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Double parton scattering in the ultraviolet: addressing the double counting problem*

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Abstract: In proton-proton collisions there is a smooth transition between the regime of double parton scattering, initiated by two pairs of partons at a large relative distance, and the regime where a single parton splits into a parton pair in one or both protons. We present a scheme for computing both contributions in a consistent and practicable way.

1 Ultraviolet behaviour of double parton scattering

The familiar factorisation formula for double parton scattering (DPS) reads

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})\,, \tag{1}$$

where C is a combinatorial factor, $\hat{\sigma}_{1,2}$ is the cross section for the first or second hard-scattering subprocess, and $F(x_1, x_2, \boldsymbol{y})$ is a double parton distribution (DPD). $\boldsymbol{y}$ denotes the transverse distance between the two partons. A field theoretical definition of $F(x_1, x_2, \boldsymbol{y})$ is naturally given by the matrix element between proton states of two twist-two operators at relative transverse distance $\boldsymbol{y}$. As explained in [1], the leading behaviour of DPDs at small $\boldsymbol{y}$ is controlled by the splitting of one parton into two, shown in figure 1a. The corresponding expression reads

$$F(x_1, x_2, \boldsymbol{y}) = \frac{1}{\boldsymbol{y}^2}\, \frac{\alpha_s}{2\pi^2}\, \frac{f(x_1 + x_2)}{x_1 + x_2}\, T\biggl(\frac{x_1}{x_1 + x_2}\biggr) \qquad \text{for small } \boldsymbol{y}\,. \tag{2}$$

For simplicity we dropped labels for the different parton species and polarisations, as we already did in (1).

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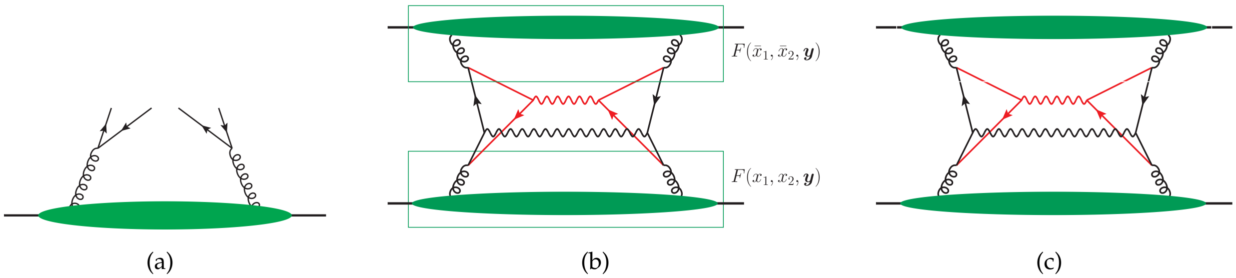

Figure 1: (a) Perturbative splitting contribution to a DPD. (b) Contribution of double perturbative splitting to DPS, also called "1 vs 1" graph. (c) Single hard scattering contribution.

Inserting the short-distance limit (2) in the cross-section formula (1) reveals an immediate problem: the integration over $\boldsymbol{y}$ diverges strongly in the ultraviolet. In fact, the approximations that lead to (1) are not valid when $\boldsymbol{y}$ becomes too small (compared with the inverse of the large momentum scale Q of the hard scattering). This unphysical ultraviolet divergence signals another problem, namely one of double counting: the graph in figure 1b shows a contribution to double parton scattering, with perturbative splitting in each DPD. Drawn as in figure 1c, the same graph gives however a contribution to single parton scattering (SPS) at higher loop order. For multi-jet production this problem was already pointed out in [2].

2 A consistent scheme

The following scheme provides a consistent treatment of single and double scattering contributions to a given process, and it removes the ultraviolet divergence in the naive double scattering formula just discussed. We regulate the DPS cross section (1) by inserting a function under the integral over DPDs,

$$\int d^2\boldsymbol{y}\, \left[\Phi(\nu y)\right]^2 F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})\,, \tag{3}$$

which is chosen such that $\Phi(u) \to 0$ for $u \to 0$ and $\Phi(u) \to 1$ for $u \gg 1$. (We take the square of Φ in order to have a closer connection to the case discussed in section 5.) This removes contributions with distances $y = |\boldsymbol{y}|$ below $1/\nu$ from what is *defined* to be double parton scattering. An appropriate choice for this cutoff scale is $\nu \sim Q$. Double and single parton scattering are then combined as

$$\sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}\,, \tag{4}$$

where σ_{DPS} is the regulated DPS cross section and σ_{SPS} the SPS cross section computed in the usual way (given by figure 1c and its crossed variants in our example). The subtraction term σ_{sub} is given by the DPS cross section with both DPDs replaced by the splitting expression (2), computed at fixed order in perturbation theory and used at all $\boldsymbol{y}$. Note that at any order in α_s, the computation of σ_{sub} is technically much simpler than the one of σ_{SPS}.

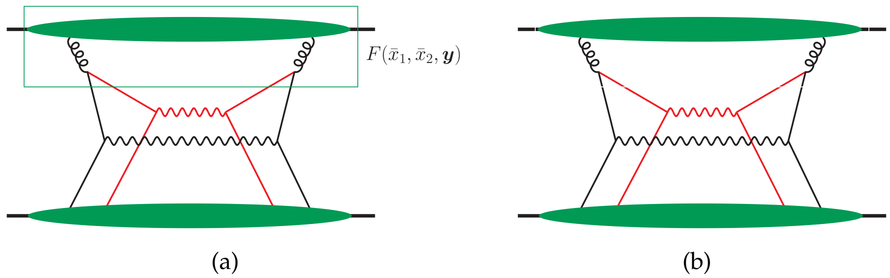

Figure 2: (a) Contribution of single perturbative splitting to DPS, also called "1 vs 2" graph. (b) Graph with a twist-two distribution for one proton and a twist-four distribution for the other.

Let us see how this construction solves the double counting problem. We work differentially in $\boldsymbol{y}$, which is Fourier conjugate to a specific transverse momentum variable as specified in [1] and can thus be given an unambiguous meaning, not only in the DPS cross section but also in the box graph of figure 1c and the associated term σ_{SPS}. For $y \lesssim 1/Q$ one has $\sigma_{\text{DPS}} \approx \sigma_{\text{sub}}$ because the perturbative approximation (2) of the DPD works well in that region. The dependence on the cutoff function $\Phi(\nu y)$ then cancels between σ_{DPS} and σ_{sub}, and one is left with $\sigma \approx \sigma_{\text{SPS}}$. For $y \gg 1/Q$ one has $\sigma_{\text{sub}} \approx \sigma_{\text{SPS}}$, because in that region the box graph can be approximated just as is done in the DPS formula. One is thus left with $\sigma \approx \sigma_{\text{DPS}}$ at large y, and the cutoff function $\Phi(y\nu) \approx 1$ does not have any effect there. The construction just explained is a special case of the general subtraction formalism discussed in chapter 10 of [3], and it works order by order in perturbation theory.

3 Splitting and intrinsic contributions to DPDs

At small $\boldsymbol{y}$ a DPD – defined as a hadronic matrix element as already mentioned – contains not only the perturbative splitting contribution described by (2) but also an "intrinsic" part in which the two partons do not originate from one and the same "parent" parton. We emphasise that our scheme does not need to distinguish these "splitting" and "intrinsic" contributions when setting up the factorisation formula for the cross section. In fact, we do not know how such a separation could be realised in a field theoretic definition valid at all $\boldsymbol{y}$. It is only when writing down a parameterisation of $F(x_1, x_2, \boldsymbol{y})$ that has the small-y limit predicted by QCD that we separate the DPD into splitting and intrinsic pieces.

If we consider the DPS cross section formula at small y and take the splitting contribution for only one of the two protons, then we obtain the "1 vs 2" contribution depicted in figure 2a, which has been discussed in detail in [4,5], [6,7] and [8]. The corresponding integral in the cross section goes like $d^2\boldsymbol{y}/\boldsymbol{y}^2$ and thus still diverges at small y if treated naively. In our regulated DPS integral (3), it gives a finite contribution with a logarithmic dependence on the cutoff scale ν.

Just as the 1 vs 1 contribution of figure 1b corresponds to the SPS graph 1c, the 1 vs 2 contribution of figure 2a corresponds to a contribution with a twist-two distribution (i.e. a usual parton density) for one proton and a twist-four distribution for the other proton, shown in figure 2b. The

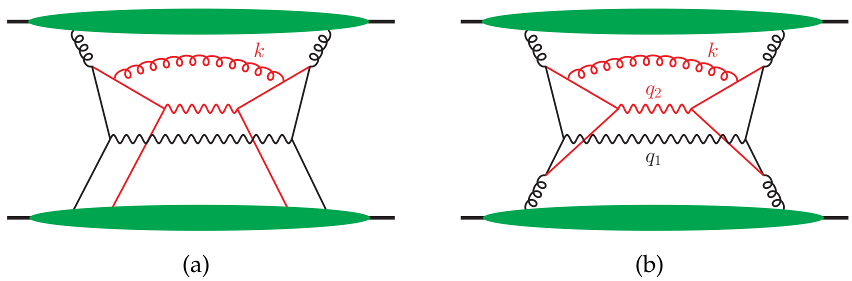

Figure 3: Graphs with additional gluon emission that give rise to DGLAP logarithms in strongly ordered kinematics, as explained in the text.

complete cross section is then obtained as

$$\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub (1vs1)}} + \sigma_{\text{SPS}} - \sigma_{\text{sub (1vs2)}} + \sigma_{\text{tw2}\,\times\,\text{tw4}}\,. \tag{5}$$

The DPS term contains the full DPDs and thus generates 1 vs 1, 1 vs 2 and the usual 2 vs 2 contributions. The terms $\sigma_{\text{sub (1vs1)}}$ and σ_{SPS} were discussed in the previous section. The term $\sigma_{\text{tw2}\,\times\,\text{tw4}}$ corresponds to figure 2b, and the associated subtraction term $\sigma_{\text{sub (1vs2)}}$ is obtained from the DPS formula by replacing one DPD with its perturbative splitting approximation (2) and the other DPD with a twist-four distribution.

Since very little is known about parton distributions of twist four, including $\sigma_{\text{tw2}\,\times\,\text{tw4}} - \sigma_{\text{sub (1vs2)}}$ in the cross section is a challenge for phenomenology. One can however show that with the choice $\nu \sim Q$ this combination is subleading in logarithms $\log(Q/\Lambda)$ compared to the 1 vs 2 part of σ_{DPS} and can hence be dropped at leading logarithmic accuracy.

4 DGLAP logarithms

As discussed in [1], the DPDs $F(x_1, x_2, \boldsymbol{y})$ are subject to homogeneous DGLAP evolution, with one DGLAP kernel for the parton with momentum fraction x_1 and another for the parton with momentum fraction x_2. One can show that the evolved distributions in the DPS cross section correctly resum large DGLAP logarithms in higher-order graphs. An example is the 1 vs 2 graph in figure 3a, which builds up a logarithm $\log^2(Q/\Lambda)$ in the region $\Lambda \ll |\boldsymbol{k}| \ll Q$, compared with the single $\log(Q/\Lambda)$ of the graph without gluon emission. Similarly, the higher-order SPS graph in figure 3b builds up a logarithm $\log(Q_2/Q_1)$ in the region $Q_1 \ll |\boldsymbol{k}| \ll Q_2$ if the scales Q_1 and Q_2 of the two hard scatters are strongly ordered among themselves. This type of logarithm is readily included in the cross section by taking separate renormalisation scales $\mu_{1,2} \sim Q_{1,2}$ for the two partons in the DPDs.

5 Extension to measured transverse momenta

So far we have discussed DPS and SPS in collinear factorisation, where the net transverse momentum $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ of the particles produced by each hard scatter is integrated over. As shown in [1], DPS can also be formulated for small measured $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ by generalising the corresponding formalism for SPS (which is e.g. documented in chapter 13 of [3]). Our scheme is readily extended to this case. The DPS cross section then involves a regularised integral

$$\int d^2\boldsymbol{y}\, d^2\boldsymbol{z}_1\, d^2\boldsymbol{z}_2\; e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2}\, \Phi(\nu y_+)\, \Phi(\nu y_-)\, F(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})\,, \tag{6}$$

where $F(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$ is a transverse-momentum dependent DPD transformed to impact parameter space. The perturbative splitting mechanism renders these distributions singular at the points $y_\pm = \left|\boldsymbol{y} \pm \frac{1}{2}(\boldsymbol{z}_1 - \boldsymbol{z}_2)\right|$, as seen in section 5.2 of [1], and the function Φ regulates the logarithmic divergences that appear in the naive DPS formula.

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