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α_s FROM GLOBAL FITS OF PARTON DISTRIBUTION FUNCTIONS*

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The status of the determination of the strong coupling constant $\alpha_s(M_Z^2)$ from deep-inelastic scattering and related hard scattering data is reviewed.

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1. Introduction

Deep-inelastic scattering (DIS) is one of the cleanest ways to measure the strong coupling constant from the scaling violations of the structure functions, which form the hadronic tensor $W_{\mu\nu}$ in the differential scattering cross section [1]. In the unpolarized case for pure photon exchange one has

$$W_{\mu\nu}(q,P) = \frac{1}{2x}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_L(x,Q^2) + \frac{2x}{Q^2}\left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu}\right) F_2(x,Q^2) \ . \quad (1)$$

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Here q denotes the 4-momentum transfer from the lepton to the nucleon with $q^2 = -Q^2$, P the nucleon momentum, and $x = Q^2/(2P.q)$ the Bjorken variable. The two structure functions $F_{L,2}(x, Q^2)$ exhibit scaling violations due to the Q^2 dependence. They contain both massless and massive quark distributions (due to charm and bottom quarks).

A comprehensive survey on the value of the strong coupling constant $\alpha_s(M_Z^2)$ as measured in different hard processes has been given by G. Altarelli [2] in 1992.[a] At this time the value of $\alpha_s(M_Z^2)$ has been known in next-to-leading order (NLO) and the deep-inelastic value turned out to be low $\alpha_s^{\rm NLO}(M_Z^2) = 0.112 \pm 0.007$. This value resulted from data by BCDMS [4, 5], a combined SLAC/BCDMS fit [6], $\rm EMC_H$ [7], $\rm CCFR_{Fe}$ [8–10], $\rm CHARM_{CaCO_3}$ [11] and NMC [12]. Except the value of NMC of 0.117 ± 0.005 all other experiments yielded values in the range of 0.108...0.114. An important test consisted in comparing the experimental results for the plot of $\partial F_2/\partial \ln(Q^2)$, cf. also [13], which failed to agree for CDHSW [14].

At this order in QCD the factorization and renormalization scale uncertainties are still large and amount to $\Delta\alpha_s(M_Z^2) = \pm 0.0050$, varying the scales $\mu_{F,R}^2 \in [Q^2/4, 4Q^2]$ [15]. In most of these analyses heavy flavor corrections have not been accounted for, as they were known to leading order only [16]. Rather they were fitted to the data using the massless NLO corrections for $N_F = 4$, using the $O(\alpha_s)$ Wilson coefficients [17] and the NLO anomalous dimensions [18], in early analyses. A few characteristic determinations of $\alpha_s(M_Z^2)$ at NLO are summarized in Table 1. Most of the values lay in the region between $\alpha_s^{\rm NLO}(M_Z^2) = 0.1150...0.1171$. In the polarized case, also

Table 1. Some NLO results until $\sim$ 2003 on $\alpha_s(M_Z^2)$ from deep inelastic scattering, including also global PDF fits. The H1 value is subject to an additional error of $+0.0009/-0.0005$ and the ZEUS value of ± 0.0018 due to model dependence. We add results from deep--inelastic scattering off polarized targets; from [27].

	$\alpha_s(M_Z^2)$	exp.	theory	Ref.
NLO				
BCDMS	0.1111	± 0.0018		[4, 5]
H1	0.1150	± 0.0017	± 0.0050	[19, 20]
CTEQ6	0.1165	± 0.0065		[21]
A02	0.1171	± 0.0015	± 0.0033	[22]
ZEUS	0.1166	± 0.0049		[23]
MRST03	0.1165	± 0.0020	± 0.0030	[24]
BB1 (pol)	0.1135	± 0.004	$+0.010$ -0.009	[25]
BB2 (pol)	0.1132	$+0.0043$ -0.0051	$+0.0056$ -0.0097	[26]

[a]Earlier summaries were given in Refs. [3].

values for $\alpha_s^{\rm NLO}(M_Z^2)$ were determined, which, however, have much larger errors, see [25, 26] for details.

Because of the large scale variation uncertainties at NLO compared to the precision of the current World data, we will discuss in the following only next-to-next-to leading order (NNLO) and next-to-next-to-next-to leading order (N^{3}LO) analyses, and refer to NLO analyses only briefly.[b]

Already the NLO analyses require a description of the heavy flavor corrections to $O(\alpha_s^2)$ [29]. The NNLO analyses rely on the massless $O(\alpha_s^2)$ Wilson coefficients [30] and the NNLO anomalous dimensions [31, 32]. The heavy flavor corrections are required to $O(\alpha_s^3)$. Their calculation is underway for scales $Q^2/m_c^2 \gtrsim 10$ in case of the structure function $F_2(x, Q^2)$ [33]. A series of moments [34] and all logarithmic corrections have been calculated [35, 36], as well as four of five contributing Wilson coefficients [36–39], which use in their representation the massless 3-loop Wilson coefficients [40]. The calculation of the last contributing Wilson coefficient is underway [41]. For an approximate NNLO representation see [42].

In Section 2 we describe NNLO and N^{3}LO non-singlet analyses and turn to combined non-singlet and singlet analyses in Section 3. We always discuss the response to individual data sets in the different fits and investigate the perturbative stability going from NLO to NNLO (and to N^{3}LO). Section 4 contains the conclusions.

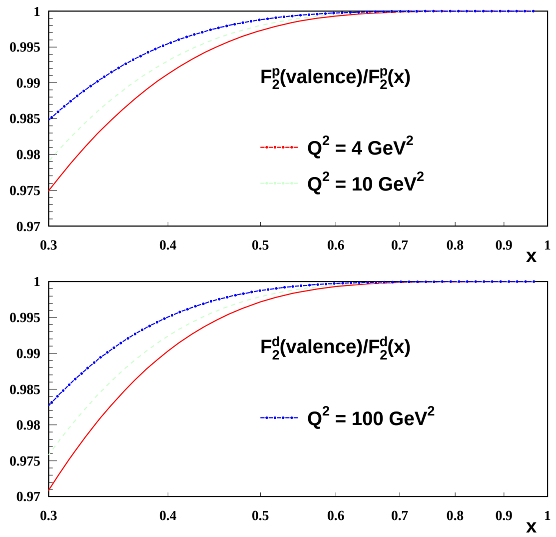

Fig. 1. The ratio $F_2^{\rm p(d),val}/F_2^{\rm p(d)}$ for $Q^2 = 4$ (red), 10 (green), 100 GeV^2 (blue) using the ABKM09 parton distribution functions [47].

[b]Recent surveys on the determination of $\alpha_s(M_Z^2)$ have been given in [28].

2. $\alpha_s(M_Z^2)$ at NNLO from Non-Singlet Data

NNLO flavor non-singlet analyses have been carried out in Refs. [27, 43–45]. They rely on the structure function differences $F_2^p(x,Q^2) - F_2^n(x,Q^2)$ and valence approximations typically in the region $x \gtrsim 0.35$. Here $F_2^n(x,Q^2)$ is measured from $F_2^d(x,Q^2)$ with account of the appropriate nuclear corrections. Also the target mass corrections [46] have to be applied. Figure 1 shows the size of sea-quark tail contributions, comparing to a consistent set of PDFs, i.e. to one whose scaling violations have a similar value of $\alpha_s(M_Z^2)$. An important issue is a necessary cut in $W^2 > 12.5$ GeV^2 and $Q^2 > 4$ GeV^2 to separate the higher twist contributions in the non-singlet case [43, 45, 48].

For the 4-loop non-singlet anomalous dimension the moments $N = 2, 3, 4$ are known [49–52]. In [43] it has been shown, that one may approximate the 4-loop anomalous dimension by the Pade-approximant

$$\gamma_n^{(3),\mathrm{approx}} = \frac{{\gamma_n^{(2)}}^2}{\gamma_n^{(1)}}, \tag{2}$$

which agrees better than by 20% for the known moments for $N_F = 3$. Furthermore, a $\pm 100\%$ error to this quantity has been assigned. In the fit this results into an uncertainty of $\delta\Lambda_{\mathrm{QCD}} = \pm 2$ MeV, far below the experimental uncertainty of $\Delta\Lambda_{\mathrm{QCD}} = \pm 26$ MeV. The 4-loop non-singlet anomalous dimension is therefore of lesser importance compared to the massless 3-loop Wilson coefficient. One has to consider also charm quark effects contributing with 2-loop order [53, 54] which are below 0.35% in the valence region. This holds similarily also for the asymptotic 3-loop corrections [38].

In the non-singlet analysis NNLO values of $\alpha_s^{\mathrm{NNLO}}(M_Z^2) = 0.1120...0.1134 \pm 0.0022$ are obtained and the corresponding NLO values are somewhat larger with $\alpha_s^{\mathrm{NLO}}(M_Z^2) = 0.1147 \pm 0.0021$, unlike the case for combined non-singlet and singlet analyses discussed in Section 3. Main results are summarized in Table 2.

The NLO results given by the experiments BCDMS and NMC are basically reproduced by the combined fit of the data from BCDMS, SLAC and NMC, as partial results. The steps from NLO to NNLO and N^{3}LO*, the * standing for the yet not completely known 4-loop non-singlet anomalous dimension, are perturbatively stable. As NNLO value we finally quote

$$\alpha_s^{\mathrm{NNLO}}(M_Z^2) = 0.1132 \pm 0.0022 \tag{3}$$

and for N^{3}LO*

$$\alpha_s^{\mathrm{N^3LO^*}}(M_Z^2) = 0.1137 \pm 0.0022\ . \tag{4}$$

The difference of these values $\Delta\alpha_s(M_Z^2) = 0.0005$ may serve as an estimate of the size of the remaining theory uncertainty.

Table 2. Comparison of the $\alpha_s(M_Z)$ values obtained by BCDMS [5] and NMC [55] at NLO with the results of the flavor non-singlet fits BBG [43] and BB [45] of the DIS flavor non-singlet world data, at NLO, NNLO, and N^{3}LO* with the response of the individual data sets, combined for the experiments BCDMS [4, 5, 56], NMC [12], and SLAC [57]; from [58].

Experiment	$\alpha_s(M_Z)$			
	NLO$_{exp}$	NLO	NNLO	N^{3}LO*
BCDMS	0.1111 ± 0.0018	0.1138 ± 0.0007	0.1126 ± 0.0007	0.1128 ± 0.0006
NMC	$0.117\ ^{+0.011}_{-0.016}$	0.1166 ± 0.0039	0.1153 ± 0.0039	0.1153 ± 0.0035
SLAC		0.1147 ± 0.0029	0.1158 ± 0.0033	0.1152 ± 0.0027
BBG		0.1148 ± 0.0019	0.1134 ± 0.0020	0.1141 ± 0.0021
BB		0.1147 ± 0.0021	0.1132 ± 0.0022	0.1137 ± 0.0022

3. $\alpha_s(M_Z^2)$ at NNLO: General Analyses

Combined non-singlet and singlet analyses at NNLO have been performed by different groups starting with the year 2000. In the following we will first compare the results of more recent analyses [58–61], in particular also w.r.t. their response to individual data sets and give a summary of the present NNLO results at the end of this section.

Table 3. Comparison of the $\alpha_s(M_Z)$ values obtained by BCDMS [5] and NMC [55] at NLO with the individual results of the fit in the present analysis at NLO and NNLO for the HERA data [62], the NMC data [12], the BCDMS data [5, 63], the SLAC data [64–69], and the DY data [70, 71]; from [58].

Experiment	$\alpha_s(M_Z)$		
	NLO$_{exp}$	NLO	NNLO
BCDMS	0.1111 ± 0.0018	0.1150 ± 0.0012	0.1084 ± 0.0013
NMC	$0.117\ ^{+0.011}_{-0.016}$	0.1182 ± 0.0007	0.1152 ± 0.0007
SLAC		0.1173 ± 0.0003	0.1138 ± 0.0003
HERA comb.		0.1174 ± 0.0003	0.1126 ± 0.0002
DY		0.108 ± 0.010	0.101 ± 0.025
ABM11 [58]		0.1180 ± 0.0012	0.1134 ± 0.0011

Table 4. Fit results of the World deep-inelastic data at NLO and NNLO using the published values of either $F_2^{\rm NMC}$ or $\sigma^{\rm NMC}$ with a correct description of the longitudinal structure function; from [72].

$\alpha_s(M_Z^2)$	with $\sigma_{\rm NMC}$	with $F_2^{\rm NMC}$	difference
NLO	0.1179(16)	0.1195(17)	$+0.0026 \simeq 1\sigma$
NNLO	0.1135(14)	0.1170(15)	$+0.0035 \simeq 2.3\sigma$
NNLO + $F_L O(\alpha_s^3)$	0.1122(14)	0.1171(14)	$+0.0050 \simeq 3.6\sigma$

An NNLO analysis has been performed on the World deep-inelastic data on proton and deuteron targets and Drell-Yan (DY) data in Ref. [58]. A summary of results is given in Table 3. The NLO analysis results in a higher value of $\alpha_s(M_Z^2) = 0.1180$. Values of this size are also obtained in other analyses, cf. [59], while the value in [61] is even higher. A shift of $\Delta\alpha_s(M_Z^2) = -0.0045$ is observed from NLO to NNLO,

Table 5. Comparison of the $\alpha_s(M_Z)$ values obtained by D0 in [75] with the ones based on including individual data sets of Tevatron jet data [76–79] into the analysis at NLO. The NNLO* fit refers to the NNLO analysis of the DIS and DY data together with the NLO and soft gluon resummation corrections (next-to-leading logarithmic accuracy) for the 1 jet inclusive data, cf. [80, 81]; from [58].

Experiment	$\alpha_s(M_Z)$		
	NLO_{exp}	NLO	NNLO*
D0 1 jet	$0.1161\ ^{+\ 0.0041}_{-\ 0.0048}$	0.1190 ± 0.0011	0.1149 ± 0.0012
D0 2 jet		0.1174 ± 0.0009	0.1145 ± 0.0009
CDF 1 jet (cone)		0.1181 ± 0.0009	0.1134 ± 0.0009
CDF 1 jet ($k_\perp$)		0.1181 ± 0.0010	0.1143 ± 0.0009
ABM11 [58]		0.1180 ± 0.0012	0.1134 ± 0.0011

Table 6. Comparison of the $\alpha_s(M_Z)$ values obtained by BCDMS [5], NMC [55], and D0 [75] at NLO with the results of NN21 [59, 60] for the fits to DIS and other hard scattering data at NLO and NNLO and the corresponding response of the different data sets analyzed; from [58].

Experiment	$\alpha_s(M_Z)$		
	NLO_{exp}	NLO	NNLO
BCDMS [5,63]	0.1111 ± 0.0018	0.1204 ± 0.0015	0.1158 ± 0.0015
NMC_p [12]		0.1192 ± 0.0018	0.1150 ± 0.0020
NMC_{pd} [89]	$0.117\ ^{+\ 0.011}_{-\ 0.016}$		0.1146 ± 0.0107
SLAC [57]		> 0.124	> 0.124
HERA I [62]		0.1223 ± 0.0018	0.1199 ± 0.0019
ZEUS H2 [90,91]		0.1170 ± 0.0027	0.1231 ± 0.0030
ZEUS F2C [92–95]		0.1144 ± 0.0060	
NuTeV [96,97]		0.1252 ± 0.0068	0.1177 ± 0.0039
E605 [70]		0.1168 ± 0.0100	
E866 [71,98,99]		0.1135 ± 0.0029	
CDF Wasy [100]		0.1181 ± 0.0060	
CDF Zrap [101]		0.1150 ± 0.0034	0.1205 ± 0.0081
D0 Zrap [102]		0.1227 ± 0.0067	
CDF R2KT [76]		0.1228 ± 0.0021	0.1225 ± 0.0021
D0 R2CON [78]	$0.1161\ ^{+\ 0.0041}_{-\ 0.0048}$	0.1141 ± 0.0031	0.1111 ± 0.0029
NN21 [59,60]		0.1191 ± 0.0006	0.1173 ± 0.0007

which is larger than in case of the non-singlet analyses, cf. Section 2. This shift is thoroughly observed for all data sets. Here the Drell-Yan data have a rather low sensitivity to α_s and serve basically to constrain the different sea-quark distributions. In the NLO fit the partial BCDMS value moves up, but takes a low value again at NNLO. The NLO NMC value comes out consistently. We would like to mention that at the time of the BCDMS and NMC experiments no proper description of the longitudinal structure function has been used. It was not available yet to $O(\alpha_s^{2(3)})$ [30, 40]. This information is, however, important and has been taken into account in the non-singlet analyses [43, 45] and discussed in detail in Ref. [72] for the combined analyses.

In case of fitting $F_2^{\rm NMC}$ and not describing $F_L(x, Q^2)$ at NNLO much larger values of $\alpha_s(M_Z^2)$ are obtained, cf. [73]. Therefore we regard it as mandatory to fit the published differential scattering cross sections using $F_L(x, Q^2)$ at $O(\alpha_s^3)$. Presently, the MMHT [74] analysis uses $F_L(x, Q^2)$ only at NLO. One should note, however, that, the values of $F_L(x, Q^2)$ at NLO and NNLO are significantly different in the small x region.

Another important issue concerns the description of higher twist contributions as discussed in the non-singlet analysis before. One has either to fit these terms within the combined analysis or to apply suitable cuts to remove part of the data being affected by these contributions. In Ref. [58] the cuts $W^2 > 12.5$ GeV^2 and $Q^2 > 2.5$ GeV^2, as used in [61], have been applied without fitting the higher twist terms. This resulted in a high value of $\alpha_s(M_Z^2) = 0.1191 \pm 0.0016$. Cutting more conservatively with $W^2 > 12.5$ GeV^2 and $Q^2 > 10$ GeV^2 led to $\alpha_s(M_Z^2) = 0.1134 \pm 0.0008$, a value well comparable to the one obtained in the ABM11 analysis [58]. We therefore recommend the latter cuts as mandatory. NNPDF [82] uses a cut of $Q^2 > 5$ GeV^2 removing part but not all of the higher twist contributions.

The ABM11 analysis considered also the effect of the Tevatron jet data on $\alpha_s(M_Z^2)$, fitting each of the individual data sets together with the deep-inelastic World data, cf. Table 5. At NLO the jet data perfectly reproduce the DIS value. As the NNLO jet cross section is not yet known, we use a NNLO* description here, referring to threshold resummation [80, 81]. We mention that the prescription deviates significantly at lower values of $p_\perp$ [83] and the approach did not account for the cone size dependence [83, 84]. Yet the inclusion of the data sets from Tevatron did not change the DIS values of $\alpha_s(M_Z^2)$ significantly at NNLO. They resulted in an enhancement to values in the range 0.1134...0.1149, differing with data set and kind of jet algorithm used. Therefore JR and ABM did not include jet data in subsequent NNLO analyses, cf. [85, 86], unlike MSTW [61], MMHT [74], NNPDF [60, 82], and CTEQ [87]. The combined analysis of the World DIS data and the jet data from hadron colliders at NNLO will only be possible if the NNLO calculation of the jet cross section is completed, cf. [88].

Because of the significant difference between $\alpha_s^{\rm NLO}(M_Z^2)$ and $\alpha_s^{\rm NNLO}(M_Z^2)$, it has to be regarded as impossible to include data sets in a NNLO fit, which can be only described at NLO as jet data in ep scattering and $p\overline{p}(p)$ scattering. Their inclusion usually leads to a larger value of $\alpha_s^{\rm NNLO}(M_Z^2)$ if compared to present pure NLO analyses. Again JR and ABM refrain from including these data, while MRST/MMHT include both types of jet data and NNPDF and CTEQ include hadron collider jet data in a significant portion in their fit, see Tables 6,7 and Ref. [87].

In the more 'global' fits also deep-inelastic data off nuclei are used, cf. Tables 6,7 and [87]. This is problematic, since it is unknown whether QCD evolution proceeds the same way within large nuclei, cf. e.g. [129]. CTEQ also includes CDHSW data, which were regarded to be not unproblematic, cf. [2]. We mention that e.g. the NuTeV νFe F_2 data request a high value of $\alpha_s(M_Z^2)$ [74].

Table 7. Comparison of the $\alpha_s(M_Z)$ values obtained by BCDMS [5], NMC [55], HERA-jet [118, 120] (see also [126, 127]), and D0 [75] at NLO with the results of the MSTW fits to DIS and other hard scattering data at NLO and NNLO and the corresponding response of the different data sets analysed, cf. Figs. 7a and 7b in [61]. Entries not given correspond to $\alpha_s(M_Z)$ central values below 0.110 or above 0.130; in case no errors are assigned these are larger than the bounds provided in form of the plots in [61, 128]; from [58].

Experiment	$\alpha_s(M_Z)$		
	NLO$_{exp}$	NLO	NNLO
BCDMS $\mu p, F_2$ [63]	0.1111 ± 0.0018	–	0.1085 ± 0.0095
BCDMS $\mu d, F_2$ [5]		0.1135 ± 0.0155	0.1117 ± 0.0093
NMC $\mu p, F_2$ [12]	$0.117\ ^{+\,0.011}_{-\,0.016}$	0.1275 ± 0.0105	0.1217 ± 0.0077
NMC $\mu d, F_2$ [12]		0.1265 ± 0.0115	0.1215 ± 0.0070
NMC $\mu n/\mu p$ [89]		0.1280	0.1160
E665 $\mu p, F_2$ [103]		0.1203	–
E665 $\mu d, F_2$ [103]		–	–
SLAC ep, F_2 [64, 104]		0.1180 ± 0.0060	0.1140 ± 0.0060
SLAC ed, F_2 [64, 104]		0.1270 ± 0.0090	0.1220 ± 0.0060
NMC,BCDMS,SLAC, F_L [12, 57, 63]		0.1285 ± 0.0115	0.1200 ± 0.0060
E886/NuSea pp, DY [99]		–	0.1132 ± 0.0088
E886/NuSea pd/pp, DY [71]		0.1173 ± 0.107	0.1140 ± 0.0110
NuTeV $\nu N, F_2$ [105]		0.1207 ± 0.0067	0.1170 ± 0.0060
CHORUS $\nu N, F_2$ [106]		0.1230 ± 0.0110	0.1150 ± 0.0090
NuTeV $\nu N, xF_3$ [105]		0.1270 ± 0.0090	0.1225 ± 0.0075
CHORUS $\nu N, xF_3$ [106]		0.1215 ± 0.0105	0.1185 ± 0.0075
CCFR [96, 97]		0.1190	–
NuTeV $\nu N \to \mu\mu X$ [96, 97]		0.1150 ± 0.0170	–
H1 ep 97-00, $\sigma_r^{\rm NC}$ [19, 20, 107, 108]		0.1250 ± 0.0070	0.1205 ± 0.0055
ZEUS ep 95-00, $\sigma_r^{\rm NC}$ [109–112]		0.1235 ± 0.0065	0.1210 ± 0.0060
H1 ep 99-00, $\sigma_r^{\rm CC}$ [20]		0.1285 ± 0.0225	0.1270 ± 0.0200
ZEUS ep 99-00, $\sigma_r^{\rm CC}$ [113]		0.1125 ± 0.0195	0.1165 ± 0.0095
H1/ZEUS $ep, F_2^{\rm charm}$ [92, 93, 114–117]		–	0.1165 ± 0.0095
H1 ep 99-00 incl. jets [118, 119]	$0.1168\ ^{+\,0.0049}_{-\,0.0035}$	0.1127 ± 0.0093	
ZEUS ep 96-00 incl. jets [120–122]	$0.1208\ ^{+\,0.0044}_{-\,0.0040}$	0.1175 ± 0.0055	
D0 II $p\bar{p}$ incl. jets [78]	$0.1161\ ^{+\,0.0041}_{-\,0.0048}$	0.1185 ± 0.0055	0.1133 ± 0.0063
CDF II $p\bar{p}$ incl. jets [76]		0.1205 ± 0.0045	0.1165 ± 0.0025
D0 II $W \to l\nu$ asym. [123]		–	–
CDF II $W \to l\nu$ asym. [124]		–	–
D0 II Z rap. [102]		0.1125 ± 0.0100	0.1136 ± 0.0084
CDF II Z rap. [125]		0.1160 ± 0.0070	0.1157 ± 0.0067
MSTW [61]		$0.1202\ ^{+\,0.0012}_{-\,0.0015}$	0.1171 ± 0.0014

The individual response in $\alpha_s(M_Z^2)$ for the data sets used in the fit of NNPDF and MRST are summarized in Tables 6,7. It is interesting to observe, that for

MRST the partial fit results to the Tevatron jet data are obtained in the range $\alpha_s(M_Z^2) = 0.1133...0.1157$ at NNLO similar to the ABM11 results, while NNPDF finds values of $0.1111, 0.1205$ and 0.1225.

In Table 8 we compare the pulls in $\alpha_s(M_Z^2)$ for the DIS and DY data sets for ABM11, BBG, NN21, and MSTW at NNLO. In case of the BCDMS data NN21 yields a higher value than obtained in the other analyses. In case of NMC the value obtained by MSTW is larger than in the other cases. For the SLAC data NNPDF obtains a large value, but MSTW obtains a lower and a larger value for *ep* and *ed* scattering, respectively. In case of the HERA data the ABM11 value is low, but those of MSTW and NNPDF are high and the DY values come out consistent between ABM11 and MSTW. Interestingly, the final values for NNPDF and MRST are very similar, but due to the above for very different reasons. They are higher than the values obtained by ABM11 and BBG by $\Delta\alpha_s(M_Z^2) = 0.0038$.

Table 8. Comparison of the pulls in $\alpha_s(M_Z)$ per data set between the ABM11 [58], BBG [43], NN21 [60] and MSTW [61] analyses at NNLO. The values in parantheses of ABM11 correspond to the case where the shape parameters are not refitted which is also the case for BBG; from [58].

Data Set	ABM11	BBG	NN21	MSTW
BCDMS	0.1128 ± 0.0020 (0.1084 ± 0.0013)	0.1126 ± 0.0007	0.1158 ± 0.0015	0.1101 ± 0.0094
NMC	0.1055 ± 0.0026 (0.1152 ± 0.0007)	0.1153 ± 0.0039	0.1150 ± 0.0020	0.1216 ± 0.0074
SLAC	0.1184 ± 0.0021 (0.1138 ± 0.0003)	0.1158 ± 0.0034	> 0.124	$\begin{cases} 0.1140 \pm 0.0060 \text{ ep} \\ 0.1220 \pm 0.0060 \text{ ed} \end{cases}$
HERA	0.1139 ± 0.0014 (0.1126 ± 0.0002)		$\begin{cases} 0.1199 \pm 0.0019 \\ 0.1231 \pm 0.0030 \end{cases}$	0.1208 ± 0.0058
DY	(0.101 ± 0.025)	–	–	0.1136 ± 0.0100
	0.1134 ± 0.0011	0.1134 ± 0.0020	0.1173 ± 0.0007	0.1171 ± 0.0014

Since the World deep inelastic data contain a large contribution due to charm it is important to determine the charm quark mass m_c correlated with $\alpha_s(M_Z^2)$ and the parameters of the parton distribution functions (PDFs), as well as the higher twist contributions in a NNLO analysis. Such analyses have been performed in [130] and [131] giving results in the $\overline{\text{MS}}$ scheme and the pole mass scheme, respectively. One may compare these values with those of the PDG [132] and precision determinations using e^+e^- data [133, 134]. The value obtained in [130] is in excellent agreement with the one given in [133, 134] and the quarkonium $1S$ state [135]. In Ref. [131] the determination has been performed in the pole mass scheme using the general mass variable flavor scheme, obtaining a very low value of $m_c^{\text{Pole}} = 1.25$ GeV with $\alpha_s(M_Z^2) = 0.1167$. The value for χ^2 turns out to be larger than that of Ref. [130], where the fixed flavor number scheme has been used. In other analyses by CTEQ [87] and NNPDF [82] pole mass values of 1.3 GeV and 1.275 GeV have been assumed as input, which are off the PDG-value.

Table 9. Comparison of the measurement of m_c in DIS, e^+e^- annihilation, and 1S quarkonium.

$m_c^{\overline{\mathrm{MS}}}$	$1.24 \pm 0.03\ ^{+0.03}_{-0.03}$	ABDLM, DIS, FFNS, $\chi^2 = 61/52$ [130]
$m_c^{\overline{\mathrm{MS}}}$	1.279 ± 0.013	e^+e^- [133]
$m_c^{\overline{\mathrm{MS}}}$	1.288 ± 0.020	e^+e^- [134]
$m_c^{\overline{\mathrm{MS}}}$	1.246 ± 0.023	quarkonium $1S$ [135]
$m_c^{\overline{\mathrm{MS}}}$	1.275 ± 0.025	PDG [132]
m_c^{Pole}	1.67 ± 0.07	PDG [132]
m_c^{Pole}	1.25	GMVFNS; $\chi^2 = 72/52$ [131]

The use of the general mass variable flavor number scheme (GMVFNS) is advocated by several fitting groups [74, 82, 87], despite it has been shown in [136] that even in the kinematic range at HERA one may work in the fixed flavor number scheme. An ideal interpolation up to $O(\alpha_s^2)$ is possible in the BMSN-scheme [137], as has been shown in [47]. This is illustrated in Figure 2. Changing from $N_F \to N_F + 1$ at the scale $\mu^2 = m^2_{N_F+1}$ is usually impossible, since it is normally near to the production threshold, where the heavy quark is not ultra-relativistic.

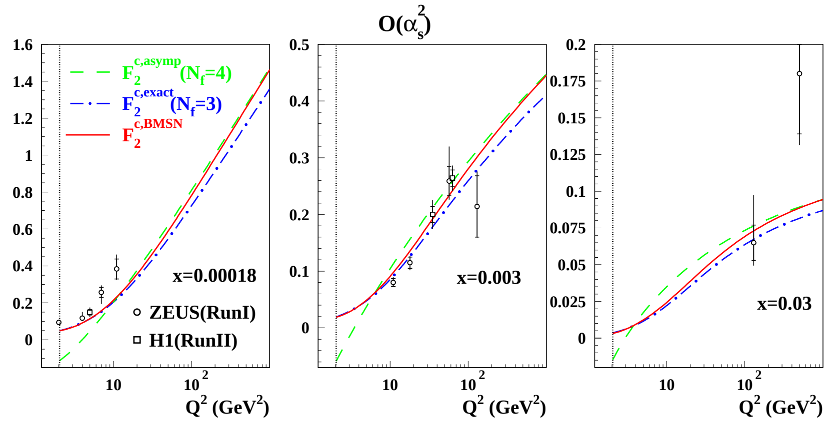

Fig. 2. Comparison of F_2^c in different schemes to H1- and ZEUS-data. Solid lines: GMVFN scheme in the BMSN prescription, dash-dotted lines: 3-flavor scheme, dashed lines: 4-flavor scheme. The vertical dotted line denotes the position of the charm-quark mass $m_c = 1.43$ GeV; from [47].

In some cases the transition of a massive quark to a massless quark at large values of Q^2 proceeds very slowly, cf. also [138]. This is illustrated in Figure 3 for the heavy quark contributions in case of the polarized Bjorken sum-rule to $O(\alpha_s^2)$, considering both the exact charm and the bottom quark contributions. Note that at low scales $\mu^2 = m_c^2$, the bottom contribution is even negative.

Let us now turn to the summary of the NNLO values for $\alpha_s(M_Z^2)$ having been determined since 2001, which we summarize in Table 10. An early determination is due J. Santiago and F. Yndurain [140], using moments of Bernstein polynomials.

Here already an error $\delta\alpha_s(M_Z^2)/\alpha_s(M_Z^2) \simeq 1\%$ has been obtained analyzing the F_2^{ep} data. Various analyses followed, mostly obtaining lower values of $\alpha_s(M_Z^2)$. We mention the MRST03 value with $\alpha_s(M_Z^2) = 0.1153 \pm 0.0020$, slightly lower than an earlier result of this group in [141]. Three values are higher with 0.1171...0.1174 by MSTW, NN21 and MMHT [60, 61, 74].

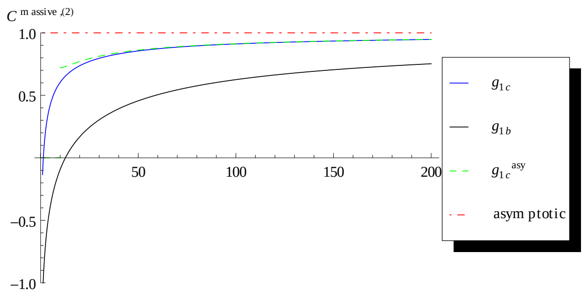

Fig. 3. Coefficient of the massive contributions to the polarized Bjorken sum rule as a function of $\xi = Q^2/m^2$ for the c- and b-contribution (full lines). Dashed line: asymptotic formula for charm. Dash-dotted line: $N_F \to N_F + 1$ transition for charm; from [139].

Table 10. $\alpha_s(M_Z^2)$ values at NNLO and N^{3}LO from QCD analyses of the deep-inelastic World data, partly including other hard scattering data.

NNLO Analyses			
		$\alpha_s(M_Z^2)$	
SY	2001	0.1166 ± 0.0013	F_2^{ep} [140]
SY	2001	0.1153 ± 0.0063	$xF_3^{\nu N}$ h. Nucl. [140]
A02	2002	0.1143 ± 0.0020	[22]
MRST03	2003	0.1153 ± 0.0020	[24]
BBG	2004(06,12)	$0.1134\,^{+0.0019}_{-0.0021}$	valence analysis, NNLO [27, 43, 45]
GRS	2006	0.112	valence analysis, NNLO [44]
AMP06	2006	0.1128 ± 0.0015	[143]
JR	2008	0.1128 ± 0.0010	dynamical approach [144]
JR	2008	0.1162 ± 0.0006	including NLO-jets [144]
ABKM	2009	0.1135 ± 0.0014	HQ: FFNS $N_f = 3$ [47]
ABKM	2009	0.1129 ± 0.0014	HQ: BMSN [47]
MSTW	2009	0.1171 ± 0.0014	[61]
Thorne	2013	0.1136	[DIS+DY+HT*] [142]
ABM11$_J$	2010	$0.1134 - 0.1149 \pm 0.0012$	Tevatron jets (NLO) incl. [145]
NN21	2011	$0.1174 \pm 0.0006 \pm 0.0001$	+h. Nucl. [59, 60]
ABM11	2013	0.1133 ± 0.0011	[58]
ABM11	2013	0.1132 ± 0.0011	(without jets) [58]
CTEQ	2013	0.1140	(without jets) [73]
JR	2014	0.1136 ± 0.0004	dyn. approach [85]
JR	2014	0.1162 ± 0.0006	standard fit [85]
CTEQ	2015	$0.1150\,^{+0.0060}_{-0.0040}$	$\Delta\chi^2 > 1$ +h. Nucl. [87]
MMHT	2015	0.1172 ± 0.0013	+h. Nucl. [74]
BBG	2006	$0.1141\,^{+0.0020}_{-0.0022}$	valence analysis, N^{3}LO

CTEQ finds a lower value of $\alpha_s(M_Z^2) = 0.1150$ quoting large errors of the size usually obtained by scale variation errors at NLO corresponding due to its $\Delta\chi^2$ assignment. On the other hand, MMHT quote much smaller errors corresponding to $\Delta\chi^2 = 7.2$, which are correspondingly smaller than those of ABM performing the analysis with $\Delta\chi^2 = 1$. We would like to highlight the difference in the central values in the ABKM analysis [47] choosing either the FFNS or the BMSN description implying a theoretical uncertainty of $\delta\alpha_s(M_Z^2) = 0.0006$. This is one of the typical theory errors which cannot be undercut easily in size. It compares to the difference of the NNLO and NNLO* values in the non-singlet analyses [43] of $\delta\alpha_s(M_Z^2) = 0.0007$. CTEQ has performed an analysis leaving out the jet data [73] and obtains $\alpha_s(M_Z^2) = 0.1140$. Likewise, R. Thorne (MRST) [142] has fitted the DIS and DY data only and restricting higher twist at low x leading to $\alpha_s^{\rm NLO}(M_Z^2) = 0.1179$ and $\alpha_s^{\rm NNLO}(M_Z^2) = 0.1136$, i.e. basically the same values as having been given in Table 3. Note that the later analysis by MMHT again reports a large value of $\alpha_s(M_Z) = 0.1172$ [74], despite the reasons for the difference to values of $\alpha_s(M_Z) \simeq 0.1136$ has been fully clarified in Ref. [58] already in 2012.

Table 11. Lower $\alpha_s(M_Z^2)$ values from other hard scattering processes in NNLO and NLO.

Other Lower α_s Values			
NNLO		$\alpha_s(M_Z^2)$	
Gehrmann et al.	2009	$0.1131\, {}^{+\,0.0028}_{-\,0.0022}$	e^+e^- thrust [146]
Abbate et al.	2010	0.1140 ± 0.0015	e^+e^- thrust [147]
Hoang et al.	2015	0.1123 ± 0.0015	C-param. dist. [148]
Alioli et al.	2015	0.1135	DY matched parton showers [149]
Bazavov et al.	2014	$0.1166\, {}^{+\,0.0012}_{-\,0.0008}$	lattice 2+1 fl. [150]
CMS	2013	$0.1151{}^{+\,0.0028}_{-\,0.0027}$	$t\bar{t}$ [151]
NLO		$\alpha_s(M_Z^2)$	
Frederix et al.	2010	$0.1156{}^{+\,0.0041}_{-\,0.0034}$	$e^+e^- \to 5$ jets [152]
H1	2009	$0.1160{}^{+\,0.0095}_{-\,0.0080}$	ep jets [127]
D0	2010	$0.1156{}^{+\,0.0041}_{-\,0.0034}$	$p\bar{p} \to$ jets [75]
ATLAS	2012	$0.1151{}^{+\,0.0093}_{-\,0.0087}$	jets [153]
CMS	2013	0.1148 ± 0.0052	3/2 jet ratio [154]

Also in a series of other hard processes a lower value of $\alpha_s(M_Z^2)$ has been measured at NNLO, and even at NLO, see Table 11. At NNLO values in the range 0.1123...0.1151 were found for thrust and the C-parameter in e^+e^- annihilation and the inclusive $t\bar{t}$ cross section by CMS. Furthermore there is a low lattice value of 0.1166, compared to other lattice determinations [155]. Further results on α_s are expected analysing the highly precise τ-data of BaBar and Belle as well as upcoming data from Belle-II. At NLO the lower values range as 0.1148...0.1160 and stem from jet measurements in e^+e^- annihilation and ep, pp and $p\bar{p}$-scattering. We clearly await the NNLO corrections for these processes for corresponding refined determinations of $\alpha_s(M_Z^2)$.

We finally would like to quote the estimated precision to measure $\alpha_s(M_Z^2)$ at LHeC [156]. For a proton beam energy of 7 TeV and an electron beam energy of 60 GeV at a luminosity of $\mathcal{L} = 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ using cuts of $Q^2 > 3.5$ GeV^2 the relative precision of $\alpha_s(M_Z^2)$ would be 0.15%, while for $Q^2 > 10$ GeV^2 0.25% would be obtained [157], quoting full experimental uncertainties. This is up to an order of magnitude better than the present accuracy. Future precise determinations of $\alpha_s(M_Z^2)$ in DIS are also expected from EIC [158].

4. Conclusions

At present NNLO analyses of the World deep-inelastic data can be supplemented only by the DY and $t\bar{t}$ data from hadron colliders as we still await the NNLO calculations for the *ep* and hadronic jet cross sections to be finished. Non-singlet analyses can be carried out at N^{3}LO yielding $\alpha_s(M_Z^2) = 0.1137 \pm 0.0022$. These analyses are free of the gluon uncertainty. The corresponding value at NNLO is $\alpha_s(M_Z^2) = 0.1132 \pm 0.0022$. Severe cuts to prevent higher twist contributions are important. Correct NNLO analyses should be based on proton and deuteron data only because of large nuclear effects for heavy nuclei, which currently cannot be controlled on the level of accuracy needed for $\alpha_s(M_Z)$ measurements of $O(1\%)$. Currently one still has to leave out jet data, since they would have to be dealt with at NLO only, leading to an upward shift of $\alpha_s(M_Z^2)$. The analyses shall deliver m_c together with α_s, reproducing the measured values of m_c determined in other high energy scattering processes. The BMSN-interpolation and the FFNS seem to be appropriate descriptions. A too early transition of charm to a massless quark in GMVFN-schemes would contradict the QCD description, since power correction terms are present in the important range at low Q^2. Assuming a fixed value of $\alpha_s(M_Z^2)$ within a PDF fit often leads to a biased determination of PDFs and other fitted parameters and the fit-result may be obtained off minimum since α_s is strongly correlated to various other fit parameters, notably to those of the gluon distribution. Various recent determinations yielded low values of $\alpha_s^{\mathrm{NNLO}}(M_Z^2)$.

NLO analyses yield values of $\alpha_s(M_Z^2)$ which are systematically larger than those at NNLO. Therefore the results of these analyses cannot be averaged. Analyses in which part of the scattering cross sections are still described at NLO cannot be regarded as NNLO analyses. We have shown that the reasons for the larger value of $\alpha_s(M_Z^2)$ obtained by MRST/MMHT [61, 74] are well understood, see also [142].

Many more QCD analyses at NNLO resulted in $\alpha_s(M_Z^2)$ lower than the present World average. This even applies to a number of NLO analyses. The next important step in the QCD analyses will be possible including the jet data from *ep* scattering and hadron collider data at NNLO. The precise knowledge of $\alpha_s(M_Z^2)$ is of instrumental importance for the understanding of various hard scattering cross sections at the LHC, notably that of Higgs-boson production [159]; for a detailed study see also [160].

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