

# $B_c^\pm$ decays into tetraquarks

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The recent observation by the D0 collaboration of a narrow structure  $X(5568)$  consisting of four different quark flavors  $bdus$ , has not been confirmed by LHCb. More data and dedicated analyses are needed to cover a larger mass range. In the tightly bound diquark model, we estimate the lightest  $bdus$ ,  $0^+$  tetraquark at a mass of about 5770 MeV, approximately 200 MeV above the reported  $X(5568)$ , and just 7 MeV below the  $B\bar{K}$  threshold. The charged tetraquark is accompanied by  $I = 1$  and  $I = 0$  neutral partners almost degenerate in mass. A  $bdus$ ,  $S$ -wave,  $1^+$  quartet at 5820 MeV is implied as well. In the charm sector,  $cdus$ ,  $0^+$  and  $1^+$  tetraquarks are predicted at 2365 MeV and 2501 MeV, about 40 – 50 MeV heavier than  $D_{s0}(2317)$  and  $D_{s1}(2460)$ .  $bdus$  tetraquarks can be searched in the hadronic debris of a jet initiated by a  $b$ . However, some of them may also be produced in  $B_c$  decays. The proposed discovery modes of  $S$ -wave tetraquarks are  $B_c \rightarrow X_{b0} + \pi$  with the subsequent decays  $X_{b0} \rightarrow B_s + \pi$ , giving rise to final states such as  $B_s\pi^+\pi^0$ . We also emphasize the importance of  $B_c$  decays as a source of bound hidden charm tetraquarks, such as  $B_c \rightarrow X(3872) + \pi$ .

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## I. INTRODUCTION

Recently, the D0 experiment reported the observation of a new narrow structure in the  $B_s^0\pi^+$  invariant mass<sup>1</sup> [1], which promptly attracted considerable attention, see [2] (but skepticism has been raised in [3]). Based on  $10.4 \text{ fb}^{-1}$  of  $p\bar{p}$  collision data at  $\sqrt{s} = 1.96 \text{ TeV}$ , this candidate resonance, dubbed  $X(5568)$ , has a mass and width given by  $M = 5568 \text{ MeV}$  and  $\Gamma = 22 \text{ MeV}$ , respectively.

A state such as  $X(5568)$  would be distinct in that a charged light quark pair cannot be created from the vacuum, leading to the unambiguous composition in terms of four valence quarks with different flavors —  $\bar{b}d\bar{s}u$  (tetraquarks with flavored quantum numbers have also been discussed in [4]).

Exciting a discovery as it would have been,  $X(5568)$  has not been confirmed by the LHCb experiment. Their analysis has been reported recently, based on  $3 \text{ fb}^{-1}$  of  $pp$  collision data at  $\sqrt{s} = 7$  and  $8 \text{ TeV}$ , yielding a data sample of  $B_s^0$  mesons 20 times higher than that of the D0 collaboration. Adding then a charged pion, the  $B_s^0\pi^+$  invariant mass shows no structure from the  $B_s^0\pi^+$  threshold up to  $M_{B_s^0\pi^+} \leq 5700 \text{ MeV}$  and an upper limit on the ratio  $\rho(X(5568)/B_s^0) < 0.016(0.018)$  @ 90 (95) % C.L. is set for  $p_T(B_s^0) > 10 \text{ GeV}$  [5].

The valence quark composition of  $X(5568)$  fits into a diquarkonium interpretation [6–10]. In this framework,

the constituents are arranged in a tightly bound diquark-antidiquark pair,  $[\bar{b}d]_{\mathbf{3}_c}[su]_{\mathbf{\bar{3}}_c}$ , both of them transforming non-trivially under color  $SU(3)$ . The possible manifestation of these compact tetraquarks follows essentially from symmetry considerations as in the original constituent quark model and their spectrum is rich. However, as outlined below, our computation of the tetraquark mass spectrum with the quark flavors  $\bar{b}d\bar{s}u$  yields significantly higher values. The lightest in this sector is the  $S$ -state,  $X_{b0}^+$ , whose mass is estimated by us to be about 5770 MeV – approximately 200 MeV heavier than the  $X(5568)$ , and below the  $B^+\bar{K}^0$  threshold by about 7 MeV.

The tetraquark mass spectrum is calculable up to a theoretical error which we estimate to be of the order of  $\pm 30 \text{ MeV}$ , judging from the discrepancies of constituent quark masses obtained from baryons and mesons (see e.g. Table I in ref. [7]). Thus,  $X_{b0}^+$  and  $X_{b0}^0$  may lie somewhat above the  $B^+\bar{K}^0$  threshold, in which case  $X_{b0}^+$  will decay, perhaps mostly, in the  $B^+\bar{K}^0$  mode, and the  $B_s^0\pi^+$  resonance signal would be reduced<sup>2</sup>. An analysis of the  $B^+K^-$  final state has been published by LHCb, based on a limited sample of  $1 \text{ fb}^{-1}$  [11].

However, it is also within the margin of errors that the actual masses of these tetraquark  $S$ -states are couple of tens of MeV below our estimates, in which case, the  $B^+\bar{K}^0$  mode is not available, and it is logical to anticipate  $X_{b0}^+$  and  $X_{b0}^0$  as resonant  $B_s\pi$  states. We pursue this possibility here. An alternative description is found

<sup>1</sup> Hereafter, adding the charged conjugated modes — e.g.  $\bar{B}_s^0\pi^-$  — is understood.

<sup>2</sup> This would be similar to the case of  $X(3278)$ , which decays predominantly in  $DD^*$  and also, appreciably, in  $J/\psi + \rho/\omega$ .

in [12].

With this hindsight, we point out that there are, in principle, two *generic different mechanisms* for producing  $X_{b0}(5770)$  in high energy  $pp$  and  $p\bar{p}$  collisions. These states can be produced as a fragmentation product of a jet initiated by a  $b$ -quark, but, subject to phase space, they can also be produced in the decays of the  $B_c^\pm$  mesons,  $B_c^\pm \rightarrow X_{b0}(5770)^{I=1} + \pi$  and  $B_c^\pm \rightarrow X_{b0}(5770)^{I=0} + \pi^\pm$  as a result of weak ( $c \rightarrow s\bar{u}$ ) decays,  $q\bar{q}$  excitation, and quark rearrangement (see Fig. 1). With the anticipated decays  $X_{b0}^\pm \rightarrow B_s^0\pi^\pm$  and  $X_{b0}^0 \rightarrow B_s^0\pi^0$ , the decay chains will lead to  $B_c^\pm \rightarrow B_s^0\pi^\pm\pi^0$  etc. A resonating structure in the  $B_s\pi$  mode can then be fished out by Dalitz analysis. This mechanism is similar to the production mechanism of many multiquark states, seen in  $B^0$  and  $B^\pm$  decays, such as  $B \rightarrow X(3872)(K, K\pi)$ , but also for the pentaquarks, such as  $P_c(4450)^+$  and  $P_c(4380)^+$ , in the decays  $\Lambda_b^0 \rightarrow (P_c(4380)^+, P_c(4450)^+)K^-$ . We recall that the dominant two-body decay mode  $B_c^\pm \rightarrow B_s^0\pi^\pm$  has been measured by LHCb, with a branching ratio of about 10% [13], and we anticipate that some of the  $B_c^\pm$ -decays to tetraquarks will be large enough to be measured.

In what follows, we present our estimates of the mass spectrum of the lowest  $S$  and  $P$ -states with the flavor quantum numbers of the state  $B_s^0\pi^+ = (\bar{b}s)(\bar{d}u)$ , having the angular momentum quantum numbers  $J^P = 0^+, 1^+$  together with their counterparts in the charm sector. This is followed by the discussion of the  $B_c^\pm$ -decays leading to some of these tetraquark states as well as the bound  $c\bar{c}$  tetraquark states  $X(3872)$  in the decays  $B_c^\pm \rightarrow X(3872) + \pi^\pm$ .

## II. SPECTRUM

Within the constituent quark model the color-spin Hamiltonian describing the interaction between the different constituents of a hadron takes the form

$$H = \sum_i m_i + 2 \sum_{i < j} \kappa_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \quad (1)$$

where  $m_i$  are the diquark constituent masses,  $\mathbf{S}_i$  the quark spins and  $\kappa_{ij}$  some effective, representation-dependent chromomagnetic couplings. The spin-spin interaction is here understood to be a contact one.

In the most recent and most successful type-II tetraquark model [8, 10], the dominant interactions are assumed to be the spin-spin interactions between quarks (antiquarks) inside the same tightly bound diquark (antidiquark). With the composition:  $[\bar{b}q]_{\mathbf{3}_c}[sq']_{\mathbf{3}_c}$  with  $q \neq q' = d, u$ , this means retaining only  $\kappa_{bq}$  and  $\kappa_{sq'}$  and the lightest states will correspond to the heavy-light diquark spins:  $S_{[bq]} = 0, 1$  and  $S_{[sq]} = 0$ . The latter case corresponds to the so called ‘good diquark’ [14], and the two resulting states have  $J^P = 0^+$  or  $1^+$ , the lightest being the  $0^+$  one. To indicate these particles, we use the

notations

$$X_{b0} = |0_{\bar{b}q}, 0_{sq'}\rangle \quad X_{b1} = |1_{\bar{b}q}, 0_{sq'}\rangle \quad (2)$$

In the above approximation, the resulting mass formula for  $S$ -wave,  $[\bar{b}q][sq']$  states is *additive in diquark energies*,

$$\begin{aligned} M(X_{bS}) &= m_{[bq]} + 2\kappa_{bq} \mathbf{S}_{\bar{b}} \cdot \mathbf{S}_q + m_{[sq]} + 2\kappa_{sq} \mathbf{S}_s \cdot \mathbf{S}_{q'} \\ &= m_{[bq]} + \kappa_{bq} \left( S(S+1) - \frac{3}{2} \right) + m_{[sq]} - \frac{3}{2}\kappa_{sq} \end{aligned} \quad (3)$$

where  $S \equiv S_{[bq]}$ .

We may compare (3) with the mass formulae of the related tetraquarks  $a_0(980)$ [6],  $Z_b(10610)$ ,  $Z'_b(10650)$ [9], obtained with the substitutions:  $b\bar{s} \rightarrow s\bar{s}$  and  $b\bar{s} \rightarrow b\bar{b}$

$$a_0(980) = |0_{\bar{s}q}, 0_{sq'}\rangle \quad (4)$$

$$M_{a_0} = 2 \left( m_{[sq]} - \frac{3}{2}\kappa_{sq} \right)$$

$$Z_b = \frac{1}{\sqrt{2}} (|1_{\bar{b}q}, 0_{bq'}\rangle - |0_{\bar{b}q}, 1_{bq'}\rangle) \quad (5)$$

$$M_{Z_b} = 2 m_{[bq]} - \kappa_{bq}$$

$$Z'_b = |1_{\bar{b}q}, 1_{bq'}\rangle_{J=1} \quad (6)$$

$$M_{Z'_b} = 2 m_{[bq]} + \kappa_{bq}$$

From Eqs. (5) and (6) and the known masses [21], we derive

$$m_{[bq]} = \frac{M(Z'_b) + M(Z_b)}{4} \simeq 5315 \text{ MeV} \quad (7a)$$

$$\kappa_{bq} = \frac{M(Z'_b) - M(Z_b)}{2} \simeq 22.5 \text{ MeV} \quad (7b)$$

In the approximation where tetraquark masses are additive in diquark energies, one finds

$$\begin{aligned} M(X_{b0}) &= \left( m_{[bq]} - \frac{3}{2}\kappa_{bq} \right)_{Z_b} + \left( m_{[sq]} - \frac{3}{2}\kappa_{sq} \right)_{a_0} = \\ &\simeq 5770 \text{ MeV} \quad (J^P = 0^+) \end{aligned} \quad (8)$$

about 200 MeV more than the  $X(5568)$  mass and just 7 MeV below the  $B^+ \bar{K}^0$ .

To be seen as resonant  $B_s\pi$  states, their masses should lie below the  $BK$  threshold. A good part of the  $B_s\pi$  invariant mass spectrum is excluded by the LHCb, but still there is a window of opportunity left unexplored so far.

As a side remark, we note that in Ref. [7] the value  $m_{[sq]} = 590$  MeV was obtained using the value  $\kappa_{sq} \simeq 64$  MeV obtained from a fit to the baryon masses, which however may be different from the spin-spin coupling inside a diquark. On the other hand,  $\kappa_{ij}$  are expected to scale inversely to the constituent quark masses and this relation is approximately verified by  $\kappa_{bq}$  and  $\kappa_{cq}$  [10] estimated from  $Z_{b,c}$  and  $Z'_{b,c}$  masses, eq. (7b)

and eq. (12b) below. If we scale  $\kappa_{sq}$  from  $\kappa_{cq}$  using the strange and charm constituent quark masses, we obtain

$$\kappa_{sq} \simeq 200 \text{ MeV} \quad (9)$$

leading to

$$m_{[sq]} \simeq 800 \text{ MeV} \quad (10)$$

The diquark mass thus obtained is close to the sum of constituent light and strange quark masses, 330 and 520 MeV, respectively.

The  $J^P = 1^+$  exotic states lies close by. From Eq. (3) we find

$$M(X_{b1}) \simeq 5820 \text{ MeV} \quad (J^P = 1^+) \quad (11)$$

The  $X_{b1}$  state is expected to decay into  $B_s^{*0}\pi^+$  followed by  $B_s^{*0} \rightarrow B_s^0\gamma$ , with a photon energy of 48 MeV in the  $B_s^*$  rest frame. Such a low energy photon escapes detection at hadron colliders, as pointed out in [1]. As a consequence of this, the observed peak of the  $X_{b1}$  would be shifted towards lower invariant masses and essentially coincide with the  $X_{b0}$  peak.

In the type-II model [8], we estimate the parameters  $m_{[cq]}$  and  $\kappa_{cq}$ , from the masses of  $Z_c(3900)$ ,  $Z'_c(4020)$  [21], obtaining

$$m_{[cq]} = \frac{M(Z'_c) + M(Z_c)}{4} \simeq 1978 \text{ MeV} \quad (12a)$$

$$\kappa_{cq} = \frac{M(Z'_c) - M(Z_c)}{2} \simeq 67 \text{ MeV} \quad (12b)$$

One might use the previous results to estimate the mass of the analogous  $X_{cS}^\pm$  expected in the charm sector and decaying into  $D_s \pi$

$$\begin{aligned} M(X_{c0}) &= m_{[cu]} + m_{[sd]} - 3/2 \kappa_{sq} - 3/2 \kappa_{cq} \\ &\simeq 2367 \text{ MeV} \end{aligned} \quad (13)$$

$$\begin{aligned} M(X_{c1}) &= m_{[cu]} + m_{[sd]} - 3/2 \kappa_{sq} + 1/2 \kappa_{cq} \\ &\simeq 2501 \text{ MeV} \end{aligned} \quad (14)$$

The estimates in Eq. (13 -14) set the exotic candidates  $X_{c0}^\pm$  just above the  $DK$  and  $D^*K$  thresholds (2363 and 2504 MeV, respectively), so that it could be useful to search also in these decay channels.

If the light diquark is in the  $S = 0$  configuration, *i.e.* it is antisymmetric in spin and color, it must also be antisymmetric in  $SU(3)_F$  (F for flavor), therefore the tetraquarks  $[\bar{Q}\bar{q}][q'q'']$ , with  $Q = b, c$  and  $q, q', q'' = u, d, s$  belong to the  $SU(3)_F$  representation:  $\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \bar{\mathbf{6}}$ .

In the charm sector, one doubly charged state is present, belonging to the  $\bar{\mathbf{6}}$ , *e.g.* with the flavor content  $[\bar{c}\bar{u}][sd] \rightarrow D_s^- \pi^-$ . In the beauty sector, doubly charged states lie in the symmetric  $\mathbf{15}$  representation of  $SU(3)_F$  (see, He and Ko in [2]), originating from the product:  $\bar{\mathbf{3}} \otimes \mathbf{6} = \mathbf{3} \oplus \mathbf{15}$ . This requires a light diquark with  $S = 1$ , the so-called ‘‘bad diquarks’’, which may be argued to have little binding [14].

At present, upper limits on the production at lepton colliders of charmed-strange doubly charged resonances have been given [15] in the  $D_s^+ \pi^+$  channel, for masses between 2.25 and 2.61 GeV.

We close this Section by considering the flavour multiplicity of the states  $X_{b0} = [\bar{b}\bar{q}][sq']$ , with  $q, q' = u, d$ , and their decay modes. These states are obviously organised in a isospin triplet and singlet, similar in structure to the scalar light tetraquarks  $a_0(980)$  and  $f_0(980)$ . The neutral  $X_{b0}$  states are similarly expected to be nearly degenerate in mass.

The isoscalar state should decay as  $X_{b0}^{(I=0)} \rightarrow B_s + \eta$  which is most likely phase space forbidden, leaving the possibility of the strong decay  $X_{b0}^{(I=0)} \rightarrow B + \bar{K}$ , a situation very similar to the decay  $f_0 \rightarrow K\bar{K}$ . Should also the latter mode be forbidden by phase space,  $X_{b0}^{(I=0)}$  has to decay by isospin violating interactions:  $X_{b0}^{(I=0)} \rightarrow B_s + \pi^0$ , which may occur due to isospin violating mixing with  $X_{b0}^{(I=1)}$  or via  $\eta - \pi^0$  mixing, similarly to  $\eta$  decay.

Similar considerations apply to the  $I = 0$   $X_{cS}^\pm$  states which estimates in Eqs. (13 -14) place only 40 - 50 MeV above the well known  $D_{s0}(2317)$  and  $D_{s1}(2460)$ . The mass difference is quite close to the theoretical error so as to suggest  $X_{cS}$  to be identified with the latter resonances, the decays into  $D_s^+ \pi^0$  arising also from isospin breaking interactions, either due to the mixing with the  $I = 1, I_3 = 0$  component or via  $\eta - \pi^0$  mixing.

### III. TETRAQUARK PRODUCTION IN WEAK DECAYS OF $B_c^\pm$ MESONS

Motivated by the observation of a large number of exotic  $XYZ$  mesons in the decays of the  $B^\pm$  and  $B^0$ -mesons, as well as the pentaquark states  $P_c(4450)^+$  and  $P_c(4380)^+$  in the decays of the  $\Lambda_b$ -baryons,  $\Lambda_b^0 \rightarrow (P_c(4380)^+, P_c(4450)^+)K^-$ , with the subsequent decays  $(P_c(4380)^+, P_c(4450)^+) \rightarrow J/\psi p$ , we anticipate production of the charged  $X_{b0}(5570)^\pm$  and neutral  $X_{b0}(5570)^0$  tetraquark states in weak decays of the  $B_c^\pm$  mesons. We also emphasize that  $B_c^\pm$ -decays are a copious, as yet unexplored, source of hidden  $c\bar{c}$  tetraquark states, via decay modes such as  $B_c^\pm \rightarrow X(3872)\pi^\pm$ . Other candidate tetraquark states in the same family but having different  $J^{PC}$  quantum numbers are, likewise, anticipated in  $B_c^\pm$  decays.

For the weak decays of  $B_c^\pm \rightarrow B_s^0 \pi^\pm$ ,  $B_c^\pm \rightarrow X_{b0}^0 \pi^\pm$ , and  $B_c^\pm \rightarrow X_{b0}^\pm \pi^0$ , the active decay at the quark level is  $c \rightarrow s\bar{u}\bar{d}$ , with the  $\bar{b}$  decay treated as a spectator. This accounts for approximately 50% of the  $B_c^\pm$  decays [16].

The effective Hamiltonian for such non-leptonic decays is

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= \frac{4G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left[ C^{(-)} \mathcal{O}^{(-)} + C^{(+)} \mathcal{O}^{(+)} \right] \quad (15) \\ \mathcal{O}^{(\pm)} &= [\bar{s}^\alpha \gamma_\mu P_L c_\alpha] [\bar{u}^\beta \gamma^\mu P_L d_\beta] \pm [\bar{s}^\alpha \gamma_\mu P_L d_\alpha] \end{aligned}$$

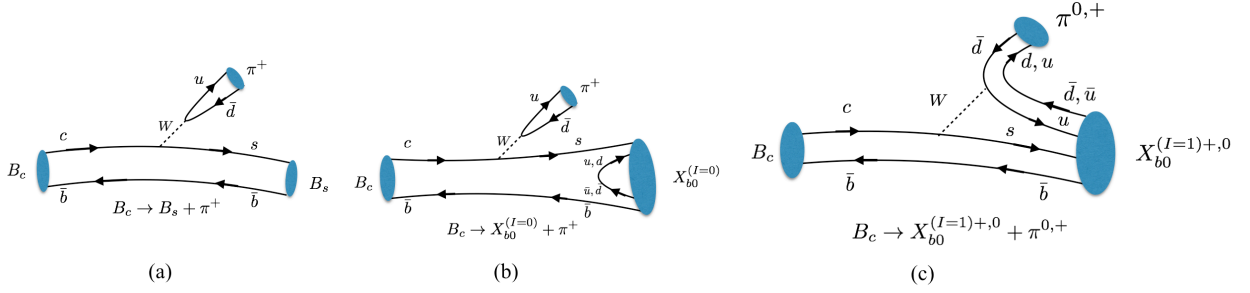


FIG. 1: (a): Leading order Feynman diagram for the decay  $B_c^+ \rightarrow B_s^0 \pi^+$  (b):  $B_c^+ \rightarrow X_{b0}^{(I=0)} + \pi^+$  (c): the corresponding diagram for the decays  $B_c^+ \rightarrow X_{b0}^{(I=1)+,0} + \pi^{0,+}$ .

where  $G_F$  is the Fermi coupling constant,  $V_{ij}$  are the CKM matrix elements,  $\alpha$  and  $\beta$  are the color indices,  $P_L = \frac{1}{2}(1 - \gamma_5)$  and  $C^{(\pm)} = (C_1 \pm C_2)/2$ ,  $C_{1,2}(\mu)$  being the Wilson coefficients at scale  $\mu = m_c, m_b$ . We have dropped QCD penguin contributions and  $C^{(\pm)}$  are QCD renormalization factors [17] computed at a momentum scale equal to the  $b$ -quark mass, with [18]

$$2C^{(-)} \simeq 1.4 \quad 2C^{(+)} \simeq 0.85 \quad (16)$$

The amplitude for  $B_c^+ \rightarrow B_s \pi^+$  can be written in the factorized form, see Fig. 1(a)

$$\mathcal{M}(B_c^+ \rightarrow B_s^0 \pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* (C^{(-)} + C^{(+)}) \tilde{M} \quad (17)$$

with  $(C^{(-)} + C^{(+)}) = C_1$

$$\begin{aligned} \tilde{M} &= \frac{f_\pi}{m_\pi^2} q^\mu \langle B_s | \bar{s} \gamma_\mu P_L c | B_c^+ \rangle \\ &= \frac{f_\pi}{m_\pi^2} [f_+(m_\pi^2)(m_{B_c}^2 - m_{B_s}^2) + f_-(m_\pi^2)m_\pi^2] \end{aligned} \quad (18)$$

Here,  $f_\pm(q^2)$  are the vector current form factors, evaluated at  $q^2 = m_\pi^2$ , which have been studied in a number of models (see, for example [19] for a comparative evaluation),  $f_\pi$  is the pion decay constant,  $f_\pi = 140$  MeV [21],  $C_1$  is the (QCD renormalized) effective Wilson coefficient, estimated to be  $C_1 \approx 1.1$ , and the second term above can be neglected, as it is multiplied by  $m_\pi^2$ . With this, the decay width can be evaluated straightforwardly

$$\Gamma(B_c^\pm \rightarrow B_s^0 \pi^\pm) = |\mathcal{M}|^2 \frac{|\mathbf{p}_\pi|}{8\pi m_{B_c}^2} \quad (19)$$

where  $|\mathbf{p}_\pi|$  is the  $\pi^\pm$  3-momentum in the rest frame of  $B_c^\pm$ -meson.

The branching ratio for  $B_c^\pm \rightarrow B_s^0 \pi^\pm$  has been measured by LHCb

$$\mathcal{B}(B_c^+ \rightarrow B_s^0 \pi^+) \frac{P(\bar{b} \rightarrow B_c^+)}{P(\bar{b} \rightarrow B_s)} = (2.37_{-0.35}^{+0.37}) \times 10^{-3} \quad (20)$$

Here,  $P(\bar{b} \rightarrow B_s)$  and  $P(\bar{b} \rightarrow B_c^+)$  are the fragmentation probabilities. The ratio of the two probabilities, *i.e.*,

the ratio of the production rates of  $B_c^+$  mesons and  $B_s$  mesons in a  $b$ -quark jet is estimated to be about 0.02, yielding a 10% branching ratio for  $B_c^+ \rightarrow B_s \pi^+$ . This is the largest branching ratio of any  $B$ -meson observed in a single channel.

The decay  $B_c^\pm \rightarrow X_{b0}^{I=0} \pi^\pm$  is expected to have a large branching ratio, as this decay amplitude, like  $B_c^\pm \rightarrow B_s^0 \pi^\pm$ , is factorizable (see Fig. 1(b)). The relevant matrix element can be written down in an analogous way to that of  $B_c^\pm \rightarrow B_s^0 \pi^\pm$ . One now needs to know the hadronic matrix element  $\langle X_{b0}^{I=0} | \bar{s} \gamma_\mu P_L c | B_c^+ \rangle$ . Recalling that  $X_{b0}^{I=0}$  has  $J^P = 0^+$ , the transition goes via the axial-vector part of the charged current, yielding an expression similar to the one for  $\mathcal{M}(B_c^\pm \rightarrow B_s^0 \pi^\pm)$  obtained above. However, in this case, the corresponding hadronic quantity, which we denote by  $f_+(m_\pi^2)^{B_c X_{b0}}$ , is unknown. This can be calculated using QCD sum rules or lattice QCD, as it involves the axial-current matrix element of a single hadron  $\rightarrow$  single hadron transition. In the diquark model at hand, it is expected to be not too different from  $f_+(m_\pi^2)^{B_c B_s^0}$ , as the heavy flavor content of the  $X_{b0}^0$  and  $B_s^0$  is the same, namely  $\bar{b}s$ .

Denoting the ratio of the two form factors as  $F(X_{b0}/B_s) \equiv f_+(m_\pi^2)^{B_c X_{b0}}/f_+(m_\pi^2)^{B_c B_s^0}$ , the relative branching ratios can be expressed as

$$\begin{aligned} \frac{\mathcal{B}(B_c^\pm \rightarrow X_{b0}^{I=0} \pi^\pm)}{\mathcal{B}(B_c^\pm \rightarrow B_s^0 \pi^\pm)} &= \\ &= F(X_{b0}/B_s)^2 \frac{(m_{B_c}^2 - m_{X_{b0}}^2)^2}{(m_{B_c}^2 - m_{B_s}^2)^2} \frac{|\mathbf{p}_\pi|^{B_c \rightarrow X_{b0} \pi}}{|\mathbf{p}_\pi|^{B_c \rightarrow B_s \pi}} \end{aligned} \quad (21)$$

With the known masses, and using our estimate  $m(X_{b0}^{I=0}) = 5.770$  GeV, we get a branching ratio of 1(2)% for the decay  $B_c^\pm \rightarrow X_{b0}^{I=0} \pi^\pm$  for an assumed value of  $F(X_{b0}/B_s)^2 = 0.5(1)$ . Given the large sample of  $B_c^\pm$  already available and in forthcoming LHC runs, this branching ratio is measurable in the decay mode  $B_c^\pm \rightarrow (B_s^0 \pi^0) \pi^\pm$ , assuming a good  $\pi^0$  detection efficiency.

We expect the corresponding branching ratio for the decay  $B_c^\pm \rightarrow X_{b0}^\pm \pi^0 \rightarrow (B_s^0 \pi^\pm) \pi^0$  to be multiplied by a factor  $C^{(-)2}/(C^{(-)} + C^{(+)})^2 \simeq 0.62$ . In fact,  $\mathcal{O}^{(-)}$  and  $\mathcal{O}^{(+)}$  contribute equally to  $B_c$  decay into  $X_{b0}^{I=0}$ , Fig. 1(b),

while only  $\mathcal{O}^{(-)}$  contributes to the decay into  $X_{b_0}^{I=1}$ , due to color antisymmetry of the final  $us$  pair, Fig. 1(c) (this is similar to the Pati and Woo argument [20] to derive the  $\Delta I = 1/2$  rule, *i.e.* flavor antisymmetry, in non-leptonic baryon decays). This pattern could be modified by non-perturbative effects, as also seen in a number of similar  $B^\pm$  and  $B^0$  decays [21].

We now discuss the  $B_c^+$  decays leading to the bound  $c\bar{c}$  tetraquarks. This requires the quark decay  $\bar{b} \rightarrow \bar{c}u\bar{d}$ , with the  $c$ -quark in  $B_c^+$  acting as a spectator quark. The benchmark decay for this class is  $B_c^\pm \rightarrow J/\psi \pi^\pm$ . Requiring now the excitation of a  $q\bar{q}$  pair, followed by quark recombination, leads to decays such as  $B_c^\pm \rightarrow X(3872)^{I=0} \pi^\pm$  and  $B_c^\pm \rightarrow X(3872)^{I=1\pm,0} \pi^{0,\pm}$ . These diagrams allow access to both the  $I = 0$  (isosinglet) and the  $I = 1$  (isotriplet) partners of the  $X(3872)$ , decaying, respectively, to  $J/\psi \omega$  and  $J/\psi \rho^0$ , as well as the decay of the charged partner  $X(3872)^\pm \rightarrow J/\psi \rho^\pm$ , in addition, possibly, to  $\bar{D}^* D$  decays. There would be enough phase space to observe the corresponding  $P$ -states as well.

Again, we expect the decay  $B_c^\pm \rightarrow X(3872)^{(I=0)} \pi^\pm$  to have a large branching ratio, which is similar to  $B_c^\pm \rightarrow J/\psi \pi^\pm$ , as both are factorizable processes and are proportional to  $C^{(-)} + C^{(+)}$ . The decays of  $B_c^\pm$  to the  $[cq][\bar{c}q']$ -tetraquarks have the potential to map out a large number of anticipated states in this sector.

#### IV. CONCLUDING REMARKS

The observation of the  $X(5568)$  by D0, with  $X(5568) \rightarrow B_s^0 \pi^\pm$ , having  $M = 5568$  MeV and  $\Gamma = 22$  MeV, has not been confirmed by LHCb. It remains to be seen if a state with the quark flavors  $b\bar{s}u\bar{d}$  exists in nature, with a different mass, decay pattern, and width. In this paper we have used the diquark-antidiquark pic-

ture to give predictions about the mass spectrum of the lowest  $S$ -state,  $X_{b_0}$  and its  $J^P = 1^+$  partners, both in charm and bottom sectors. Our estimates set the mass of the lowest such state in the  $b$ -quark sector at around 5770 MeV, somewhat below the  $BK$  threshold. Within the errors of our approach,  $X_{b_0}^+$  could lie just above this threshold, and one has to look for it in the decay  $X_{b_0}^+ \rightarrow B^+ \bar{K}^0$ . However,  $X_{b_0}^+$  may as well reveal itself as a resonating  $B_s \pi$  state, or not manifest at all, if below threshold, as discussed in [12].

Here we propose to search tetraquark states in the decays of the  $B_c^\pm$  mesons,  $B_c^\pm \rightarrow X_{b_0}^0 \pi^\pm$  and  $B_c^\pm \rightarrow X_{b_0}^\pm \pi^0$  and have argued that some of these decay modes may have a large branching ratio. This requires a good  $\pi^0$ -detection efficiency, which we advocate to improve in hadron collider experiments, such as the LHCb. The two detached vertices (of the  $B_c^\pm$  and  $B_s^0$ ) may help in reducing the background.

So far, only a handful of  $B_c^\pm$  decays have been observed [21], and it is worthwhile to put in a dedicated effort to increase this database. Apart from the possibility of observing the tetraquark states of the  $B_s \pi$  variety, we anticipate several bound  $c\bar{c}$  tetraquark states, which emerge from the decay  $B_c^+ \rightarrow (c\bar{c})u\bar{d}$ , followed by a  $q\bar{q}$  excitation from the vacuum. These would lead to decays such as  $B_c^\pm \rightarrow X(3872)^0 \pi^\pm$  and  $B_c^\pm \rightarrow X(3872)^\pm \pi^0$ , as well as to other related tetraquark states. They should be searched for at the LHC, and also at Belle-II, if the  $e^+e^-$  center of mass energies could reach the  $B_c^+ B_c^-$  threshold.

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