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Heavy quark symmetry and weak decays of the b-baryons in pentaquarks with a $c\bar{c}$ component

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Abstract

The discovery of the baryonic states $P_c^+(4380)$ and $P_c^+(4450)$ by the LHCb collaboration in the process $pp \to b\bar{b} \to \Lambda_b^0 X$, followed by the decay $\Lambda_b^0 \to J/\psi\, p\, K^-$ has evoked a lot of theoretical interest. These states have the minimal quark content $c\bar{c}uud$, as suggested by their discovery mode $J/\psi\ p$, and the preferred J^P assignments are $\frac{5}{2}^+$ for the $P_c^+(4450)$ and $\frac{3}{2}^-$ for the $P_c^+(4380)$. In the compact pentaquark hypothesis, in which they are interpreted as hidden charm diquark-diquark-antiquark baryons, the assigned spin and angular momentum quantum numbers are $P_c^+(4380) = \{\bar{c}[cu]_{s=1}[ud]_{s=1}; L_{\mathcal{P}} = 0, J^{\rm P} = \frac{3}{2}^-\}$ and $P_c^+(4450) = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 1, J^{\rm P} = \frac{5}{2}^+\}$. The subscripts denote the spin of the diquarks and $L_{\mathcal{P}} = 0, 1$ are the orbital angular momentum quantum numbers of the pentaquarks. We point out that in the heavy quark limit, the spin of the light diquark in heavy baryons becomes a good quantum number, which has consequences for the decay $\Lambda_b^0 \to J/\psi\, p\, K^-$. With the quantum numbers assigned above for the two pentaquarks, this would allow only the higher mass pentaquark state $P_c^+(4450)$ having $[ud]_{s=0}$ to be produced in Λ_b^0 decays, whereas the lower mass state $P_c^+(4380)$ having $[ud]_{s=1}$ is disfavored, requiring a different interpretation. Pentaquark spectrum is rich enough to accommodate a $J^P = \frac{3}{2}^-$ state, which has the correct light diquark spin $\{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 0, J^{\rm P} = \frac{3}{2}^-\}$ to be produced in Λ_b^0 decays. Assuming that the mass difference between the charmed pentaquarks which differ in the orbital angular momentum L by one unit is similar to the corresponding mass difference in the charmed baryons, $m[\Lambda_c^+(2625); J^P = \frac{3}{2}^-] - m[\Lambda_c^+(2286); J^P = \frac{1}{2}^+] \simeq 341$ MeV, we estimate the mass of the lower pentaquark $J^P = 3/2^-$ state to be about 4110 MeV and suggest to reanalyze the LHCb data to search for this third state. Extending these considerations to the pentaquark states having a $c\bar{c}$ pair and three light quarks (u, d, s) in their Fock space, we present the spectroscopy of the S- and P-wave states in an effective Hamiltonian approach. Some of these pentaquarks can be produced in weak decays of the b-baryons. Combining heavy quark symmetry and the $SU(3)_F$ symmetry results in strikingly simple relations among the decay amplitudes which are presented here.

I. INTRODUCTION

The discovery of the charmonium-like resonance $X(3872)$ in B-meson decays $B \to X(3872)\ K$, followed by the decay $X(3872) \to J/\psi\pi^+\pi^-$, reported by the Belle collaboration in 2003 [1], subsequently confirmed by the D0 [2], CDF [3] and Babar Collaborations [4], has proved to be the harbinger of a new quarkonium-like spectroscopy. Since then, well over two dozen such hidden $c\bar{c}$ states, both neutral and charged, have been reported. Very recently, observation of four structures in the $J/\psi\phi$ mass spectrum in the decays $B^+ \to J/\psi\ \phi\ K^+$ have been reported by LHCb, yielding two $J^P = 1^+$ states, $X(4140)$ and $X(4274)$, and two $J^P = 0^+$ states $X(4500)$ and $X(4700)$ [5]. So far, three states $Y_b(10890)$ [6], $Z_b^\pm(10610)$ and $Z_b^\pm(10650)$ [7] have also been discovered having a $b\bar{b}$ pair in their valence compositions. All these hadrons are distinct by the presence of a $c\bar{c}$ (or a $b\bar{b}$) quark pair in addition to light degrees of freedom (a light $q\bar{q}$ pair or gluons) in their Fock space. Collectively called the X, Y, Z states, they have prompted a lot of theoretical interest in their interpretations [8] - [20]. In 2015, LHCb reported the first observation of two hidden charm pentaquark states $P_c^+(4380)$ and $P_c^+(4450)$ in the decay $\Lambda_b^0 \to J/\psi\ p\ K^-$ [21], having the masses $4380 \pm 8 \pm 29$ MeV and $4449.8 \pm 1.7 \pm 2.5$ MeV, and widths $205 \pm 18 \pm 86$ MeV and $39 \pm 5 \pm 19$ MeV, with the preferred spin-parity assignments $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^+$, respectively. These states have the quark composition $c\bar{c}uud$, and like their tetraquark counterparts X, Y, Z, they lie close in mass to several (charm meson-baryon) thresholds. This has led to a number of theoretical proposals for their interpretation, which include rescattering-induced kinematical effects [22], open charm-baryon and charm-meson bound states [23], and baryocharmonia [24]. They have also been interpreted as compact pentaquark hadrons with the internal structure organized as diquark-diquark-anti-charm quark [25, 26] or as diquark-triquark [27, 28].

In this work we follow the compact pentaquark interpretation. The basic idea is that highly correlated diquarks play a key role in the physics of multiquark states [8, 29, 30]. Since quarks transform as a triplet $\mathsf{3}$ of color $SU(3)$, the diquarks resulting from the direct product $\mathsf{3} \otimes \mathsf{3} = \bar{\mathsf{3}} \oplus \mathsf{6}$, are thus either a color anti-triplet $\bar{\mathsf{3}}$ or a color sextet $\mathsf{6}$. Of these only the color $\bar{\mathsf{3}}$ configuration is kept, as suggested by perturbative arguments. Both spin-1 and spin-0 diquarks are, however, allowed. In the case of a diquark $[qq']$ consisting of two light quarks, the spin-0 diquarks are believed to be more tightly bound than the spin-1, and this hyperfine splitting has implications for the spectroscopy. For the heavy-light diquarks, such as $[cq]$ or $[bq]$, this splitting is suppressed by $1/m_c$ for a $[cq]$ or by $1/m_b$ for a $[bq]$ diquark, and hence both spin-configurations are treated at par. For the pentaquarks, the mass spectrum will depend upon how the five quarks, i.e., the 4 quarks and an antiquark, are dynamically structured. A diquark-triquark picture, in which the two observed pentaquarks consist of a rapidly separating pair of a color-$\bar{\mathsf{3}}$ $[cu]$ diquark and a color-$\mathsf{3}$ triquark $\bar{\theta} = \bar{c}[ud]$, has been presented in [27]. A "Cornell"-type non-relativistic linear-plus-coulomb potential [31] is used to determine the diquark-triquark separation R and the ensuing phenomenology is worked out.

We prefer to keep the basic building blocks of the pentaquarks to be quarks and diquarks, and follow here the template in which the 5q baryons, such as the two P_c states, are assumed to be four quarks, consisting of two highly correlated diquark pairs, and an antiquark. For the present discussion, it is an anti-charm quark $\bar{c}$ which is correlated with the two diquarks $[cq]$ and $[q'q'']$, where q, q', q'' can be u or d. The tetraquark formed by the diquark-diquark $([cq]_{\bar{\mathsf{3}}}[q'q'']_{\bar{\mathsf{3}}})$ is a color-triplet object, following from $\bar{\mathsf{3}} \times \bar{\mathsf{3}} = \bar{\mathsf{6}} + \mathsf{3}$, with orbital and spin quantum numbers, denoted by $L_{\mathcal{Q}\mathcal{Q}}$ and $S_{\mathcal{Q}\mathcal{Q}}$, which combines with the color-anti-triplet $\bar{\mathsf{3}}$ of the $\bar{c}$ to form an overall color-singlet pentaquark, with the corresponding quantum numbers $L_{\mathcal{P}}$ and $S_{\mathcal{P}}$. This is shown schematically in Fig. 1.

An effective Hamiltonian based on this picture is constructed, extending the underlying tetraquark Hamiltonian developed for the X, Y, Z states [8]. We explain how the various input parameters in this Hamiltonian are determined. Subsequently, we work out the mass spectrum of the low-lying S- and P-wave pentaquark states, with a $c\bar{c}$ and three

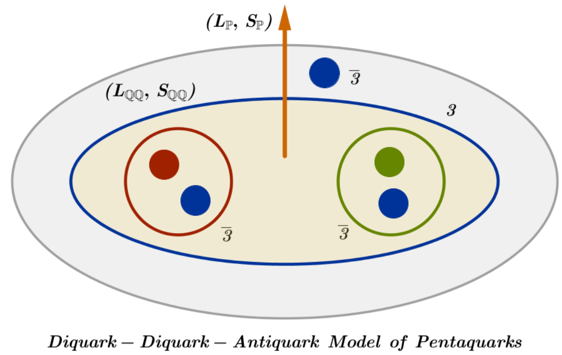

FIG. 1: $SU(3)$-color quantum numbers of the diquarks, tetraquark and antiquark are indicated, together with the orbital and spin quantum numbers of the tetraquark and pentaquark [32]

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light quarks (u, d, s) in their Fock space.

The pentaquark states reported by the LHCb are produced in Λ_b^0 decays, $\Lambda_b^0 \to \mathcal{P}^+ \; K^-$, where $\mathcal{P}$ denotes a generic pentaquark state, a symbol we use subsequently in this work. We take a closer look at the dynamics of Λ_b decays. In particular, we point out that QCD has a symmetry in the heavy quark limit, i.e., for $m_b \gg \Lambda_{\rm QCD}$, b-quark becomes a static quark and the light diquark spin becomes a good quantum number, constraining the states which can otherwise be produced in b-baryon decays. The consequences of heavy quark symmetry are well known, starting from the early uses in the decays of the heavy mesons (B, B^* etc.) [33, 34], for the heavy meson spectroscopy [35], and in heavy baryon decays [36–38]. The extent to which heavy quark symmetry holds can be judged from the data on the semileptonic decays $\Lambda_b^0 \to \Lambda_c^+ \ell^- \bar{\nu}_\ell$, which is a $j^P = 0^+ \to j^P = 0^+$ transition, for which a branching ratio $\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \ell^- \bar{\nu}_\ell) = (6.2^{+1.4}_{-1.2})\%$ is listed in the PDG [39]. The corresponding decay $\Lambda_b^0 \to \Sigma_c^+ \ell^- \bar{\nu}_\ell$, involving a $j^P = 0^+ \to j^P = 1^+$ transition, is non-existent. The decays $\Lambda_b^0 \to \Sigma(2455)^0 \pi^+ \ell^- \bar{\nu}_\ell$ and $\Lambda_b^0 \to \Sigma(2455)^{++} \pi^- \ell^- \bar{\nu}_\ell$, facilitating an $0^+ \to 1^+$ transition, are highly suppressed, $(1/2\Gamma(\Lambda_b^0 \to \Sigma(2455)^0 \pi^+ \ell^- \bar{\nu}_\ell) + 1/2\Gamma(\Lambda_b^0 \to \Sigma(2455)^{++} \pi^- \ell^- \bar{\nu}_\ell)/\Gamma(\Lambda_b^0 \to \Lambda_c^+ \ell^- \bar{\nu}_\ell) = 0.054 \pm 0.022^{+0.021}_{-0.018}$ [39]. For the non-leptonic decays, one finds, for example, $\mathcal{B}(\Lambda_b^0 \to \Sigma_c^0(2455)\pi^+\pi^-)/\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \pi^-) \simeq 0.1$, indicating an order of magnitude suppression of the $j^P = 0^+ \to j^P = 1^+$ transition. Whether the heavy quark symmetry holds in b-baryon decays to pentaquarks is, of course, a dynamical question and we currently lack data to test it, but it is worthwhile to work out its implications for the interpretation of the LHCb data and the pentaquark phenomenology, in general.

In the pioneering work by Maiani *et al.* [25] on the pentaquark interpretation of the LHCb data on $\Lambda_b^0 \to J/\psi \; p \; K^-$ decay, heavy quark symmetry is not invoked. The assigned internal quantum numbers are: $P_c^+(4450) = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 1, J^{\rm P} = \frac{5}{2}^+\}$ and $P_c^+(4380) = \{\bar{c}[cu]_{s=1}[ud]_{s=1}; L_{\mathcal{P}} = 0, J^{\rm P} = \frac{3}{2}^-\}$. Taking into account the mass differences due to the orbital angular momentum and the light diquark spins, the observed mass difference between the two P_c^+ states of about 70 MeV is reproduced. The crucial assumption is that the two diagrams for the decay $\Lambda_b^0 \to J/\psi \; p \; K^-$ in Fig. 1 in [25], in which the ud-spin in Λ_b^0 goes over to the $[ud]$-diquark spin in the pentaquark, Fig. 1(A), and the one in which the ud-spin is shared among the final state pentaquark and a meson, generating a light diquark $[ud]$ having spin-0 and spin-1, Fig. 1(B), are treated at par. This, as we discussed in the previous paragraph, is at variance with the data on $b \to c$ baryonic decays, and also with the heavy quark symmetry.

We argue here that the b-baryon decays to pentaquarks having a $c\bar{c}$ component are also subject to the selection rules following from the heavy quark symmetry. In particular, this implies a dynamical suppression of Fig. 1(B) in [25].

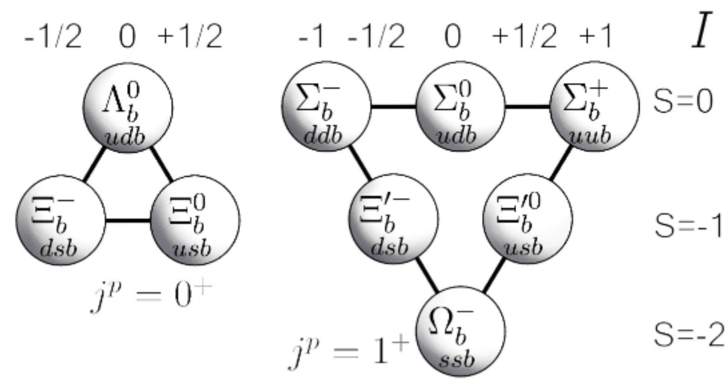

FIG. 2: The $SU(3)_F$ flavor multiplets of the ground-state bottom baryons. The $J^P = \frac{1}{2}^+$ triplet (with the light-quark spin $j^P = 0^+$) is shown on the left, and the $J^P = \frac{1}{2}^+$ sextet (with the light-quark spin $j^p = 1^+$) is shown on the right. The isospin I and strangeness S of each b-baryon state are specified (taken from [40]).

With this additional symmetry as a diagnostic tool, we analyze the two observed pentaquark states in Λ_b^0 decays and find that only the higher mass sate $P_c^+(4450) = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 1, J^{\rm P} = \frac{5}{2}^+\}$ is allowed to be produced in Λ_b^0 decays, but the lower-mass state $P_c^+(4380) = \{\bar{c}[cu]_{s=1}[ud]_{s=1}; L_{\mathcal{P}} = 0, J^{\rm P} = \frac{3}{2}^-\}$, in which the spin-0 (ud) diquark in Λ_b^0 is broken, is disfavored in this limit. Interestingly, the fractions of the total sample in the decay $\Lambda_b^0 \to J/\psi\, p\, K^-$ due to $P_c^+(4380)$ and $P_c^+(4450)$ are reported by the LHCb [21] as $(8.4 \pm 0.7 \pm 4.2)\%$ and $(4.1 \pm 0.75 \pm 1.1)\%$, respectively. Thus, another theoretical interpretation may be required to accommodate the state $P_c^+(4380)$ in the LHCb data.

Spectrum of the multiquark states is, however, rich, as also presented here for the S- and P-wave pentaquark states having a hidden $c\bar{c}$ pair and three light $(u,\, d,\, s)$ quarks. We find that there indeed is a lower-mass $J^P = \frac{3}{2}^-$ pentaquark state with the quantum numbers $\{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 0, J^{\rm P} = \frac{3}{2}^-\}$ present in the spectrum, which has the correct light diquark spin to be produced in the decay $\Lambda_b^0 \to J/\psi\, p\, K^-$, compatible with the heavy quark symmetry. Assuming that the mass difference in the charmed pentaquarks differing by the orbital angular momentum $\Delta L = 1$ is similar to the corresponding mass difference in the charmed baryons, $m[\Lambda_c^+(2625); J^P = \frac{3}{2}^-] - m[\Lambda_c^+(2286); J^P = \frac{1}{2}^+] \simeq 341$ MeV, we predict the mass of the lower pentaquark state to be about 4110 MeV. Estimates based on the parameters of the $c\bar{c}$ tetraquarks in an effective Hamiltonian approach, which we detail in this paper, yield a nominally larger but compatible value for the mass of this state, around 4130 MeV. We suggest to search for the lower mass $P_c^+(J^P = \frac{3}{2}^-)$ state decaying into $J/\psi\, p$ in the LHCb data on $\Lambda_b^0 \to J/\psi\, pK^-$. A renewed fit of the LHCb data by allowing a third resonance is called for.

In addition to the $\Lambda_b^0 = (udb)$, which is the lightest of the b-baryons in which the light quark pair ub has $j^P = 0^+$, there are two others in this $SU(3)_F$ triplet with strangeness $S = -1$, $\Xi_b^0(5792) = (usb)$, having isospin $I = I_3 = 1/2$ and $\Xi_b^-(5794) = (dsb)$, having isospin $I = -I_3 = 1/2$. Likewise, there are six b-baryons with the light quark pair having $j^P = 1^+$, with $S = 0$ ($\Sigma_b^- = (ddb)$, $\Sigma_b^0 = (udb)$, $\Sigma_b^+ = (uub)$), $S = -1$ ($\Xi_b' = (dsb)$, $\Xi_b'^0 = (usb)$), and one with $S = -2$ ($\Omega_b^- = (ssb)$.) These bottom baryon multiplets are shown in Fig. 2.

Weak decays of some of these b-baryons are expected to produce pentaquark states with a hidden $c\bar{c}$ pair and three light quarks, which form baryons present in the octet and decuplet representations of $SU(3)_F$. The observed pentaquarks $P_c^+(4450)$ and the one being proposed here $P_c^+(4110)$ belong to the $SU(3)_F$ octet. Examples of the bottom-strange b-baryon respecting $\Delta I = 0$, $\Delta S = -1$ are [25]:

$$\Omega_b^- \to K^-(J/\psi\, \Xi^0),\ K^0(J/\psi\, \Xi^-),$$

which correspond to the formation of the pentaquarks in the $SU(3)_F$ decuplet representation with the spin configuration $\mathcal{P}_{10}(\bar{c}\,[cu]_{s=0,1}\,[ss]_{s=1})$. Thus, while calculating the decay amplitudes of these transitions is a formidable challenge, $SU(3)_F$ symmetry can be used to relate various decays. This was already pointed out by Maiani *et al.* [25], and these relations have been worked out in great detail subsequently [41, 42]. They provide a very useful guide for the future pentaquark searches. We point out that imposing the heavy quark symmetry brings clarity in this analysis, reducing the number of unknown matrix elements and providing a better understanding of why some topological diagrams are disfavored. In addition, as already pointed out, heavy quark symmetry reduces the number of pentaquark states which can be reached in b-baryon decays. We revisit the decays of the b-baryons to pentaquarks and pseudoscalar mesons, which were considered in [41, 42] without this symmetry constraint.

This paper is organized as follows. In section II, we give the effective Hamiltonian used to work out the pentaquark mass spectrum and specify the values of the various input parameters. Section III contains our predictions for the pentaquark masses having the quark flavors $\bar{c}[cq][q'q'']$, with $q, q', q'' = u, d, s$, assuming isospin symmetry. In section IV, we discuss decays of the b-baryons into pentaquarks, obeying the heavy quark symmetry, which is followed by the $SU(3)_F$ relations among b-baryon decays and their numerical evaluation in section V. We conclude in section VI.

II. PENTAQUARK SPECTRUM IN AN EFFECTIVE HAMILTONIAN FRAMEWORK

We calculate the mass spectrum of the pentaquarks by assuming that the underlying structure is given by $\bar{c}[cq][q'q'']$, extending the effective Hamiltonian proposed for the tetraquark spectroscopy by Maiani *et al.* [43]. The resulting Hamiltonian for pentaquarks is described in terms of the constituent diquarks masses, $m_{[cq]}$, $m_{[q'q'']}$, the spin-spin interactions between the quarks in each diquark shell, and the spin-orbit and orbital angular momentum of the tetraquarks. To this are added the charm quark mass m_c, the spin-orbit and the orbital terms of the pentaquarks. Thus, in this picture, there are two flux tubes, with the first stretched between the the two diquarks and the second between the tetraquark and the charm antiquark, with each string having its L quantum number.

$$H = H_{[\mathcal{Q}\mathcal{Q}']} + H_{\bar{c}[\mathcal{Q}\mathcal{Q}']} + H_{S_\mathcal{P} L_\mathcal{P}} + H_{L_\mathcal{P} L_\mathcal{P}}, \tag{1}$$

where the diquarks $[cq]$ and $[q'q'']$ are denoted by $\mathcal{Q}$ and $\mathcal{Q}'$ having masses $m_\mathcal{Q}$ and $m_{\mathcal{Q}'}$, respectively. $L_\mathcal{P}$ and $S_\mathcal{P}$ are the orbital angular momentum and the spin of the pentaquark state, and the quantities $A_\mathcal{P}$ and $B_\mathcal{P}$, defined below, parametrize the strength of their spin-orbit and orbital angular momentum couplings, respectively. The individual terms in the Hamiltonian (1) are

$$H_{[\mathcal{Q}\mathcal{Q}']} \;=\; m_\mathcal{Q} + m_{\mathcal{Q}'} + H_{SS}(\mathcal{Q}\mathcal{Q}') + H_{SL}(\mathcal{Q}\mathcal{Q}') + H_{LL}(\mathcal{Q}\mathcal{Q}'),\;\; H_{S_\mathcal{P} L_\mathcal{P}} \;=\; 2A_\mathcal{P}(\mathbf{S}_\mathcal{P}\cdot\mathbf{L}_\mathcal{P}),\;\; H_{L_\mathcal{P} L_\mathcal{P}} \;=\; B_\mathcal{P}\tfrac{L_\mathcal{P}(L_\mathcal{P}+1)}{2}, \tag{2}$$

and the other terms are

$$\begin{aligned} H_{SS}(\mathcal{Q}\mathcal{Q}') &= 2(\mathcal{K}_{cq})_{\bar{3}}(\mathbf{S}_c\cdot\mathbf{S}_q) + 2(\mathcal{K}_{q'q''})_{\bar{3}}(\mathbf{S}_{q'}\cdot\mathbf{S}_{q''}), && (3)\\ H_{\bar{c}[\mathcal{Q}\mathcal{Q}']} &= m_c + 2\,\mathcal{K}_{\bar{c}c}(\mathbf{S}_{\bar{c}}\cdot\mathbf{S}_c) + 2\mathcal{K}_{\bar{c}q}(\mathbf{S}_{\bar{c}}\cdot\mathbf{S}_q) + 2\mathcal{K}_{\bar{c}q'}(\mathbf{S}_{\bar{c}}\cdot\mathbf{S}_{q'}) + 2\mathcal{K}_{\bar{c}q''}(\mathbf{S}_{\bar{c}}\cdot\mathbf{S}_{q''}), && (4)\end{aligned}$$

where $(\mathcal{K}_{cq})_{\bar{3}}$ and $(\mathcal{K}_{q'q''})_{\bar{3}}$ are the couplings corresponding to spin-spin interactions between the quarks inside the diquarks. Finally, the spin and orbital angular momentum couplings of the tetraquark are

$$H_{SL}(\mathcal{Q}\mathcal{Q}') = 2A_{\mathcal{Q}\mathcal{Q}'}\mathbf{S}_{\mathcal{Q}\mathcal{Q}'}\cdot\mathbf{L}_{\mathcal{Q}\mathcal{Q}'},\;\; H_{LL} = B_{\mathcal{Q}\mathcal{Q}'}\frac{L_{\mathcal{Q}\mathcal{Q}'}(L_{\mathcal{Q}\mathcal{Q}'}+1)}{2}. \tag{5}$$

In the earlier model proposed by Maiani *et al.* [8], in addition to the coupling between quarks inside the diquark, the couplings between the quarks of the two diquarks were also included. This extends the $H_{SS}(\mathcal{Q}\mathcal{Q}')$ part given above by including four additional spin-spin terms,

$$H_{SS}(\mathcal{Q}\mathcal{Q}') = 2(\mathcal{K}_{cq})_{\bar{3}}(\mathbf{S}_c\cdot\mathbf{S}_q)+2(\mathcal{K}_{q'q''})_{\bar{3}}(\mathbf{S}_{q'}\cdot\mathbf{S}_{q''})+2(\mathcal{K}_{cq'})_{\bar{3}}(\mathbf{S}_c\cdot\mathbf{S}_{q'})+2(\mathcal{K}_{cq''})_{\bar{3}}(\mathbf{S}_c\cdot\mathbf{S}_{q''})+2(\mathcal{K}_{qq'})_{\bar{3}}(\mathbf{S}_q\cdot\mathbf{S}_{q'})+2(\mathcal{K}_{qq''})_{\bar{3}}(\mathbf{S}_q\cdot\mathbf{S}_{q''}). \tag{6}$$

This then accounts for all possible spin-spin interactions; we have taken all the couplings to be positive.

The mass formula for the pentaquark state with the ground state tetraquark ($L_{\mathcal{Q}\mathcal{Q}'} = 0$) can be written as

$$M = M_0 + \frac{B_{\mathcal{P}}}{2} L_{\mathcal{P}}(L_{\mathcal{P}}+1) + 2A_{\mathcal{P}} \frac{J_{\mathcal{P}}(J_{\mathcal{P}}+1) - L_{\mathcal{P}}(L_{\mathcal{P}}+1) - S_{\mathcal{P}}(S_{\mathcal{P}}+1)}{2} + \Delta M \tag{7}$$

where $M_0 = m_{\mathcal{Q}} + m_{\mathcal{Q}'} + m_c$ and ΔM is the mass term that arises from different spin-spin interactions. With the tetraquark in $L_{\mathcal{Q}\mathcal{Q}'} = 1$, one has to add the two terms given above with their coefficients $A_{\mathcal{Q}\mathcal{Q}'}$ and $B_{\mathcal{Q}\mathcal{Q}'}$. In this work, we restrict ourselves to the S-wave tetraquarks.

For $L_{\mathcal{P}} = 0$, we have classified the states in terms of the diquarks spins, $S_{\mathcal{Q}}$ and $S_{\mathcal{Q}'}$; the spin of anti-charm quark is $S_{\bar{c}} = 1/2$. There are four S-wave pentaquark states for $J^P = \frac{3}{2}^-$ and a single state with $J^P = \frac{5}{2}^-$. For $J^P = \frac{3}{2}^-$, we have the following states[1]:

$$\begin{aligned}
|0_{\mathcal{Q}}, 1_{\mathcal{Q}'}, \frac{1}{2}_{\bar{c}}; \frac{3}{2}\rangle_1 &= \frac{1}{\sqrt{2}}[(\uparrow)_c(\downarrow)_q - (\downarrow)_c(\uparrow)_q](\uparrow)_{q'}(\uparrow)_{q''}(\uparrow)_{\bar{c}} \\
|1_{\mathcal{Q}}, 0_{\mathcal{Q}'}, \frac{1}{2}_{\bar{c}}; \frac{3}{2}\rangle_2 &= \frac{1}{\sqrt{2}}[(\uparrow)_{q'}(\downarrow)_{q''} - (\downarrow)_{q'}(\uparrow)_{q''}](\uparrow)_c(\uparrow)_q(\uparrow)_{\bar{c}} \\
|1_{\mathcal{Q}}, 1_{\mathcal{Q}'}, \frac{1}{2}_{\bar{c}}; \frac{3}{2}\rangle_3 &= \frac{1}{\sqrt{6}}(\uparrow)_c(\uparrow)_q\{2(\uparrow)_{q'}(\uparrow)_{q''}(\downarrow)_{\bar{c}} - [(\uparrow)_{q'}(\downarrow)_{q''} + (\downarrow)_{q'}(\uparrow)_{q''}](\uparrow)_{\bar{c}}\} \\
|1_{\mathcal{Q}}, 1_{\mathcal{Q}'}, \frac{1}{2}_{\bar{c}}; \frac{3}{2}\rangle_4 &= \sqrt{\frac{3}{10}}[(\uparrow)_c(\downarrow)_q + (\downarrow)_c(\uparrow)_q](\uparrow)_{q'}(\uparrow)_{q''}(\uparrow)_{\bar{c}} - \sqrt{\frac{2}{15}}(\uparrow)_c(\uparrow)_q\{(\uparrow)_{q'}(\uparrow)_{q''}(\downarrow)_{\bar{c}} \\
&\quad +[(\uparrow)_{q'}(\downarrow)_{q''} + (\downarrow)_{q'}(\uparrow)_{q''}](\uparrow)_{\bar{c}}\},
\end{aligned} \tag{8}$$

and the spin representation corresponding to $J^P = \frac{5}{2}^-$ state is[2]

$$|1_{\mathcal{Q}}, 1_{\mathcal{Q}'}, \frac{1}{2}_{\bar{c}}; \frac{5}{2}\rangle = (\uparrow)_c(\uparrow)_q(\uparrow)_{q'}(\uparrow)_{q''}(\uparrow)_{\bar{c}}\,. \tag{9}$$

Using the basis vector defined in Eq. (8), the corresponding mass splitting matrix ΔM may be obtained as

$$\Delta M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix}. \tag{10}$$

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[1] For a similar classification in the diquark-triquark picture, see[28].

[2] In the following, we shall suppress the $\frac{1}{2}_{\bar{c}}$ quantum number, as it is the same for all the states discussed here.

TABLE I: S (P)- wave pentaquark states $\mathcal{P}_{X_i}$ ($\mathcal{P}_{Y_i}$) and their spin- and orbital angular momentum quantum numbers. The subscripts $\mathcal{Q}$ and $\mathcal{Q}'$ represent the heavy $[cq]$ and light $[q'q'']$ diquarks, respectively. In the expressions for the masses of the $\mathcal{P}_{Y_i}$ states, the terms $M_{\mathcal{P}_{X_i}} = M_0 + \Delta M_i$ with $i = 1, ..., 5$.

Label	$\|S_{\mathcal{Q}}, S_{\mathcal{Q}'}; L_{\mathcal{P}}, J^P\rangle_i$	Mass	Label	$\|S_{\mathcal{Q}}, S_{\mathcal{Q}'}; L_{\mathcal{P}}, J^P\rangle_i$	Mass
$\mathcal{P}_{X_1}$	$\|0_{\mathcal{Q}}, 1_{\mathcal{Q}'}, 0; \frac{3}{2}^-\rangle_1$	$M_0 + \Delta M_1$	$\mathcal{P}_{Y_1}$	$\|0_{\mathcal{Q}}, 1_{\mathcal{Q}'}, 1; \frac{5}{2}^+\rangle_1$	$M_{\mathcal{P}_{X_1}} + 3A_{\mathcal{P}} + B_{\mathcal{P}}$
$\mathcal{P}_{X_2}$	$\|1_{\mathcal{Q}}, 0_{\mathcal{Q}'}, 0; \frac{3}{2}^-\rangle_2$	$M_0 + \Delta M_2$	$\mathcal{P}_{Y_2}$	$\|1_{\mathcal{Q}}, 0_{\mathcal{Q}'}, 1; \frac{5}{2}^+\rangle_2$	$M_{\mathcal{P}_{X_2}} + 3A_{\mathcal{P}} + B_{\mathcal{P}}$
$\mathcal{P}_{X_3}$	$\|1_{\mathcal{Q}}, 1_{\mathcal{Q}'}, 0; \frac{3}{2}^-\rangle_3$	$M_0 + \Delta M_3$	$\mathcal{P}_{Y_3}$	$\|1_{\mathcal{Q}}, 1_{\mathcal{Q}'}, 1; \frac{5}{2}^+\rangle_3$	$M_{\mathcal{P}_{X_3}} + 3A_{\mathcal{P}} + B_{\mathcal{P}}$
$\mathcal{P}_{X_4}$	$\|1_{\mathcal{Q}}, 1_{\mathcal{Q}'}, 0; \frac{3}{2}^-\rangle_4$	$M_0 + \Delta M_4$	$\mathcal{P}_{Y_4}$	$\|1_{\mathcal{Q}}, 1_{\mathcal{Q}'}, 1; \frac{5}{2}^+\rangle_4$	$M_{\mathcal{P}_{X_4}} + 3A_{\mathcal{P}} + B_{\mathcal{P}}$
$\mathcal{P}_{X_5}$	$\|1_{\mathcal{Q}}, 1_{\mathcal{Q}'}, 0; \frac{5}{2}^-\rangle_5$	$M_0 + \Delta M_5$	$\mathcal{P}_{Y_5}$	$\|1_{\mathcal{Q}}, 1_{\mathcal{Q}'}, \frac{1}{2}_{\bar{c}}, 1; \frac{5}{2}^+\rangle_5$	$M_{\mathcal{P}_{X_5}} - 2A_{\mathcal{P}} + B_{\mathcal{P}}$

where, the different entries of the above matrix can be written in terms of spin-spin couplings as follow:

$$\begin{aligned}
m_{11} &= \frac{1}{2}((\mathcal{K}_{q'q''})_{\bar{3}} - 3(\mathcal{K}_{cq})_{\bar{3}} + \mathcal{K}_{\bar{c}q'} + \mathcal{K}_{\bar{c}q''}),\\
m_{12} &= m_{21} = \frac{1}{2}((\mathcal{K}_{cq'})_{\bar{3}} - (\mathcal{K}_{cq''})_{\bar{3}} - (\mathcal{K}_{qq'})_{\bar{3}} + (\mathcal{K}_{qq''})_{\bar{3}}),\\
m_{13} &= m_{31} = \frac{1}{2\sqrt{3}}((\mathcal{K}_{cq'})_{\bar{3}} + (\mathcal{K}_{cq''})_{\bar{3}} - (\mathcal{K}_{qq'})_{\bar{3}} - (\mathcal{K}_{qq''})_{\bar{3}} + 2\mathcal{K}_{\bar{c}q} - 2\mathcal{K}_{\bar{c}c}),\\
m_{14} &= m_{41} = \frac{\sqrt{15}}{6}((\mathcal{K}_{cq'})_{\bar{3}} + (\mathcal{K}_{cq''})_{\bar{3}} - (\mathcal{K}_{qq'})_{\bar{3}} - (\mathcal{K}_{qq''})_{\bar{3}} - \mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}),\\
m_{22} &= \frac{1}{2}((\mathcal{K}_{cq})_{\bar{3}} - 3(\mathcal{K}_{q'q''})_{\bar{3}} + \mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}),\\
m_{23} &= m_{32} = \frac{1}{2\sqrt{3}}(-(\mathcal{K}_{cq'})_{\bar{3}} + (\mathcal{K}_{cq''})_{\bar{3}} - (\mathcal{K}_{qq'})_{\bar{3}} + (\mathcal{K}_{qq''})_{\bar{3}}),\\
m_{24} &= m_{42} = \frac{\sqrt{15}}{6}(-(\mathcal{K}_{cq'})_{\bar{3}} + (\mathcal{K}_{cq''})_{\bar{3}} - (\mathcal{K}_{qq'})_{\bar{3}} + (\mathcal{K}_{qq''})_{\bar{3}}),\\
m_{33} &= \frac{1}{6}(2((\mathcal{K}_{cq'})_{\bar{3}} + (\mathcal{K}_{cq''})_{\bar{3}} + (\mathcal{K}_{qq'})_{\bar{3}} + (\mathcal{K}_{qq''})_{\bar{3}}) + 3(\mathcal{K}_{cq})_{\bar{3}} + 3(\mathcal{K}_{q'q''})_{\bar{3}}\\
&\quad -\mathcal{K}_{\bar{c}q} - \mathcal{K}_{\bar{c}c} - 6\mathcal{K}_{\bar{c}q'} - 6\mathcal{K}_{\bar{c}q''}),\\
m_{34} &= m_{43} = \frac{\sqrt{5}}{3}(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c} - \frac{1}{2}(\mathcal{K}_{cq'})_{\bar{3}} - \frac{1}{2}(\mathcal{K}_{cq''})_{\bar{3}} - \frac{1}{2}(\mathcal{K}_{qq'})_{\bar{3}} - \frac{1}{2}(\mathcal{K}_{qq''})_{\bar{3}}),\\
m_{44} &= \frac{1}{6}(3(\mathcal{K}_{cq})_{\bar{3}} + 3(\mathcal{K}_{q'q''})_{\bar{3}} - 2(\mathcal{K}_{cq'})_{\bar{3}} - 2(\mathcal{K}_{cq''})_{\bar{3}} - 2(\mathcal{K}_{qq'})_{\bar{3}} - 2(\mathcal{K}_{qq''})_{\bar{3}}\\
&\quad -2\mathcal{K}_{\bar{c}q} - 2\mathcal{K}_{\bar{c}c} + 3\mathcal{K}_{\bar{c}q'} + 3\mathcal{K}_{\bar{c}q''}).
\end{aligned} \tag{11}$$

The couplings $(\mathcal{K}_{cq'})_{\bar{3}}$, $(\mathcal{K}_{cq''})_{\bar{3}}$, $(\mathcal{K}_{qq'})_{\bar{3}}$, $(\mathcal{K}_{qq''})_{\bar{3}}$ given in the above expressions correspond to the spin-spin interactions between the quarks of the two diquarks in Model I [8], which are set to zero in the later version, Model II [43].

The masses for the four S-wave pentaquark states with $J^P = \frac{3}{2}^-$ and a single state with $J^P = \frac{5}{2}^-$ are given in Table I, where we label the states as $\mathcal{P}_{X_i}$. The corresponding five P-wave pentaquark states with $L_{\mathcal{P}} = 1$ and $J^P = \frac{5}{2}^+$ are labeled as $\mathcal{P}_{Y_i}$ in Table I. Here ΔM_i (i runs from 1 to 4) are the mass splitting terms that arise after the diagonalizing the 4×4 matrix (10), whereas ΔM_5 is given by

$$\Delta M_5 = \frac{1}{2}(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c} + \mathcal{K}_{\bar{c}q'} + \mathcal{K}_{\bar{c}q''} + (\mathcal{K}_{cq'})_{\bar{3}} + (\mathcal{K}_{cq''})_{\bar{3}} + (\mathcal{K}_{qq'})_{\bar{3}} + (\mathcal{K}_{qq''})_{\bar{3}} + (\mathcal{K}_{cq})_{\bar{3}} + (\mathcal{K}_{q'q''})_{\bar{3}}). \tag{12}$$

III. *S*- AND *P*-WAVE PENTAQUARK SPECTRUM WITH A $c\bar{c}$ PAIR AND THREE LIGHT QUARKS

To evaluate numerically the pentaquark mass spectrum, we use the values of the input parameters extracted from the hidden $c\bar{c}$ states (X, Y, Z) [8], where the coupling of the heavy-light diquark $(\mathcal{K}_{cq})_{\bar{3}}$ ($q = u\ , d$) and its mass $m_{\mathcal{Q}}$

TABLE II: Constituent quark masses derived from the $L = 0$ mesons and baryons.

Constituent quark	q	s	c	b
Mass (MeV) [Mesons]	305	490	1670	5008
Mass (MeV) [Baryons]	362	546	1721	5050

TABLE III: Spin-Spin couplings for quark-antiquark and the quark-quark pairs in the color singlet and triplet states from the known mesons.

quark-antiquark	$q\bar{q}$	$s\bar{q}$	$s\bar{s}$	$c\bar{q}$	$c\bar{s}$	$c\bar{c}$	quark-quark	qq	sq	ss	cq	cs
$(\mathcal{K}_{ij})_0$(MeV)	318	200	129	70	72	59	$(\kappa_{ij})_{\bar{3}}$ (MeV)	103	64	126	22	25

are estimated to be 110 MeV and 1980 MeV, respectively. Corresponding values for the other diquarks couplings $(\kappa_{ij})_{\bar{3}}$ are summarized in Table III. As we are working with the pentaquark states in the hidden charm sector, we fixed the value of the spin-orbit coupling from the corresponding coupling in the hidden charm tetraquark sector, yielding $A_{\mathcal{P}} = 52$ MeV [43].

For the pentaquarks, consisting of an anti-charm quark and a tetraquark $[\mathcal{Q}\mathcal{Q}']$, various spin-spin couplings $(\mathcal{K}_{ij})_0$ and $(\mathcal{K}_{ij})_{\bar{3}}$ enter in the mass formulae, whose values are summarized in Table III. Fixing these couplings, we are left with one unknown parameter $B_{\mathcal{P}}$, involving the pentaquark orbital angular momentum. To estimate this, we identify the state $|1_{\mathcal{Q}}, 0_{\mathcal{Q}'}, 1; \frac{5}{2}^+\rangle_2$ as the pentaquark state $\frac{5}{2}^+$ having a mass 4450 MeV and $L_{\mathcal{P}} = 1$. This yields 160 MeV and 220 MeV for $B_{\mathcal{P}}$ in the type I and type II models, respectively.

With these input values, we have calculated the mass spectrum for the different pentaquark states which have a $c\bar{c}$ pair and three light quarks belonging to $SU(3)_F$. The results are shown in Figs. 3 and 4, for the parameters in Model I, and in Figs. 5 and 6, for Model II. The labels c_i used in these figures specify the quark flavor content of the pentaquark states, explicitly given in Table IV. The corresponding numerical values for the masses and their errors in both the type I and type II models are given in Tables V and VI for $\mathcal{P}_{X_i}$ and $\mathcal{P}_{Y_i}$ states, respectively. Obviously, the errors are of parametric origin, assuming the effective Hamiltonian, and not from the assumed form of the Hamiltonian. The errors shown arise from the uncertainties in the spin-spin couplings of the quarks inside the diquarks, the masses of the diquarks, and also the couplings of the anti-charm quark with the quarks in the colored tetraquark states. To estimate these errors, we have used the couplings from the hidden charm and the hidden bottom tetraquark states. As an example, by using the states $Z_b^+(10610)$ and $Z_b^+(10650)$, the heavy-light diquark coupling $(\mathcal{K}_{bq})_{\bar{3}}$ is estimated to be 23 MeV [44]. With the relation $(\mathcal{K}_{q'q''})_{\bar{3}} = (\mathcal{K}_{bq})_{\bar{3}} \times m_b/m_{q'}$, and a similar one involving the coupling $(\mathcal{K}_{bc})_{\bar{3}}$, we can get two different estimates of the quark-quark couplings inside the light diquark. With this, and the uncertainties on the masses of the diquarks, we have calculated the errors in the masses of the pentaquark states which are given in the Tables V and VI. Typical parametric errors in this way are about $\pm$ 50 MeV.

TABLE IV: Quark contents (with $q = u$ or d) and the corresponding flavor labels c_i $(i = 1, ..., 5)$ used in the text for the pentaquark states.

Quark contents	$\bar{c}[cq][qq]$	$\bar{c}[cq][sq]$	$\bar{c}[cs][qq]$	$\bar{c}[cs][sq]$	$\bar{c}[cq][ss]$
Label	c_1	c_2	c_3	c_4	c_5

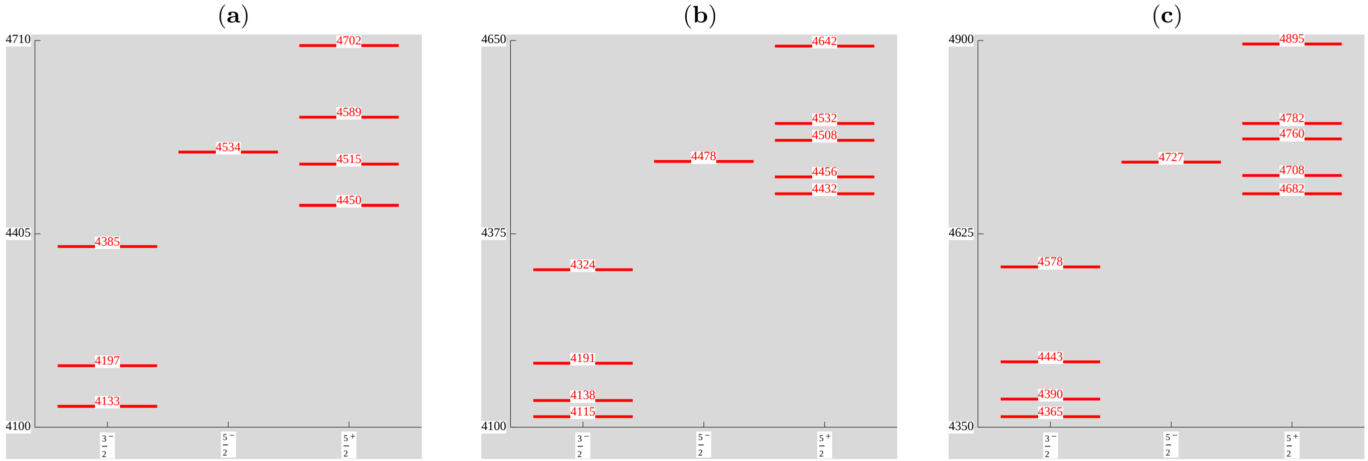

FIG. 3: Mass Spectrum (in MeV) of the lowest S- and P-wave pentaquark states in the diquark-diquark-antiquark picture for the charmonium sector using the tetraquark type-I model. Here, (a) c_1, (b) c_2 and (c) c_3 are the labels for the pentaquarks with their quark contents given in Table IV.

TABLE V: Masses of the hidden charm S-wave pentaquark states $\mathcal{P}_{X_i}$ (in MeV) formed through different diquark-diquark-anti-charm quark combinations in type I and type II models of tetraquarks. The quoted errors are obtained from the uncertainties in the input parameters in the effective Hamiltonian.The masses given in the parentheses are for the input values taken from the type II model.

$\mathcal{P}_{X_i}$	$\mathcal{P}_{X_1}$	$\mathcal{P}_{X_2}$	$\mathcal{P}_{X_3}$	$\mathcal{P}_{X_4}$	$\mathcal{P}_{X_5}$
c_1	4133 ± 55 (4072 ± 40)	4133 ± 55 (4133 ± 55)	4197 ± 55 (4300 ± 40)	4385 ± 55 (4342 ± 40)	4534 ± 55 (4409 ± 40)
c_2	4115 ± 58 (4031 ± 43)	4138 ± 47 (4172 ± 47)	4191 ± 53 (4262 ± 43)	4324 ± 47 (4303 ± 43)	4478 ± 47 (4370 ± 43)
c_3	4365 ± 55 (4304 ± 55)	4390 ± 42 (4365 ± 40)	4443 ± 49 (4532 ± 40)	4578 ± 43 (4574 ± 40)	4727 ± 42 (4641 ± 40)
c_4	4313 ± 47 (4263 ± 43)	4382 ± 45 (4404 ± 47)	4434 ± 51 (4494 ± 43)	4568 ± 46 (4535 ± 43)	4721 ± 45 (4602 ± 43)
c_5	4596 ± 47 (4577 ± 43)	4664 ± 46 (4596 ± 47)	4721 ± 51 (4810 ± 43)	4853 ± 46 (4851 ± 43)	5006 ± 45 (4918 ± 47)

Entries in Table V in the first row, corresponding to the flavor label c_1 (representing the quark content $\bar{c}[cq][qq]$ with $q = u$, d) show that indeed a state $\mathcal{P}_{X_4}$ is predicted with the mass 4385 (4342) MeV in the type I (II) model, having the quantum numbers $|1_\mathcal{Q}, 1_{\mathcal{Q}'}, 0; \frac{3}{2}^-\rangle$. This agrees with the mass of the observed state $P_c^+(4380)$, and the spin assignment matches with the proposal by Maiani *et al.* [25]. Likewise, identifying the state $P_c^+(4550)$, having $J^P = \frac{5}{2}^+$ with the state $\mathcal{P}_{Y_2}$ in the first row of Table VI having the quantum numbers $|1_\mathcal{Q}, 0_{\mathcal{Q}'}, 1; \frac{5}{2}^+\rangle$, also fits well with the one given by Maiani *et al.* [25]. The point where we differ is that with the heavy quark symmetry imposed, the state $\mathcal{P}_{X_4}$ is unlikely to be produced in Λ_b decays, as it has the wrong light-diquark spin. On the other hand, we do have a lower mass state $\mathcal{P}_{X_2}$, having the correct flavor and spin quantum numbers $|1_Q, 0_{\mathcal{Q}'}, 0; \frac{3}{2}^-\rangle$, with a mass of about 4130 MeV, which we expect to be produced in Λ_b decays. One could argue that our mass estimates following from the assumed effective Hamiltonian are in error by a larger amount than what we quote. However, as already stated, the mass difference between the $J^P = \frac{5}{2}^+$ and $J^P = \frac{3}{2}^-$ pentaquarks, having the right quantum numbers $|1_\mathcal{Q}, 0_{\mathcal{Q}'}, 1; \frac{5}{2}^+\rangle$ and $|1_\mathcal{Q}, 0_{\mathcal{Q}'}, 0; \frac{3}{2}^-\rangle$ is expected to be around 340 MeV, yielding a mass for the lower-mass $J^P = \frac{3}{2}^-$ pentaquark state of about 4110 MeV. The two estimates are compatible with each other, and we advocate to search for this state in the LHCb data. Among the ten states listed in the first rows of Tables V and VI, only the ones called $\mathcal{P}_{X_2}$ and $\mathcal{P}_{Y_2}$ are allowed as the Λ_b decay products.

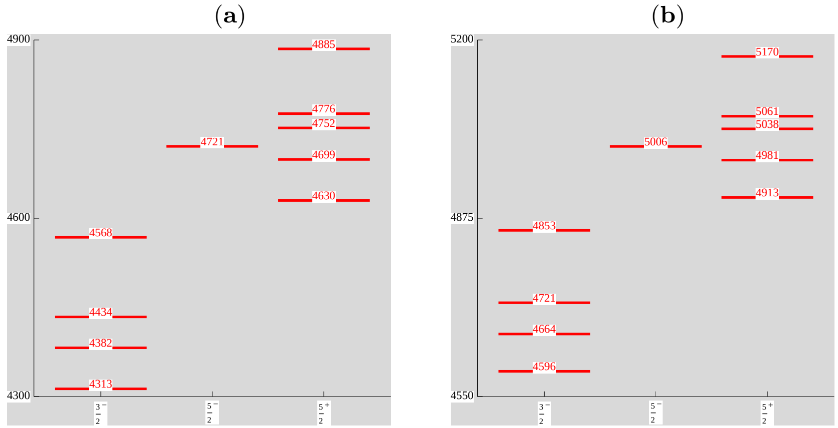

FIG. 4: Mass Spectrum (in MeV) of the lowest S-and P-wave pentaquark states in the diquark-diquark-antiquark picture for the charmonium sector using the tetraquark type-I model. Here, (a) c_4 and (b) c_5 are the labels for the pentaquarks with their quark contents given in Table IV.

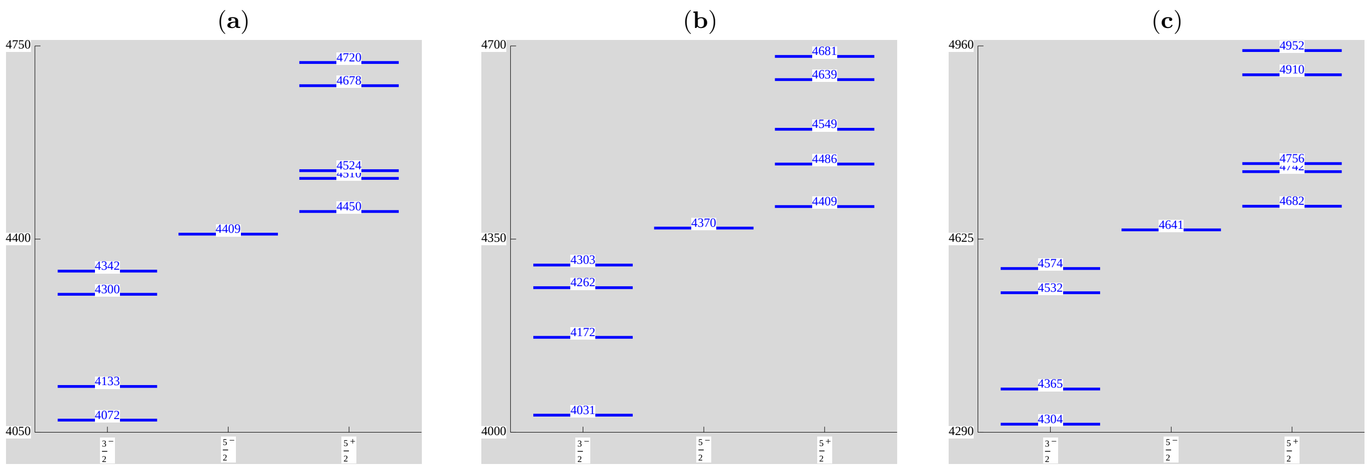

FIG. 5: Mass Spectrum (in MeV) of the lowest S-and P-wave pentaquark states in the diquark-diquark-antiquark picture for the charmonium sector using the tetraquark type-II model. Here, (a) c_1, (b) c_2 and (c) c_3 are labels for the pentaquarks with their quark contents given in Table IV.

IV. WEAK DECAYS OF THE b-BARYONS INTO PENTAQUARK STATES

In the previous sections, we have worked out the spectroscopy of the hidden charm S- and P-wave pentaquarks with their flavor structure displayed in Table IV. There are fifty such states anticipated having masses estimated to lie in the range 4100 - 5100 MeV. They can, in principle, be produced in prompt production processes at the LHC. In practice, only a few have a chance to be detected in experiments due to their significantly lower cross sections, as compared to the corresponding ones for tetraquarks, and the formidable experimental background. As most of the multiquark states (X, Y, Z, P_c) have been observed in the decays of the B mesons and Λ_b, we anticipate that some of the pentaquark states discussed earlier can be measured in b-baryon decays. However, only those states obeying the flavor constraints following from the weak Hamiltonian and having the internal spin quantum numbers compatible with the heavy quark symmetry will actually be produced in b-baryon decays. The state $P_c^+(4450)$ discovered by the LHCb fits all the criterion to be a pentaquark state, and we expect another $J^P = 3/2^-$ pentaquark around 4110 MeV,

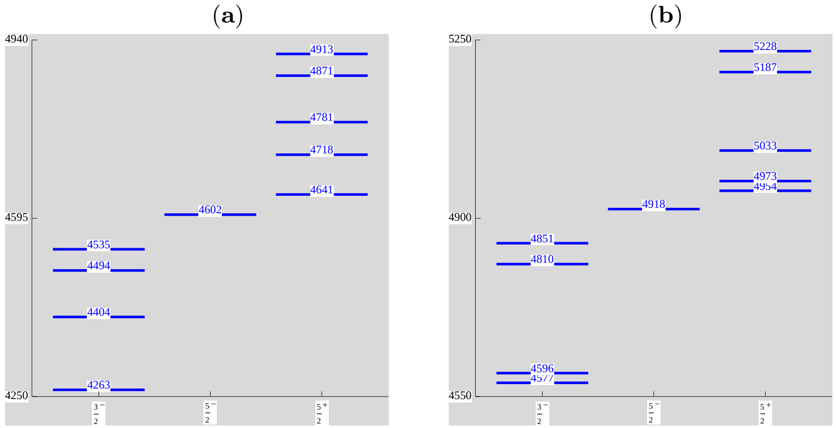

FIG. 6: Mass Spectrum (in MeV) of the lowest S-and P-wave pentaquark states in the diquark-diquark-antiquark picture for the charmonium sector using the tetraquark type-II model. Here, (a) c_4 and (b) c_5 are labels for the pentaquarks with their quark contents given in Table IV.

TABLE VI: Masses of the hidden charm P-wave pentaquark states $\mathcal{P}_{Y_i}$ (in MeV) formed through different diquark-diquark-anti-charm quark combinations in type I and type II models of tetraquarks. The quoted errors are obtained from the uncertainties in the input parameters in the effective Hamiltonian.The masses given in the parentheses are for the input values taken from the type II model.

$\mathcal{P}_{Y_i}$	$\mathcal{P}_{Y_1}$	$\mathcal{P}_{Y_2}$	$\mathcal{P}_{Y_3}$	$\mathcal{P}_{Y_4}$	$\mathcal{P}_{Y_5}$
c_1	4450 ± 57 (4450 ± 44)	4450 ± 57 (4510 ± 57)	4515 ± 57 (4678 ± 44)	4702 ± 58 (4720 ± 44)	4589 ± 56 (4524 ± 41)
c_2	4432 ± 61 (4409 ± 47)	4456 ± 50 (4549 ± 51)	4508 ± 56 (4639 ± 47)	4642 ± 50 (4681 ± 47)	4532 ± 48 (4486 ± 45)
c_3	4682 ± 57 (4682 ± 44)	4708 ± 46 (4742 ± 57)	4760 ± 52 (4910 ± 44)	4895 ± 47 (4952 ± 44)	4782 ± 44 (4756 ± 41)
c_4	4603 ± 51 (4641 ± 47)	4699 ± 49 (4781 ± 51)	4752 ± 54 (4871 ± 47)	4885 ± 49 (4913 ± 47)	4776 ± 47 (4718 ± 45)
c_5	4913 ± 51 (4954 ± 47)	4981 ± 49 (4973 ± 51)	5038 ± 54 (5187 ± 47)	5170 ± 49 (5228 ± 47)	5061 ± 47 (5033 ± 47)

but a number of other pentaquarks are anticipated in b-baryon decays. Heavy quark symmetry reduces the otherwise allowed decay topologies (diagrams) and simplifies the discussion of various contributions. $SU(3)_F$ symmetry can be invoked to relate some of these allowed decay rates, following [41, 42], resulting in a number of predictions to be tested in the future.

The decay amplitudes of interest can be generically written as

$$\mathcal{A} = \langle \mathcal{P}\mathcal{M} \left| H^W_{\text{eff}} \right| \mathcal{B} \rangle \,, \tag{13}$$

where, H^W_{eff} is the effective weak Hamiltonian inducing the Cabibbo-allowed $\Delta I = 0, \Delta S = -1$ transition $b \to c\bar{c}s$, and the Cabibbo-suppressed $\Delta S = 0$ transition $b \to c\bar{c}d$.

$$H^W_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left[V_{cb} V^*_{cq} (c_1 O^{(q)}_1 + c_2 O^{(q)}_2) \right] . \tag{14}$$

Here, G_F is the Fermi coupling constant, V_{ij} are the CKM matrix elements, and c_i are the Wilson coefficients of the operators $O^{(q)}_1$ $(q = d,\ s)$, defined as

$$O^{(q)}_1 = (\bar{q}_\alpha c_\beta)_{V-A} (\bar{c}_\alpha b_\beta)_{V-A}; \quad O^{(q)}_2 = (\bar{q}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta b_\beta)_{V-A}, \tag{15}$$

where α and β are $SU(3)$ color indices, and $V - A = 1 - \gamma_5$ reflects that the charged currents are left-handed.

In (13), $\mathcal{B}$ is a flavor anti-triplet b-baryon with the light-quark spin $j^P = 0^+$ (the flavor sextet b-baryons with $j^P = 0^+$ are denoted by $\mathcal{C}$),

$$\mathcal{B}_{ij}\left(\bar{3}\right) = \begin{pmatrix} 0 & \Lambda_b^0 & \Xi_b^0 \\ -\Lambda_b^0 & 0 & \Xi_b^- \\ -\Xi_b^0 & -\Xi_b^- & 0 \end{pmatrix}, \qquad \mathcal{C}_{ij}\left(6\right) = \begin{pmatrix} \Sigma_b^+ & \frac{\Sigma_b^0}{\sqrt{2}} & \frac{\Xi_b'^0}{\sqrt{2}} \\ \frac{\Sigma_b^0}{\sqrt{2}} & \Sigma_b^- & \frac{\Xi_b'^-}{\sqrt{2}} \\ \frac{\Xi_b'^0}{\sqrt{2}} & \frac{\Xi_b'^-}{\sqrt{2}} & \Omega_b^- \end{pmatrix}.$$

$\mathcal{M}$ is a light pseudoscalar meson in the $SU(3)_F$ octet, and $\mathcal{P}$ is the final state pentaquark state

$$\mathcal{M}_i^j = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}, \qquad \mathcal{P}_i^j\left(J^P\right) = \begin{pmatrix} \frac{P_{\Sigma^0}}{\sqrt{2}} + \frac{P_\Lambda}{\sqrt{6}} & P_{\Sigma^+} & P_p \\ P_{\Sigma^-} & -\frac{P_{\Sigma^0}}{\sqrt{2}} + \frac{P_\Lambda}{\sqrt{6}} & P_n \\ P_{\Xi^-} & P_{\Xi^0} & -\frac{P_\Lambda}{\sqrt{6}} \end{pmatrix}.$$

The hidden-charm pentaquark states are classified by their J^P quantum numbers and their light-quark contents, usually specified by the corresponding light baryons indicated by a subscript. Thus, the two states $P_c^+(J^P = \frac{3}{2}^-)$ and $P_c^+(J^P = \frac{5}{2}^+)$, having the flavor content $c\bar{c}uud$, are denoted here by the components $P_p(J^P = \frac{3}{2}^-)$ and $P_p(J^P = \frac{5}{2}^+)$. For the $J^P = 3/2^-$, also a decuplet is present $\mathcal{P}_{ijk}$ (symmetric in all indices) and they can be listed as: $\mathcal{P}_{111} = P_{\Delta_{10}^{++}}$, $\mathcal{P}_{112} = P_{\Delta_{10}^{+}}/\sqrt{3}$, $\mathcal{P}_{122} = P_{\Delta_{10}^{0}}/\sqrt{3}$, $\mathcal{P}_{222} = P_{\Delta_{10}^{-}}$, $\mathcal{P}_{113} = P_{\Sigma_{10}^{+}}/\sqrt{3}$, $\mathcal{P}_{123} = P_{\Sigma_{10}^{0}}/\sqrt{6}$, $\mathcal{P}_{223} = P_{\Sigma_{10}^{-}}/\sqrt{3}$, $\mathcal{P}_{133} = P_{\Xi_{10}^{0}}/\sqrt{3}$, $\mathcal{P}_{233} = P_{\Xi_{10}^{-}}/\sqrt{3}$ and $\mathcal{P}_{333} = P_{\Omega_{10}^{-}}$ [42].

With these definitions, the tree amplitudes for the anti-triplet b-baryon decays into an octet pentaquark and a pseudoscalar meson, denoted by $\mathcal{A}_{t8}^J$, can be decomposed as [42]

$$\begin{aligned} \mathcal{A}_{t8}^J\left(q\right) \;=\; & T_1^J \left\langle \mathcal{P}_l^k \mathcal{M}_k^l \left| H\left(\bar{3}\right)^i \right| \mathcal{B}_{i'i''} \right\rangle \varepsilon^{ii'i''} + T_2^J \left\langle \mathcal{P}_j^k \mathcal{M}_k^i \left| H\left(\bar{3}\right)^j \right| \mathcal{B}_{i'i''} \right\rangle \varepsilon^{ii'i''} \\ & + T_3^J \left\langle \mathcal{P}_k^i \mathcal{M}_j^k \left| H\left(\bar{3}\right)^j \right| \mathcal{B}_{i'i''} \right\rangle \varepsilon^{ii'i''} + T_4^J \left\langle \mathcal{P}_{j'}^{i'} \mathcal{M}_j^i \left| H\left(\bar{3}\right)^{i''} \right| \mathcal{B}_{jj'} \right\rangle \varepsilon_{ii'i''} \\ & + T_5^J \left\langle \mathcal{P}_{j'}^{i'} \mathcal{M}_j^i \left| H\left(\bar{3}\right)^j \right| \mathcal{B}_{i''j'} \right\rangle \varepsilon_{ii'i''} + T_6^J \left\langle \mathcal{P}_j^{i'} \mathcal{M}_{j'}^i \left| H\left(\bar{3}\right)^j \right| \mathcal{B}_{i''j'} \right\rangle \varepsilon_{ii'i''}, \end{aligned}$$

where the superscript J represents the spin of the final state pentaquark, $J = \frac{3}{2}$ or $J = \frac{5}{2}$.

For the case of an anti-triplet b-baryon decaying into a pentaquark state, the relevant contributions arise with the coefficients T_1^J, T_2^J, T_3^J and T_5^J, in which the spin of the initial state diquark in the decaying b-baryon remains the same in the final state pentaquark states. The contributions with the coefficients T_4^J and T_6^J arise only if the initial light diquark spin is shared by the pentaquark and the meson in the final state as shown in Figs. 7. These contributions are suppressed by the heavy quark symmetry. The coefficients T_1^J and T_2^J are expected to be smaller than T_3^J and T_5^J, as they involve higher Fock states of the pentaquarks (such as five quarks and two antiquarks). The remaining dominant external W-emission diagrams with the coefficients T_3^J and T_5^J are shown in Figs. 7 (a) and (b), respectively. Noting that the amplitudes of the anti-triplet b-baryons decaying into an octet pentaquark and a pseudoscalar meson enter in the combination $2T_3^J + T_5^J$, we denote this by T_8^J. So, for this class of decays there is just one amplitude for a given J^P of the pentaquarks. Similarly, in the case of the sextet b-baryons decaying into decuplet pentaquark states, the only relevant diagram with external W-emission comes with $T_5^{s\ J}$. The internal W-emission diagrams with the coefficients $T_4^{s\ J}$ and $T_6^{s\ J}$, shown in Fig. 7 (c) and (d), respectively, arise only if the initial light diquark spin is shared by the pentaquark and the meson in the final state. These contributions are again suppressed by the heavy quark symmetry. The amplitude for the sextet b-baryons decaying into a decuplet pentaquark and a meson is:

$$\mathcal{A}_{t10}^J\left(q\right) = T_5^{s\ J} \left\langle \mathcal{P}_{kj'i''} \mathcal{M}_l^k \left| H\left(\bar{3}\right)^l \right| \mathcal{C}_{i''j'} \right\rangle. \tag{16}$$

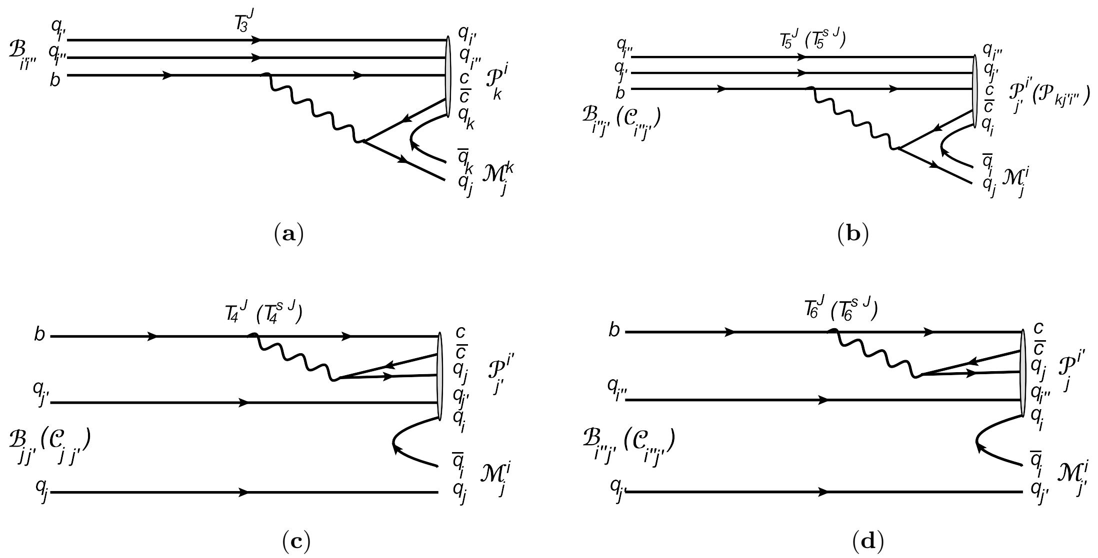

FIG. 7: Decays of the anti-triplet ($\mathcal{B}_{ij}$) and sextet ($\mathcal{C}_{ij}$) b-baryons into a hidden charm octet pentaquark state $\mathcal{P}_i^j$ and an octet of pseudoscalar meson $\mathcal{M}_i^j$. Figures (a) and (b) correspond to the case when the spin of initial state light diquark remains unchanged and is transferred to the pentaquark state. In Figures (c) and (d), the spin of the initial state light diquark is shared between the pentaquark and the meson in the final state and their contribution is suppressed in the heavy quark limit.

TABLE VII: $SU(3)$ amplitudes corresponding to the $\Delta S = 0$ transitions for the Λ_b, Ξ_b^0, Ξ_b^- and Ω_b^- decays. The symbols $\{X_i; Y_i\}_{c_i}$ with $i = 1, ..., 5$ represent the different $\{\mathcal{P}_{X_i}; \mathcal{P}_{Y_i}\}$ states with c_i defining the flavor contents of the pentaquark states. For the decays of Λ_b, Ξ_b^0, Ξ_b^- (Ω_b^-) the pentaquark states are octet (decuplet).

Decay Mode	Amplitude	Decay Mode	Amplitude
$\Lambda_b \to P_p^{\{X_2\,;Y_2\}_{c_1}} \pi^-$	T_8^J	$\Xi_b^- \to P_{\Xi^-}^{\{X_2\,;Y_2\}_{c_4}} K^0$	T_8^J
$\Lambda_b \to P_n^{\{X_2\,;Y_2\}_{c_1}} \eta_8$	$\frac{1}{\sqrt{6}} T_8^J$	$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\,;Y_2\}_{c_2}} \eta_8$	$\frac{1}{\sqrt{6}} T_8^J$
$\Lambda_b \to P_n^{\{X_2\,;Y_2\}_{c_1}} \pi^0$	$-\frac{1}{\sqrt{2}} T_8^J$	$\Xi_b^- \to P_{\Lambda^0}^{\{X_2\,;Y_2\}_{c_2}} \pi^-$	$\frac{1}{\sqrt{6}} T_8^J$
$\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\,;Y_2\}_{c_2}} \eta_8$	$-\frac{1}{6} T_8^J$	$\Xi_b^- \to P_{\Sigma^0}^{\{X_2\,;Y_2\}_{c_2}} \pi^-$	$\frac{1}{\sqrt{2}} T_8^J$
$\Xi_b^0 \to P_{\Sigma^0}^{\{X_2\,;Y_2\}_{c_2}} \eta_8$	$\frac{1}{\sqrt{12}} T_8^J$	$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\,;Y_2\}_{c_2}} \pi^0$	$-\frac{1}{\sqrt{2}} T_8^J$
$\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\,;Y_2\}_{c_2}} \pi^0$	$\frac{1}{\sqrt{12}} T_8^J$	$\Xi_b^0 \to P_{\Sigma^0}^{\{X_2\,;Y_2\}_{c_2}} \pi^0$	$-\frac{1}{2} T_8^J$
$\Omega_b^- \to P_{\Xi_{10}^-}^{\{X_3\,;Y_3\}_{c_5}} \pi^0$	$\frac{1}{\sqrt{6}} \left(-T_5^{s\;J}\right)$	$\Omega_b^- \to P_{\Xi_{10}^0}^{\{X_3\,;Y_3\}_{c_5}} \pi^-$	$\frac{1}{\sqrt{3}} T_5^{s\;J}$

Also, in this class of decays, there is only one amplitude per J^P due to heavy quark symmetry. An example is the decay of of the $\Omega_b^- = b(ss)$, which is an $SU(3)_F$ sextet with the (ss) quarks having $j^p = 1^+$. Due to heavy quark symmetry, Ω_b^- decay will produce a decuplet pentaquark, but not an octet, though the SU(3) algebra admits the octet $3 \times 6 = 8 + 10$. In particular, the amplitude $\mathcal{A}_{s8}^J$ given in Eq. (13) in [42] is not allowed in the heavy quark symmetry limit. This underscores the use of the heavy quark symmetry in b-baryonic decays to pentaquarks.

V. *SU*(3) RELATIONS FOR THE PENTAQUARK STATES

Relations among the various decay amplitudes using $SU(3)_F$ symmetry have been derived in [42]. In terms of the internal and external W-boson classification, they are given in [41]. In particular, in [42], they are written including all possible topologies. However, heavy quark symmetry suppresses some contributions as we have argued in the previous

TABLE VIII: $SU(3)$ amplitudes corresponding to the $\Delta S = 1$ transitions for the Λ_b, Ξ_b^0, Ξ_b^- and Ω_b^- decays. The symbols $\{X_i;Y_i\}_{c_i}$ are the same as defined in Table VII.

Decay Mode	Amplitude	Decay Mode	Amplitude
$\Lambda_b \to P_p^{\{X_2\,;Y_2\}_{c_1}} K^-$	T_8^J	$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\,;Y_2\}_{c_2}} \bar{K}^0$	T_8^J
$\Lambda_b \to P_n^{\{X_2\,;Y_2\}_{c_1}} \bar{K}^0$	T_8^J	$\Xi_b^- \to P_{\Xi^-}^{\{X_2\,;Y_2\}_{c_2}} \eta_8$	$-\frac{2}{\sqrt{6}}T_8^J$
$\Lambda_b \to P_{\Sigma^0}^{\{X_2\,;Y_2\}_{c_3}} \pi^0$	0	$\Xi_b^- \to P_{\Lambda^0}^{\{X_2\,;Y_2\}_{c_2}} K^-$	$\frac{1}{\sqrt{6}}T_8^J$
$\Lambda_b \to P_{\Lambda^0}^{\{X_2\,;Y_2\}_{c_3}} \eta_8$	$\frac{2}{3}T_8^J$	$\Xi_b^- \to P_{\Sigma^0}^{\{X_2\,;Y_2\}_{c_2}} K^-$	$\frac{1}{\sqrt{2}}T_8^J$
$\Xi_b^0 \to P_{\Sigma^+}^{\{X_2\,;Y_2\}_{c_2}} K^-$	$-T_8^J$	$\Xi_b^0 \to P_{\Xi^0}^{\{X_2\,;Y_2\}_{c_2}} \eta_8$	$\frac{2}{\sqrt{6}}T_8^J$
$\Omega_b^- \to P_{\Xi_{10}^0}^{\{X_3\,;Y_3\}_{c_5}} K^-$	$\frac{1}{\sqrt{3}}T_5^{s\ J}$	$\Omega_b^- \to P_{\Xi_{10}^-}^{\{X_3\,;Y_3\}_{c_5}} \bar{K}^0$	$\frac{1}{\sqrt{3}}T_5^{s\ J}$

section. Combining $SU(3)_F$-symmetry with the heavy quark symmetry , these relations involving pentaquarks with a definite J^P are given in Tables VII and VIII. They are strikingly predictive and we write them below, though they can all be read from these tables.

The $SU(3)$ relations involving different anti-triplet b-baryon decays into an octet pentaquark state and a meson from the Cabibbo-suppressed part of the weak Hamiltonian $H^W(\Delta S = 0)$ are:

$$\begin{aligned}
&\mathcal{A}_{t8}^J(\Lambda_b \to P_p^{\{X_2\,;Y_2\}_{c_1}}\pi^-) = -\sqrt{2}\mathcal{A}_{t8}^J(\Lambda_b \to P_n^{\{X_2\,;Y_2\}_{c_1}}\pi^0) = \sqrt{6}\mathcal{A}_{t8}^J(\Lambda_b \to P_n^{\{X_2\,;Y_2\}_{c_1}}\eta_8)\ ,\\
&\mathcal{A}_{t8}^J(\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\,;Y_2\}_{c_2}}\pi^0) = -\mathcal{A}_{t8}^J(\Xi_b^0 \to P_{\Sigma^0}^{\{X_2\,;Y_2\}_{c_2}}\eta_8)\ ,\\
&\mathcal{A}_{t8}^J(\Xi_b^- \to P_{\Xi^-}^{\{X_2\,;Y_2\}_{c_4}}K^0) = \sqrt{6}\mathcal{A}_{t8}^J(\Xi_b^- \to P_{\Sigma^-}^{\{X_2\,;Y_2\}_{c_2}}\eta_8) = \sqrt{6}\mathcal{A}_{t8}^J(\Xi_b^- \to P_{\Lambda^0}^{\{X_2\,;Y_2\}_{c_2}}\pi^-)\ ,\\
&\mathcal{A}_{t8}^J(\Xi_b^- \to P_{\Xi^-}^{\{X_2\,;Y_2\}_{c_4}}K^0) = \sqrt{2}\mathcal{A}_{t8}^J(\Xi_b^- \to P_{\Sigma^0}^{\{X_2\,;Y_2\}_{c_2}}\pi^-) = -\sqrt{2}\mathcal{A}_{t8}^J(\Xi_b^- \to P_{\Sigma^-}^{\{X_2\,;Y_2\}_{c_2}}\pi^0)\ ,
\end{aligned}\tag{17}$$

whereas for the $\Delta S = 1$, the relations can be written as follows

$$\begin{aligned}
&\mathcal{A}_{t8}^J(\Lambda_b \to P_p^{\{X_2\,;Y_2\}_{c_1}}K^-) = \mathcal{A}_{t8}^J(\Lambda_b \to P_n^{\{X_2\,;Y_2\}_{c_1}}\bar{K}^0)\ ,\\
&\mathcal{A}_{t8}^J(\Xi_b^0 \to P_{\Sigma^+}^{\{X_2\,;Y_2\}_{c_2}}K^-) = -\sqrt{\frac{3}{2}}\mathcal{A}_{t8}^J(\Xi_b^0 \to P_{\Xi^0}^{\{X_2\,;Y_2\}_{c_2}}\eta_8) = \mathcal{A}_{t8}^J(\Xi_b^- \to P_{\Sigma^-}^{\{X_2\,;Y_2\}_{c_2}}\bar{K}^0),\\
&= \sqrt{2}\mathcal{A}_{t8}^J(\Xi_b^- \to P_{\Sigma^0}^{\{X_2\,;Y_2\}_{c_2}}K^-)\ .
\end{aligned}\tag{18}$$

Likewise, the amplitudes corresponding to the $\Delta S = 0$ transitions of the b-baryon sextet (Ω_b^-) decaying into a decuplet pentaquark and a meson are related:

$$-\sqrt{2}\mathcal{A}_{t10}^J(\Omega_b^- \to P_{\Xi_{10}^-}^{\{X_3\,;Y_3\}_{c_5}}\pi^0) = \mathcal{A}_{t10}^J(\Omega_b^- \to P_{\Xi_{10}^0}^{\{X_3\,;Y_3\}_{c_5}}\pi^-),\tag{19}$$

and for $\Delta S = 1$, they corresponding ones are:

$$\mathcal{A}_{t10}^J(\Omega_b^- \to P_{\Xi_{10}^0}^{\{X_3\,;Y_3\}_{c_5}}K^-) = \mathcal{A}_{t10}^J(\Omega_b^- \to P_{\Xi_{10}^-}^{\{X_3\,;Y_3\}_{c_5}}\bar{K}^0)\ .\tag{20}$$

Here, as before, $\mathcal{P}^{X_i}$ and $\mathcal{P}^{Y_i}$ have $J = \frac{3}{2}$ and $J = \frac{5}{2}$, respectively.

The two-body decay rate in the center-of-mass frame can be expressed as

$$\Gamma \propto |q_{cm}||\mathcal{A}|^2 \propto |q_{cm}|^{2L+1},\tag{21}$$

where $|\mathcal{A}|$ is the amplitude of the respective decay mode, L is the relative orbital angular momentum of the final state particles, and q_{cm} is the center-of-mass momentum, defined as

$$\begin{aligned}
|q_{cm}| &= q_{\mathcal{P}} = \sqrt{E_{\mathcal{P}}^2 - m_{\mathcal{P}}^2},\\
E_{\mathcal{P}} &= \frac{m_{\mathcal{B}}^2 + m_{\mathcal{P}}^2 - m_{\mathcal{M}}^2}{2m_{\mathcal{B}}},
\end{aligned}\tag{22}$$

TABLE IX: Estimate of the ratios of the decay widths $\Gamma(\mathcal{B}(\mathcal{C}) \to \mathcal{P}^{5/2}\mathcal{M})/\Gamma(\Lambda_b^0 \to P_p^{5/2}K^-)$ for $\Delta S = 1$ transitions [21].

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{5/2}K^-)$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{5/2}K^-)$
$\Lambda_b \to P_p^{\{Y_2\}c_1} K^-$	1	$\Xi_b^- \to P_{\Sigma^-}^{\{Y_2\}c_2} \bar{K}^0$	2.07
$\Lambda_b \to P_n^{\{(Y_2\}c_1} \bar{K}^0$	1	$\Xi_b^0 \to P_{\Sigma^+}^{\{Y_2\}c_2} K^-$	2.07
$\Lambda_b \to P_{\Lambda^0}^{\{Y_2\}c_3} \eta'$	0.03	$\Lambda_b \to P_{\Lambda^0}^{\{Y_2\}c_3} \eta$	0.19
$\Xi_b^- \to P_{\Sigma^0}^{\{Y_2\}c_2} K^-$	1.04	$\Xi_b^- \to P_{\Lambda^0}^{\{Y_2\}c_2} K^-$	0.34
$\Omega_b^- \to P_{\Xi_{10}^-}^{\{Y_3\}c_5} \bar{K}^0$	0.14	$\Omega_b^- \to P_{\Xi_{10}^0}^{\{Y_3\}c_5} K^-$	0.14

TABLE X: Estimate of the ratios of decay widths $\Gamma(\mathcal{B}(\mathcal{C}) \to \mathcal{P}^{5/2}\mathcal{M})/\Gamma(\Lambda_b^0 \to P_p^{5/2}K^-)$ for $\Delta S = 0$ transitions. These transitions are suppressed by a factor $|V_{cd}^*/V_{cs}^*|^2$ compared to $\Delta S = 1$ transitions [21].

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{5/2}K^-)$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{5/2}K^-)$
$\Lambda_b \to P_p^{\{Y_2\}c_1} \pi^-$	0.08	$\Lambda_b \to P_n^{\{Y_2\}c_1} \pi^0$	0.04
$\Lambda_b \to P_n^{\{Y_2\}c_1} \eta$	0.01	$\Lambda_b \to P_n^{\{Y_2\}c_1} \eta'$	0
$\Xi_b^- \to P_{\Xi^-}^{\{Y_2\}c_4} K^0$	0.02	$\Xi_b^- \to P_{\Sigma^0}^{\{Y_2\}c_2} \pi^-$	0.08
$\Xi_b^- \to P_{\Sigma^-}^{\{Y_2\}c_2} \eta$	0.02	$\Xi_b^- \to P_{\Sigma^-}^{\{Y_2\}c_2} \eta'$	0.01
$\Xi_b^- \to P_{\Sigma^-}^{\{Y_2\}c_2} \pi^0$	0.08	$\Xi_b^0 \to P_{\Sigma^0}^{\{Y_2\}c_2} \pi^0$	0.04
$\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\ (Y_2)\}c_2} \eta$	0.01	$\Xi_b^0 \to P_{\Lambda^0}^{\{Y_2\}c_2} \eta'$	0.01
$\Xi_b^0 \to P_{\Lambda^0}^{\{Y_2\}c_2} \pi^0$	0.01	$\Omega_b^- \to P_{\Xi_{10}^-}^{\{Y_3\}c_5} \pi^0$	0.01
$\Omega_b^- \to P_{\Xi_{10}^0}^{\{Y_3\}c_5} \pi^-$	0.02		

where, $m_\mathcal{B}$, $m_\mathcal{P}$ and $m_\mathcal{M}$ are the masses of the initial state b-baryon, final state pentaquark and pseudoscalar meson, respectively. Using this formula, the decay ratios $\Gamma(\mathcal{B}(\mathcal{C}) \to \mathcal{P}^{5/2}\mathcal{M})/\Gamma(\Lambda_b^0 \to P_p^{5/2}K^-)$ for $\Delta S = 1$ and $\Delta S = 0$ transitions are summarized in the Tables IX and X, respectively.

In line with the arguments presented here, we have argued that the state $P_c(4380)$ with $J^P = 3/2^-$ is an unlikely candidate for a pentaquark. Rather, it is the state with a lower mass, $P_c(4110)$ with $J^P = 3/2^-$, which has the correct angular momentum and light diquark spin to be produced in Λ_b decays. The ratios $\Gamma(\mathcal{B}(\mathcal{C}) \to \mathcal{P}^{3/2}\mathcal{M})/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1}}K^-)$ for $\Delta S = 1$ and $\Delta S = 0$ transitions are given in Tables XI and XII, respectively. In working out the numbers, we have used the pentaquark mass spectrum worked out by us. The small rates for the $\Delta S = 0$ transitions compared to the $\Delta S = 1$ reflect Cabibbo-suppression. In particular, $\Gamma(\Lambda_b^0 \to \mathcal{P}_c(4450)^+\pi^-)/\Gamma(\Lambda_b^0 \to \mathcal{P}_c(4450)^+K^-) \simeq 0.08$, and a similar number for the ratio involving the lower-mass $J^P = 3/2^-$ state.[3]

In the numerical estimates for the Ω_b^- decays, we have assumed $T_5^{s\ J} = T_8^J$, which is expected as both of them are tree-amplitudes. However, this is only a ball-park estimate, as the corresponding amplitudes, which are independent quantities, may differ substantially. Many of the predictions presented here for the ratios can be sharpened by calculating the $SU(3)_F$-breaking and including the sub-leading contributions in $1/m_b$. More data on pentaquark states is required to estimate them. Absolute decay rates, on the other hand, require a reliable computation of the amplitudes T_8^J and $T_5^{s\ J}$. This, however, is a daunting task, way beyond the theoretical tools available currently.

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[3] First evidence of tetraquark- and pentaquark-structures in the Cabibbo-suppressed channel $\Lambda_b^0 \to J/\psi\ p\ \pi^-$ is reported recently by LHCb [45].

TABLE XI: Estimate of the ratios of decay widths $\Gamma(\mathcal{B}(\mathcal{C}) \to \mathcal{P}^{3/2}\mathcal{M})/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1}} K^-)$ for $\Delta S = 1$ transitions. Note that we have used the pentaquark masses worked out in this work.

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1} K^-})$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1}} K^-)$
$\Lambda_b \to P_p^{\{X_2\}_{c_1}} K^-$	1	$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\}_{c_2}} \bar{K}^0$	1.38
$\Lambda_b \to P_n^{\{X_2\}_{c_1}} \bar{K}^0$	1	$\Xi_b^0 \to P_{\Sigma^+}^{\{X_2\}_{c_2}} K^-$	1.38
$\Lambda_b \to P_{\Lambda^0}^{\{X_2\}_{c_3}} \eta'$	0.17	$\Lambda_b \to P_{\Lambda^0}^{\{X_2\}_{c_3}} \eta$	0.22
$\Xi_b^- \to P_{\Sigma^0}^{\{X_2\}_{c_2}} K^-$	0.69	$\Xi_b^- \to P_{\Lambda^0}^{\{X_2\}_{c_2}} K^-$	0.23
$\Omega_b^- \to P_{\Xi_{10}^-}^{\{X_3\}_{c_5}} \bar{K}^0$	0.24	$\Omega_b^- \to P_{\Xi_{10}^0}^{\{X_3\}_{c_5}} K^-$	0.24

TABLE XII: Estimate of the ratios of the decay widths $\Gamma(\mathcal{B}(\mathcal{C}) \to \mathcal{P}^{3/2}\mathcal{M})/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1}} K^-)$ for $\Delta S = 0$ transitions. These transitions are suppressed by a factor $|V_{cd}^*/V_{cs}^*|^2$ compared to $\Delta S = 1$ transitions. Other input values are the same as in Table XI.

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1} K^-})$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1}} K^-)$
$\Lambda_b \to P_p^{\{X_2\}_{c_1}} \pi^-$	0.06	$\Lambda_b \to P_n^{\{X_2\}_{c_1}} \pi^0$	0.03
$\Lambda_b \to P_n^{\{X_2\}_{c_1}} \eta$	0.01	$\Lambda_b \to P_n^{\{X_2\}_{c_1}} \eta'$	0.01
$\Xi_b^- \to P_{\Xi^-}^{\{X_2\}_{c_4}} K^0$	0.02	$\Xi_b^- \to P_{\Sigma^0}^{\{X_2\}_{c_2}} \pi^-$	0.03
$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\}_{c_2}} \eta$	0.02	$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\}_{c_2}} \eta'$	0.01
$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\}_{c_2}} \pi^0$	0.04	$\Xi_b^0 \to P_{\Sigma^0}^{\{X_2\}_{c_2}} \pi^0$	0.02
$\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\}_{c_2}} \eta$	0	$\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\}_{c_2}} \eta'$	0
$\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\}_{c_2}} \pi^0$	0.01	$\Omega_b^- \to P_{\Xi_{10}^-}^{\{X_3\}_{c_5}} \pi^0$	0.01
$\Omega_b^- \to P_{\Xi_{10}^0}^{\{X_3\}_{c_5}} \pi^-$	0.02		

VI. CONCLUDING REMARKS

We have studied the mass spectrum of the pentaquarks $\bar{c}[cq][q'q'']$ where q, q', q'' can be u, d, and s quarks, in the diquark-diquark-antiquark approach, using an effective Hamiltonian. The various input parameters are fixed from the experimentally measured states X, Y, Z, under the assumption that they are diquark-antidiquark states. The determination of the spin-spin couplings in a diquark differs between the type I [8] and type II [43] models, and we have worked out the numerics in both cases. Our mass estimates for the pentaquark states agree with the ones given in [25], using their spin and angular-momentum assignments: $P_c^+(4380) = \{\bar{c}[cu]_{s=1}[ud]_{s=1}; L_{\mathcal{P}} = 0, J^{\mathrm{P}} = \frac{3}{2}^-\}$ and $P_c^+(4450) = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 1, J^{\mathrm{P}} = \frac{5}{2}^+\}$. They correspond to the S-wave pentaquark state $\mathcal{P}_{X_4}$ and the P-wave pentaquark state $\mathcal{P}_{Y_2}$, respectively, in our work and given in Table I. We have argued that as the state $\mathcal{P}_{X_4}$ has a light diquark with spin 1, $[ud]_{s=1}$, heavy-quark symmetry suppresses the decay $\Lambda_b^0 \to \mathcal{P}_{X_4} K^-$. Hence, identifying the lower-mass state $P_c^+(4380)$ with the state $\mathcal{P}_{X_4}$, as done in [25] is, in our opinion, problematic. In other words, only those b-baryon decays in which the spin of the light diquark is transferred to the spin of the light diquark in the pentaquark go unhindered.

However, there is a $J^P = \frac{3}{2}^-$ state in the spectrum, called $\mathcal{P}_{X_2} = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 0, J^{\mathrm{P}} = \frac{3}{2}^-\}$ in Table I, which has the right diquark-spin to be produced in Λ_b decays. The mass difference between the $J^P = \frac{5}{2}^+$ pentaquark state $P_c(4450)$ and the $J^P = \frac{3}{2}^-$ pentaquark state $\mathcal{P}_{X_2}$ can be determined by the corresponding mass difference in the charm-baryon sector $m[\Lambda_c^+(2625); J^P = \frac{3}{2}^-] - m[\Lambda_c^+(2286); J^P = \frac{1}{2}^+] \simeq 341$ MeV. Using the effective Hamiltonian, and fixing the parameters from the tetraquark states, we estimate this mass difference to be about 320 MeV, thus yielding the mass of the $J^P = \frac{3}{2}^-$ pentaquark in the range 4110 - 4130 MeV. We urge the LHCb collaboration to reanalyze their data to search for this lower-mass $J^P = \frac{3}{2}^-$ pentaquark state decaying into $J/\psi\, p$.

The pentaquark spectrum with hidden $c\bar{c}$ and three light quarks is very rich. Restricting to the lowest S- and P-waves, there are 50 such states in the mass range 4100 - 5100 MeV. They can, in principle, be produced in prompt processes at the LHC but face unfavorable rates and formidable background. A small fraction of these pentaquarks having the correct flavor and spin/angular-momentum quantum numbers can be produced in b-baryonic decays and their searches appear very promising. We have studied these decays in the second part of our paper, following closely earlier analysis along the same lines [41, 42]. However, there is one difference between our work and theirs in that we use the heavy quark symmetry to classify the various topologies (diagrams), and have kept only those transitions which are allowed by the heavy-quark-spin conservation. This substantially reduces the number of allowed amplitudes and results in a number of relations which can be tested in the future. Relative rates for certain decay channels are calculated and we find that there are good chances to discover yet other pentaquark states in the decays of the b baryons Ξ_b^0 , Ξ_b^- and Ω_b. In the meanwhile, we await for more data and renewed analysis of the current LHCb data to test some of the predictions presented here, in particular, the existence of a $J^P = 3/2^-$ pentaquark state having a mass around 4110 MeV in the Dalitz analysis of the decay $\Lambda_b \to J/\psi p K^-$.

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