

Prospects of discovering stable double-heavy tetraquarks at a Tera- Z factory

Ahmed Ali,^{1,*} Alexander Ya. Parkhomenko,^{2,†} Qin Qin,^{3,‡} and Wei Wang^{4,§}

¹*Deutsches Elektronen-Synchrotron DESY, D-22607 Hamburg, Germany*

²*Department of Theoretical Physics, P. G. Demidov Yaroslavl State University, Sovietskaya 14, 150003 Yaroslavl, Russia*

³*Theoretische Physik 1, Naturwissenschaftlich-Technische Fakultät, Universität Siegen, Walter-Flex-Strasse 3, D-57068 Siegen, Germany*

⁴*INPAC, Shanghai Key Laboratory for Particle Physics and Cosmology, MOE Key Laboratory for Particle Physics, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China*

(Dated: May 8, 2018)

Motivated by a number of theoretical and phenomenological considerations, predicting a deeply bound double-heavy tetraquark $T_{[\bar{u}\bar{d}]}^{\{bb\}}$, with the quark content $(bb\bar{u}\bar{d})$, and the related ones $T_{[\bar{u}\bar{s}]}^{\{bb\}}$ and $T_{[\bar{d}\bar{s}]}^{\{bb\}}$, we explore the potential of their discovery at Tera- Z factories being contemplated at the future circular e^+e^- colliders CEPC and FCC-ee. Using the process $Z \rightarrow b\bar{b}b\bar{b}$, which has been measured at LEP, we calculate, employing the Monte Carlo generators MadGraph5_aMC@NLO and Pythia6, the phase space configuration in which the bb (or $\bar{b}\bar{b}$) pair is likely to fragment as a diquark. In a jet-cone, defined by an invariant mass interval $m_{bb} < M_{T_{[\bar{q}\bar{q}']}^{\{bb\}}} + \Delta M$, the sought-after tetraquarks $T_{[\bar{q}\bar{q}']}^{\{bb\}}$, ($q \neq q' = u, d, s$) as well as the double-bottom baryons, $\Xi_{bb}^0(bbu)$, $\Xi_{bb}^-(bbd)$, and $\Omega_{bb}^-(bbs)$, are expected to be produced as the fragments of the $(bb)_{\text{jet}}$. Using the heavy quark-heavy diquark symmetry, which relates the relative probabilities of $(bb)_{\text{jet}} \rightarrow T_{[\bar{q}\bar{q}']}^{\{bb\}} + X$ and $(bb)_{\text{jet}} \rightarrow (\Xi_{bb}^0, \Xi_{bb}^-, \Omega_{bb}^-) + X$, we estimate $\mathcal{B}(Z \rightarrow T_{[\bar{u}\bar{d}]}^{\{bb\}} + \bar{b}\bar{b}) = (1.4_{-0.5}^{+1.1}) \times 10^{-6}$, and about a half of this for the corresponding branching ratios for the other two tetraquarks $T_{[\bar{u}\bar{s}]}^{\{bb\}}$ and $T_{[\bar{d}\bar{s}]}^{\{bb\}}$. We also present an estimate of their lifetimes using the heavy quark expansion, which yields $\tau(T_{[\bar{q}\bar{q}']}^{\{bb\}}) \simeq 800$ fs. Measuring the tetraquark masses would require decays, such as $T_{[\bar{u}\bar{d}]}^{\{bb\}-} \rightarrow B^- D^- \pi^+$, $T_{[\bar{u}\bar{d}]}^{\{bb\}-} \rightarrow J/\psi \bar{K}^0 B^-$, $T_{[\bar{u}\bar{d}]}^{\{bb\}-} \rightarrow J/\psi K^- \bar{B}^0$, $T_{[\bar{u}\bar{s}]}^{\{bb\}-} \rightarrow \Xi_{bc}^0 \Sigma^-$, and $T_{[\bar{d}\bar{s}]}^{\{bb\}0} \rightarrow \Xi_{bc}^0 \bar{\Sigma}^0$, with subsequent decay chains in exclusive non-leptonic final states. We estimate a couple of the decay widths and find that the product branching ratios do not exceed 10^{-5} . Hence, a good fraction of these modes will be required for a discovery of $T_{[\bar{q}\bar{q}']}^{\{bb\}}$ at a Tera- Z factory.

PACS numbers:

I. INTRODUCTION

The experimental discovery of exotic, hidden-charm and hidden-beauty states, has opened a new field in hadron spectroscopy. The exotic states, called X , Y , Z and P_c , have been analysed in a number of theoretical models. They have been claimed to be hybrid quarkonia, hadron molecules, coupled-channel or threshold effects, and multi-quark states, see [1–5] for recent reviews and extensive references therein. Their dynamics is very much an open question with the models dividing

themselves approximately in two classes, those reflecting residual QCD long-distance effects, dominated by meson exchanges, and those reflecting genuine short-distance forces, induced by gluon exchanges. A particular realisation of the latter class of models assumes that heavy baryons and tetraquarks may be viewed as diquark-quark and diquark-antidiquark objects, respectively, with the diquarks having well-defined color, spin and flavor quantum numbers. Indeed, if tetraquarks, which are stable against strong and radiative decays, could be found in experiments, this would provide an irrefutable evidence of compact diquarks as building blocks of hadronic matter.

The objects of our interest in this paper are the double-bottom $J^P = 1^+$ tetraquarks $T_{[\bar{u}\bar{d}]}^{\{bb\}}$, and the related ones $T_{[\bar{u}\bar{s}]}^{\{bb\}}$ and $T_{[\bar{d}\bar{s}]}^{\{bb\}}$ collected in Fig.1, which have evoked lately a lot of theoretical and phenomenological inter-

*Electronic address: ahmed.ali@desy.de

†Electronic address: parkh@uniyar.ac.ru

‡Electronic address: qin@physik.uni-siegen.de

§Electronic address: wei.wang@sjtu.edu.cn

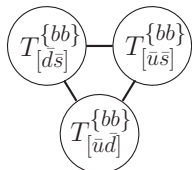


FIG. 1: $SU(3)_F$ -triplet of double-bottom tetraquark states. The spin-parity quantum numbers are $J^P = 1^+$.

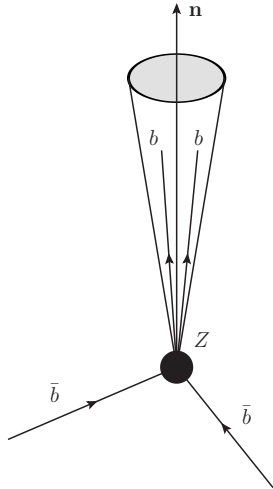


FIG. 2: Production of $\bar{b}b\bar{b}b$ in Z -boson decays with the b -quark pair forming the bb -jet.

est [6–11], though the possibility of stable multiquark states was already pointed out a long time ago [12, 13]. Likewise, estimates of the tetraquark masses with two heavy quarks carried out in quark models well over a decade ago also predicted stable tetraquarks [14].

Concentrating on the $T_{[\bar{u}\bar{d}]}^{\{bb\}}$ state, which is a $J^P = 1^+$, $I = 0$ tetraquark, consisting of the S -wave bound axial-vector $\{bb\}$ diquark and the scalar light $[\bar{u}\bar{d}]$ antiquark, its mass is pitched at (10389 ± 21) MeV [6], lying about 215 MeV below the BB^* threshold. Other estimates are somewhat higher in mass, with a Q -value of -121 MeV [7], -189 ± 10 MeV [8], and -60^{+30}_{-38} MeV [9]. In all likelihood, $T_{[\bar{u}\bar{d}]}^{\{bb\}}$ is deeply bound against strong decays and will decay weakly. Mass estimates of the other two bound tetraquarks $T_{[\bar{u}\bar{s}]}^{\{bb\}}$ and $T_{[\bar{d}\bar{s}]}^{\{bb\}}$ likewise are below their corresponding mesonic thresholds, and hence they are also expected to decay weakly. Since no weakly decaying multiquark state has so far been observed, their observation would herald a new era in genuine multiquark physics.

At a Z -factory, the underlying partonic process for the searches of such tetraquarks is

$$Z \rightarrow \bar{b}b\bar{b}b, \quad (1)$$

which has been measured at LEP, having the branching

ratio [15]:

$$\mathcal{B}(Z \rightarrow \bar{b}b\bar{b}b) = (3.6 \pm 1.3) \times 10^{-4}. \quad (2)$$

The production of a double-bottom tetraquark, $T_{[\bar{q}\bar{q}']}^{\{bb\}}$, where the light antiquark has the flavor content $[\bar{u}\bar{d}]$, $[\bar{u}\bar{s}]$, or $[\bar{d}\bar{s}]$,¹ is a non-perturbative process, and it involves the formation of the bb -diquark and its fragmentation, producing a bb -diquark jet. The conducive configuration for the diquark formation is the one where the two b -quarks (or two b -antiquarks) are almost collinear, satisfying $\theta_{bb} \leq \delta$, where θ_{bb} is the angle between the b -quark momenta and δ is the cone apex angle, and have a small relative velocity, i. e., their energies E_{b1} and E_{b2} differ by a small amount, $(E_{b1} - E_{b2}) / (E_{b1} + E_{b2}) \leq \epsilon$, with $(\epsilon, \delta) \ll 1$, with an example shown in Fig. 2. The bb -diquark jet can be defined, like a quark jet, by a jet resolution parameter, such as the invariant mass, or a Serman-Weinberg jet cone [16]. A judicious choice of these cut-off parameters forces the two b -quarks to overlap in the phase space, which then fragment as a diquark. A tetraquark of the commensurate quark flavor is formed by picking up a light antiquark pair $\bar{u}\bar{d}$, $\bar{u}\bar{s}$, or $\bar{d}\bar{s}$ from the debris of the jet, which consists of mostly soft pions or kaons. Likewise, the bb -diquark will also fragment into double-bottom baryons, $\Xi_{bb}^0(bbu)$, $\Xi_{bb}^-(bbd)$, and $\Omega_{bb}^-(bbs)$, shown in the right-hand frame in Fig. 3. Outside the bb jet-cone, no double-bottom hadrons (tetraquarks or double-bottom baryons) will be produced. Hence, the hadronic texture of the bb -diquark jet is anticipated to be different from the fragmentation of two b -quark jets, whose fragmentation products are B -mesons or b -baryons.

We calculate the branching fraction

$$\mathcal{B}(Z \rightarrow (bb)_{\text{jet}}(\Delta M) + \bar{b} + b), \quad (3)$$

by defining the $(bb)_{\text{jet}}$ with a cut-off on the bb -pair invariant mass, ΔM . An estimate of ΔM can be obtained from the inclusive B_c -meson production in Z -boson decays. Using the inclusive cross section $\sigma(e^+e^- \rightarrow Z \rightarrow \bar{b}b\bar{c}c)$, obtained via MadGraph [17] and Pythia6 [18], and the inclusive B_c -meson cross section $\sigma(e^+e^- \rightarrow Z \rightarrow B_c + b + \bar{c})$, obtained by using the NRQCD-based calculations [19], one can evaluate the fraction $f(\bar{c}b \rightarrow B_c)$ for the $\bar{c}b$ -pair fusion into the B_c mesons². The phase space of the fragmentation process to produce the B_c -meson is limited by $m_{\bar{c}b} < M_{\text{cut}}^{\bar{c}b}$, with $M_{\text{cut}}^{\bar{c}b}$ being the maximum value of the invariant mass in which the $\bar{c}b$ -fusion takes place. Beyond this cut, the b - and \bar{c} -quarks fragment independently. We use $M_{\text{cut}}^{\bar{c}b}$ to estimate ΔM , which yields us the partial branching ratio in Eq. (3). The details of this calculation are given in Section II.

¹ Throughout this paper, the charged conjugation is assumed.

² We would have liked to use the LEP data for this process, but it is too sparse for a quantitative analysis.

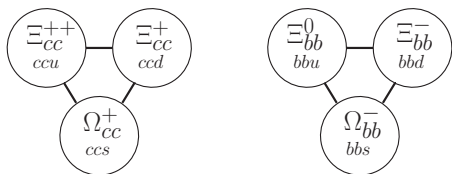


FIG. 3: $SU(3)_F$ -triplets of double-charm (left diagram) and double-bottom (right diagram) baryons. Of these, the Ξ_{cc}^{++} (ccu) baryon has been measured by the LHCb Collab. [23].

The final step in the calculation of the branching ratio

$$\mathcal{B}(Z \rightarrow T_{[\bar{q}\bar{q}']}^{\{bb\}} + X), \quad (4)$$

is to estimate the probability of the diquark-jet $(bb)_{\text{jet}}(\Delta M)$ to fragment into $T_{[\bar{q}\bar{q}']}^{\{bb\}} + X$. There are essentially two possibilities: The bb -diquark jet will either fragment into double-bottom tetraquarks, shown in Fig. 1, or into the double-bottom baryons, shown in the right panel in Fig. 3. This involves estimating the relative probability of emitting a light (anti)-diquark $[\bar{u}\bar{d}]$, $[\bar{u}\bar{s}]$, or $[\bar{d}\bar{s}]$ from the vacuum in the presence of a bb -diquark color source to that of picking up a light quark q ($q = u, d, s$) from the $q\bar{q}$ -pair produced in the similar situation. The two probabilities are related by heavy quark–heavy diquark symmetry.

This probability is similar to the relative probability that a b -quark fragments into a b -baryon, such as $\Lambda_b = bud$, which involves picking up the diquark $[ud]$ from the vacuum, to that of a b -quark fragmenting into a heavy-light meson $B^- = b\bar{u}$ (or $\bar{B}^0 = b\bar{d}$). Denoting these fractions by f_{Λ_b} and f_{B_u} (f_{B_d}), respectively, we need to know the ratio $f_{\Lambda_b}/(f_{B_u} + f_{B_d})$. As is well-known, this ratio has been measured differently at the Fermilab-Tevatron [20]: $f_{\Lambda_b}/(f_{B_u} + f_{B_d}) = 0.281 \pm 0.012_{-0.056}^{+0.058+0.128}$, at the LHC by LHCb Collab. [21], which finds the significant dependence of this ratio on p_T^3 , $f_{\Lambda_b}/(f_{B_u} + f_{B_d}) = (0.404 \pm 0.017 \pm 0.027 \pm 0.105) [1 - (0.031 \pm 0.004 \pm 0.003) \times p_T \text{ (GeV)}]$, and by LEP experiments from Z -boson decays $f_{\Lambda_b}/(f_{B_u} + f_{B_d}) = 0.11 \pm 0.02$ [15]. The dynamical reason behind these variations is not clear, but as we are estimating the Z -boson branching ratios, we shall use the average from the LEP experiments.

We also present an estimate of the total lifetimes of the tetraquarks $T_{[\bar{q}\bar{q}']}^{\{bb\}}$ based on the heavy quark expansion. As each of the two b -quarks in the tetraquark will decay independently, the decay widths of $T_{[\bar{q}\bar{q}']}^{\{bb\}}$ should be approximately about twice that of the b -quark, yielding $\tau(T_{[\bar{q}\bar{q}']}^{\{bb\}}) \simeq 800$ fs. The details are given in Section III.

The tell-tale signatures of the double-bottom tetraquarks are decays into “wrong-sign” heavy mesons,

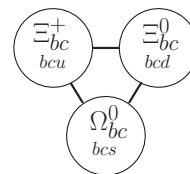


FIG. 4: $SU(3)_F$ -triplet of the bottom-charmed baryons.

such as $T_{[\bar{u}\bar{d}]}^{\{bb\}-} \rightarrow B^- D^+ \pi^-$, $\bar{B}^0 D^0 \pi^-$ [6, 7, 24, 25]. Also interesting are the three-body decays $T_{[\bar{u}\bar{s}]}^{\{bb\}-} \rightarrow B^- (J/\psi, \psi') \phi$, $B_s^- (J/\psi, \psi') K^-$ involving the J/ψ - or ψ' -mesons, and into bottom-charmed baryons (see Fig. 4), such as $T_{[\bar{u}\bar{d}]}^{\{bb\}-} \rightarrow \Xi_{bc}^0 \bar{p}$, $\Omega_{bc}^0 \bar{\Lambda}_c^-$. These baryons also remain to be discovered, and their discovery channels have been recently presented in Refs. [26, 27]. This underscores the huge potential of the Tera- Z colliders in mapping out the landscape of the double-heavy baryons and double-heavy tetraquarks.

The rest of this work is organised as follows. In Sec. II, we present an estimate of the branching ratios of double-bottom tetraquarks in Z -boson decays. Lifetimes of the tetraquarks are discussed in Sec. III. In Secs. IV and V, we discuss the discovery modes for the $T_{[\bar{u}\bar{d}]}^{\{bb\}}$, $T_{[\bar{u}\bar{s}]}^{\{bb\}}$ and $T_{[\bar{d}\bar{s}]}^{\{bb\}}$ tetraquarks, respectively. A special type of decays induced by W -exchange diagrams is discussed in Sec. VI. A brief conclusion is given in the last section.

II. PRODUCTION OF DOUBLE-BOTTOM TETRAQUARKS IN Z -BOSON DECAYS

As already stated, at a Z -factory, double-heavy tetraquarks $T_{[\bar{q}\bar{q}']}^{\{bb\}}$ are anticipated to be produced in the partonic process $Z \rightarrow b\bar{b}b\bar{b}$, with the two b -quarks (or two b -antiquarks) moving collinearly and having a small relative velocity. A reasonable cut on the invariant mass of the two b -quarks ensures these constraints.

To calculate the branching ratio in (4), we invoke the decay $Z \rightarrow B_c + X$, which has been measured at LEP. The idea is to calculate the fraction of the $c\bar{b}$ quarks which hadronise into the B_c -mesons, $f(c\bar{b} \rightarrow B_c)$, in the underlying process $Z \rightarrow b\bar{b}c\bar{c}$. Theoretically, the B_c production cross section has been calculated at the leading order (LO) in the NRQCD framework, yielding: $\sigma(e^+e^- \rightarrow Z \rightarrow B_c + b + \bar{c}) = (5.19_{-2.42}^{+6.22})$ pb [19]. The central (upper, lower) value corresponds to the input values of the quark masses $m_b = 4.9$ GeV and $m_c = 1.5$ GeV ($m_b = 5.3$ GeV and $m_c = 1.2$ GeV; $m_b = 4.5$ GeV and $m_c = 1.8$ GeV). With the same input, we have generated three sets of 10000 LO parton showered $e^+e^- \rightarrow b\bar{b}c\bar{c}$ events at the Z -boson mass using MadGraph [17] and Pythia6 [18], varying the bottom and charmed quark mass values. (The mass pa-

³ The LHCb analysis has been updated in Ref. [22].

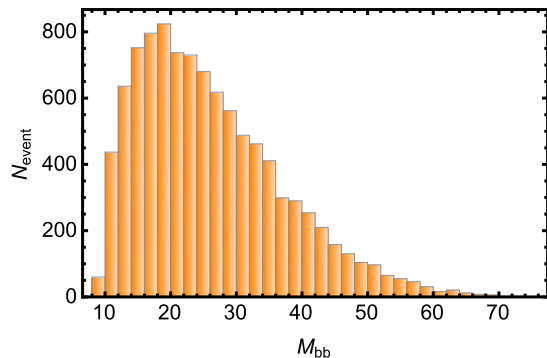


FIG. 5: The bb -quark-pair invariant-mass distribution for the process $e^+e^- \rightarrow Z \rightarrow (bb)_{\text{jet}} + \bar{b} + \bar{b}$ obtained by generating 10^4 events using MadGraph and Pythia6 at the NLO accuracy. The $(bb)_{\text{jet}}$ is defined by the interval $M_{bb}(\Delta M)$.

parameters used by MadGraph are the pole masses. The difference between the $1S$ mass value used in [19] and the pole mass of a heavy quark is of the α_s^2 order (see (1) of [28]), which can be neglected safely.) The cross section $\sigma(e^+e^- \rightarrow Z \rightarrow b\bar{b}c\bar{c})$, evaluated using MadGraph yields 64.50 pb for the central values of the quark masses ($m_b = 4.9$ GeV, $m_c = 1.5$ GeV), 76.79 pb for the upper ($m_b = 5.3$ GeV, $m_c = 1.2$ GeV) and 56.75 pb for the lower values of the masses ($m_b = 4.5$ GeV and $m_c = 1.8$ GeV). This yields $f(c\bar{b} \rightarrow B_c) = 8.05\%$, 14.86% and 4.88%, respectively⁴. With the $m_{c\bar{b}}$ distribution of the generated events, we find that the invariant mass cuts $m_{c\bar{b}} < M_{B_c} + 2.82$ GeV, $M_{B_c} + 4.22$ GeV and $M_{B_c} + 2.22$ GeV give the correct ratios $f(c\bar{b} \rightarrow B_c)$ in the three cases. All the results are collected in Table I.

TABLE I: Central, lower and upper values for the cross sections $\sigma(B_c b\bar{c})$ and $\sigma(b\bar{b}c\bar{c})$, the fraction $f(c\bar{b} \rightarrow B_c)$, and the invariant mass upper bound ΔM for the process $e^+e^- \rightarrow b\bar{b}c\bar{c}$.

Quark masses	$\sigma(B_c b\bar{c})$ [pb]	$\sigma(b\bar{b}c\bar{c})$ [pb]	$f(c\bar{b} \rightarrow B_c)$	ΔM [GeV]
central	5.19	64.50	8.05%	2.82
upper	11.41	76.79	14.86%	4.22
lower	2.77	56.75	4.88%	2.22

We then generate 10000 showered $e^+e^- \rightarrow b\bar{b}b\bar{b}$ events at the Z -boson mass as the centre-of-mass energy with MadGraph [17] and Pythia6 [18] at the NLO of QCD, with the b -quark pair invariant mass distribution

⁴ The NLO MadGraph result for the cross section is enhanced by more than a factor of 2 compared with the LO result, but as the corresponding NRQCD calculation is done only in the LO approximation, we use the LO simulation to be consistent with Ref. [19].

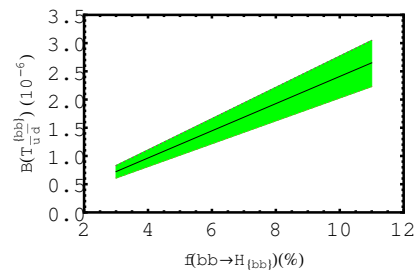


FIG. 6: The branching ratio $\mathcal{B}(Z \rightarrow T_{[\bar{u}d]}^{\{bb\}} + \bar{b}\bar{b})$ plotted against the fraction $f(bb \rightarrow H_{\{bb\}})$.

played in Fig. 5. With the cross sections by MadGraph, the branching ratio $\mathcal{B}(Z \rightarrow b\bar{b}b\bar{b})$ is obtained to be 3.23×10^{-4} , which is consistent with the world average $(3.6 \pm 1.3) \times 10^{-4}$ [15].

Next, we employ the same kinematics cuts, namely $M_{bb} \leq M_{T_{[\bar{u}d]}^{\{bb\}}} + 2.82$ GeV, $M_{T_{[\bar{u}d]}^{\{bb\}}} + 4.22$ GeV, and $M_{T_{[\bar{u}d]}^{\{bb\}}} + 2.22$ GeV, and find that 5.73% (5.91%), 9.88% (10.30%) and 3.98% (4.02%) of the $e^+e^- \rightarrow Z \rightarrow b\bar{b}b\bar{b}$ events fragment into a hadron with the bb ($\bar{b}\bar{b}$) quark pair. The numerical differences between the charged conjugated processes reflect statistical fluctuations, which will disappear in simulations with higher statistics. We average them and list the results of the ratios $f(bb \rightarrow H_{\{bb\}})$ in Table II.

TABLE II: The fraction of the events $f(bb \rightarrow H_{\{bb\}})$ in the simulation for the process $e^+e^- \rightarrow Z \rightarrow b\bar{b}b\bar{b}$, which survive the indicated invariant mass cuts on the bb and $\bar{b}\bar{b}$ invariant masses.

ΔM (GeV)	2.82	4.22	2.22
$f(bb \rightarrow H_{\{bb\}})$	5.8%	10.1%	4.0%

Thus, our simulation yields

$$f(bb \rightarrow H_{\{bb\}}) = (5.8_{-1.8}^{+4.3})\%. \quad (5)$$

The uncertainty in the fragmentation fraction is sizeable, and, within $\pm 1\sigma$, it lies in the range (4 – 10)%. The double-bottom hadrons $H_{\{bb\}}$ include the double-heavy tetraquarks $T_{[\bar{q}q]}^{\{bb\}}$ and the double-bottom baryons $\Xi_{bb}^0(bbu)$, $\Xi_{bb}^-(bbd)$, and $\Omega_{bb}^-(bbs)$.

The product of $\mathcal{B}(Z \rightarrow b\bar{b}b\bar{b})$, given in Eq. (2), $f(bb \rightarrow H_{\{bb\}})$, calculated by us and given in Eq. (5), and the fraction $f(bb \rightarrow T_{[\bar{u}d]}^{\{bb\}} + X)/f(bb \rightarrow \Xi_{bb} + X)$, which we assume is close to f_{Λ_b}/f_B , derived from $f_{\Lambda_b}/(f_{B_u} + f_{B_d}) = 0.11 \pm 0.02$ [15], yields

$$\mathcal{B}(Z \rightarrow T_{[\bar{u}d]}^{\{bb\}} + \bar{b}\bar{b}) = (1.4_{-0.5}^{+1.1}) \times 10^{-6}. \quad (6)$$

We show the dependence of the branching ratio $\mathcal{B}(Z \rightarrow T_{[\bar{u}\bar{d}]}^{\{bb\}} + X)$ on $f(bb \rightarrow H_{\{bb\}})$ in Fig. 6. The same estimate applies to the charged conjugated process. We expect approximately half of this number for the production of $T_{[\bar{u}\bar{s}]}^{\{bb\}}$ and $T_{[\bar{d}\bar{s}]}^{\{bb\}}$ in Z -decays.

The branching ratios for the double-bottom baryons (summed over the states) are estimated as

$$\mathcal{B}(Z \rightarrow (\Xi_{bb}^0, \Xi_{bb}^-, \Omega_{bb}^-) + X) : \mathcal{B}(Z \rightarrow T_{[\bar{q}\bar{q}']}^{\{bb\}} + X) \approx 5.8 : 1. \quad (7)$$

Thus, we anticipate about six times as many double-bottom baryons as the double-bottom tetraquarks in the Z -boson decays.

III. LIFETIMES

Before discussing the weak decays, we present an estimate of the lifetimes, the inverse of total decay rates. The decay width of $T_{[\bar{q}\bar{q}']}^{\{bb\}}$ into an inclusive final state X can be expressed as a phase space integral over the matrix element squared of the effective Hamiltonian sandwiched between the initial $|T_{[\bar{q}\bar{q}']}^{\{bb\}}\rangle$ and final $\langle X|$ states. Summing over all final states one has:

$$\Gamma(T_{[\bar{q}\bar{q}']}^{\{bb\}} \rightarrow X) = \frac{1}{2m_T} \sum_X \int \prod_i \left[\frac{d^3 \vec{p}_i}{(2\pi)^3 2E_i} \right] \quad (8)$$

$$\times (2\pi)^4 \delta^{(4)}(p_T - \sum_i p_i) \frac{1}{3} \sum_\lambda |\langle X | \mathcal{H}_{\text{eff}}^{\text{ew}} | T_{[\bar{q}\bar{q}']}^{\{bb\}} \rangle|^2,$$

where m_T and p_T^μ are the mass and four-momentum of the tetraquark, respectively. The factor $1/3$ results after averaging over the polarization states λ of the tetraquark with the spin-parity $J^P = 1^+$. The effective electroweak Hamiltonian $\mathcal{H}_{\text{eff}}^{\text{ew}}$ is governing the weak decays to be given in the following discussions. Using the optical theorem, one can rewrite the total decay width as

$$\Gamma(T_{[\bar{q}\bar{q}']}^{\{bb\}} \rightarrow X) = \frac{1}{2m_T} \frac{1}{3} \sum_\lambda \langle T_{[\bar{q}\bar{q}']}^{\{bb\}} | \mathcal{T} | T_{[\bar{q}\bar{q}']}^{\{bb\}} \rangle, \quad (9)$$

with the transition operator \mathcal{T} as the time-ordered non-local double insertion of the effective Hamiltonian:

$$\mathcal{T} = \text{Im} \int d^4x T [\mathcal{H}_{\text{eff}}^{\text{ew}}(x), \mathcal{H}_{\text{eff}}^{\text{ew}}(0)]. \quad (10)$$

The heavy-quark expansion, the operator product expansion in essence, greatly simplifies the decay widths of inclusive decays. Integrating over the off-shell intermediate states in Eq. (10) results in a set of local operators with increasing dimensions. Higher dimensional operators are suppressed by inverse powers of the heavy-quark mass. (For a recent review see Ref. [29].) Up to dimen-

sion 6, we have the transition operator expanded as [29]:

$$\mathcal{T} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{\text{CKM}}|^2 \left[c_{3,b} (\bar{b}b) + \frac{c_{5,b}}{m_b^2} (\bar{b}g_s \sigma_{\mu\nu} G^{\mu\nu} b) \right. \\ \left. + 2 \frac{c_{6,b}}{m_b^3} (\bar{b}q)_\Gamma (\bar{q}b)_\Gamma + \dots \right], \quad (11)$$

where G_F is the Fermi constant and V_{CKM} are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. The coefficients $c_{i,b}$ are the corresponding short-distance coefficients from the semileptonic and non-leptonic heavy quark decays. These short-distance coefficients have been precisely calculated using QCD perturbation theory [29].

The total decay width is then determined by matrix elements of these operators, and at the leading order in $1/m_b$, only the $\bar{b}b$ operator contributes:

$$\Gamma(T_{[\bar{q}\bar{q}']}^{\{bb\}}) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{\text{CKM}}|^2 c_{3,b} \frac{1}{3} \sum_\lambda \frac{\langle T_{[\bar{q}\bar{q}']}^{\{bb\}} | \bar{b}b | T_{[\bar{q}\bar{q}']}^{\{bb\}} \rangle}{2m_T} \quad (12)$$

The matrix element

$$\frac{1}{3} \sum_\lambda \frac{\langle T_{[\bar{q}\bar{q}']}^{\{bb\}} | \bar{b}b | T_{[\bar{q}\bar{q}']}^{\{bb\}} \rangle}{2m_T} \quad (13)$$

corresponds to the bottom-quark number in the $T_{[\bar{q}\bar{q}']}^{\{bb\}}$ state and is twice the matrix element for B -meson and Λ_b -baryon. Accordingly, we expect that the lifetime of the double-bottom tetraquark is one half of that of the B -meson [15]:

$$\tau(T_{[\bar{q}\bar{q}']}^{\{bb\}}) \sim \frac{1}{2} \times 1.6 \times 10^{-12} \text{ s} = 0.8 \text{ ps}. \quad (14)$$

This is approximately twice the corresponding lifetime $\tau(T_{[\bar{q}\bar{q}']}^{\{bb\}}) \simeq 367 \text{ fs}$ estimated in [6].

IV. WEAK DECAYS OF $T_{[\bar{u}\bar{d}]}^{\{bb\}}$

If a state is below the lowest hadronic threshold, its decay width is determined by weak interactions. The decay modes are dominated by flavor-changing charged currents in the effective Hamiltonian [30]:

$$\mathcal{H}_{\text{eff}}^{(cc)} = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \{ C_1 [\bar{c}_\alpha \gamma_\mu P_L b^\alpha] [\bar{d}_\beta \gamma^\mu P_L u^\beta] \quad (15)$$

$$+ C_2 [\bar{c}_\beta \gamma_\mu P_L b^\alpha] [\bar{d}_\alpha \gamma^\mu P_L u^\beta] \}$$

$$+ \frac{4G_F}{\sqrt{2}} V_{cb} V_{cs}^* \{ C_1 [\bar{c}_\alpha \gamma_\mu P_L b^\alpha] [\bar{s}_\beta \gamma^\mu P_L c^\beta]$$

$$+ C_2 [\bar{c}_\beta \gamma_\mu P_L b^\alpha] [\bar{s}_\alpha \gamma^\mu P_L c^\beta] \} + \text{h. c.}$$

Here, α and β are the quark color indices, and $P_L = (1 - \gamma_5)/2$ is the left-handed projector. The coefficients C_1 and C_2 are determined after matching the

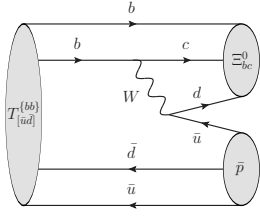


FIG. 7: Feynman diagrams which yield two-body baryonic final states in $T_{[u\bar{d}]}^{\{bb\}-}$ decays due to the b -quark decay $b \rightarrow c + d + \bar{u}$ (left panel) and $b \rightarrow c + s + \bar{c}$ (right panel).

effective theory and the Standard Model, being scale-dependent quantities.

We start by considering the two-body decays of $T_{[u\bar{d}]}^{\{bb\}}$ and continue by taking up the three-body ones which can be divided into the purely mesonic and baryonic decays.

A. Two-Body Baryonic Decays

As the doubly bottom tetraquark $T_{[u\bar{d}]}^{\{bb\}}$ (10482)⁵ consists of two b -quarks and two antiquarks, \bar{u} and \bar{d} , the weak decay of one of the b -quarks results into two possible Cabibbo-allowed decay channels at the quark level: $b \rightarrow c + d + \bar{u}$ or $b \rightarrow c + s + \bar{c}$. From three quarks and three antiquarks one can easily construct baryon and antibaryon, as shown in Fig. 7. Baryonic states produced in these modes have in addition to the well-known antibaryons, also the so-far undiscovered bottom-charmed baryons shown in Fig. 4. They are being searched for at the LHC and are also a part of the experimental programme at future electron-positron colliders. Amplitudes of these decays are non-factorisable as a quark and antiquark produced in the weak transition hadronise into baryon and anti-baryon, respectively (see Fig. 7). Taking into account the axial-vector nature of the tetraquark, with $J^P = 1^+$, the general form of the decay amplitude induced by the $b \rightarrow c + d + \bar{u}$ quark channel is as follows:

$$\begin{aligned} \mathcal{M}(T_{[u\bar{d}]}^{\{bb\}-} \rightarrow \Xi_{bc}^0 \bar{p}) &= \bar{v}(p_p) \left[f_1^{\Xi_{bc}^0 \bar{p}} q_\mu + f_2^{\Xi_{bc}^0 \bar{p}} \gamma_\mu \right. \\ &+ f_3^{\Xi_{bc}^0 \bar{p}} \sigma_{\mu\nu} \frac{q^\nu}{m_T} + g_1^{\Xi_{bc}^0 \bar{p}} \gamma_5 q_\mu + g_2^{\Xi_{bc}^0 \bar{p}} \gamma_\mu \gamma_5 \\ &\left. + g_3^{\Xi_{bc}^0 \bar{p}} \sigma_{\mu\nu} \gamma_5 \frac{q^\nu}{m_T} \right] u(p_{\Xi_{bc}^0}) \varepsilon_T^\mu(p_T), \end{aligned} \quad (16)$$

where $u(p_{\Xi_{bc}^0})$ and $v(p_p)$ are the wave-functions of the Ξ_{bc}^0 -baryon and antiproton with the four-momenta $p_{\Xi_{bc}^0}$ and p_p , respectively, $q = p_{\Xi_{bc}^0} - p_p$, $\varepsilon_T^\mu(p_T)$ is the polarisation vector of the axial-vector tetraquark with the four-momentum $p_T = p_{\Xi_{bc}^0} + p_p$ and mass m_T . The $f_i^{\Xi_{bc}^0 \bar{p}}$ and

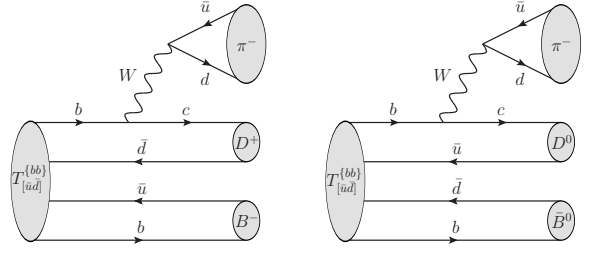


FIG. 8: Feynman diagrams which yield three-body mesonic final states in $T_{[u\bar{d}]}^{\{bb\}-}$ decays due to the b -quark decay $b \rightarrow c + d + \bar{u}$.

$g_i^{\Xi_{bc}^0 \bar{p}}$, with $i = 1, 2, 3$, are two sets of form factors, to be evaluated at a known kinematic point. Similar amplitude corresponds to the $T_{[u\bar{d}]}^{\{bb\}-} \rightarrow \Omega_{bc}^0 \bar{\Lambda}_c^-$ decay described by the right diagram in Fig. 7.

Inspired by the B -meson decay data [15]:

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-) &= (2.52 \pm 0.13) \times 10^{-3}, \\ \mathcal{B}(\bar{B}^0 \rightarrow D^+ D_s^-) &= (7.2 \pm 0.8) \times 10^{-3}, \end{aligned} \quad (17)$$

one infers that the branching fractions for $T_{[u\bar{d}]}^{\{bb\}-} \rightarrow \Xi_{bc}^0 \bar{p}$ and $T_{[u\bar{d}]}^{\{bb\}-} \rightarrow \Omega_{bc}^0 \bar{\Lambda}_c^-$ decays might also reach the same magnitude, i. e., of order of 10^{-3} .

One needs to further reconstruct the bottom-charmed baryons Ξ_{bc}^0 and Ω_{bc}^0 . Including the decay chain $\Xi_{bc}^0 \rightarrow \Lambda_b K^- \pi^+$, we find that the two-body baryonic decay modes of the double-bottom tetraquarks are expected to have branching fractions of order of 10^{-6} .

B. Three-body Mesonic Decay Modes of $T_{[u\bar{d}]}^{\{bb\}-}$

1. Open-Charm Final States

There are some Feynman diagrams with the help of which the three-body purely mesonic final states with the open charm in the $T_{[u\bar{d}]}^{\{bb\}-}$ decays can be identified. They are shown in Fig. 8. The factorisable amplitudes of these decays can be written as follows:

$$\begin{aligned} \mathcal{M}(T_{[u\bar{d}]}^{\{bb\}-} \rightarrow B^- D^+ \pi^-) &= i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1^{\text{eff}} f_\pi p_\pi^\mu \quad (18) \\ &\times \langle (BD)_{J_{BD}}^0(p_{BD}) | \bar{d} \gamma_\mu (1 - \gamma_5) u | T_{[u\bar{d}]}^{\{bb\}-}(p_T) \rangle, \end{aligned}$$

$$\begin{aligned} \mathcal{M}(T_{[u\bar{d}]}^{\{bb\}-} \rightarrow \bar{B}^0 D^0 \pi^-) &= i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1^{\text{eff}} f_\pi p_\pi^\mu \quad (19) \\ &\times \langle (BD)_{J_{BD}}^0(p_{BD}) | \bar{s} \gamma_\mu (1 - \gamma_5) c | T_{[u\bar{d}]}^{\{bb\}-}(p_T) \rangle, \end{aligned}$$

where $a_1^{\text{eff}} = C_1 + C_2/N_c$, with $N_c = 3$ being the number of quark colors, and the standard definition of the π -meson leptonic decay constant is used:

$$\langle \pi^-(p_\pi) | \bar{d}(0) \gamma^\mu (1 - \gamma_5) u(0) | 0 \rangle = i f_\pi p_\pi^\mu. \quad (20)$$

⁵ The mass assignment is according to the predictions of Ref. [7].

Next, we need to parametrise the transition matrix elements from the tetraquark state to the double-meson one, having the total angular momentum J_{BD} and zero electric charge (specified by the superscript index), in terms of the form factors.

For the case of the D - and B -mesons, which are pseudoscalar particles, the total momentum J_{BD} is completely determined by the angular momentum L_{BD} of the system. For the BD -system in the S -wave, the transition matrix elements:

$$\langle (BD)_0^0(p_{BD}) | \bar{c}(0)\gamma_\mu b(0) | T_{[\bar{u}\bar{d}]}^{\{bb\}^-}(p_T) \rangle, \quad (21)$$

$$\langle (BD)_0^0(p_{BD}) | \bar{c}(0)\gamma_\mu\gamma_5 b(0) | T_{[\bar{u}\bar{d}]}^{\{bb\}^-}(p_T) \rangle, \quad (22)$$

define the S -wave generalised form factors.

One can also expect a production of $(B^*D)_{J_{BD}}^0$ or $(BD^*)_{J_{BD}}^0$ pairs in the S -wave and the total angular momentum ($J_{B^*D} = 1$ or $J_{BD^*} = 1$) is determined by the spin of the vector meson ($S_{B^*} = 1$ or $S_{D^*} = 1$). This requires another set of S -wave generalised form factors.

Currently, we lack a reliable dynamical approach to calculate the generalised form factors, which prevents us to reliably predict the branching fractions for $T_{[\bar{u}\bar{d}]}^{\{bb\}^-} \rightarrow B^- D^+ \pi^-$ and $T_{[\bar{u}\bar{d}]}^{\{bb\}^-} \rightarrow \bar{B}^0 D^0 \pi^-$ decays. However, as after the b -quark weak decay, the emitted charmed quark and three spectators have an invariant mass larger than the $B^- D^+$ and $\bar{B}^0 D^0$ thresholds, we expect the split into two hadrons will not cause any dynamical suppression. In this case, the transition $T_{[\bar{u}\bar{d}]}^{\{bb\}^-} \rightarrow (B^- D^+, \bar{B}^0 D^0)$ is expected to be comparable with the $B \rightarrow D$ transition. Accordingly, their branching fractions can reach the value of 10^{-3} as well.

From Fig. 8, one may speculate that the final $B^- D^+$ and $\bar{B}^0 D^0$ mesons arise from an intermediate bottom-charmed tetraquark state $T_{\bar{u}\bar{d}}^{bc}$. In the case of a scalar ($J^P = 0^+$) tetraquark, the transition amplitude is governed by the four form factors:

$$\begin{aligned} \langle T_{\bar{u}\bar{d}}^{bc}(p_{T'}) | \bar{c}\gamma_\mu b | T_{[\bar{u}\bar{d}]}^{\{bb\}^-}(p_T) \rangle &= \frac{-2V(q^2)}{m_T + m_{T'}} \epsilon_{\mu\nu\rho\sigma} \epsilon_T^\nu p_T^\rho p_{T'}^\sigma, \\ \langle T_{\bar{u}\bar{d}}^{bc}(p_{T'}) | \bar{c}\gamma^\mu\gamma_5 b | T_{[\bar{u}\bar{d}]}^{\{bb\}^-}(p_T) \rangle &= 2i m_T A_0(q^2) \frac{\epsilon_T \cdot q}{q^2} q^\mu \\ &+ i(m_T + m_{T'}) A_1(q^2) \left[\epsilon_T^\mu - \frac{\epsilon_T \cdot q}{q^2} q^\mu \right] \\ &- \frac{iA_2(q^2)(\epsilon_T \cdot q)}{m_T + m_{T'}} \left[(p_T^\mu + p_{T'}^\mu) - \frac{m_T^2 - m_{T'}^2}{q^2} q^\mu \right], \quad (23) \end{aligned}$$

where $q^\mu = p_T^\mu - p_{T'}^\mu$, ϵ_T^μ is the polarization vector of the axial-vector $T_{[\bar{u}\bar{d}]}^{\{bb\}^-}$ tetraquark, and m_T and $m_{T'}$ are the masses of $T_{[\bar{u}\bar{d}]}^{\{bb\}^-}$ and $T_{\bar{u}\bar{d}}^{bc}$, respectively. The decay width for the $T_{[\bar{u}\bar{d}]}^{\{bb\}^-} \rightarrow T_{\bar{u}\bar{d}}^{bc} \pi^-$ channel is then evaluated as

$$\Gamma(T_{[\bar{u}\bar{d}]}^{\{bb\}^-} \rightarrow T_{\bar{u}\bar{d}}^{bc} \pi^-) = 2 \times \frac{1}{3} \frac{|\vec{p}_T|}{8\pi m_T^2} \sum_\lambda |\mathcal{M}|^2, \quad (24)$$

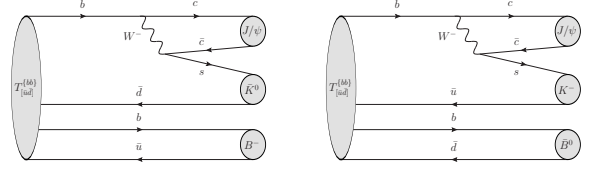


FIG. 9: Feynman diagrams which yield three-body mesonic final states in $T_{[\bar{u}\bar{d}]}^{\{bb\}^-}$ decays due to the b -quark decay $b \rightarrow c + s + \bar{c}$.

with the decay amplitude

$$\mathcal{M} = i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1^{\text{eff}} f_\pi \times 2i m_T A_0(q^2 = 0) (\epsilon_T \cdot q) \quad (25)$$

The factor 2 arises since there are two b -quarks in the initial state, while the factor $1/3$ denotes the spin average. Neglecting the π -meson mass, the decay width can be written as follows:

$$\begin{aligned} \Gamma(T_{[\bar{u}\bar{d}]}^{\{bb\}^-} \rightarrow T_{\bar{u}\bar{d}}^{bc} \pi^-) &= \frac{G_F^2 m_T^3}{48\pi} |V_{ud} V_{cb}^*|^2 f_\pi^2 \\ &\times (a_1^{\text{eff}})^2 \left[1 - \frac{m_{T'}^2}{m_T^2} \right]^3 |A_0(q^2 = 0)|^2, \quad (26) \end{aligned}$$

where $G_F = 1.166 \times 10^{-5}$ GeV $^{-2}$, $f_\pi \simeq 130$ MeV, $V_{ud} = 0.974$, and $|V_{cb}| = 40.5 \times 10^{-3}$ [15]. The effective coefficient a_1^{eff} is a scale-dependent quantity but in a wide range of the energy scale its value is close to unity, so we take $a_1^{\text{eff}} = 1$ in estimates. Using $m_T = 10.5$ GeV, the $\Xi_{bb} \rightarrow \Xi_{bc}$ transition form factor, $A_0(q^2 = 0) = 0.44$ [31], and $m_{T'} = m_T + m_c - m_b \sim 7.2$ GeV, we obtain the estimate of the partial decay width:

$$\Gamma(T_{[\bar{u}\bar{d}]}^{\{bb\}^-} \rightarrow T_{\bar{u}\bar{d}}^{bc} \pi^-) \simeq 7.9 \times 10^{-16} \text{ GeV} \simeq 1.2 \text{ ns}^{-1}, \quad (27)$$

and the branching fraction with account of the lifetime (14):

$$\mathcal{B}(T_{[\bar{u}\bar{d}]}^{\{bb\}^-} \rightarrow T_{\bar{u}\bar{d}}^{bc} \pi^-) \simeq 1.0 \times 10^{-3}. \quad (28)$$

If one uses the $B_c \rightarrow J/\psi$ transition form factor instead, $A_0(q^2 = 0) = 0.53$ [32], the results can be enhanced by approximately 50%. As the intermediate state, $T_{\bar{u}\bar{d}}^{bc}$, will subsequently turn into the $B^- D^+$ and $\bar{B}^0 D^0$ states with equal probabilities, and thus we expect

$$\begin{aligned} \mathcal{B}(T_{[\bar{u}\bar{d}]}^{\{bb\}^-} \rightarrow B^- D^+ \pi^-) &= \mathcal{B}(T_{[\bar{u}\bar{d}]}^{\{bb\}^-} \rightarrow \bar{B}^0 D^0 \pi^-) \\ &\simeq 0.5 \times 10^{-3}. \quad (29) \end{aligned}$$

2. Hidden-Charm Final States

There are some Feynman diagrams in which the hidden-charm mesons, such as J/ψ and ψ' , can be produced. The corresponding Feynman diagrams are shown

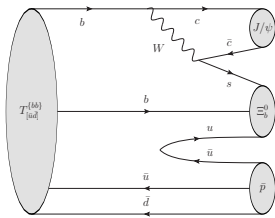


FIG. 10: Feynman diagrams which yield three-body baryonic final states in $T_{[\bar{u}d]}^{\{bb\}-}$ decays due to the b -quark decay $b \rightarrow c + s + \bar{c}$.

in Fig. 9. Decay channels include

$$\begin{aligned} T_{[\bar{u}d]}^{\{bb\}} &\rightarrow J/\psi \bar{K}^0 B^-, \\ T_{[\bar{u}d]}^{\{bb\}} &\rightarrow J/\psi K^- \bar{B}^0, \end{aligned} \quad (30)$$

whose CKM factors are $V_{cb}^* V_{cs}$. Their decay branching ratios can be comparable with the $B \rightarrow J/\psi K$ decays [15]:

$$\begin{aligned} \mathcal{B}(B^+ \rightarrow J/\psi K^+) &= (1.026 \pm 0.031) \times 10^{-3}, \\ \mathcal{B}(\bar{B}^0 \rightarrow J/\psi \bar{K}^0) &= (8.73 \pm 0.32) \times 10^{-4}. \end{aligned} \quad (31)$$

So, it reasonable also to expect branching fractions of the $T_{[\bar{u}d]}^{\{bb\}}$ tetraquark decays (30) at the level of few 10^{-4} .

C. Three-body baryonic decay modes of $T_{[\bar{u}d]}^{\{bb\}-}$

The other type of diagrams with hidden-charm have a light- and a b -baryon in the final state. A representative example of these decays $T_{[c\bar{d}]}^{\{bb\}-} \rightarrow \Xi_c^0 \bar{p} J/\psi$, which can be measured at the colliders, is presented in Fig. 10. As the two-body baryonic decays discussed earlier, the corresponding amplitudes are non-factorisable as the quark pair produced in the weak decay is divided and hadronised into a baryon and meson. An account of a quark pair picked up from the vacuum makes the theoretical analysis even more complicated and we postpone such a discussion for future.

V. WEAK DECAYS OF $T_{[\bar{u}s]}^{\{bb\}}(10643)^-$ AND $T_{[\bar{d}s]}^{\{bb\}}(10643)^0$

As mentioned earlier, in the case of the double-bottom tetraquarks having the strange antiquark, $T_{[\bar{u}s]}^{\{bb\}}(10643)^-$ and $T_{[\bar{d}s]}^{\{bb\}}(10643)^0$ ⁶, their masses are closer to the corresponding hadronic threshold, $\bar{B}^* \bar{B}_s$, but are still

estimated to lie below it. In particular, these tetraquark states are predicted to be below the threshold (10691 MeV) by 48 MeV in Ref. [7]. If this is true, their decay modes are dominated by flavor-changing charged currents in the effective Hamiltonian (15). Like the decays of the tetraquark $T_{[\bar{u}d]}^{\{bb\}}$, they have corresponding two-body and three-body decays, which can be divided into the purely mesonic and baryonic modes. They are briefly discussed below.

A. Two-Body Baryonic Decays

As the double-bottom tetraquark $T_{[\bar{u}s]}^{\{bb\}}(10643)^-$ consists of two b -quarks and two antiquarks, \bar{u} and \bar{s} , the weak decay of one of the b -quarks results into two possibilities: $b \rightarrow c + d + \bar{u}$ or $b \rightarrow c + s + \bar{c}$. From three quarks and three antiquarks one can easily construct baryon and anti-baryon. Golden modes are $T_{[\bar{u}s]}^{\{bb\}-} \rightarrow \Xi_{bc}^0 \bar{\Sigma}^-$ and $T_{[\bar{u}s]}^{\{bb\}-} \rightarrow \Omega_{bc}^0 \bar{\Xi}_c^-$. For $T_{[\bar{d}s]}^{\{bb\}}(10643)^0$, one can replace the spectator u -antiquark by the d -antiquark which results the decay channels: $T_{[\bar{d}s]}^{\{bb\}0} \rightarrow \Xi_{bc}^0 (\bar{\Lambda}^0, \bar{\Sigma}^0)$ and $T_{[\bar{d}s]}^{\{bb\}0} \rightarrow \Omega_{bc}^0 \bar{\Xi}_c^0$.

B. Three-body Mesonic Decay Modes

1. Open-Charm Final State

Three-body mesonic final states with the open charm include the following decay modes $T_{[\bar{u}s]}^{\{bb\}-} \rightarrow B^- D_s^+ \pi^-$, $T_{[\bar{u}s]}^{\{bb\}-} \rightarrow \bar{B}_s^0 D^0 \pi^-$, for the charged strange double-bottom tetraquark and $T_{[\bar{d}s]}^{\{bb\}0} \rightarrow \bar{B}^0 D_s^+ \pi^-$, $T_{[\bar{d}s]}^{\{bb\}0} \rightarrow \bar{B}_s^0 D^+ \pi^-$ for the neutral one.

2. Hidden-Charm Final State

There are channels in which the hidden-charm mesons like J/ψ , ψ' , and etc. can be produced:

$$\begin{aligned} T_{[\bar{u}s]}^{\{bb\}-} &\rightarrow J/\psi \phi B^-, & T_{[\bar{d}s]}^{\{bb\}0} &\rightarrow J/\psi \phi \bar{B}^0, \\ T_{[\bar{u}s]}^{\{bb\}-} &\rightarrow J/\psi K^- \bar{B}_s^0, & T_{[\bar{d}s]}^{\{bb\}0} &\rightarrow J/\psi \bar{K}^0 \bar{B}_s^0. \end{aligned} \quad (32)$$

The final states mentioned here are well reconstructed at both electron-positron and hadron colliders.

C. Three-body baryonic decay modes

The other type of diagrams correspond to three-body decays with a light- and bottom baryon in the final state. The most interesting processes could be $T_{[\bar{u}s]}^{\{bb\}-} \rightarrow$

⁶ The mass assignment is according to the predictions of Ref. [7].

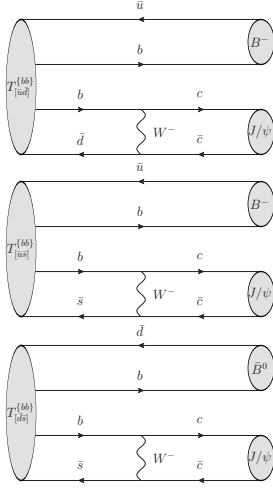


FIG. 11: The W -exchange diagrams resulting into two-body mesonic decays with the production of the B - and J/ψ -mesons in $T_{[u\bar{d}]}^{\{bb\}}$ decays (upper frame), $T_{[u\bar{s}]}^{\{bb\}}$ decays (middle frame), and $T_{[d\bar{s}]}^{\{bb\}}$ decays (lower frame).

$$\Xi_b^0 \bar{\Sigma}^- J/\psi, T_{[u\bar{s}]}^{\{bb\}-} \rightarrow \Xi_b^- \bar{\Lambda} J/\psi, T_{[d\bar{s}]}^{\{bb\}0} \rightarrow \Xi_b^0 \bar{\Sigma}^0 J/\psi$$

and $T_{[d\bar{s}]}^{\{bb\}0} \rightarrow \Xi_b^- \bar{\Sigma}^+ J/\psi$.

VI. W -EXCHANGE DIAGRAMS

The W -exchange diagrams result into two-body mesonic decays, some of which can be of interest at the LHC. The use of W -exchange decay modes of double-heavy baryons has been advocated in Ref. [33]. The decays with the J/ψ -meson production are the most likely. So, the processes which can be searched for are $T_{[u\bar{d}]}^{\{bb\}}(10482)^- \rightarrow B^- J/\psi$, $T_{[u\bar{s}]}^{\{bb\}}(10643)^- \rightarrow B^- J/\psi$, and $T_{[d\bar{s}]}^{\{bb\}}(10643)^0 \rightarrow \bar{B}^0 J/\psi$. The corresponding diagrams for these processes are presented in Fig. 11.

The decay amplitude to leading order can be factorized. For the $T_{[u\bar{s}]}^{\{bb\}}(10643)^- \rightarrow B^- J/\psi$ decay, as an example, one can write it as follows:

$$\mathcal{M}(T_{[u\bar{s}]}^{\{bb\}-} \rightarrow B^- J/\psi) = \frac{G_F}{\sqrt{2}} V_{cd} V_{cs}^* a_2^{\text{eff}} m_{J/\psi} f_{J/\psi} \varepsilon_\psi^{*\mu} \times \langle B^-(p_B) | \bar{s} \gamma_\mu (1 - \gamma_5) b | T_{[u\bar{s}]}^{\{bb\}-}(p_T, \varepsilon_T) \rangle, \quad (33)$$

where $a_2^{\text{eff}} = C_2 + C_1/N_c$ is the effective Wilson coefficient and the matrix element between the vacuum and J/ψ -meson is parametrised in terms of the decay constant:

$$\langle J/\psi(p_\psi, \varepsilon_\psi) | \bar{c} \gamma^\mu P_L c | 0 \rangle = \frac{1}{2} m_{J/\psi} f_{J/\psi} \varepsilon_\psi^{*\mu}. \quad (34)$$

The general decomposition of the transition matrix element can be written in the form similar to the $B \rightarrow A$ transition, where A is an axial-vector meson [34].

An advantage of these decay modes is that there are only two mesons in the final state. The J/ψ final state can be easily reconstructed in the $\mu^+ \mu^-$ channel, and the bottom meson can be studied in its decays into two-body final states. For the $T_{[u\bar{s}]}^{\{bb\}}(10643)^- \rightarrow B^- J/\psi$, and $T_{[d\bar{s}]}^{\{bb\}}(10643)^0 \rightarrow \bar{B}^0 J/\psi$, the CKM factors in the transition are $V_{cb} V_{cs}^*$, and thus they can have sizeable branching fractions. Without any additional power suppression, we expect the branching ratios for these two modes to be of order of 10^{-3} . For the $T_{[u\bar{d}]}^{\{bb\}}(10643)^- \rightarrow B^- J/\psi$ decay, the decay width is suppressed due the CKM factor V_{cd} , leading to a smaller branching fraction by approximately the factor 25.

VII. CONCLUSIONS

We have presented the estimate of the double-bottom hadron production in Z -boson decays. These include the tetraquarks $T_{[u\bar{d}]}^{\{bb\}}$, $T_{[u\bar{s}]}^{\{bb\}}$ and $T_{[d\bar{s}]}^{\{bb\}}$, and the double-bottom baryons, $\Xi_{bb}^0(bb u)$, $\Xi_{bb}^-(bb d)$, and $\Omega_{bb}^-(bbs)$, which are expected to be produced as the jet-fragments of the (bb) -diquark jet in the process $e^+ e^- \rightarrow Z \rightarrow b\bar{b}b\bar{b}$. Using the Monte Carlo generators MadGraph5_aMC@NLO and Pythia6, we estimate the branching ratios $\mathcal{B}(Z \rightarrow T_{[u\bar{d}]}^{\{bb\}} + \bar{b}\bar{b}) = (1.4_{-0.5}^{+1.1}) \times 10^{-6}$, and about the same for the sum of the branching ratios $\mathcal{B}(Z \rightarrow T_{[u\bar{s}]}^{\{bb\}} + X)$ and $\mathcal{B}(Z \rightarrow T_{[d\bar{s}]}^{\{bb\}} + X)$. The summed branching ratios of the double-bottom baryons is also estimated compared to the double-bottom tetraquarks: $\mathcal{B}(Z \rightarrow (\Xi_{bb}^0, \Xi_{bb}^-, \Omega_{bb}^-) + X) : \mathcal{B}(Z \rightarrow T_{[q\bar{q}]}^{\{bb\}} + X) \approx 5.8 : 1$. Thus, the double-bottom baryons will be produced about six times more frequently in Z -boson decays.

The lifetimes of the tetraquarks $T_{[q\bar{q}]}^{\{bb\}}$ are estimated, and for the $SU(3)_F$ -multiplet they are expected to be approximately similar, about one half of the B -meson lifetimes, $\tau \simeq 0.8$ ps. We also discussed some signature decay modes of these tetraquarks. These include the two-body baryonic decays, such as $T_{[u\bar{d}]}^{\{bb\}-} \rightarrow \Xi_{bc}^0 \bar{p}$ and $T_{[u\bar{d}]}^{\{bb\}-} \rightarrow \Omega_{bc}^0 \bar{\Lambda}_c^-$, whose branching ratios are estimated to be of order of 10^{-3} . Likewise, three-body mesonic decays of $T_{[u\bar{d}]}^{\{bb\}-}$ into final states, such as $B^- D^+ \pi^-$ and $B^0 D^0 \pi^-$, are estimated, and their branching ratios may also reach order of 10^{-3} . Of particular interest in this class of decays are those which have hidden-charm mesons in the final states, such as $J/\psi \bar{K}^0 B^-$ and $J/\psi K^- \bar{B}^0$. The decay branching ratios $\mathcal{B}(T_{[u\bar{d}]}^{\{bb\}-} \rightarrow J/\psi \bar{K}^0 B^-)$ and $\mathcal{B}(T_{[u\bar{d}]}^{\{bb\}-} \rightarrow J/\psi K^- \bar{B}^0)$ may be comparable to that of $B \rightarrow J/\psi K$, which are also of order of 10^{-3} [15]. The decays of the other two members of the $SU(3)_F$ -triplet, $T_{[u\bar{s}]}^{\{bb\}-}$ and $T_{[d\bar{s}]}^{\{bb\}0}$ are also discussed, and the order of magnitude

of their signature decay modes are also presented. Finally, we emphasise the so-called W -exchange diagrams, which give rise to potentially interesting two-body decays $T_{[\bar{u}d]}^{\{bb\}}(10482)^- \rightarrow B^- J/\psi$, $T_{[\bar{u}s]}^{\{bb\}}(10643)^- \rightarrow B^- J/\psi$, and $T_{[\bar{d}s]}^{\{bb\}}(10643)^0 \rightarrow \bar{B}^0 J/\psi$. We estimate that the product branching ratios in many of the decay modes discussed here are not expected to exceed 10^{-5} . Hence, several of these decay modes will be required to measure the masses of the tetraquarks to establish them firmly. Thus, we recommend to aim for an integrated luminosity of 10^{12} Z -bosons at the Tera- Z factories being contemplated now. They will greatly advance the physics of the double-heavy hadrons, both in the mesonic and baryonic sectors.

Acknowledgements

We would like to thank Marek Karliner, Xiang Liu, Sheldon Stone, Fu-Sheng Yu and Zhi-Jie Zhao for help-

ful discussions. We thank Cai-Dian Lü, the Center for Future High Energy Physics, and IHEP, Beijing, for their generous hospitality. This work is supported in part by DFG Forschergruppe FOR 1873 “Quark Flavour Physics and Effective Field Theories”, the National Natural Science Foundation of China under Grant Nos. 11575110, 11655002, 11735010, and the Natural Science Foundation of Shanghai under Grant No. 15DZ2272100, This research is partially supported by the “YSU Initiative Scientific Research Activity” (Project No. AAAA-A16-116070610023-3).

-
- [1] A. Ali, J. S. Lange and S. Stone, *Prog. Part. Nucl. Phys.* **97**, 123 (2017) doi:10.1016/j.ppnp.2017.08.003 [arXiv:1706.00610 [hep-ph]].
- [2] A. Esposito, A. Pilloni and A. D. Polosa, *Phys. Rept.* **668**, 1 (2016) doi:10.1016/j.physrep.2016.11.002 [arXiv:1611.07920 [hep-ph]].
- [3] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, *Phys. Rept.* **639**, 1 (2016) doi:10.1016/j.physrep.2016.05.004 [arXiv:1601.02092 [hep-ph]].
- [4] F. K. Guo, C. Hanhart, U. G. Meissner, Q. Wang, Q. Zhao and B. S. Zou, *Rev. Mod. Phys.* **90**, no. 1, 015004 (2018) doi:10.1103/RevModPhys.90.015004 [arXiv:1705.00141 [hep-ph]].
- [5] S. L. Olsen, T. Skwarnicki and D. Zieminska, *Rev. Mod. Phys.* **90**, no. 1, 015003 (2018) doi:10.1103/RevModPhys.90.015003 [arXiv:1708.04012 [hep-ph]].
- [6] M. Karliner and J. L. Rosner, *Phys. Rev. Lett.* **119**, no. 20, 202001 (2017) doi:10.1103/PhysRevLett.119.202001 [arXiv:1707.07666 [hep-ph]].
- [7] E. J. Eichten and C. Quigg, *Phys. Rev. Lett.* **119**, no. 20, 202002 (2017) doi:10.1103/PhysRevLett.119.202002 [arXiv:1707.09575 [hep-ph]].
- [8] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, *Phys. Rev. Lett.* **118**, no. 14, 142001 (2017) doi:10.1103/PhysRevLett.118.142001 [arXiv:1607.05214 [hep-lat]].
- [9] P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer and M. Wagner, *Phys. Rev. D* **96**, no. 5, 054510 (2017) doi:10.1103/PhysRevD.96.054510 [arXiv:1704.02383 [hep-lat]].
- [10] T. Mehen, *Phys. Rev. D* **96**, no. 9, 094028 (2017) doi:10.1103/PhysRevD.96.094028 [arXiv:1708.05020 [hep-ph]].
- [11] A. Czarnecki, B. Leng and M. B. Voloshin, *Phys. Lett. B* **778**, 233 (2018) doi:10.1016/j.physletb.2018.01.034 [arXiv:1708.04594 [hep-ph]].
- [12] J. P. Ader, J. M. Richard and P. Taxil, *Phys. Rev. D* **25**, 2370 (1982). doi:10.1103/PhysRevD.25.2370
- [13] A. V. Manohar and M. B. Wise, *Nucl. Phys. B* **399**, 17 (1993) doi:10.1016/0550-3213(93)90614-U [hep-ph/9212236].
- [14] D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha, *Phys. Rev. D* **76**, 114015 (2007) doi:10.1103/PhysRevD.76.114015 [arXiv:0706.3853 [hep-ph]].
- [15] C. Patrignani *et al.* [Particle Data Group], *Chin. Phys. C* **40**, no. 10, 100001 (2016). doi:10.1088/1674-1137/40/10/100001
- [16] G. F. Sterman and S. Weinberg, *Phys. Rev. Lett.* **39**, 1436 (1977). doi:10.1103/PhysRevLett.39.1436
- [17] J. Alwall *et al.*, *JHEP* **1407**, 079 (2014) doi:10.1007/JHEP07(2014)079 [arXiv:1405.0301 [hep-ph]].
- [18] T. Sjostrand, S. Mrenna and P. Z. Skands, *JHEP* **0605**, 026 (2006) doi:10.1088/1126-6708/2006/05/026 [hep-ph/0603175].
- [19] Z. Yang, X. G. Wu, G. Chen, Q. L. Liao and J. W. Zhang, *Phys. Rev. D* **85**, 094015 (2012) doi:10.1103/PhysRevD.85.094015 [arXiv:1112.5169 [hep-ph]].
- [20] T. Aaltonen *et al.* [CDF Collaboration], *Phys. Rev. D* **77**, 072003 (2008) doi:10.1103/PhysRevD.77.072003 [arXiv:0801.4375 [hep-ex]].
- [21] R. Aaij *et al.* [LHCb Collaboration], *Phys. Rev. D* **85**, 032008 (2012) doi:10.1103/PhysRevD.85.032008 [arXiv:1111.2357 [hep-ex]].
- [22] R. Aaij *et al.* [LHCb Collaboration], *JHEP* **1408**, 143 (2014) doi:10.1007/JHEP08(2014)143 [arXiv:1405.6842 [hep-ex]].
- [23] R. Aaij *et al.* [LHCb Collaboration], *Phys. Rev. Lett.* **119**, no. 11, 112001 (2017) doi:10.1103/PhysRevLett.119.112001 [arXiv:1707.01621 [hep-ex]].

- [24] A. Esposito, M. Papinutto, A. Pilloni, A. D. Polosa and N. Tantalo, *Phys. Rev. D* **88**, no. 5, 054029 (2013) doi:10.1103/PhysRevD.88.054029 [arXiv:1307.2873 [hep-ph]].
- [25] S. Q. Luo, K. Chen, X. Liu, Y. R. Liu and S. L. Zhu, *Eur. Phys. J. C* **77**, no. 10, 709 (2017) doi:10.1140/epjc/s10052-017-5297-4 [arXiv:1707.01180 [hep-ph]].
- [26] W. Wang, Z. P. Xing and J. Xu, *Eur. Phys. J. C* **77**, no. 11, 800 (2017) doi:10.1140/epjc/s10052-017-5363-y [arXiv:1707.06570 [hep-ph]].
- [27] Y. J. Shi, W. Wang, Y. Xing and J. Xu, *Eur. Phys. J. C* **78**, no. 1, 56 (2018) doi:10.1140/epjc/s10052-018-5532-7 [arXiv:1712.03830 [hep-ph]].
- [28] A. H. Hoang, *Nucl. Phys. Proc. Suppl.* **86**, 512 (2000) doi:10.1016/S0920-5632(00)00613-7 [hep-ph/9909356].
- [29] A. Lenz, *Int. J. Mod. Phys. A* **30**, no. 10, 1543005 (2015) doi:10.1142/S0217751X15430058 [arXiv:1405.3601 [hep-ph]].
- [30] A. V. Manohar and M. B. Wise, *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.* **10**, 1 (2000).
- [31] W. Wang, F. S. Yu and Z. X. Zhao, *Eur. Phys. J. C* **77**, no. 11, 781 (2017) doi:10.1140/epjc/s10052-017-5360-1 [arXiv:1707.02834 [hep-ph]].
- [32] W. Wang, Y. L. Shen and C. D. Lu, *Phys. Rev. D* **79**, 054012 (2009) doi:10.1103/PhysRevD.79.054012 [arXiv:0811.3748 [hep-ph]].
- [33] R. H. Li, C. D. Lu, W. Wang, F. S. Yu and Z. T. Zou, *Phys. Lett. B* **767**, 232 (2017) doi:10.1016/j.physletb.2017.02.003 [arXiv:1701.03284 [hep-ph]].
- [34] R. H. Li, C. D. Lu and W. Wang, *Phys. Rev. D* **79**, 094024 (2009) doi:10.1103/PhysRevD.79.094024 [arXiv:0902.3291 [hep-ph]].