Addressing the strong CP problem with quark mass ratios

J. L. Díaz-Cruz[a, 1],   W. G. Hollik[b, 2],   and   U. J. Saldaña-Salazar[a, 3]

a Facultad de Ciencias Físico-Matemáticas, Benemérita Universidad Autónoma de Puebla,
C.P. 72570, Puebla, Pue., México.
b Deutsches Elektronen-Synchrotron (DESY)
Notkestrasse 85, D-22607 Hamburg, Germany

Abstract

The strong CP problem is one of many puzzles in the theoretical description of elementary particles physics that still lacks an explanation. Solutions to that problem usually comprise new symmetries or fields or both. The main problem seems to be how to achieve small CP in the strong interactions despite large CP violation in weak interactions. Observation of CP violation is exclusively through the Higgs–Yukawa interactions. In this letter, we show that with minimal assumptions on the structure of mass (Yukawa) matrices the strong CP problem does not exist in the Standard Model and no extension to solve this is needed. However, to solve the flavor puzzle, models based on minimal SU(3) flavor groups leading to the proposed flavor matrices are favored.

[1] jldiaz@fcfm.buap.mx
[2] w.hollik@desy.de
[3] ulisesjesus@protonmail.ch

Introduction.—Problems are the nourishment of theoretical physics. Besides several other problems that still wait to be finally solved, the strong CP problem seems to be one of the simplest to be elucidated. The problem itself resides in the interplay of non-perturbative effects in Quantum Chromodynamics (QCD) and CP violation (CPV) in weak interactions (basically the Higgs–Yukawa sector in the Standard Model). Curiously, the majority of present day solutions to many of the problems in high energy physics obey the tendency of always going beyond the Standard Model (SM); the strong CP problem seems to share the same tendency. However, here we follow a different philosophy and carefully scrutinize the structure of the SM, offering an alternative solution to the strong CP problem. Our point of view might be defined as pragmatic as we only study the mass matrices along with the bi-unitary transformations diagonalizing them. Now, before moving to the details of our treatment let us first briefly discuss what the strong CP problem is. A comprehensive review of this problem can be found e. g. in [1] and similar references.

The θ parameter of QCD parametrizes the non-equivalence of possible QCD vacua as for non-abelian gauge fields there can be non-vanishing winding numbers defined as

$$n = \frac{g^2}{32\pi^2} \int \mathrm{d}^4 x F^a_{\mu\nu} \tilde{F}^{a\mu\nu}, \tag{1}$$

and therefore an effective action

$$S_{\text{eff}} = \int \mathrm{d}^4 x \mathcal{L} + i\theta n. \tag{2}$$

The axial anomaly introduces via

$$\partial^\mu j^5_\mu = \frac{g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu}, \tag{3}$$

effectively a change in S_{eff} by rotations of the quark fields with $\exp(i\gamma_5 \frac{\theta_q}{2})$ that shifts the gauge field θ parameter as

$$\theta \rightarrow \bar{\theta} = \theta - \theta_q. \tag{4}$$

The same transformation affects quark mass terms as $m\bar{q}_L q_R \rightarrow e^{i\theta_q} m\bar{q}_L q_R$ and may conversely be used to trace CP violating effects stemming from the masses. Due to this property, we may identify the physical remaining phase after such rephasing with the axial phase

transformation and be left with

$$\bar{\theta} = \theta + \arg\det\left(\boldsymbol{M}_{\mathrm{u}}\right) + \arg\det\left(\boldsymbol{M}_{\mathrm{d}}\right) = \theta + \arg\det\left(\boldsymbol{M}_{\mathrm{u}}\boldsymbol{M}_{\mathrm{d}}\right). \tag{5}$$

The parameter $\bar{\theta}$ violates CP and induces an electric dipole moment for the neutron, so bounds are roughly $\bar{\theta} < 10^{-10}$ [2]. Such a huge cancellation between those two contributions in Eq. (5) is to be seen as a fine-tuning problem as they are conceptually independent. The strong CP problem now manifests itself in the question why $\bar{\theta}$ is so small although CPV in weak interactions has been found to be rather large (large, of course, compared to $\bar{\theta}$, not on absolute grounds).

Interpreting θ as a Lagrangian parameter, it is the only parity violating term in the QCD Lagrangian (and because charge conjugation is conserved, the θ-term explicitly violates CP). Imposing global CP-invariance (though parity is enough), then $\theta = 0$ and the problem reduces to understand why $\arg\det\left(\boldsymbol{M}_{\mathrm{u}}\boldsymbol{M}_{\mathrm{d}}\right)$ is such a small number (or why it should exactly vanish).

Popular solutions to this problem are besides the possibility of having one massless quark (typically the u-quark but the same holds for a massless d-quark), the introduction of at least one new symmetry (like an axial U(1) or Peccei–Quinn [3] symmetry) that gets spontaneously (or softly [4, 5]) broken and comprises a light pseudo Nambu–Goldstone boson, the axion [6, 7].[1] A third way to compass the problem is via mechanisms worked out by Nelson [12] and Barr [13, 14], for a recent review see [15]. The principal requirement for this mechanism is a vanishing $\arg\det\left(\boldsymbol{M}_q\right)$ and a way to spontaneously break CP in the context of Grand Unified Theories, alternatively spontaneously [16] or softly broken parity [17] (or a combination of all of them [18, 19]). Another approach with spontaneous CPV is the one involving discrete flavor symmetries [20]. For last, generalized P-invariance in left-right symmetric theories can also provide valuable methods on computing approximately $\bar{\theta}$ through the corresponding right-handed quark mixing matrix [21–23].

In the course of this letter, we show that the vanishing $\arg\det\left(\boldsymbol{M}_q\right)$ is automatic in the SM imposing a minimal constraint on the relevant phases despite having sufficiently large CPV in flavor physics. The CPV in the weak interactions is unrelated to the relevant phases for $\arg\det\left(\boldsymbol{M}_q\right)$ and may only give a small finite contribution at higher orders as $\theta = 0$ at tree-level [24]. In the following, we give an explicit example of symmetry structures of mass

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[1]Axions and axion-like particles (ALPs) have a very rich phenomenology, summarized e. g. in [8], with an ongoing experimental effort to detect them (as the ALPS experiment at DESY [9, 10] and future facilities like ALPS-II or SHiP at CERN [11]).

matrices that have the desired property and start by finding the minimal assumptions for the mass matrices to fulfill that.

Disentangling weak and strong CPV.—A complete knowledge of the quark mass matrices, M_u and M_d, tackles down the flavor puzzle in the SM and finally gives the solution to the strong CP problem. Strong and weak CPV are of different origins, as we will show, and large CPV in the weak sector therefore does not necessarily imply strong CPV in spite of the fact that both reside in the same mass matrices

- weak CPV: the Jarlskog invariant $J_q \sim \mathrm{Im}\left[\det([M_u M_u^\dagger, M_d M_d^\dagger])\right]$, see [25],

- strong CPV: the θ_q-term from above $\theta_q \equiv -\arg\det\left(M_u M_d\right)$.

For "complete knowledge" of mass matrices, it is sufficient to set up each matrix in terms of known parameters, where a full theory of flavor is still lacking. By the freedom of relying on an effective description of the mass matrices, we encompass the strong CP problem by an ansatz to understand the flavor puzzle.

In the following, we will assume that all quark masses are different from zero, as suggested by lattice calculations [26]. Our minimal requirement for the mass matrices follows very obviously from the usual Singular Value Decomposition

$$M_q = L_q^\dagger \Sigma_q R_q, \tag{6}$$

with L_q and R_q unitary transformations and $\Sigma_q = \mathrm{diag}(m_{q_1}, m_{q_q}, m_{q_3})$ a positive diagonal matrix, $m_{q_i} > 0$. Consequently, we find

$$\arg\det\left(M_u M_d\right) = \arg\det\left(L_u^\dagger \Sigma_u R_u L_d^\dagger \Sigma_d R_d\right) = \arg\left(\det L_u^\dagger \det R_u \det L_d^\dagger \det R_d\right), \tag{7}$$

after using the well-known property of the determinant, $\det(AB) = \det A \det B$, and the args of the diagonal matrices vanish (as they are real and positive).

In the limit of vanishing mass matrices, the SM Lagrangian (the kinetic terms) obeys a $U(n)$ symmetry for each gauge multiplet (i.e. left-handed quark doublets and the up- and down-type right-handed singlets; in total $U(n)^3$) and n generations. The maximal freedom to rotate these fields is parametrized hence by $U(n)$ transformations U with $\det U = e^{i\phi}$, such that

$$\arg\det\left(M_u M_d\right) = \alpha_R^{(u)} - \alpha_L^{(u)} + \alpha_R^{(d)} - \alpha_L^{(d)}, \tag{8}$$

where $\arg\det\left(K_q\right) = \alpha_K^{(q)}$ and K either L or R. If the left and right unitary rotations L_q and R_q, respectively, were *special* unitary rotations, Eq. (8) would vanish trivially. [Recall that

unitary transformations are equal to special unitary transformations times a global phase, $U(n) = U(1) \otimes SU(n)$.] The global phases give rise to strong CPV and consequently θ_q is sensitive to these global phases only and insensitive to the complex structure of the underlying SU(3) transformations responsible for flavor mixing phenomena. Weak CPV on the other hand has exactly the opposite relation: it is sensitive to the complex nature of the special bi-unitary transformations and insensitive to global phases (a property exactly expressed by Jarlskog invariant which is known as a rephasing invariant). Even in the absence of a global phase that violates CP strongly, we can have arbitrarily large weak CPV: the existence or non-existence of strong CPV is completely unrelated to the existence or non-existence of weak CPV as can be seen from the two family case which has no weak CPV while showing in principle strong CPV. Note that the existence of these global U(1)-phases and the invariance of the SM field content under a particular combination of such rephasings is known as the (accidental) conservation of baryon number, which, however, is accidental.

The vanishing $\arg \det \left(\boldsymbol{M}_{\mathrm{u}} \boldsymbol{M}_{\mathrm{d}} \right)$ is contrary to the common folklore automatic without imposing arbitrary assumptions. Our approach to solve the strong CP problem reduces essentially to explain why $\alpha_L^{(q)} = \alpha_R^{(q)}$ while, simultaneously, explain the observed amount of weak CPV, $\delta_{\mathrm{CP}}^{\mathrm{CKM}} = (1.19 \pm 0.15)$ rad, see [27]. Before we move to an explicit realization of our findings, let us briefly summarize what we have so far: even if all quarks are massive, we have a solution to the strong CP problem without imposing any new symmetries. The requirement $\arg \det \left(\boldsymbol{M}_{\mathrm{u}} \boldsymbol{M}_{\mathrm{d}} \right) = 0$ can be achieved by ensuring $\alpha_L^{(q)}$ and $\alpha_R^{(q)}$ for $q = u, d$ to be zero or even simpler, by requiring them to be equal; the minimal way for the former scenario would be to propose SU(3) transformations for the diagonalization of the mass matrices which conversely means that any flavor model based on SU(3) gives a solution to the strong CP problem. Finally, we can still have (arbitrarily large) CPV in weak interactions as this is unrelated to strong CPV. The main task is somewhat to reduce the arbitrariness in complex phases that are generally allowed for the mass matrices and give a restrictive prescription for weak CPV.

Weak CPV and quark mass ratios.—Let us discuss the origin of weak CPV. We want to construct the desired solution in a minimalistic way without introducing new fields and later see whether the conditions we find are related to underlying symmetries (that can then be used to build a model of flavor). This minimalistic solution relates the entries of the Cabibbo–Kobayashi–Maskawa (CKM) matrix entirely to quark mass ratios and dictates the exact position of weak CPV in that parametrization [28].

A very famous expression of a mixing angle as a function of a mass ratio was provided by

the well-known $\tan\theta_C \approx \sqrt{m_d/m_s}$ for the Cabibbo angle θ_C [29]. In the following, we name this expression the Gatto–Sartori–Tonin (GST) relation although the original expression of Ref. [29] is quite different (we adopt to the usual language in the literature). Based on this finding, a parametrization of the fermion mixing matrices was proposed that only uses the mass ratios as input [28]. Besides the phenomenological observation $m_i \ll m_j$ for $i < j$ with masses of the i-th and j-th generation, a crucial assumption behind this parametrization is that the Euler rotations can be individually expressed by $\tan\theta_{ij} = \sqrt{m_i/m_j}$. Likewise, symmetrical structures in the mass matrices have been detected that lead to exactly this kind of mixing matrices [30]. In that view, the final quark mixing matrix can be decomposed into a chain of successive rotations where each planar SU(2) rotation can then be written as

$$U'_{ij}(\mu_{ij}, \delta_{ij}) = \begin{pmatrix} \frac{1}{\sqrt{1+\mu_{ij}}} & \sqrt{\frac{\mu_{ij}}{1+\mu_{ij}}}\, e^{-i\delta_{ij}} \\ -\sqrt{\frac{\mu_{ij}}{1+\mu_{ij}}}\, e^{i\delta_{ij}} & \frac{1}{\sqrt{1+\mu_{ij}}} \end{pmatrix}, \tag{9}$$

with $\mu_{ij} = m_i/m_j$ and an *a priori* arbitrary complex phase $\delta_{ij} \in [0, 2\pi)$. We identify $\sin\theta_{ij} = \sqrt{\frac{\mu_{ij}}{1+\mu_{ij}}}$ and $\cos\theta_{ij} = \frac{1}{\sqrt{1+\mu_{ij}}}$. Defining the CKM-matrix

$$V_{\mathrm{CKM}} = L_{\mathrm{u}} L_{\mathrm{d}}^{\dagger}, \tag{10}$$

with the U(3) transformations $L_{\mathrm{u,d}}$ defined via Eq. (6), we have in the formulation of [28] four mass ratios entering the game and six phases from which three can be removed by choosing the up-type mass matrix real.[2] From the remaining three, only *one maximally CP-violating phase* sitting in the 1-2 rotation is needed to fully reproduce the CKM-phase, details may be found in [28]. It was also pointed out in Ref. [31] that the same follows for certain 1-3 texture zero mass matrices. The approach of [28] is however more general as it does not rely on specific texture zeros but merely on symmetrical structures à la Ref. [30].

Using this mass ratios parametrization, we similarly compute the Kobayashi–Maskawa CP-phase in terms of mass ratios. In the standard parametrization, the most recent global fit obtains for it $\delta_{\mathrm{CP}}^{\mathrm{CKM}} = (1.19 \pm 0.15)$ rad [27]. In the following, we want to estimate the corresponding theoretical value. After imposing individual rotations of the type (9), we can finally build up a quark mixing matrix that has non-vanishing CPV (even though the two

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[2]This rephasing should not introduce a new strong CP-phase as we only shuffle complex entries from M_{u} to M_{d}. Moreover, any global phase does not play a role for weak CP-violation as the relevant objects are the left-hermitian products $M_{\mathrm{u,d}} M_{\mathrm{u,d}}^{\dagger}$.

family case lacks a viable source for CPV as the remaining phase can be absorbed in the complex spinors).

The procedure introduced in Ref. [28] gives a mixing matrix which cannot be directly compared to the conventional parametrization. In order to do that, we first need to rephase both the up and down type quark fields

$$\tilde{V}_{\mathrm{CKM}} = \boldsymbol{\chi}_{\mathrm{u}} V_{\mathrm{CKM}} \boldsymbol{\chi}_{\mathrm{d}}^{\dagger}, \tag{11}$$

in such a way that we are able to produce the following structure

$$\tilde{V}_{\mathrm{CKM}} \sim \begin{pmatrix} \mathrm{Re} & \mathrm{Re} & \mathcal{C} \\ \mathcal{C} & \mathcal{C} & \mathrm{Re} \\ \mathcal{C} & \mathcal{C} & \mathrm{Re} \end{pmatrix}, \tag{12}$$

where $\boldsymbol{\chi}_q = \mathrm{diag}(e^{i\phi_q}, 1, 1)$ and Re and $\mathcal{C}$ mean real and complex entries. After rephasing, we get the following expression for the Kobayashi–Maskawa CP-phase

$$\delta_{\mathrm{CP}}^{\mathrm{q}} \approx \arctan\left[\sqrt{\frac{\mu_{ds}(1+\mu_{ds})}{\mu_{uc}(1+\mu_{uc})}}\right] \approx (1.38 \pm 0.10)\,\mathrm{rad}, \tag{13}$$

which after insertion of the values of the quark mass ratios, $\mu_{ds} = 0.051 \pm 0.001$ and $\mu_{uc} = 0.0021 \pm 0.0001$, we find it to be in agreement to the experimental value.[3] Hence, we have proven that inside the SM the amount of weak CPV can be calculated by means of the quark mass ratios.

Strong CPV and flavored mass matrices.—Finally, we show how to avoid strong CPV and achieve $\theta_q = 0$. For that we need to extend the previous treatment to include global phases. We start by finding out the form of the mass matrices which are implied by only taking special unitary transformations.

The mass matrix which gets diagonalized by a transformation of the type (9) can be obtained very easily in the two family case and can be generalized to $n > 2$ generations according to [30]. In Ref. [30], mass matrices are constructed in such a way to allow the sequential diagonalization of [28] without preference to any of the families. The basic assumption behind this approach is that Higgs–Yukawa interactions (or conversely mass matrices if one does not specify the mass generating mechanism directly) are symmetric

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[3]The quark masses have been treated as running $\overline{\mathrm{MS}}$ masses evaluated at the weak scale ($Q^2 = M_Z^2$), numbers are taken from App. A in Ref. [28].

under permutations of the fermion fields. This permutation symmetry is then supposed to be broken stepwise as $S_{3L} \otimes S_{3R} \to S_{2L} \otimes S_{2R} \to S_{2A} \oplus S_{2S}$, where the last step proceeds to a sum of anti-symmetric and symmetric permutation matrices of two objects.

We exemplarily study the two-family case where the mass matrix originated in the sequential breakdown of permutation symmetries [30] is given in a preferred basis as

$$M = \begin{pmatrix} 0 & \sqrt{m_1 m_2} e^{-i\delta_m} \\ -\sqrt{m_1 m_2} e^{i\delta_m} & m_2 - m_1 \end{pmatrix}. \tag{14}$$

Now, we want to map this structure resulting in a GST relation to the most general case of a 2×2 mass matrix. The GST relation gives Eq. (9) as the corresponding unitary transformation. A general U(2) matrix has two more parameters that can be expressed as an additional phase on the diagonal and an overall phase factor,

$$U = e^{i\phi/2} \begin{pmatrix} \cos\theta e^{i\eta} & \sin\theta e^{-i\delta} \\ -\sin\theta e^{i\delta} & \cos\theta e^{-i\eta} \end{pmatrix}, \tag{15}$$

such that $\det U = e^{i\phi}$. According to Eq. (15), the relevant left rotation of a generic mass matrix should also have the form

$$L = e^{i\alpha_L/2} \begin{pmatrix} \cos\theta_L e^{i\beta_L} & \sin\theta_L e^{-i\delta_L} \\ -\sin\theta_L e^{i\delta_L} & \cos\theta_L e^{-i\beta_L} \end{pmatrix}. \tag{16}$$

The same expression follows for the right transformation R with $L \leftrightarrow R$ in Eq. (16) and the individual entries of the mass matrices can be expressed via

$$M = e^{i\frac{\alpha_R - \alpha_L}{2}} \begin{pmatrix} \cos\theta_L e^{-i\beta_L} & -\sin\theta_L e^{-i\delta_L} \\ \sin\theta_L e^{i\delta_L} & \cos\theta_L e^{i\beta_L} \end{pmatrix} \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \cos\theta_R e^{i\beta_R} & \sin\theta_R e^{-i\delta_R} \\ -\sin\theta_R e^{i\delta_R} & \cos\theta_R e^{-i\beta_R} \end{pmatrix}, \tag{17}$$

and thus

$$
\begin{aligned}
M_{11} &= e^{i\frac{\alpha_R - \alpha_L}{2}} \left[e^{-i(\beta_L - \beta_R)} m_1 \cos\theta_L \cos\theta_R + e^{-i(\delta_L - \delta_R)} m_2 \sin\theta_L \sin\theta_R \right], \\
M_{12} &= e^{i\frac{\alpha_R - \alpha_L}{2}} \left[e^{-i(\beta_L + \delta_R)} m_1 \cos\theta_L \sin\theta_R - e^{-i(\beta_R + \delta_L)} m_2 \cos\theta_R \sin\theta_L \right], \\
M_{21} &= e^{i\frac{\alpha_R - \alpha_L}{2}} \left[e^{i(\beta_R + \delta_L)} m_1 \cos\theta_R \sin\theta_L - e^{i(\beta_L + \delta_R)} m_2 \cos\theta_L \sin\theta_R \right], \\
M_{22} &= e^{i\frac{\alpha_R - \alpha_L}{2}} \left[e^{i(\beta_L - \beta_R)} m_2 \cos\theta_L \cos\theta_R + e^{i(\delta_L - \delta_R)} m_1 \sin\theta_L \sin\theta_R \right].
\end{aligned}
\tag{18}
$$

Matching this set of equations to the matrix form of Eq. (14) reduces the freedom of the

U(2) rotations as the structure is dictated by the simple symmetry patterns. We find the conditions

$$\alpha_L = \alpha_R, \qquad \beta_L - \beta_R = \delta_L - \delta_R = 0, \qquad \theta_R = -\theta_L. \qquad (19)$$

The first condition on the global phase is our essential solution to the strong CP problem and the reason why Eq. (8) vanishes. (Of course, this is trivial as Eq. (14) exhibits no global phase.) The second condition gives the off-diagonal phase in Eq. (14) as $\delta_m = \delta_{L(R)} + \beta_{L(R)}$. And the third condition plainly says that left and right unitary transformations rotate in opposite directions. Actually, there is no need for prohibiting a global phase for the mass matrix as the left rotations L (that are phenomenologically important) are defined via the object $MM^\dagger$. Still, the phases of L and M would be unrelated.

The constraints of Eqs. (19) provide very valuable information especially on the *right-handed* rotations coded in R resulting in the clear prediction that mixing angles of the right-handed sector have exactly the same magnitude as the known left-handed (i. e. CKM) ones. The future detection of right-handed currents may be a razor to finally rule the proposed description out, although we do not predict right-handed currents at all! As side remark, let us note that recent investigations on minimal left-right symmetric models hint toward the same conclusion of $V_{\mathrm{CKM}}^R = V_{\mathrm{CKM}}^L$ [22, 23]. Surprisingly, we do no get exact equality but rather find for the right-handed sector the angles $\theta_{12}^{\mathrm{CKM},R} = \theta_{12}^{\mathrm{CKM},L}$, $\theta_{23}^{\mathrm{CKM},R} = \theta_{23}^{\mathrm{CKM},L}$, and $\theta_{13}^{\mathrm{CKM},R} \approx \theta_{13}^{\mathrm{CKM},L}/10$, which results from the intricate structure of V_{CKM} in Ref. [28].

Conclusions.—We have addressed the strong CP problem within the SM by fully modeling the quark mixing matrix in terms of mass ratios as recently proposed [28]. The assessment of the strong CP problem basically consists in explaining why the amount of strong CPV stemming from the quark masses, here denoted as $\theta_q = -\arg\det(M_\mathrm{u} M_\mathrm{d})$, should be zero, while simultaneously a large value (compared to θ_q) of weak CPV appears, which is coded in the Jarlskog invariant J_q of the experimentally measured (fitted) CKM-matrix. In this letter, we have showed that this problem is, on general grounds, not a problem inside the SM. Without the need of introducing new fields and/or symmetries, we have studied the general properties of the source of strong CPV, θ_q. The complex phases implied by θ_q are entirely unrelated to the phases of weak CPV. Splitting the generational freedom of the gauge kinetic terms as $U(3) = U(1) \otimes SU(3)$, it can be clearly seen that arbitrary U(1) factors lead to $\theta_q \neq 0$ while the SU(3) nature is responsible for $J_q \neq 0$. The specific pattern of Yukawa matrices needed to build the proper hierarchies for the fermion masses as well as the correct mixing matrix may be produced by spontaneous breakdown of the maximal flavor group as

proposed in Ref. [32]. For the weak CPV we showed that in the recently proposed fermion mass ratios parametrization [28] the leading contribution to the CKM-phase is given by

$$\delta_{\text{CP}}^{\text{q}} \approx \arctan\left[\sqrt{\frac{\mu_{ds}(1+\mu_{ds})}{\mu_{uc}(1+\mu_{uc})}}\right] \approx (1.38 \pm 0.10) \,\text{rad}, \tag{20}$$

which is in agreement to the observed value $\delta_{\text{CP}}^{\text{CKM}} = (1.19 \pm 0.15)$ rad. The absence of the strong CPV is guaranteed by imposing a very minimal condition on the mass matrices such that $\theta_q = \sum_{q=u,d} \alpha_R^{(q)} - \alpha_L^{(q)} = 0$ if $\alpha_L^{(q)} = \alpha_R^{(q)}$, though the basic constraint is much weaker. (This gets important in the context of Grand Unification when up- and down-quark mass matrices are related to each other.) It has been shown that minimal symmetrical requirements on the Higgs–Yukawa interactions according to [30] lead to the given constraint and non-trivial CKM-mixing. As consequence of this, the mixing of the right-handed sector is fixed and predicts for the right-handed CKM-matrix $\theta_{12}^{\text{CKM},R} = \theta_{12}^{\text{CKM},L}$, $\theta_{23}^{\text{CKM},R} = \theta_{23}^{\text{CKM},L}$, and $\theta_{13}^{\text{CKM},R} \approx \theta_{13}^{\text{CKM},L}/10$. This fingerprint can be tested in future experiments within a variety of extensions of the Standard Model.

Acknowledgments

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