

Higher-spin currents in the Gross-Neveu model at $1/n^2$

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ABSTRACT: We calculate the anomalous dimensions of higher-spin currents, both singlet and non-singlet, in the Gross - Neveu model at the $1/n^2$ order. It was conjectured that in the critical regime this model is dual to a higher-spin gauge theory on AdS_4 . The *AdS/CFT* correspondence predicts that the masses of higher-spin fields correspond to the scaling dimensions of the singlet currents in the Gross - Neveu model.

KEYWORDS: Gross-Neveu, conformal symmetry, higher spins, AdS/CFT

In honour of Alexander A. Andrianov's 70th birthday

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1 Introduction

Since its introduction in 1974 [1] the Gross-Neveu (GN) model serves as a toy model for studies of many interesting physical phenomena. In particular, the GN model in $2 < d < 4$ dimensions possesses the Wilson-Fisher fixed point. In the critical regime this model enjoys scale and conformal invariance and provides an example of nontrivial conformal field theory (CFT). Various critical indices in the GN model were calculated both in the $2 + \epsilon$ and $1/N$ expansions (here N is the number of the fermion field components), see e.g. Refs. [2–11]. Due to its simplicity, the GN model together with the $O(N)$ symmetric φ^4 model presents an ideal playground for testing new techniques in CFTs [12–24].

In this work we compute the anomalous dimensions γ_s of the specific composite operators, known as higher-spin currents:

$$J_s \equiv J_{\mu_1 \dots \mu_s} = \bar{q}^a \gamma_{\mu_1} \partial_{\mu_2} \dots \partial_{\mu_s} q^a + \dots \quad (1.1)$$

Here s is the spin and q is the N -component Dirac fermion while the ellipses stand for the total derivative terms and symmetrization over all indices and subtraction of traces are implied. In the free field approximation all the higher-spin currents are conserved and give rise to an infinite-dimensional symmetry, known as the higher-spin symmetry, which is then broken by interactions, see e.g. Refs. [13, 17, 18, 25].

At the $1/N$ order the anomalous dimension of the current J_s have been calculated in 1977 by Muta and Popović [26]. In this work we extend this calculation to the next order using the technique developed in [27–29]. As a nontrivial check of our result we verify that the structure of an asymptotic expansion of the anomalous dimension as a function of the conformal spin, $j = s - 1 + (d + \gamma_s)/2$, agrees with the predictions of Refs. [30–32].

Another surge of recent interest to the GN model comes from studies of the AdS/CFT correspondence [33–35]. It was conjectured in [36] that the critical $O(N)$ -vector model is dual to the

higher-spin gauge theory in AdS_4 . Shortly after, the conjecture was extended to the GN model and its super-symmetric extensions [37, 38]. Some non-trivial tests of this conjecture were performed at the level of three-point functions [39] and at the level of one-loop determinants, see [40, 41] for a discussion and references. Remarkably, the same AdS_4 higher-spin theory should be dual both to free and interacting models, depending on the boundary conditions. Moreover, there is a continuous transition between fermionic and bosonic versions of these CFT's, i.e. the three-dimensional bosonization [25, 42, 43], which on the AdS side is accounted for by a free parameter of the higher-spin theory and on the CFT side is realized via coupling to the Chern-Simons sector [42]. While the duality at the tree level is better supported, the most subtle effects should come from the loops. According to [44], see also [17, 45, 46], radiative corrections on the AdS side should generate masses δm^2 of the higher-spin fields that from the CFT point of view correspond to the anomalous dimensions γ_s of the higher-spin currents

$$m_s^2 = m_0^2(s) + \delta m_s^2, \quad m_0^2(s) = (d + s - 2)(s - 2) - s, \quad \delta m_s^2 = \gamma_s (d - 4 + 2s + \gamma_s), \quad (1.2)$$

The masses are measured in the units of the cosmological constant. Therefore, our results should be equivalent to two-loop computations in the higher-spin theory. More specifically, one should be able to extract γ_s from the logarithmic corrections to the near boundary behaviour of higher-spin fields.

The paper is organized as follows: In Section 2 we recall the definition of the GN model and briefly review the method to compute the critical exponents. The section 3 contains our results and some details of calculations for the anomalous dimensions of the higher-spin currents at the next-to-leading order in $1/N$. The renormalized propagators can be found in Appendix A, while some of the numerical values of the anomalous dimensions are collected in Appendix B.

2 GN model in d dimensions

The GN model with $U(N)$ symmetry describes a system of d ($d \equiv 2\mu$)-dimensional N -component Dirac fermions, $q(\bar{q}) \equiv \{q^a(\bar{q}^a), a = 1, \dots, N\}$. Its action (in Euclidean space) takes the form ¹ [1]

$$S = - \int d^d x \left[\bar{q} \not{\partial} q + \frac{g}{2N} (\bar{q} q)^2 \right]. \quad (2.1)$$

To generate a systematic $1/N$ expansion it is convenient to introduce an auxiliary scalar field σ and rewrite GN action (2.1) in the following form:

$$S = - \int d^d x \left[\bar{q} \not{\partial} q + \sigma \bar{q} q - \frac{N}{2g} \sigma^2 \right]. \quad (2.2)$$

At a certain value of the coupling $g = g_*$ the system undergoes the second order phase transition [51]. For $g < g_*$ the expectation value of σ field vanishes, $\sigma_0 = \langle \sigma \rangle = 0$, and the fermions are massless, while for $g > g_*$ $\langle \sigma \rangle \neq 0$ and fermions acquire mass, $m = \langle \sigma \rangle$ at the leading order. At the critical point, $g = g_*$, the correlators of the fields q, \bar{q}, σ exhibit power law behaviour and, as it can be shown, the model enjoys scale and conformal invariance [6]. Critical exponents are usually calculated with the help of the self-consistency equations [52] or the conformal bootstrap [53] methods (see Ref. [54] for a review). However, it turns out that for the analysis of the operators with nontrivial tensor structure it is more convenient to use another approach described below.

¹The discussion of the issues related to renormalization of this model in $2 + \epsilon$ dimensions can be found in Refs. [11, 47–50].

In the infrared region (IR) (momenta much less than the cutoff Λ) the dominant contribution to the propagator of the σ field in the leading order comes from the fermion loop [51]

$$D_\sigma(p) = -\frac{1}{n}b(\mu)/(p^2)^{\mu-1}, \quad D_\sigma(x) = -\frac{1}{n}B(\mu)/x^2. \quad (2.3)$$

Here $n = N \times \text{tr } \mathbb{1}$, where $\text{tr } \mathbb{1}$ is a trace of the unit matrix in the space of d -dimensional spinors² and the normalization factors are

$$b(\mu) = (4\pi)^\mu \frac{\Gamma(2\mu-1)}{\Gamma^2(\mu)\Gamma(1-\mu)}, \quad B(\mu) = \frac{4\Gamma(2\mu-1)}{\Gamma^2(\mu)\Gamma(\mu-1)\Gamma(1-\mu)}. \quad (2.4)$$

For practical calculations it is convenient to use a simplified (massless) version of the GN model which is critically equivalent to (2.2). The action of the model is given by the following expression [27, 54]

$$S' = - \int d^d x \left[\bar{q} \not{\partial} q - \frac{1}{2} \sigma L \sigma + \sigma \bar{q} q + \frac{1}{2} \sigma L \sigma \right]. \quad (2.5)$$

The kernel L is the inverse σ -propagator (2.3), $L^{-1} = D_\sigma$. It has the form

$$L(x) = \text{tr } D_q(x) D_q(-x) = -n \left(\frac{\Gamma(\mu)}{2\pi^\mu} \right)^2 \frac{1}{(x^2)^{2\mu-1}}, \quad D_q(x) = -\frac{\Gamma(\mu)}{2\pi^\mu} \frac{\not{x}}{[x^2]^\mu}, \quad (2.6)$$

where $D_q(x)$ is the fermion propagator.

The first two terms in (2.5) are considered as the free part of the action, S_0 , and the remaining ones – as an interaction, S_{int} . The last term in (2.5) cancels diagrams with insertions of simple fermion loops in the σ -lines. Of course, for giving a sense to the model (2.5) it is necessary to introduce a regularization. Indeed, it can be easily checked that the vertex $\sigma \bar{q} q$ diverges logarithmically in any dimensions. A regularization preserving the masslessness of propagators, that is important for practical calculations, was proposed in [27]. Namely, in order to make diagrams finite it is sufficient to change the kernel L in the free part of the action, S_0 , as follows

$$L(x) \rightarrow L_\Delta(x) = L(x)(M^2 x^2)^{-\Delta} C^{-1}(\Delta) \sim x^{-2(2\mu-1+\Delta)}. \quad (2.7)$$

Here M is the scale parameter and $C(\Delta)$ is an arbitrary function regular at $\Delta = 0$ such that $C(0) = 1$. The choice of the function $C(\Delta)$ affects only the normalization of correlators but not their scaling dimensions.

The UV divergences appear in diagrams as poles in Δ and are removed by the corresponding counterterms. The renormalized action (2.5) takes the form

$$S'_R = - \int d^d x \left[Z_1 \bar{q} \not{\partial} q - \frac{1}{2} \sigma L_\Delta \sigma + Z_2 \sigma \bar{q} q + \frac{1}{2} \sigma L \sigma \right]. \quad (2.8)$$

The model is not, however, multiplicatively renormalized, i.e. $S'_R(q, \sigma) \neq S'(q_0, \sigma_0)$. This means that the anomalous dimensions of the fields or composite operators are not related directly to the corresponding renormalization factors. The multiplicative renormalizability can be restored in the

²This trace does not have (and does not require) an exact expression in terms of d (for the integer dimensions $d = 2, 3, 4$ it is usually assumed that $\text{tr } \mathbb{1} = 2, 2, 4$).

extended model by introducing two new charges [27],

$$S'_R(u, v) = - \int d^d x \left[Z_1(u, v) \bar{q} \not{\partial} q - \frac{u}{2} \sigma L_\Delta \sigma + Z_2(u, v) \sigma \bar{q} q + \frac{v}{2} \sigma L \sigma \right], \quad (2.9)$$

so that $S'_R(q, \sigma, u, v) = S'(q_0, \sigma_0, u_0, v_0)$. Obviously, for $u = v = 1$ the extended model coincides with the model (2.8). Since the model (2.9) is multiplicatively renormalizable the scale dependence of the Green functions is described by the renormalization group equations (RGEs). For instance, let $\{\mathcal{O}_i\}$ be a set of operators which mix under renormalization. The RGE for the r -point 1PI functions with the insertion of the operators \mathcal{O}_i takes the form

$$\left([M\partial_M + \beta_u \partial_u + \beta_v \partial_v - n_\Phi \gamma_\Phi] \delta^{ik} + \gamma_{\mathcal{O}}^{ik} \right) \Gamma_k(u, v; p, p_1, \dots, p_r) = 0, \quad (2.10)$$

where $n_\Phi = (n_q + n_{\bar{q}}) \gamma_q + n_\sigma \gamma_\sigma$ and the RG functions are defined in the standard way

$$\beta_u = M\partial_M u, \quad \beta_v = M\partial_M v, \quad \gamma_\Phi = M\partial_M \ln Z_\Phi, \quad \gamma_{\mathcal{O}} = -M\partial_M \mathbf{Z} \mathbf{Z}^{-1}. \quad (2.11)$$

Here $Z_q = Z_1^{1/2}$, $Z_\sigma = Z_2 Z_1^{-1}$ and the matrix \mathbf{Z} enters the definition of the renormalized operator, $\mathcal{O}_i^R(\Phi) = \mathbf{Z}_{ik} \mathcal{O}_k(\Phi_0)$. In an arbitrary subtraction scheme the term

$$(\beta_u \partial_u + \beta_v \partial_v) \Gamma_i(u, v; \{p\}) \Big|_{u=v=1} = -2\gamma_\sigma (\partial_u + \partial_v) \Gamma_i(u, v; \{p\}) \Big|_{u=v=1} \neq 0, \quad (2.12)$$

which implies that the RG functions $\gamma_\Phi, \gamma_{\mathcal{O}}$ are not true anomalous dimensions that determine the scale dependence of the correlators. Let us stress that the correlators in the model (2.8) and in the extended model (2.9) at $u = v = 1$ have certain scaling dimensions, namely

$$\left([M\partial_M - n_\Phi \bar{\gamma}_\Phi] \delta^{ik} + \bar{\gamma}_{\mathcal{O}}^{ik} \right) \Gamma_k(u = v = 1; \{p\}) = 0, \quad (2.13)$$

but, in general, $\bar{\gamma}_\Phi \neq \gamma_\Phi$, $\bar{\gamma}_{\mathcal{O}} \neq \gamma_{\mathcal{O}}$. It was shown in [27] that in the MOM scheme the renormalized Green functions depend only on the difference of the charges u and v , $\Gamma_k(u, v; \{p\}) = \Gamma_k(u-v; \{p\})$. This implies that the term with β -functions in the RGE (2.10) vanishes and, hence, $\bar{\gamma}_\Phi = \gamma_\Phi^{MOM}$, $\bar{\gamma}_{\mathcal{O}} = \gamma_{\mathcal{O}}^{MOM}$. Unfortunately the calculations in the MOM scheme is hardly feasibly beyond the leading order. In the most suitable for practical calculations MS-scheme, (Z -factors are given by series in $1/\Delta$) in general, $\bar{\gamma}_{\mathcal{O}} \neq \gamma_{\mathcal{O}}$. However as it was shown in [29] the difference is of order $1/n^3$

$$\gamma_\Phi^* = \bar{\gamma}_\Phi - \gamma_\Phi^{\text{MS}} = O(1/n^3), \quad \gamma_{\mathcal{O}}^* = \bar{\gamma}_{\mathcal{O}} - \gamma_{\mathcal{O}}^{\text{MS}} = O(1/n^3). \quad (2.14)$$

Thus, the anomalous dimensions in the MS scheme up to $1/n^2$ order inclusively can be calculated with the help of Eqs. (2.11). Taking into account that

$$\beta_u = 2u(\Delta - \gamma_\sigma), \quad \beta_v = -2v\gamma_\sigma \quad (2.15)$$

one derives (from now on we consider only the MS scheme and omit the label MS)

$$\gamma_{\mathcal{O}} = -2 \left(\Delta u \partial_u - \gamma_\sigma (u \partial_u + v \partial_v) \right) \mathbf{Z} \mathbf{Z}^{-1} = -2u \partial_u \mathbf{Z}^{(1)}(u, v) \Big|_{u=v=1} \quad (2.16)$$

where

$$\mathbf{Z} = 1 + \sum_{k>0} \Delta^{-k} \mathbf{Z}^{(k)}. \quad (2.17)$$

Taking into account that there is no derivative with respect v in (2.16) one can put $v = 1$ from the very beginning³ arriving to the final expression for anomalous dimensions [29]

$$\gamma_{\mathcal{O}} = -2u\partial_u \mathbf{Z}^{(1)}(u, 1) \Big|_{u=1}. \quad (2.18)$$

Taking into account that the charge u appears only in the σ field propagator

$$D_{\sigma}(x) = -\frac{1}{u} \times \frac{1}{n} B(\mu) C(\Delta) \frac{M^{2\Delta}}{(x^2)^{1-\Delta}}, \quad (2.19)$$

and $-u\partial_u$ counts a number of σ lines in diagrams one concludes that the contributions of each diagram to the \mathbf{Z} factor and to the anomalous dimension, $\gamma_{\mathcal{O}}$, differ by a factor $2n_{\sigma}$, where n_{σ} is the number of internal σ lines in the diagram.

Equations (2.16), (2.18) have a striking resemblance to the analogous expressions in the MS scheme in the dimensional regularization. Being a variation of the standard RG technique, this approach is rather effective for the analysis of composite operators with a nontrivial tensor structure. A more detailed discussion can be found in Refs. [29, 55] and a generalization to gauge theories in Refs. [56, 57].

On finishing the review we recall known results for the anomalous dimensions of the fermion and auxiliary fields [7–10]. We adopt the standard notations [54]

$$\eta = 2\gamma_q, \quad \gamma_{\sigma} = -\eta - \kappa. \quad (2.20)$$

The index η is known with $1/n^3$ [9, 10] and κ with $1/n^2$ [7, 9] accuracy. We need the first two coefficients in the expansion for η , $\eta = \sum_{k \geq 1} \eta_k/n^k$,

$$\eta_1 = -B(\mu)/2\mu = -\frac{2\Gamma(2\mu-1)}{\Gamma(\mu+1)\Gamma(\mu)\Gamma(\mu-1)\Gamma(1-\mu)}, \quad (2.21)$$

$$\eta_2 = \eta_1^2 \frac{1}{2} \frac{1}{(\mu-1)^2} \left[\frac{(\mu-1)^2}{\mu} + 3\mu + 4(\mu-1) + 2(\mu-1)(2\mu-1)\Psi(\mu) \right], \quad (2.22)$$

where $\Psi(\mu) = \psi(2\mu-1) - \psi(1) + \psi(2-\mu) - \psi(\mu)$, and only the first one for κ :

$$\kappa_1 = \eta_1 \frac{\mu}{\mu-1}, \quad \gamma_{\sigma}^{(1)} = -\frac{2\mu-1}{\mu-1} \eta_1. \quad (2.23)$$

3 Higher-spin currents

We consider the higher-spin (traceless and symmetric) operators bilinear in fermionic fields:

- the scalar (singlet) operators

$$\mathcal{O}_s = \bar{q} \gamma_{\mu_1} \partial_{\mu_2} \dots \partial_{\mu_s} q + \dots \quad (3.1)$$

- the adjoint (non-singlet) operators

$$\mathcal{O}_s^A = \bar{q} t^A \gamma_{\mu_1} \partial_{\mu_2} \dots \partial_{\mu_s} q + \dots \quad (3.2)$$

Here t^A are the generators of the $SU(N)$ group and summation over isotopic indices is always implied, ($\bar{q}q = \bar{q}^a q^a$). It is assumed that Lorentz indices are symmetrized and traces subtracted so

³In this case, $v = 1$, there are no diagrams with an insertion of the simple fermion loop into the σ lines (it is exactly cancelled by the term $1/2\sigma L\sigma$ in the action (2.9)).

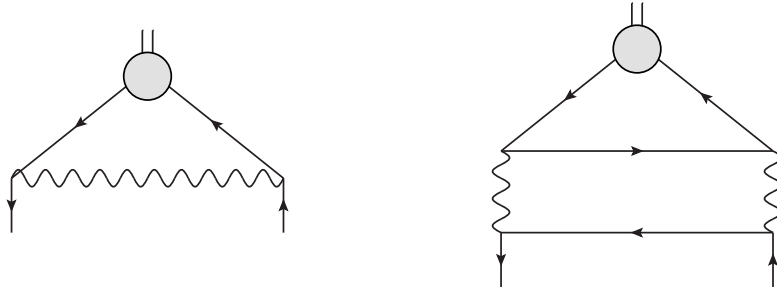


Figure 1. The leading order diagrams for the correlator $\langle \mathcal{O}(0)q(p)\bar{q}(p) \rangle$. The right diagram contributes only to the correlator of singlet operators of even spin.

that s is the spin of the operator. The ellipses stand for the total derivatives which can be neglected if one is interested in the anomalous dimensions only. The renormalized operators take the form

$$[\mathcal{O}_s] = \mathcal{Z}_s \mathcal{O}_s, \quad [\mathcal{O}_s^A] = \mathcal{Z}_s^A \mathcal{O}_s^A. \quad (3.3)$$

Up to $1/n^2$ terms inclusively the anomalous dimensions are given by

$$\gamma(s) = \eta - 2u\partial_u \mathcal{Z}_s^{(1)} \Big|_{u=1}, \quad \gamma_A(s) = \eta - 2u\partial_u \mathcal{Z}_s^{A,(1)} \Big|_{u=1}, \quad (3.4)$$

where $\mathcal{Z}^{(1)}$ is a simple pole in the corresponding renormalization factor $\mathcal{Z} = 1 + \mathcal{Z}^{(1)}/\Delta + O(1/\Delta^2)$.

For odd s the anomalous dimensions of singlet and non-singlet operators coincide, $\gamma_s = \gamma_s^A$. Therefore, from now on, it will be tacitly implied that spin s is even for the singlet currents.

As usual, the renormalization factor \mathcal{Z} is extracted from the correlator of the operator with fermion fields at zero momentum transfer, $\langle \mathcal{O}(0)q(p)\bar{q}(p) \rangle$. Calculating the leading order ($1/n$) diagrams shown in Fig. 1 we reproduce the result of Muta and Popovic [26]

$$\gamma_A(s) = \frac{1}{n} \eta_1 \left(1 - \frac{\mu(\mu-1)}{(s+\mu-1)(s+\mu-2)} \right) + O(1/n^2), \quad (3.5a)$$

$$\gamma(s) = \frac{1}{n} \eta_1 \left(1 - \frac{\mu(\mu-1)}{(s+\mu-1)(s+\mu-2)} \left(1 + \frac{\Gamma(2\mu-1)s!}{\Gamma(2\mu-3+s)(\mu-1)} \right) \right) + O(1/n^2). \quad (3.5b)$$

It can be easily checked that the conserved currents, the spin-one and spin-two singlet (the energy-momentum tensor) currents, have vanishing anomalous dimensions, $\gamma_A(1) = 0$ and $\gamma(2) = 0$.

The anomalous dimensions (determined only for integer s) define analytic functions of complex variable (spin) s , which at integer points coincide with the corresponding anomalous dimensions. It is well known that such a continuation should be done separately for even and odd s . Thus the Eqs. (3.5) define three analytic functions, $\gamma_A^\pm(s)$ and $\gamma(s)$, where γ_A^+, γ reproduce the corresponding anomalous dimensions for even s , and $\gamma_A^-(s)$ for odd s . Obviously, in the $1/n$ order, $\gamma_A^+(s) = \gamma_A^-(s)$.

In CFTs it is more natural to consider anomalous dimensions as functions of the conformal spin j defined as

$$j = \frac{1}{2}(\Delta_s + s) = \mu - 1 + s + \frac{1}{2}\gamma(s), \quad (3.6)$$

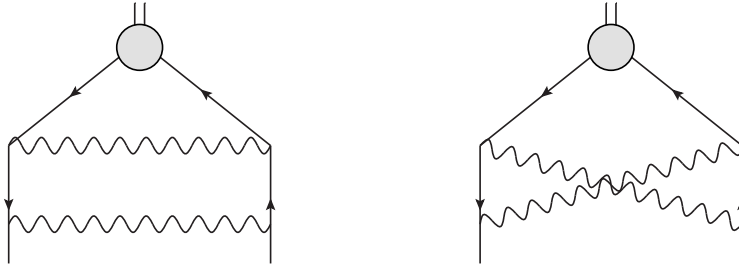


Figure 2. The $1/n^2$ order diagrams for the non-singlet currents.

where s is the spin and Δ_s is the scaling dimension of an operator. For a given anomalous dimension $\gamma(s)$ let us define a function $f(j)$ as follows

$$f(j) = f\left(\mu - 1 + s + \frac{1}{2}\gamma(s)\right) = \gamma(s). \quad (3.7)$$

It was noticed in [30] that in all known examples the large j expansion of the function $f(j)$ has a rather specific structure. Let us consider the f -functions for the anomalous dimensions (3.5). They take the form

$$\begin{aligned} \gamma_A^\pm(s) = f_A^\pm(j) &= \frac{1}{n}\eta_1 \left(1 - \frac{\mu(\mu-1)}{j(j-1)}\right) + O(1/n^2), \\ \gamma(s) = f(j) &= \frac{1}{n}\eta_1 \left(1 - \frac{\mu(\mu-1)}{j(j-1)} \left(1 + \frac{\Gamma(2\mu-1)\Gamma(j-\mu+2)}{\mu-1\Gamma(j+\mu-2)}\right)\right) + O(1/n^2). \end{aligned} \quad (3.8)$$

At the leading order the functions f_A^\pm are invariant under $j \rightarrow 1-j$. The singlet function $f(j)$ is given by the sum of two terms one of which is invariant under $j \rightarrow 1-j$, but another one, $\sim \Gamma(j-\mu+2)/\Gamma(j+\mu-2)$, is not. The asymptotic expansion for this term has the form

$$\left(j - \frac{1}{2}\right)^{-2(\mu-1)} \sum_{k \geq 0} \frac{a_k}{(j(j-1))^k}. \quad (3.9)$$

Therefore, up to the prefactor the series is invariant under $j \rightarrow 1-j$. It was argued in [30–32] that a generic contribution to the asymptotic expansion of $f(j)$ has the structure (3.9) where the coefficients a_k are allowed to be a function of $\ln(j-1/2)$.

Taking these findings into account we also present our results for the anomalous dimensions as functions of conformal spin. Besides that the corresponding expressions have a simpler form the very possibility to bring results to the form (3.9) provides a nontrivial check of calculations.

3.1 Non-singlet currents at $1/n^2$

Diagrams contributing to the renormalization of the non-singlet current at $1/n^2$ order comprise the diagrams with the self-energy (SE) and vertex corrections to the leading order diagram (shown in the l.h.s. of Fig. 1) plus two additional diagrams shown in Fig. 2. The calculations are straightforward so that we present the answer only. It is worth emphasizing that any diagram containing a fermion loop with odd number of attached σ lines vanishes, which is exactly the feature that makes computations in the GN model more feasible than in the $O(N)$ σ -model.

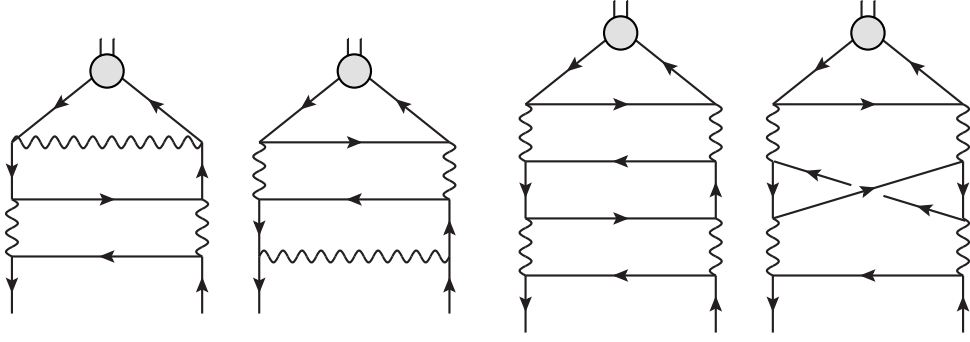


Figure 3. The $1/n^2$ order diagrams for the singlet currents.

3.1.1 Anomalous dimensions

The full conformal dimension of the non-singlet currents can be written as

$$\begin{aligned}\Delta_s &= \Delta_s^{(0)} + \frac{1}{n}\gamma^{(1)}(s) + \frac{1}{n^2}\gamma^{(2)}(s) + \dots \\ &= 2\mu + s - 2 + \frac{1}{n}\left(\eta_1 + \gamma_s^{(1)}\right) + \frac{1}{n^2}\left(\eta_2 + \gamma_s^{(2)}\right) + O\left(\frac{1}{n^3}\right),\end{aligned}\quad (3.10)$$

where η_1 and η_2 are given in Eqs. (2.21) and (2.22), respectively, and the anomalous dimension $\gamma_s^{(1)}$ takes the form

$$\gamma_s^{(1)} = -\eta_1 \frac{\mu(\mu-1)}{(\mu+s-1)(\mu+s-2)}. \quad (3.11)$$

For the $1/n^2$ order anomalous dimension $\gamma_s^{(2)}$ we found

$$\begin{aligned}\gamma_s^{(2)} &= \gamma_s^{(1)}\eta_1 \left\{ \psi(s+\mu-2) - \psi(\mu+1) + \frac{2\mu-1}{\mu-1} [\psi(2\mu-1) + \psi(-\mu) - \psi(\mu) - \psi(1)] \right. \\ &\quad - \frac{\mu(\mu-1)}{2(s+\mu-1)(s+\mu-2)} \left(1 - \frac{1}{s+\mu-1} - \frac{1}{s+\mu-2} \right) + \frac{\mu}{(\mu-1)(s+\mu-2)} + \frac{1}{2} \frac{\mu}{\mu-1} \\ &\quad \left. - \frac{\mu}{\mu-1} [\psi(\mu) - \psi(s+\mu-2)] \right\} - \frac{\eta_1^2}{2} \mu^2 \left(1 - \frac{(\mu-1)^2}{(s+\mu-1)(s+\mu-2)} \right) R_s(\mu),\end{aligned}\quad (3.12)$$

where $R_s(\mu)$ is (cf. with Eq. (5.11) in [29])

$$R_s(\mu) = \int_0^1 d\alpha (1-\alpha)^{\mu-2} \int_0^1 d\beta (1-\beta)^{\mu-2} (1-\alpha-\beta)^{s-1}. \quad (3.13)$$

The last term in (3.12), the only contribution which can not be expressed in terms of Euler's ψ -function, comes entirely from the diagram in the r.h.s. in Fig 2. Finally, as a simple consistency check one can verify that $\gamma^{(2)}(s=1) \equiv \gamma_{s=1}^{(2)} + \eta_2 = 0$.

3.2 Singlet currents at $1/n^2$

In order to find the anomalous dimensions of the singlet currents at $1/n^2$ order one has to calculate diagrams shown in Fig. 3 and the diagrams with all possible self-energy and vertex insertions to the rightmost diagram in Fig. 1. The calculation does not bring about any troubles and can be easily performed with the help of the standard methods, see Ref. [54] for a review.

3.2.1 Anomalous dimensions

The anomalous dimensions of the singlet currents with odd spins are equal to those of non-singlet ones. The scaling dimensions of the currents with even spins can be written as

$$\Delta_s = 2\mu + s - 2 + \frac{1}{n} \left(\eta_1 + \gamma_s^{(1)} + \Delta\gamma_s^{(1)} \right) + \frac{1}{n^2} \left(\eta_2 + \gamma_s^{(2)} + \Delta\gamma_s^{(2)} \right) + O\left(\frac{1}{n^3}\right). \quad (3.14)$$

The indices $\eta_{1,2}$ are defined in Eqs. (2.21), (2.22), $\gamma_s^{(1)}$, $\gamma_s^{(2)}$ – in Eqs. (3.11), (3.12), respectively, and $\Delta\gamma_s^{(1)}$ is the additional shift in (3.5b) for the singlet currents at order $1/n$:

$$\Delta\gamma_s^{(1)} = -\eta_1 \frac{\mu\Gamma(2\mu-1)\Gamma(s+1)}{(\mu+s-2)(\mu+s-1)\Gamma(s+2\mu-3)}. \quad (3.15)$$

For the second order correction $\Delta\gamma_s^{(2)}$ we found

$$\begin{aligned} \Delta\gamma_s^{(2)} = & \eta_1 \Delta\gamma_s^{(1)} \left\{ 2 \frac{2\mu-1}{\mu-1} \left(\psi(2\mu-1) + \psi(-\mu) - \psi(\mu) - \psi(1) \right) \right. \\ & - \frac{1}{2} \left[3 \left(\psi(2\mu+s-3) - \psi(\mu+s-2) + \psi(2-\mu) - \psi(2) \right) + \psi(s) - \psi(1) + \frac{1}{s+2\mu-3} \right. \\ & \left. \left. + \psi(\mu+s-2) - \psi(\mu) + 4 \frac{2\mu-1}{\mu(\mu-1)} + 1 \right] \right. \\ & \left. - \frac{\mu}{\mu-1} \left[\psi(s) + \psi(2\mu-2+s) - 2\psi(s+\mu-1) + \psi(2-\mu) + \psi(\mu) - 2\psi(1) \right] \right\} \quad (3.16) \\ & - \frac{1}{2} \gamma_s^{(1)} \Delta\gamma_s^{(1)} \left\{ \frac{\mu+s-2}{(\mu-1)(2\mu+s-3)} + 2 \left(-1 + \frac{1}{\mu+s-1} + \frac{1}{\mu+s-2} \right) \right. \\ & \left. + \psi(2\mu-3+s) - \psi(\mu) + \psi(1-\mu) - \psi(s) \right\} \\ & - \frac{1}{2} (\Delta\gamma_s^{(1)})^2 \left\{ \frac{1}{s(2\mu+s-3)} + \left(-1 + \frac{1}{\mu+s-1} + \frac{1}{\mu+s-2} \right) \right. \\ & \left. + \psi(2\mu+s-3) - \psi(\mu) + \psi(2-\mu) - \psi(s+1) \right. \\ & \left. - \frac{(\mu+s-1)(\mu+s-2)}{s(\mu-1)(s+2\mu-3)} \left[\psi(s+2\mu-3) + \psi(s+1) - 2\psi(s+\mu-1) + \psi(\mu) - \psi(1) - J_s(\mu) \right] \right\}, \end{aligned}$$

where the function $J_s(\mu)$ is defined as

$$J_s(\mu) = \frac{\Gamma(s+\mu-1)}{s!\Gamma(\mu-2)} \int_0^1 d\alpha \alpha^{2\mu-4+s} \int_0^1 d\beta \frac{\beta^{\mu-2}(1-\beta)^s}{1-\alpha\beta}. \quad (3.17)$$

Again, the contribution in the last line in (3.16), which can not be expressed in terms of ψ functions only, is due the rightmost diagram in Fig. 3. Finally, taking into account that

$$J_2(\mu) = \frac{1}{2} \left(-2S_1(\mu-2) + 2S_1(2\mu-2) + \frac{1+\mu}{1-\mu} \right). \quad (3.18)$$

one can check that the stress-tensor is conserved, i.e. $\eta_2 + \gamma_{s=2}^{(2)} + \Delta\gamma_{s=2}^{(2)} = 0$.

3.3 Conformal spin expansion

Below we check that the anomalous dimensions of singlet and non-singlet currents can be cast into the form (3.9) when expressed as functions of conformal spin.

3.3.1 Conformal spin expansion of non-singlet currents

In order to express the anomalous dimensions in terms of conformal spin we write

$$\gamma_A^\pm(s) = f_A^\pm(j) = \eta \left(1 + f_1(j) \right) + \eta^2 f_2^\pm(j), \quad (3.19)$$

where $\eta = \eta_1/n + \eta_2/n^2 + \dots$ and j is defined in Eq. (3.6). Note that the first term in (3.19) contains both $1/n$ and $1/n^2$ terms. For the functions f_1 and f_2^\pm we get

$$\begin{aligned} f_1(j) &= -\frac{\mu(\mu-1)}{j(j-1)}, \quad (3.20) \\ f_2^\pm(j) &= \frac{1}{2} \frac{\mu^2 - \mu + 1}{\mu(\mu-1)} f_1^2(j) + f_1(j) \left\{ \frac{2\mu-1}{\mu-1} (\psi(j) - \psi(\mu)) + \frac{\mu-1}{2\mu} - \frac{1}{2(\mu-1)^2} \right\} \\ &\quad - \frac{1}{2} \mu^2 \left[1 - \frac{(\mu-1)^2}{j(j-1)} \right] \left(R^+(j, \mu) \mp R^-(j, \mu) \right), \quad (3.21) \end{aligned}$$

where

$$R^-(j, \mu) = \Gamma^2(\mu-1) \frac{\Gamma(j+1-\mu)}{\Gamma(j+\mu-1)}, \quad R^+(j, \mu) = \frac{1}{j} \int_0^1 du \bar{u}^{j-2} {}_2F_1 \left(\begin{matrix} 2-\mu, 1 \\ j+1 \end{matrix} \middle| -\frac{u}{\bar{u}} \right). \quad (3.22)$$

The functions $R^\pm(j, \mu)$ are related to the function $R_s(\mu)$ as follows:

$$R^+(j_s, \mu) + (-1)^{s-1} R^-(j_s, \mu) = R_s(\mu), \quad (3.23)$$

where $j_s = s + \mu - 1$. Note also, that in this formulation the spin-one current conservation is equivalent to the constraint $f_2^-(\mu) = 0$.

One can verify that the large j behavior of anomalous dimensions (3.19) agrees with the predictions of Refs. [30–32]. It is easy to see for all terms except, may be, $R^+(j, \mu)$. For this function one can use the Mellin - Barnes representation for hypergeometric function to get asymptotic expansion

$$R^+(j, \mu) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dt \frac{\Gamma(2-\mu+t)\Gamma^2(1+t)\Gamma(-t)\Gamma(j-t-1)}{\Gamma(2-\mu)\Gamma(j+t+1)} \underset{j \rightarrow \infty}{\simeq} \sum_{k \geq 0} c_k \frac{\Gamma(j-k-1)}{\Gamma(j+k+1)}, \quad (3.24)$$

with $c_k = (-1)^k k! \Gamma(2-\mu+k) / \Gamma(2-\mu)$.

3.3.2 Conformal spin expansion of singlet currents

We can rewrite the answer for the singlet current in the form

$$\gamma(s) = f(j) = \eta \left(1 + f_1(j) + \Delta f_1(j) \right) + \eta^2 \left(f_2^+(j) + \Delta f_2(j) \right) + O(1/n^3), \quad (3.25)$$

where $j = s + \mu - 1 + \gamma(s)/2$, the functions $f_1(j)$, $f_2^+(j)$ are defined by Eqs. (3.20), (3.21) and

$$\Delta f_1(j) = f_1(j) \frac{\Gamma(2\mu-1)}{\mu-1} \frac{\Gamma(j-\mu+2)}{\Gamma(j+\mu-2)}. \quad (3.26)$$

For the function $\Delta f_2(j)$ we obtained

$$\begin{aligned}
\Delta f_2(j) = \Delta f_1(j) & \left\{ - \left[\psi(j) - \psi(\mu) \right] - \frac{2\mu - 1}{\mu - 1} \left(\Psi(j, \mu) - \Psi(\mu, \mu) \right) - \frac{1}{2} \left(\psi(2 - \mu) - \psi(\mu) \right) \right. \\
& - \frac{1}{2} \frac{1}{j(j-1)} + \frac{(2\mu - 3)(3\mu - 1)}{2(\mu - 1)(j - \mu + 1)(j + \mu - 2)} + \frac{1}{(\mu - 1)^2} + \frac{1}{2\mu(\mu - 1)} - 1 \\
& - \frac{1}{2} f_1(j) \left(\psi(1 - \mu) - \psi(\mu - 1) - 2 + \frac{2\mu - 3}{(j - \mu + 1)(j + \mu - 2)} \right) \\
& - \frac{1}{2} \Delta f_1(j) \left(\psi(2 - \mu) - \psi(\mu) - 1 + \frac{1}{(j - \mu + 1)(j + \mu - 2)} \right. \\
& \left. - \frac{j(j-1)}{(\mu - 1)(j - \mu + 1)(j - 2 + \mu)} \left[\Psi(j, \mu) + \psi(\mu) - \psi(1) - J(j, \mu) \right] \right) \left. \right\}, \quad (3.27)
\end{aligned}$$

where

$$\Psi(j, \mu) = \psi(j - 2 + \mu) + \psi(j + 2 - \mu) - 2\psi(j) \quad (3.28)$$

and $J(j, \mu)$ is an analytic continuation of the function $J_s(\mu)$, defined in (3.17), to non-integer spins: $J(j_s, \mu) = J_s(\mu)$, for $j_s = s + \mu - 1$:

$$\begin{aligned}
J(j, \mu) &= \frac{\Gamma(j)}{\Gamma(\mu - 2)\Gamma(j + 2 - \mu)(j + \mu - 2)} \int_0^1 du u^{\mu-2} \bar{u}^{j-\mu} {}_2F_1 \left(\begin{matrix} 1, 1 \\ j + \mu - 1 \end{matrix} \middle| -\frac{u}{\bar{u}} \right) \\
&= \frac{1}{\Gamma(\mu - 2)} \frac{\Gamma(j - 2 + \mu)}{\Gamma(j + 2 - \mu)} \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dt \Gamma^2(t + 1) \Gamma(\mu + t) \Gamma(-t) \frac{\Gamma(j - \mu + 1 - t)}{\Gamma(j + \mu - 1 + t)}. \quad (3.29)
\end{aligned}$$

It can be checked that $f_2^+(j) + \Delta f_2(j)$ vanishes for $j = \mu + 1$, so that the anomalous dimension of the energy-momentum tensor, $\gamma(s = 2) = 0$, as it should be. Next, taking into account that for large j

$$\Psi(j, \mu) - \Psi(1 - j, \mu) = O(e^{-\pi|\text{Im}j|})$$

and, as it follows from Eq. (3.29),

$$J(j, \mu) \underset{j \rightarrow \infty}{\simeq} \sum_{n \geq 0} c_n(\mu) \frac{\Gamma(j - 2 + \mu)}{\Gamma(j + 2 - \mu)} \frac{\Gamma(j - \mu + 1 - n)}{\Gamma(j + \mu - 1 + n)} \quad (3.30)$$

we conclude that the asymptotic expansion of the function $f(j)$ for large j agrees with the predictions of Ref. [32].

3.4 Three dimensions and higher-spin masses

The case of three dimensions is of the most interest. First of all, we give the expression for the index η in $d = 3$ model

$$\eta_1 = \frac{8}{3\pi^2}, \quad \eta = \frac{\eta_1}{n} \left(1 + \frac{28}{3n} \eta_1 + \dots \right). \quad (3.31)$$

The full order $1/n^2$ anomalous dimension of the non-singlet currents $\gamma_s^{(2)}$ can be simplified to

$$\gamma_s^{(2)} = \frac{3\eta_1^2}{4(4s^2-1)} \left\{ \frac{3\pi(-1)^s(2s^2-1)}{s} - \frac{2(24s+9)}{4s^2-1} - \frac{48s}{(4s^2-1)^2} + 2 \right. \\ \left. - 32\log(2) + \frac{3(2s^2-1)}{s} \left[S_1\left(\frac{s}{2} - \frac{3}{4}\right) - S_1\left(\frac{s}{2} - \frac{1}{4}\right) \right] - 16S_1\left(s - \frac{3}{2}\right) \right\}, \quad (3.32)$$

where $S_1(j) \equiv \psi(j+1) - \psi(1)$. The singlet anomalous dimensions can also be simplified:

$$\Delta\gamma_s^{(2)} = \frac{3\eta_1^2}{4s^2-1} \left\{ -\frac{3(144s^3+92s^2-28s-15)}{2(4s^2-1)^2} + \frac{11}{2} - 8s - 2(14s+3)\log(2) \right. \\ \left. - 2(7s+3)S_1\left(s - \frac{3}{2}\right) + 2(8s+3)S_1(s-1) + 3 \left[S_1\left(\frac{s-1}{2}\right) - S_1\left(\frac{s-2}{2}\right) \right] \right\}. \quad (3.33)$$

The conservation of the stress-tensor corresponds to $\eta_2 + \gamma_{s=2}^{(2)} + \Delta\gamma_{s=2}^{(2)} = 0$.

Using the above results for the anomalous dimensions of the currents in the critical $d=3$ GN model one can derive masses of the higher-spin gauge fields (1.2) in the dual AdS_4 model. Plugging the first order anomalous dimensions and (3.32), (3.33) into (1.2) one gets

$$\delta m_s^2 = \frac{2}{n}\eta_1(s-2) + \frac{\eta_1^2}{n^2} \frac{1}{s(1+2s)} \left\{ \frac{9}{4}\pi(2s^2-1) + \frac{s(224s^3-244s^2+88s-317)}{3(2s-1)} + \right. \\ \left. + \frac{9}{4}(2s^2-1) \left[S_1\left(\frac{s}{2} - \frac{3}{4}\right) - S_1\left(\frac{s}{2} - \frac{1}{4}\right) \right] + 9s \left[S_1\left(\frac{s-1}{2}\right) - S_1\left(\frac{s-2}{2}\right) \right] + \right. \\ \left. + 6s(8s+3)S_1(s-1) - 6s(7s+5)S_1\left(s - \frac{3}{2}\right) - 42s(2s+1)\log(2) \right\}. \quad (3.34)$$

One can see that the graviton remains massless, $\delta m_{s=2}^2 = 0$, as it should be. For large spin the mass-spin dependence for the higher-spin fields, Eq. (1.2), can be written in the form

$$\delta m_s^2 = 2\eta(s-2) \left(1 + \eta\kappa_1(s) + \dots \right). \quad (3.35)$$

Thus in the leading order this dependence takes the form of linear Regge trajectory with the slope $\alpha' = 1/2\eta$. Note that the same linear mass squared spin dependence holds also in the $O(N)$ model [45]. The deviation from the linear trajectory, $\kappa_1(s)$, is due to the next-to-leading corrections,

$$\kappa_1(s) = \frac{1}{2s-1} + \frac{2s-1}{2(s-2)} \left(f_2^+(s+1/2) + \Delta f_2(s+1/2) \right). \quad (3.36)$$

The correction $\kappa_1(s)$ is positive for even spins, see Fig. 4, and vanishes as $\kappa_1(s) = \frac{3}{2} \ln s/s + \dots$ for large spin.

4 Summary

We have calculated the $1/n^2$ corrections to the scaling dimensions of the (non)singlet higher-spin currents in the GN model. As nontrivial checks we found that the spin-one non-singlet current and the spin-two stress-tensor current are conserved and the asymptotic expansion in terms of conformal spin agrees with the results of Refs. [30–32].

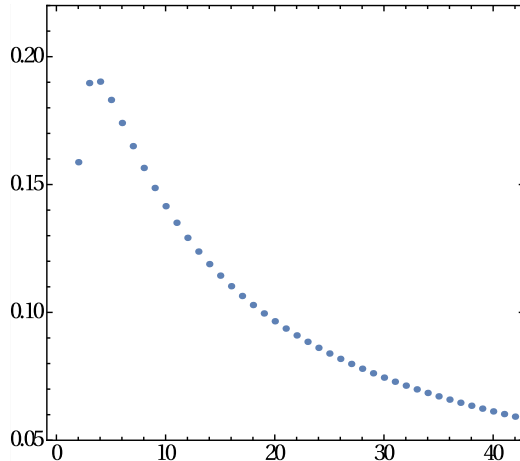


Figure 4. The function $z_1(2k)$, Eq. (3.36).

In three dimensions the anomalous dimensions can be considerably simplified. Some of the numerical values can be found in Appendix B, which should facilitate comparison with other methods, e.g. the numerical bootstrap along the lines of [58–60].

Given the anomalous dimensions of the singlet currents we also computed the loop corrections to the masses of higher-spin fields in the four-dimensional higher-spin theory dual to the GN model, known as Type-B. At the leading order the mass spin dependence has the form of a linear Regge trajectory while the next-to-leading correction gives rise to deviation from linearity.

As was observed in [61, 62], there is some diagrammatic dictionary between all the Feynman-Witten graphs at order $1/N^k$ in the bulk and Feynman graphs at the same order on the CFT side. We note that the graphs of certain topologies present in the bosonic $O(N)$ -model are absent in the Gross-Neveu model since they contain traces of odd number of γ -matrices. This fact indicates that the Type-B higher-spin theory should enjoy some hidden simplicity as compared to the type-A case, which is dual to the bosonic $O(N)$ model. Also, as it is clear already from the order $1/n$ results, there are several contributions to the anomalous dimensions which differ by their large spin asymptotic. It would be interesting to understand this effect from the bulk side.

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A Renormalized Propagators

For completeness of exposition we give here expressions for the renormalized propagators in the $1/n$ order assuming that the normalization factor $C(\Delta)$ is chosen $C(\Delta) = 1$. The propagators take the form

$$D_q^{-1}(p) = i\not{p} \left(p^2/\overline{M}^2\right)^{-\gamma_q} A_q(\mu), \quad D_\sigma^{-1}(p) = -nb^{-1}(\mu)p^{2(\mu-1)} \left(p^2/\overline{M}^2\right)^{-\gamma_\sigma} A_\sigma(\mu), \quad (\text{A.1})$$

where $\overline{M} = 2M$, $b(\mu)$ is defined by Eq. (2.4) and

$$\begin{aligned} A_q(\mu) &= 1 + \gamma_q/\mu(\mu - 1) + O(1/n^2), \\ A_\sigma(\mu) &= 1 + \gamma_\sigma \left(\psi(2\mu - 1) + \psi(-\mu) - \psi(\mu) - \psi(1) \right) + O(1/n^2). \end{aligned} \quad (\text{A.2})$$

B Numerical Values

Since the formulas for the anomalous dimensions are quite complicated we list below few numerical values in one of the most interesting cases of three dimensions. The order $1/n$ results are due to [26] and we collect the order $1/n^2$ coefficients only. It is worth stressing that we give below the full anomalous dimensions at order $1/n^2$, i.e. η_2 is included. It is convenient to measure the anomalous dimensions in the units of $\eta_1^2 = 64/(9\pi^4)$. The spin-one current is always conserved, $\gamma_A^2(1) = 0$, and for few other we find

$$\gamma_A^{(2)}(2) = \frac{1104}{125} \approx 8.832 \quad \gamma_A^{(2)}(3) = \frac{912896}{128625} \approx 7.09734 \quad (\text{B.1})$$

$$\gamma_A^{(2)}(4) = \frac{1324432}{138915} \approx 9.53412 \quad \gamma_A^{(2)}(5) = \frac{154300672}{18866925} \approx 8.17837 \quad (\text{B.2})$$

The anomalous dimensions of the singlet currents with odd spins are the same as for the non-singlet ones. Below are the full anomalous dimensions at order $1/n^2$ for even spins currents. Stress-tensor is conserved, i.e. $\gamma_S^2(2) = 0$, and for a few others we have in the units of η_1^2

$$\gamma_S^{(2)}(4) = \frac{16600}{3087} \approx 5.37739 \quad \gamma_S^{(2)}(6) = \frac{12495584}{1816815} \approx 6.87774 \quad (\text{B.3})$$

$$\gamma_S^{(2)}(8) = \frac{145039504}{19144125} \approx 7.57619 \quad \gamma_S^{(2)}(10) = \frac{133304287652}{16712124975} \approx 7.9765 \quad (\text{B.4})$$

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