the particles under the action of a given electromagnetic fields) is performed in the absolute time world-picture. The first, Lorentz's prerelativistic way, consists in a 'translation' of Maxwell's electrodynamics to the absolute time world-picture. The second, Einstein's way, consists in a 'translation' of the particle tracking results to the electromagnetic, standardly synchronized world-picture.

One great achievement of Lorentz (1895) was to introduce the idea of 'local time': when considering some event which happens at time t and place (x, y, z) in the rest frame from the viewpoint of a frame moving parallel to x with velocity v_x , one should not only change the space coordinates and time according to Galilean formula: $x' = x - v_x t$, $t' = t$, but one should also use another time, the 'local time', given by $t' = t - xv_x/c^2$. Note that Lorentz used the notion of 'true time' for the Galilean time t , while the 'local time' in the absolute time world-picture was introduced with the aid of a mathematical trick. Actually Lorentz found that the solution of the electrodynamic problem in the 'true' time world-picture can be obtained with minimal efforts by formally desynchronizing the 'true' time and using it without changing Maxwell's equations.

The full logic of this mathematical trick relies explicitly on the formal symmetry properties of Maxwell's equations. Indeed, it is immediately seen

by direct calculations that a shift of time is necessary in order to preserve the d’Alambertian up to the second order terms: $\partial^2 F / \partial (ct)^2 - \nabla^2 F = \partial^2 F / \partial (ct')^2 - \nabla'^2 F + O[v_x^2/c^2]$. The effect of this transformation is just a dislocation in the timing of processes. Take the case of a propagating plane wave. A transformation whose velocity is transverse to the direction of propagation of the wave has the effect of rotating the wave front. For $v_x^2/c^2 \ll 1$ the rotating turns out to be just the angle v_x/c radians. This effect gives the aberration angle of v_x/c radians in accordance with the experimental results of the LCLS beam splitting experiment.

There is also a second outstanding puzzle concerning the beam splitting experiment at the LCLS. In accordance with theory and numerical simulations, if a microbunched beam is at perfect undulator resonance without kick, then after the kick the same microbunched beam must be at perfect resonance in the kicked direction. In contrast, experimental results clearly show that there is a red shift in the resonance wavelength in the kicked direction. The maximum power of the coherent radiation after the kick is reached when the undulator is detuned to be resonant to the lower longitudinal velocity after the kick [1].

We point out that one can easily quantitatively explain this experimental puzzle with the help of Lorentz’s prerelativistic theory. Lorentz demonstrated that the principle of Galilean relativity (i.e. the Galilean law of addition of velocities) would be expected not only in mechanics, but also in first-order experiments involving optics and electrodynamics. If the relativity principle is true, the physical laws should appear unvaried in two inertial frames, S_0 and S . In frame S_0 after the kick, beam velocity components are $(v_x, 0, v_z)$, where $v_z = \sqrt{v^2 - v_x^2}$ and v is a beam velocity along the z -axis (i.e. the undulator axis) upstream of the kicker. Consider a reference frame S moving with uniform motion at speed v_x along the x -axis of the frame S_0 . In the frame S , the beam velocity components are $(0, 0, v_z)$. If all inertial frames are equivalent, the frame S and S_0 can only be distinguished from one another by their relative speed v_x . We remark that, according to Galilean transformation simultaneity is absolute. Therefore, in the inertial frame S , the wavefront normal and the electrons motion have the same direction along the z -axis. A radiating setup in the frame S reproduces the situation upstream of the kicker, and coherent undulator radiation is emitted along the normal to the microbunching wavefront (i.e. along the z -axis). However, it is clear from the above that if a microbunched beam is at perfect resonance without kick, then after the kick the same microbunched beam cannot be at perfect resonance in the frame S at new longitudinal velocity v_z . A red shift in the resonance wavelength due to the difference between longitudinal velocities appears, which cannot be neglected in practical situations. The rule for computing the result in the lab frame S_0 is simple. One takes the velocity of light with respect to the frame S and adds it vectorially to

the velocity of the frame S with respect to lab frame. The direction of the resulting vector is the apparent direction of the source light as measured at the observer position in the laboratory frame. The rule conforms to the principle of Galilean kinematics. The apparent readjusting of microbunch front orientation and red shift in the resonance wavelength that was observed at the LCLS beam splitting experiment can be quantitatively explained with the help of Galilean relativity.

In particle tracking, the simultaneity along the x direction has absolute character, it is independent of the kick. According to the theory of relativity, however, using the standard synchronization procedure in the laboratory frame, we can establish a criterion for the simultaneity of two events, which is based on the invariance of the speed of light. Two events are simultaneous in a reference frame if they are coincident with the arrival of light signals previously emitted from a position at equal distance from both events. Before the kick, the light signals are emitted from a place equidistant from the positions along x axis where the events happened. After the kick, instead, the place where the light signal is emitted is not equidistant to the positions where the events happened. It is immediately understood that the simultaneity of events is no longer absolute (i.e. independent of the kick), as a result of the invariance of the speed of light. In fact, light signals do not arrive simultaneously at each electron in laboratory frame: the electrons have time to move from their positions equidistant from the source because the signal propagates with finite speed. This reasoning is in analogy with Einstein's train-embankment thought experiment. Finally, the time t' under standard synchronization in the laboratory frame is readily obtained by introducing the offset factor xv_x/c^2 and substituting $t' = t - xv_x/c^2$. This expression forms the Lorentz transformation for time in the first order approximation. We can conclude that Lorentz's 'local time' is the time read from the clocks that are synchronized using light signals. The relativity of simultaneity term xv_x/c^2 of the Lorentz transformation for time is nothing more than an artifact that arises from the standard synchronization procedure. This term is only conventional and can be eliminated. Therefore, with suitable clock synchronization, Lorentz's time can be reduced to Galilean time.

4 Conclusions

It is beyond doubt that different types of clocks synchronization simply provide different time coordinates to describe the same reality. In addition, the words 'time', 'speed' and 'simultaneity', which we use to attribute a precise physical meaning, actually refer to different notions when different types of clocks are used. Since different descriptions, made with various types of clocks, are mathematically equivalent, this latter issue is mainly question of

language. Nonetheless it is an important one and likely to originate severe misunderstandings because the physical concepts underlying each of these description are quite different. Debates around the LCLS beam splitting experiment are related to the problem of using the same words to designate different concepts. For this reason, it is of major importance to know with what kind of clocks one ends up after performing synchronization .

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