DESY 18-063

arXiv:1805.02659v2 [hep-th] 18 Sep 2018

Pole N-flation

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Abstract: A second order pole in the scalar kinetic term can lead to a class of inflation models with universal predictions referred to as pole inflation or α-attractors. While this kinetic structure is ubiquitous in supergravity effective field theories, realising a consistent UV complete model in e.g. string theory is a non-trivial task. For one, one expects quantum corrections arising in the vicinity of the pole which may spoil the typical attractor dynamics. As a conservative estimate of the range of validity of supergravity models of pole inflation we employ the weak gravity conjecture (WGC). We find that this constrains the accessible part of the inflationary plateau by limiting the decay constant of the axion partner. For the original single complex field models, the WGC does not even allow the inflaton to reach the inflationary plateau region. We analyze if evoking the assistance of N scalar fields from the open string moduli helps addressing these problems. Pole N-flation improves radiative control by reducing the required range of each individual field. However, the WGC bound prohibiting pole inflation for a single such field *persists* even for a collective motion of N such scalars if we impose the lattice WGC as its strongest form. Finally, we outline steps towards an embedding of pole N-flation in type IIB string theory on fibred Calabi-Yau manifolds.

Contents

1 Introduction

The paradigm of cosmological inflation seemingly explains the origin of spatial homogeneity and isotropy, as well as the seeding process for cosmic structure formation. However, its physical origin remains unclear. High precision studies of the cosmic microwave background have revealed the primordial curvature perturbation to have extremely simple statistics: gaussian to very high precision and describable by just two numbers, the amplitude and tilt of its power spectrum [1]. It is therefore important to identify what microphysical mechanism could be at the origin of this observation. One possibility is that there is a symmetry present in the underlying theory which ultimately forbids contributions to the inflationary potential capable of giving rise to features in the observed data, *i.e.* a fundamental reason why the inflation potential must take a simple form. A more recent suggestion is that a simple spectrum could be an emergent property of some type of large N dynamics, be it through a large number of terms contributing to a single scalar field potential [2–4] or by the interaction of a very large number of scalar fields [5, 6]. A third possibility is that there is some structure in the underlying theory which makes inflation insensitive to a broad array of microphysical details. This would essentially give rise to a universality class, where a diverse range of models result in the same predictions for observable quantities.

A particularly dramatic example of this third possibility is the universality displayed by the class of models termed 'pole inflation' [7, 8] (or 'α-attractors' for a special subclass thereof [9, 10]). This class of models is defined by the presence of a pole in the kinetic term, such as

$$\mathcal{L}_{kin} = -\frac{3\alpha}{4}\frac{(\partial\tau)^2}{\tau^2}\,, \quad \alpha = \mathcal{O}(1)\,, \tag{1.1}$$

which renders the dynamics of inflation insensitive to the details of a generic scalar potential, provided this can be expanded around the pole as[1]

$$V = V_0 - a_1\tau + \mathcal{O}(\tau^2)\,. \tag{1.2}$$

These models lead to a universal prediction given by $n_s = 1-2/N_e$ and $r = 12\alpha/N_e^2$, where N_e is the number of e-folds of expansion between the pivot scale leaving the horizon and the end of inflation. These predictions are in remarkable agreement with observations and are fully determined by the order of the pole (which sets the deviation of n_s from perfect scale invariance; for general pole of order p, $n_s = 1 - p/N_e$ - see e.g. [8, 11]), its residue (which controls the amplitude of primordial gravity waves), and the value of N_e, which depends on details of post-inflationary physics.

The kinetic structure of this class of models is of special interest as they can result from logarithmic Kähler potentials in 4D $\mathcal{N} = 1$ supergravity effective scenarios, which are abundant in the context of string compactifications (a more detailed discussion is given in Sec. 2). Schematically, one can have

$$K = -3\alpha\ln(T+\bar{T}) \qquad \text{or} \qquad K = -3\alpha\ln(1-\Phi\bar{\Phi})\,, \tag{1.3}$$

where the first hermitian function is defined in the 'half plane' $T+\bar{T} > 0$, while the domain of the second one is the 'unit disk' $\Phi\bar{\Phi} < 1$.[2] The corresponding component of the Kähler metrics are given by

$$K_{T\bar{T}} = \frac{3\alpha}{(T+\bar{T})^2} \qquad \text{or} \qquad K_{\Phi\bar{\Phi}} = \frac{3\alpha}{(1-\Phi\bar{\Phi})^2}\,, \tag{1.4}$$

with subscripts denoting partial derivatives. Just as eq. (1.1), they have a second-order pole in the real part of T and radial direction of Φ, respectively, such that these fields can act as the inflaton. Interestingly in this context, the presence of the kinetic pole has the geometric interpretation of the existence of a boundary in moduli space. Note that, as both T and Φ are complex, the inflaton always comes together with a partner scalar degree of freedom which we will argue to be axionic.

While this class of models arises in 4D $\mathcal{N} = 1$ supergravity (see also [14, 15]), it is important to understand if it corresponds to a low-energy effective description that can consistently be embedded in a quantum theory of gravity, like string theory. Various consistency requirements such as the convergence of the higher instanton corrections, the weak gravity conjecture or the swampland conjectures [16–18] place strong bounds on the nature of effective theories that can be embedded in quantum gravity.

[1] The invariance of the kinetic term (1.1) under the inversion symmetry $\tau \to 1/\tau$ makes this model equivalent to one also with negligible kinetic term and a scalar potential $V = V_0 - a_1/\tau + \mathcal{O}(1/\tau^2)$.

[2] Note that these two Kähler potentials are related via the holomorphic relation $\Phi = \frac{T-1}{T+1}$ and a Kähler transformation (see e.g. [12, 13]).

A classic example of the importance of this type of bound is provided by natural inflation [19], where a single axion in a shift-symmetric potential is responsible for the cosmological dynamics. In the effective field theory description, the axionic shift-symmetry protects the potential from radiative corrections such that large periodicities, *i.e.* potentials with a decay constant $f \gg M_{\rm P}$, can ensure radiative stability even for field displacements larger than $M_{\rm P}$. However, it is not obvious that such symmetries admit completions in quantum gravity. Specifically, the weak gravity conjecture places a bound on the axion decay constant as $f \lesssim M_{\rm P}$. But compatibility with observations suggests the axion in natural inflation to have a decay constant $f \gtrsim 5M_{\rm P}$.

Proposals to evade this contradiction fall into two types: either they realize an axionic approximate shift symmetry via monodromy (by coupling to a 4-form field strength) [20–23], or they use assistance effects driven by several axions participating during inflation. The second case can arise through the tuned alignment of two axions [24], by arranging a hierarchy of the decay constants [25–27], or as a generic assistance effect driven by a large number of axions termed 'N-flation' [28–32]. In the latter, the total inflaton field range $\Delta\varphi$ arises through the collective displacements of individual axions, each of them satisfying the constraints of the WGC: $\Delta\phi_i \sim f_i \lesssim M_{\rm P}$. In the simplest setup of N-flation with N axions with roughly similar decay constants, it is easy to see that the field displacements are related by $\Delta\varphi \sim \sqrt{N}\Delta\phi_i$, such that for large enough N, super-Planckian inflaton displacements seem to be allowed. It turns out that generalizing the WGC bounds to theories with multiple axions is more subtle than simply implementing the bound $f \lesssim M_{\rm P}$ for each individual axion. Implementing WGC bounds for this case implies using the *convex hull condition* [33–36]. Which version of the WGC applies is still an open issue. But for both lattice WGC [37] and milder formulations of this conjecture, one finds a bound on the collective axionic motion forbidding the $\sqrt{N}$ enhancement with respect to the single field case. This will be further discussed in section 4.

The situation for a single field driving pole inflation is morally similar. The universal predictions of this scenario arise when the non-canonical field approaches the kinetic pole, or from a supergravity perspective, the boundary of the moduli space. However, on generic grounds, precisely in this regime we expect numerous quantum corrections to grow large, thus potentially leading to a loss of control of the setup. We can argue both in 4D effective field theory [38] and in string theory [39–42] for the appearance of such dangerous terms. In this paper, we explore the possibility of using the collective behaviour of a large number N of moduli-like scalar fields to alleviate some of these problems, in what we call *pole N-flation*. Specifically, we propose a scenario where the approach to the pole is achieved by the assistance effect of many fields, such that each field is individually further away from the boundary by a factor of $\sqrt{N}$. This may suppress some of the generically expected loop contributions which grow large when individual fields approach the boundary.

As a conservative estimate for the domain of validity of the effective description of both pole inflation and pole N-flation, we make use of the WGC constraints. In both cases the moduli of the simplest supergravity models are associated with axionic partners, allowing us to implement bounds on their periodicities as conditions for the consistency with ultra violet physics. We find that imposing WGC bounds on the axionic periodicities directly translates into the impossibility of getting close the boundary, and therefore to a finite inflationary plateau in canonically normalized variables. This has dramatic consequences for the viability of pole inflation in general. This bound is very stringent for the case of a single superfield, and one might have hoped for a weakening of the WGC-imposed bound when many disk-variable provide a collective 'pole N-flation' mode. However, it turns out that for very strong forms of the WGC such as the lattice version, the convex hull condition conspires much like in axion N-flation to erase any N-enhancement. Only the milder forms of the WGC allow for pole N-flation, again drawing a close parallel to the situation for axion N-flation.

The outline of the paper is the following: in §2 we scan the typical kinetic structures derived from string theory and identify the most natural for a scenario with a pole due the collective behaviour of many fields. In §3 we will study a supergravity toy model of pole N-flation, describing the ellipsoid structure of the pole and the subsequent universality behaviour. We use these results in §4 to establish contact with the WGC and the swampland conjectures, and derive a bound on the field range in pole N-flation. With the aim of embedding these ideas in string theory, in §5 we develop an explicit scenario based on type IIB string theory on fibered Calabi-Yau manifolds. We draw our conclusions in §6. Throughout the paper, we will work in reduced Planck mass units ($M_{\rm P} = 1$).

2 Kinetic poles in string theory

In order to search for setups with $N \gg 1$ fields and second order kinetic poles, it is illuminating to analyse the structure of kinetic terms in 4D $\mathcal{N} = 1$ supergravity derived from string theory. String compactifications on Calabi-Yau manifolds generically produce Kähler potentials containing both closed and open string moduli, with the exact number of such moduli given by the underlying geometry and the amount of D-branes. We can classify the possible Kähler potentials as follows:

1. In perturbative string theory, in the large volume and large complex structure limit, there are at most 3 volume and 3 complex structure moduli which describe the total Calabi-Yau manifold. Together with the axio-dilaton, these corresponds to a maximum of 7 chiral fields, which lead to the tree-level Kähler potential

$$K = -3\sum_{i=1}^{n} \alpha_i \ln\left(T_i + \bar{T}_i\right) \quad , n \leq 7 \quad \text{and} \quad 1 \leq 3\sum_{i=1}^{n}\alpha_i \leq 7\,, \tag{2.1}$$

   with the parameters α_i depending on the number of fields (see e.g. [43] in the context of α-attractors).

2. There are open string moduli describing brane positions (e.g. D3-branes), and/or open string matter fields as well as gauge fields. Their number is usually subject to tadpole bounds and can be as large as $\mathcal{O}(10^4)$. They appear as a contribution to the volume moduli Kähler with the schematic form

$$K = -3\alpha \ln\left(T + \bar{T} - \sum_{i=1}^{N} a_i \Phi_i \bar{\Phi}_i\right)\,, \tag{2.2}$$

where the parameter α depends on the specific configuration of the bulk geometry and the Kähler moduli.

3. Finally, the moduli space of Calabi-Yau manifolds contains singular regions, most easily seen as the conifold points of complex structure moduli space. Near such singularities, the corresponding moduli acquire a Kähler potential of non-polynomial form inside the primary logarithm. For complex structure moduli near a conifold point this generically implies a Kähler potential of the form

$$K_{c.s.} = -\ln\left(f(u_i, \bar{u}_{\bar{\imath}}) - z\bar{z}\ln z\bar{z}\right) , \tag{2.3}$$

where z denotes the complex structure moduli parametrizing the vicinity of the conifold singularity, and the u_i denote the remaining other complex structure moduli of the Calabi-Yau. The number of such conifold regions in a given Calabi-Yau can be quite large, easily of the order of a few tens.

From this short list we see that a realization of pole inflation involving assistance effects of a large number of fields cannot arise from the large-volume or large-complex-structure type of closed string moduli described in the first class of the list, as their number is intrinsically limited. The study of the third class in the list would require an in-depth analysis of multi-conifold complex structure Kähler potentials and their dependence on non-conifold complex structure moduli. This analysis is beyond the scope of the present paper and we leave it for future work. In this paper we therefore explore the second class, where a large number of open string moduli fields with Kähler potentials of the form eq. (2.2) lead to the pole N-flation scenario.

3 The pole N-flation picture

3.1 Kinetic structure and universality

We start our analysis by looking at the Kähler potential given by eq. (2.2). Once the Kähler modulus T is stabilized, the relevant dynamics in the EFT is described by

$$K = -3\alpha\ln\left(1 - \sum_{i=1}^{N} A_i\ \Phi_i\bar{\Phi}_i\right) , \tag{3.1}$$

where the new coefficients are rescaled by the VEV of T, such that $A_i = a_i/\langle T + \bar{T}\rangle$. The corresponding kinetic term is given by

$$-\sum_{i,j=1}^{N} K_{\Phi_i\bar{\Phi}_j}\ \partial\Phi^i\partial\bar{\Phi}^j = -3\alpha\sum_{i,j=1}^{N}\left[\frac{A_iA_j\ \bar{\Phi}_i\Phi_j}{\left(1-\sum_{k=1}^{N}A_k\ \Phi_k\bar{\Phi}_k\right)^2} + \frac{A_i\ \delta_{ij}}{1-\sum_{k=1}^{N}A_k\ \Phi_k\bar{\Phi}_k}\right]\partial\Phi^i\partial\bar{\Phi}^j\,, \tag{3.2}$$

which has a pole for $R^2 \equiv \sum_k A_k\ \Phi_k\bar{\Phi}_k = 1$, which is the equation of an ellipsoid in field space with N independent radii of length directly related to the brane contributions A_i. As can be seen by the form of the denominator, the N fields collectively contribute to reach the boundary without any Φ_k reaching the boundary itself.[3] Therefore, when all fields equally contribute to inflation, the displacement of each individual field will be reduced by a factor of $\sqrt{N}$. This property will protect the model against radiative corrections which grow as the fields individually approach the pole.

To further understand how the presence of these fields affects the model and its dynamics, we make the following change of variables

$$\begin{aligned}\Phi_i &= \frac{R}{\sqrt{A_i}}\Omega_i(\psi_\beta)e^{i\theta_i} \\ \bar{\Phi}_i &= \frac{R}{\sqrt{A_i}}\Omega_i(\psi_\beta)e^{-i\theta_i}\,,\end{aligned} \tag{3.3}$$

where $\Omega_i(\psi_\beta)$ is the spherical angular element such that $\sum_i \Omega_i^2(\psi_\beta) = 1$. Here the index $\beta = 1, \cdots, N-1$. As will be discussed in §4, the angles θ_i can be associated with axions and will play a crucial role in our understanding of the UV consistency of the setup.

In these variables the line element becomes

$$-3\alpha\left[\frac{1}{(1-R^2)^2}\,\partial R\partial R + \frac{R^2}{1-R^2}\sum_i(\partial_\beta\Omega_i)^2\,\partial\psi_\beta\partial\psi^\beta + \sum_{ij} G_{ij}\,\partial\theta_i\partial\theta_j\right], \tag{3.4}$$

where $\partial_\beta \equiv \partial/\partial\psi_\beta$ and

$$G_{ij} = \frac{R^4}{(1-R^2)^2}\Omega_i^2\Omega_j^2 + \frac{R^2}{1-R^2}\delta_{ij}\Omega_i^2\,. \tag{3.5}$$

In this form, the field-space metric has useful features. First, it is independent of the axionic variables θ_i. It is also diagonal in the variables R and ψ_β. The mixed terms associated to $\partial R\partial\psi_\beta$ vanish due to $\sum_i \Omega_i\partial_\beta\Omega_i = 1/2\ \partial_\beta\sum_i\Omega_i^2 = 0$, and so do the terms $\partial\psi_\beta\partial\psi_\gamma$ for $\beta \neq \gamma$ due to trigonometric relation $\sum_i \partial_\beta\Omega_i\partial_\gamma\Omega_i = 0$, proved in Appendix A.

One can easily identify R as the variable with a kinetic pole of second order. Upon canonically normalizing the kinetic term, one has

$$R = \tanh\frac{\varphi}{\sqrt{6\alpha}}\,, \tag{3.6}$$

such that the boundary at $R \to 1$ is equivalent to $\varphi \to \infty$. Writing the system in this canonical variable, just like in the single-field pole inflation case, makes evident how the model is stable with respect to considerable deformations of the inflaton scalar potential. The potential can be generated by means of several mechanisms: via an inflaton-dependent superpotential (with stabilizer superfield [9, 45] or without it [10]), by Kähler [46, 47], loop [38–41] or higher-derivative [48] corrections. A generic expansion looks like

$$V = \sum_{ij,p\geq 1} b_{ij,p}(\Phi_i\bar{\Phi}_j + c.c.)^p = \sum_{ij,p\geq 1} b_{ij,p}\left[2\frac{\Omega_i\Omega_j}{\sqrt{A_iA_j}}\cos(\theta_i-\theta_j)\ R^2\right]^p, \tag{3.7}$$

[3] This is unlike systems where each field has its own distinct pole as in the case studied in ref. [44]. The corresponding Kähler potential would be sum-separable of the form of eq. (2.1). Note that, in this case, the inflationary predictions are strictly related to the specific direction in field space.

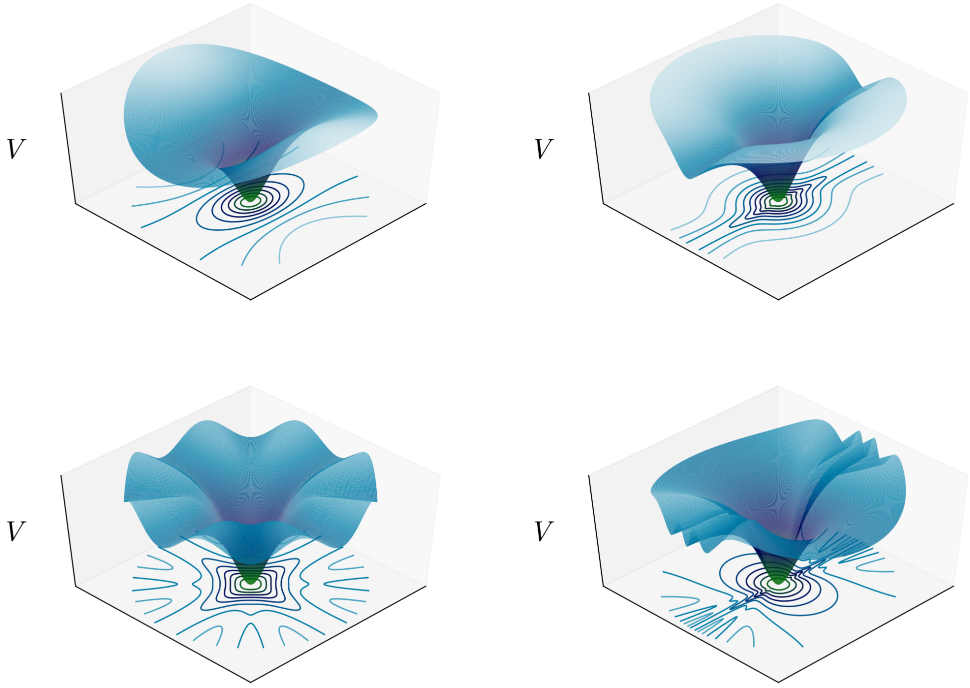

Figure 1. Potential for the $N = 2$ case plotted in the polar coordinates $\{\varphi, \psi\}$, with angles θ_1 and θ_2 minimized. The top-left plot shows the potential eq. (3.7) up to terms with $p = 1$, in the case of equal A_i. The top-right plot shows the same potential but for $A_1 = 1$ and $A_2 = 0.2$. The bottom-left plot shows the potential with terms up to $p = 4$, in the case of equal A_i. The bottom-right plot shows the same potential but for $A_1 = 1$ and $A_2 = 0.2$. We can see that the elliptical structure of the model makes valleys in the potential bundle-up. While different radial directions have plateaus with different amplitudes, the exponential fall-off has the same signature for all initial conditions, or in other words, values of the angular coordinate ψ_β. The oscillating plateau of the circular case were already noted for a real 2-disk α-attractor (without supergravity) in [49].

with constant coefficients $b_{ij,p}$. We therefore see that in the vicinity of the pole the scalar potential decomposes into an exponential fall-off from a de Sitter plateau as

$$V = V_0\left(\theta_i, \psi_\beta\right) - e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\, V_1(\theta_i, \psi_\beta) + \mathcal{O}\left(e^{-2\sqrt{\frac{2}{3\alpha}}\varphi}\right), \tag{3.8}$$

with V_0 and V_1 functions of the angular variables as dictated by eq. (3.7). It is interesting to note that the residue of the pole does not depend on either A_i or Ω_i, leading the exponential plateau to have a universal nature. The slope of the exponential fall-off is therefore not affected by the particular radial direction in field space. The amplitude of the plateau is exclusively determined by the angular and axionic directions (ψ_β, θ_i). This effect can be observed in fig. 1.

If inflation occurs purely in the R direction, the observable predictions of this model, regardless of the inclusion of multiple fields, retain the universality properties extensively discussed in the literature (see e.g. [7–9, 45]) for the single field case,

$$n_s = 1 - \frac{2}{N_e}, \qquad r = \frac{12\alpha}{N_e^2}, \tag{3.9}$$

where N_e is the number of e-folds of expansion between the pivot scale leaving the horizon and the end of inflation. However, the angular and axionic fields might play a role in the inflationary dynamics leading to multifield effects that can modify the predictions. To assess this, we need to study the hierarchies in the mass spectrum.

3.2 Scaling of mass spectrum

The hierarchies in the mass spectrum are intimately related to the eigenvalues of the field space metric given by eq. (3.4) and eq. (3.5). Using the canonically normalised field φ we define the parameter ϵ as a measure of proximity to the moduli boundary:

$$R = \tanh(\varphi/\sqrt{6\alpha}) \simeq 1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi} \equiv 1 - \epsilon\,. \tag{3.10}$$

We can then see that the line element scales with the proximity to the boundary as

$$-\frac{1}{2}(\partial\varphi)^2 - \frac{3\alpha}{2\epsilon}\sum_i(\partial_\beta\Omega_i)^2\,\partial\psi_\beta\partial\psi^\beta - 3\alpha\sum_{ij}\left[\frac{1}{4\epsilon^2}\Omega_i^2\Omega_j^2 + \frac{1}{2\epsilon}\delta_{ij}\Omega_i^2\right]\partial\theta_i\partial\theta_j\,. \tag{3.11}$$

where we have neglected higher order contributions in ϵ.

In general, it is not straightforward to compute the eigenvalues of this kinetic metric but we can consider specific field configurations which simplify the situation. We are particularly interested in the regime where all the fields Φ_i contribute equally when approaching the boundary, *i.e.* when all the branes are equally displaced from the origin and the inflationary dynamics is determined by their collective motion. In this maximally multifield case, this corresponds to the choice $\Omega_i^2 = 1/N$ for any i. In this case, following Appendix A, the metric element for the angular coordinate ψ_β is

$$-\frac{3\alpha}{2\epsilon}\sum_i(\partial_\beta\Omega_i)^2 = -\frac{3\alpha(N-\beta+1)}{2N\epsilon}, \tag{3.12}$$

which ranges from $3\alpha/2\epsilon$ to $3\alpha/N\epsilon$.

Also in this regime, close to the boundary, the metric for the axionic fields θ_i eq. (3.5) can be written as

$$G_{ij} = v^2 J_{ij} + \delta_{ij} v \quad , \quad v = \frac{R^2}{N(1-R^2)} \simeq \frac{1}{2N\epsilon}\,, \tag{3.13}$$

where J_{ij} is the all-ones matrix (a square matrix with all entries equal to 1). The eigenvalues of this metric are easy to compute: the matrix J_{ij} has rank one, with one single non-zero eigenvalue equal to $\mathrm{Tr}(J_{ij}) = N$. The identity matrix is invariant under any transformation and therefore G_{ij} has all but one eigenvalue equal to v. We denote the corresponding eigenvectors of the $N-1$ equal eigenvalues, Θ_a, with $a = 1, \ldots, N-1$. The last eigenvalue, corresponding to what we define as the ϑ direction, is

$$Nv^2 + v = \frac{1}{N}\frac{R^2}{(1-R^2)^2} \simeq \frac{1}{4N\epsilon^2}\,. \tag{3.14}$$

At the point in field space where the metric has these eigenvalues, we can locally canonically normalize the angular fields by defining

$$\begin{aligned}\hat{\psi}_\beta &\equiv \sqrt{\frac{3\alpha(N-\beta+1)}{N\epsilon}}\psi_\beta \\ \hat{\Theta}_a &\equiv \sqrt{\frac{3\alpha}{N\epsilon}}\Theta_a \\ \hat{\vartheta} &\equiv \sqrt{\frac{3\alpha}{2N\epsilon^2}}\vartheta\end{aligned} \tag{3.15}$$

and write the potential near the pole as

$$V = V_0\left(\hat{\Theta}_a, \hat{\vartheta}, \hat{\psi}_\beta\right) - \epsilon\, V_1\left(\hat{\Theta}_a, \hat{\vartheta}, \hat{\psi}_\beta\right) + \mathcal{O}\left(\epsilon^2\right) . \tag{3.16}$$

Owing to the local canonical normalization of the kinetic terms, the mass spectrum therefore scales as

$$\begin{aligned}|V_{\varphi\varphi}| &\sim \epsilon V_1 \sim \epsilon V_0 \\ \left|V_{\hat{\psi}_\beta\hat{\psi}_\beta}\right| &\sim \frac{N\epsilon}{N-\beta+1}V_0 \\ \left|V_{\hat{\Theta}_a\hat{\Theta}_a}\right| &\sim N\epsilon V_0 \\ \left|V_{\hat{\vartheta}\hat{\vartheta}}\right| &\sim N\epsilon^2 V_0\end{aligned} \tag{3.17}$$

for a generic scalar potential where usually $\mathcal{O}(V_0) \approx \mathcal{O}(V_1)$. The mass scaling of the $N-1$ elliptical angular fields $\hat{\psi}_\beta$ ranges from ϵV_0 to $N\epsilon V_0$, *i.e.* from the same scaling as the light radial field φ to the scaling of the heavier axionic fields $\hat{\Theta}_a$. We should therefore allow for the possibility that some of these fields might contribute to the inflationary dynamics. These effects might lead to multifield deviations from the simplest predictions given by eq. (3.9). The $N-1$ axions $\hat{\Theta}_a$, in the large N limit, correspond to a heavier sector which we do not expect to contribute significantly to the dynamics. The single $\hat{\vartheta}$ direction, in the limit of $\varphi \gg 1$, is exponentially lighter than all other sectors and becomes a true spectator; in this deep plateau limit the dynamics of $\hat{\vartheta}$ is frozen in deep slow-roll and will resemble the case of the single angular field of ref. [50]. The kinetic scaling of the mass spectrum of the theory for a configuration where all fields contribute equally to inflation is illustrated in Figure 2.

4 The fate of inflationary plateaus - weak gravity strikes back

Effective field theories arising from a theory of quantum gravity are constrained by consistency conditions such as the Weak Gravity Conjecture (WGC) [17] or the Swampland Distance Conjecture (SDC) [18, 51–53]. The SDC states that whenever one moves an infinite distance in moduli space, an infinite tower of states becomes massless causing the break of the effective description. We will comment on the connection between the SDC and α-attractor models later in this section, but for now we focus on enforcing consistency arguments coming from the weak gravity conjecture.

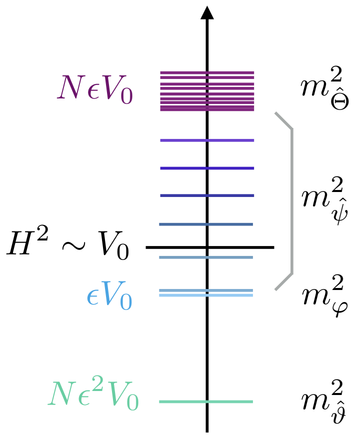

Figure 2. Mass hierarchies of eq. (3.17) in the deep plateau limit, $\varphi \gg 1$, and large N limit.

The WGC arose as a proposal to argue that black holes should always be able to evaporate via Hawking radiation, such that the final state of any charged black hole would be able to decay and leave no remnant. For this to happen it is necessary that any theory of quantum gravity has at least a fundamental object fulfilling the condition

$$1 \lesssim \frac{M_P}{m} q \tag{4.1}$$

such that the decay process is possible for any black hole. A number of attempts were made to further constrain the particular object fulfilling the WGC condition. These have given rise to several versions of the WGC. In particular, the strong form of the WGC requires the lightest particle on the spectrum to be the one fulfilling the above condition, whereas more loose forms do not impose further conditions on the particular particle fulfilling the condition. It is still unknown which version of the conjecture (if any) is the right one and it is not our intention to provide any new insight in this direction, so we just refer the interested reader to [34–37, 54–78] for extensive discussion. Our purpose, instead, will be to apply the WGC constraints to the α-attractor models as a consistency requirement to allow for a string theory embedding of these effective field theories.

Since string theory contains fundamental charged objects of different dimension, first note that the above argument can be extended to other black objects of different dimensionality. This implies one is not completely free concerning the assignment of charges and tensions of the fundamental objects of any theory. For our purposes we will be interested in how the WGC applies to instantons. In this case, the WGC sets the bound

$$\mathcal{S}_E \lesssim \frac{M_\text{P}}{f} n \ , \tag{4.2}$$

where $\mathcal{S}_E$ is the euclidean action of the instanton, f the decay constant of the axion coupled to it and n the instanton number/charge. This bound will be our starting point to set the limitations of α-attractors. To do so while working in a controlled effective field theory, we want to guarantee a controlled instanton expansion, which in turn implies $\mathcal{S}_E > 1$. Recalling that we are taking $M_{\rm P} = 1$, the above WGC bound, when applied to the single complex field case (single axion), simplifies to

$$f \lesssim n \ , \tag{4.3}$$

with n being the instanton charge. This can be seen as the loose form of the WGC bound. Furthermore, string theory includes fundamental objects with all possible charges (see [79] for some important progress in this direction). In particular, this means that there will always exist an instanton with $n = 1$ which is the relevant object for the strong form of the WGC, such that the strong bound now reads

$$f \lesssim 1 \ , \tag{4.4}$$

in agreement with evidence from string theory compactifications [16].

For effective field theories including multiple axions, the above argument needs to be extended. In the black hole picture this corresponds to a theory with multiple $U(1)$ gauge factors, first studied in ref. [33]. The key factor in this case is that black holes can be charged under more than a single $U(1)$ factor at the same time, and thus the WGC constraints cannot be implemented by considering each of these $U(1)$ factors separately. One needs to consider the lattice of charge-to-mass ratios of the theory such that the single $U(1)$ bound extends to the so-called *convex hull condition* on this lattice. This condition was first translated to the axion language in ref. [34]. Before discussing this condition in more detail, we note that as in the single axion case, there exist several versions of the WGC on this multiple axion case depending on which instantons are considered to be relevant.

We start by considering the generalization of the strong version to multiple axions, the so-called lattice WGC [37], and then give a more loose version.

The convex-hull condition is nothing but a base independent (in axion space) generalization of eq. (4.4) for each axion decay constant. In order to achieve base independence, we proceed by considering a N-dimensional polygon whose edges are located at positions

$$\pm\,(0, ..., 0, 1/f_i, 0, ..., 0) \quad , \quad i = 1, ..., N \ , \tag{4.5}$$

f_i being the decay constants of the N axions in the model under consideration. The condition eq. (4.4) applied to each of these vectors requires all edges to sit at least at distance 1 from the origin. This is a base dependent statement, so demanding instead that the whole polygon contains the unit ball gives rise to base independence. This geometric requirement is quite simple but not very practical. Luckily, it can be translated into a more practical and intuitive inequality [34]:

$$\sum_i f_i^2 \lesssim 1 \ . \tag{4.6}$$

Again, we find a constraint on the possible decay constants of axions, but this time the constraint involves all axions relevant for the WGC at the same time. We can now easily extend this condition to more loose versions of the WGC. If the instantons relevant for the WGC have charge $n_i \neq 1$, then one should replace $f_i \to f_i/n_i$ in the analysis above. For simplicity, we may consider that the instanton number of all relevant instantons is the same, $n_i \to n_{eff}$. In this case, the convex hull inequality is

$$\sum_i f_i^2 \lesssim n_{eff}^2 \ . \tag{4.7}$$

Since we are setting bounds on axion decay constants, we next argue that the fields θ_i in §3 are indeed axions. It is nowadays standard in supergravity to denominate axions those fields that do not appear in the Kähler potential and whose potential is periodic. In order to apply WGC arguments, these conditions are necessary but not sufficient: one needs to argue that the axion potential can only be generated by instantons. In order to do so, recall that the string theory picture corresponding to our inflationary setup corresponds to the motion of D3-branes, whose position we parametrized by the fields Φ_i. It is known that in standard type IIB compactifications *à la* GKP [80, 81], the radial part $|\Phi_i|$ of the position moduli is massless at the perturbative level unless supersymmetry is broken, while their axion phases θ_i remain flat directions always in perturbation theory. So, it is necessary to also include non-perturbative objects on the compactification in order to stabilize the θ_i. The inclusion of Euclidean D3-branes will indeed generate a potential and stabilize these moduli. In the 4D supergravity language these instantons generate a non-perturbative superpotential [82, 83]

$$W_{np} = F(\Phi_i, z_I)e^{-T_J}, \tag{4.8}$$

T_J being the superfield describing the size of Σ_J where the ED3-brane is wrapped and F a holomorphic function of the D3-brane moduli as well as other geometric moduli z_I, such as the complex structure ones. Note here that holomorphy of the superpotential will ensure the potential to be compatible with the axion periodicity arising from our choice of polar coordinates $\theta_i \sim \theta_i + 2\pi$. Using this fact, we conclude that the potential of the θ_i is indeed generated by instantons and compatible with the usual identification coming from each axion living on a $\mathbf{S}^1$.[4]

As a final step before applying WGC constraints, note that it is always possible to rescale the axions $\hat{\theta}_i = f_i\theta_i$, such that if the periodicity of θ_i is given by $\theta_i \sim \theta_i + 2\pi$, then for $\hat{\theta}_i$ it is $\hat{\theta}_i \sim \hat{\theta}_i + 2\pi f_i$. Therefore the decay constant changes with rescaling, and so does the axion kinetic term. In the single field case, the relevant scale for WGC arguments is the one giving rise to a canonical kinetic term for the axion, or equivalently, the square root of the prefactor in the kinetic term for an axion with periodicity $\theta_i \sim \theta_i + 2\pi$. In the multiple axion case, we need to apply the same criterion to each axion in a base where there is no kinetic mixing. Due to our initial coordinate choice where $\theta_i \sim \theta_i + 2\pi$ for all axions, this task can easily be carried out: we just need to compute the eigenvalues of the field space metric. In fact, after a change of coordinates the kinetic Lagrangian will be

[4]We note that the 4D low-energy effective description of instantons from string theory reduces in many cases to the Giddings-Strominger type gravitational instantons [84, 85]. Expanding the axion on a n-instanton background provides the usual axion-instanton term enforcing $\theta_i \sim \theta_i + 2\pi$ for the path integral to be well defined.

$-\sum_i \frac{f_i^2}{2}\partial_\mu\theta_i\partial^\mu\theta_i$ which is canonically normalized by the change $\hat{\theta}_i = f_i\theta_i$, such that f_i are the decay constants of interest for WGC arguments.

We are now ready to start studying the consequences for the single (complex) field case. In this case, the Kähler metric is

$$K_{\Phi\bar{\Phi}} = \frac{3\alpha}{(1-\Phi\bar{\Phi})^2}\,, \tag{4.9}$$

where $\Phi\bar{\Phi} = \phi^2$, and leads to an axionic partner of ϕ with a decay constant

$$f^2(\phi) = 6\alpha\frac{\phi^2}{(1-\phi^2)^2}\ . \tag{4.10}$$

As the field ϕ approaches the moduli boundary, the decay constant of the axionic partner diverges; this behaviour has been noticed in ref. [86]. The WGC therefore constraints the maximum displacement in the radial direction. To understand how stringent this constraint is, it is interesting to compare the WGC bound $f^2 \lesssim n^2$ with the slow-roll condition $\epsilon_{\rm SR} < 1$. Assuming inflation occurs primarily in the radial direction and following eq. (3.4), we have that in the slow-roll limit

$$\epsilon_{\rm SR} \simeq \frac{1}{h^2}\left(\frac{V_\phi}{V}\right)^2\,, \quad h^2 = 3\alpha\frac{1}{(1-\phi^2)^2}\,. \tag{4.11}$$

Taking the potential to be expanded as in eq. (3.7) and dominated by $\mathcal{O}(\phi^2)$ terms, inflation occurs when

$$3\alpha\frac{\phi^2}{(1-\phi^2)^2} > 1\,, \tag{4.12}$$

which comparing with eq. (4.10) is equivalent to $f^2 \gtrsim \mathcal{O}(1)$. This implies that obtaining sufficient inflation in this scenario, while remaining compatible with the WGC, requires n in eq. (4.3) to be quite large. If instead one considers the strong form of the WGC where $n = 1$, then α-attractors turn out to be unable to provide enough inflation. This observation is independent of the value of α. From here we conclude that the simplest supergravity α-attractor model, when embedded and coupled to quantum gravity, is in direct conflict with consistency requirements coming from the strong form of the weak gravity conjecture, which in turn is supported by evidence from string theory [16].

We now apply a similar argument to the N field case studied above. We saw in §3 that, taking polar coordinates for the complex field, the field-space metric on the axionic sector is not diagonal. In order to apply the WGC bound we choose a configuration where the inflationary dynamics is carried out by the collective motion of all D3-branes, *i.e.* when all fields contribute equally and all angular functions Ω_i have the same value $\Omega_i^2 = 1/N$. Diagonalizing the field space metric in this configuration gives rise to the axionic fields Θ_a , $a = 1\ldots N-1$, and ϑ with diagonal kinetic terms

$$-\sum_a \frac{f_a^2}{2}\partial_\mu\Theta_a\partial^\mu\Theta_a - \frac{f_\vartheta^2}{2}\partial_\mu\vartheta\partial^\mu\vartheta\ . \tag{4.13}$$

The decay constants are given by eqs. (3.13) and (3.14)[5]

$$f_a^2 = \frac{6\alpha R^2}{N(1-R^2)} \quad , \quad f_\vartheta^2 = \frac{6\alpha R^2}{N(1-R^2)^2} \ . \tag{4.14}$$

To study the bound the WGC sets on this model, we plug these expressions in eq. (4.6) to find:

$$f_\vartheta^2 + (N-1)f_a^2 = \frac{1}{N}\left(\frac{6\alpha R^2}{(1-R^2)^2} - \frac{6\alpha R^2}{(1-R^2)}\right) + \frac{6\alpha R^2}{(1-R^2)} \lesssim n_{eff}^2 \ . \tag{4.15}$$

When $N = 1$ this expression reduces to the result derived above, whereas for large N only the last term is relevant. Naïvely we might have expected that the inclusion of the assistance of N fields could relax the WGC bound allowing the system to be sufficiently displaced in R while keeping away from the moduli boundary. However, we can see by the N-independence of the last term that this does not happen. In fact, it turns out that the large N limit leads to a slightly weaker bound on the maximum allowed value for R, but the gain turns out to be negligibly small. This puts pole N-flation in a situation similar to the single field case: strong forms of the WGC are incompatible with the production of sufficient inflation. Only weaker forms where the relevant instantons have large enough n_{eff} would be able to provide enough e-folds to render these models viable.

We would like to contrast this with the case of axion N-flation. As previously discussed in the literature (see e.g. [35]), large displacements in axion N-flation are incompatible with the lattice WGC: the base independence described by the convex hull condition results in the cancellation of the proposed $\sqrt{N}$ enhancement in the diagonal direction in axion space, such that maximum allowed displacement is independent of direction. As we argued above, we also found little gain in this regard when applying the lattice WGC to pole N-flation. Yet, when applying a looser form of this conjecture, the constraints on both models are rather different. If higher instanton numbers $n_{eff} > 1$ are the relevant ones for the WGC as in eq. (4.7), there exists the possibility of larger axion decay constants. In the case of pole N-flation this relaxes the bound on the radial field displacement such that inflation can happen in the plateau. In the case of axion N-flation, however, one finds that higher harmonics in the axion potential compensate any gain obtained by relaxing eq. (4.6) into eq. (4.7), and the bound on field displacement remains effectively the same [16, 34].

As a final remark, we would like to make a connection between these results and the swampland distance conjecture (SDC). Consider a very loose version of the WGC, with $n_{eff} \gg 1$, thus allowing the inflaton to get deep into the plateau. In this case when the inflaton approaches the boundary (when $R \to 1$) not only do the decay constants grow exponentially according to eq. (4.14), but also the masses of the axions become exponentially small as can be seen in eq. (3.17). As the plateau itself arises from a kinetic

[5] We note here that, in α-attractor models, inflation does not involve an active axion or linear combination of axions. Therefore, the only input needed from the axionic sector of the theory are the periodicities of the canonically normalized axions. This is unlike models where inflation occurs in the axionic sector, such as N-flation [29], where it is necessary to have information about the instanton numbers on each cycle in order to compute the potential and the fundamental field-space domain of the axions; see e.g. [57, 87] for a discussion of models of this type.

term with a 2nd order pole, this behaviour shows certain similarity to recent arguments in favor of the SDC provided in ref. [88, 89] as reaching the 2nd order pole of the metric on moduli space makes the axions in our setup exponentially light.

As a consequence of these observations, we conclude that the WGC rules out the *infinite* inflationary plateaus of pure supergravity α-attractor models based on disk variable-type kinetic terms with 2nd order poles. This suggests that Pole N-flation with infinite plateaus does not admit a UV completion in string theory.

5 Towards pole N-flation in type IIB string theory

As we have seen in the previous sections, the Kähler potential (2.2) becomes singular exactly at the same point where the kinetic Lagrangian develops a pole. This fact poses strict limitations on the possibility of realizing the pole inflation scenario within a supergravity framework. Indeed, the F-term scalar potential $V = e^K(\dots)$ will in general not be regular at this point in field space and the inflationary plateau will be easily spoiled. To avoid this situation, one can tune the superpotential such as to cancel the pole induced by the exponential pre-factor in V, but this appears to be a quite non-generic and model-dependent situation. But even granted this possibility, we will interpret the field approaching the pole as a shrinking volume of extra dimensions in string theory. If this volume is the total volume of the extra dimensions, sending this to zero will send perturbative corrections soaring in magnitude and thus compromising control.

A rather more appealing alternative is to find a class of models where the form of K has a regular behaviour while still inducing a pole in the corresponding kinetic structure. Interestingly, stabilizing the overall volume of fibred Calabi-Yau (CY) geometries [90–92] using the Large Volume Scenario (LVS) mechanism [93] provides a large class of string models with a Kähler potential with the desired properties.

In the following, we will review the main characteristics of this framework and show how to embed the pole N-flation picture therein. We will also discuss moduli stabilization of this setup, pointing out its limitations given the current status of knowledge on quantum corrections. Finally, we will provide an analysis of the model's dynamics and cosmological predictions.

5.1 Pole N-flation from fibred Calabi-Yau manifolds

A large fraction of CY manifolds are $K3$-fibred. This means that the positive part of the CY volume takes the form

$$\mathcal{V} = \kappa_{122} v^1 (v^2)^2 \,, \tag{5.1}$$

in terms of the 2-cycle volumes v^i, and κ_{122} is the intersection number between the 2-cycles on the given Calabi-Yau manifold. The 4-cycle volumes τ_i are related to the 2-cycles by

$$\tau_i = \frac{\partial \mathcal{V}}{\partial v^i} \,, \tag{5.2}$$

allowing us to write the volume of a $K3$-fibred CY in terms of the 4-cycle volumes as

$$\mathcal{V} \sim \sqrt{\tau_1} \tau_2 \,. \tag{5.3}$$

The corresponding Kähler potential then takes the form

$$K = -2\ln\mathcal{V} = -\ln(T_1+\bar{T}_1) - 2\ln(T_2+\bar{T}_2)\,. \tag{5.4}$$

where we have introduced the volume moduli T_i, which are related to the 4-cycle volumes by means of $2\tau_j = T_j + \bar{T}_j$ while their axionic partners are $2c_j = (T_j - \bar{T}_j)/i$.

Now assume the CY to possess a warped near-conifold region. Assume further that the 4-cycle Σ_2^4 with volume τ_2 reaches somewhat into the warped region. This is not particularly restrictive, as we can stabilize part of the complex structure moduli using flux near conifold points for a large fraction of all $K3$-fibred CYs. Finally, place a number N of $D3$-branes at the IR end of the warped region.

The Kähler potential for models in this class will look like

$$K = -\ln\left(T_1+\bar{T}_1\right) - 2\ln\left(T_2+\bar{T}_2 - R^2\right)\,, \tag{5.5}$$

where we define as before

$$R^2 \equiv \sum_{i=1}^{N} A_i\Phi_i\bar{\Phi}_i\,, \tag{5.6}$$

with Φ_i parametrizing the positions of the $D3$-branes. The O7-orientifolding enforces the relation between 2-cycle volumes, 4-cycle volumes and D3-brane coordinates such that [81, 94]

$$\tau_1 = (v^2)^2 \quad , \quad \tau_2 = v^1v^2 + \frac{R^2}{2} \quad \Rightarrow \quad v^2 = \sqrt{\tau_1} \quad , \quad v^1 = \frac{1}{\sqrt{\tau_1}}(\tau_2 - \frac{R^2}{2})\,. \tag{5.7}$$

The corresponding expression for the CY volume now reads

$$\mathcal{V} \sim \sqrt{\tau_1}(\tau_2 - \frac{1}{2}R^2)\,. \tag{5.8}$$

An alternative construction might instead shift τ_1 by the D3-brane Kähler potential $R^2/2$. In this case, the CY volume would become $\mathcal{V} \sim \tau_2\sqrt{\tau_1 - R^2/2}$.

The LVS scheme of volume stabilization can now proceed if we assume the total CY to have a third pure blow-up Kähler modulus τ_3, such that the CY volume becomes

$$\mathcal{V} \sim \sqrt{\tau_1}(\tau_2 - \frac{1}{2}R^2) - \lambda_3\tau_3^{3/2}\,. \tag{5.9}$$

We therefore include the leading order type IIB α'-correction into the Kähler potential

$$K = -2\ln(\mathcal{V} + \xi/2)\,, \tag{5.10}$$

and τ_3 acquires an ED3 instanton contribution in the superpotential, in addition to the constant piece from 3-form fluxes, such that

$$W = W_0 + Ae^{-2\pi T_3}\,. \tag{5.11}$$

This setup will stabilize the modulus τ_3 and the whole leading-order volume combination $\mathcal{V}_0 \equiv \sqrt{\tau_1}(\tau_2 - \frac{1}{2}R^2)$ at VEVs with a relation

$$\langle\mathcal{V}_0\rangle \sim e^{2\pi\langle\tau_3\rangle}\,. \tag{5.12}$$

In order to reproduce the pole N-flation dynamics, schematically encoded by eq. (3.1), we would like to stabilize the modulus τ_2 *separately*. For this purpose, we first observe that the scales of LVS stabilization operate at $\mathcal{O}(\mathcal{V}^{-3})$. This rules out the possibility of stabilizing τ_2 supersymmetrically *à la* KKLT, by adding a non-perturbative effect to W. The resulting potential terms from the KKLT mechanism would indeed appear at $\mathcal{O}(\mathcal{V}^{-2})$ and eventually spoil the LVS mechanism.

Hence, we need to stabilize τ_2 perturbatively, presumably using an interplay of string loop corrections and higher-order F-term contributions to the scalar potential, which operate starting at $\mathcal{O}(\mathcal{V}^{-10/3})$. However, in the known simple cases, where we can compute some of the string loop corrections to K and the F^4-terms in the scalar potential [48], these depend on the 2-cycle volumes v^i [38–41]. Therefore, looking at expressions (5.7), these corrections do not affect τ_2 individually but rather τ_1 and the whole combination $\tau_2 - R^2/2$.

At this point, we content ourselves with merely pointing out as a challenge the need to explicate a perturbative stabilization mechanism which will stabilize τ_2 just by itself. From now on, we will simply assume that such stabilization for τ_2 exists.

As a final remark, we wish to point out that we could have instead looked at the case where the whole combination $\tilde{\tau}_2 \equiv \tau_2 - R^2/2$ is given a potential and is stabilized by string loop corrections such as those discussed above. For those models, one can show that the structure of the kinetic terms, in terms of τ_1, $\tilde{\tau}_2$, R and the angular variables ψ_α, θ_i, reduces to $\mathcal{L}_{kin.} = -\frac{1}{4\tau_1^2}(\partial\tau_1)^2 - \frac{1}{2\tilde{\tau}_2^2}(\partial\tilde{\tau}_2)^2 - \frac{1}{\tilde{\tau}_2}(\partial R)^2 + \mathcal{L}_{kin.}(\partial\psi_\alpha, \partial\theta_i)$. If the potential only has contributions of the type discussed above, this setup resembles precisely the original fibre inflation setup [90] (see also [91, 95]) in terms of the effective half-plane variables $\tau_1, \tilde{\tau}_2$ [96] – except for the extra $2N$ massless spectator fields: $2N-1$ angular fields ψ_α and θ_i and one field direction given by a linear combination of R and τ_2 orthogonal to $\tilde{\tau}_2$. If in general these $2N$ fields are also given a potential, we expect a rich mass spectrum and possible multifield phenomenology in analogy with §3. In this work we do not study this type of model, focusing instead on the stabilized τ_2 case.

5.2 Dynamics of fibred pole N-flation and universal predictions

The effective Kähler potential of fibred pole N-flation reads

$$K = -\ln\left(T_1 + \bar{T}_1\right) - 2\ln\left(2\langle\tau_2\rangle\right) - 2\ln\left(1 - \frac{R^2(\Phi_i, \bar{\Phi}_j)}{2\langle\tau_2\rangle}\right), \tag{5.13}$$

once we assume stabilization of τ_2. Note that the last contribution is identical to eq. (3.1), with $3\alpha = 2$ and up to a multiplicative factor in R. Therefore in the following analysis we can employ the results derived in §3.

The kinetic Lagrangian of the dynamical degrees of freedom is given by

$$-K_{T_2\bar{T}_2}\big|_{\tau_2=\langle\tau_2\rangle}(\partial c_2)^2 - K_{T_1\bar{T}_1}\partial T_1\partial\bar{T}_1 - K_{\Phi_i\bar{\Phi}_i}\partial\Phi_i\partial\bar{\Phi}_j\,. \tag{5.14}$$

Applying the LVS procedure for volume stabilization forces $2\tau_1 = T_1 + \bar{T}_1$ to be a function of R, such as

$$\tau_1(R) = \frac{\mathcal{V}_0^2}{\langle\tau_2\rangle^2}\,\frac{1}{\left(1 - \frac{R^2}{2\langle\tau_2\rangle}\right)^2}, \tag{5.15}$$

with $\mathcal{V}_0$ being the stabilized volume. This implies an additional contribution to the total kinetic term of R of the form

$$-\frac{1}{(2\tau_1)^2}\left(\frac{\partial\tau_1}{\partial R}\right)^2(\partial R)^2 = -\frac{R^2}{\langle\tau_2\rangle^2}\frac{1}{\left(1-\frac{R^2}{2\langle\tau_2\rangle}\right)^2}(\partial R)^2\,. \tag{5.16}$$

Therefore, after volume stabilization, the field-space metric for the radial direction is determined by eq. (3.4) together with the contribution of the D3-branes from eq. (5.16) (with the R properly rescaled):

$$-\left[\frac{R^2}{\langle\tau_2\rangle^2}\frac{1}{\left(1-\frac{R^2}{2\langle\tau_2\rangle}\right)^2}+\frac{1}{\langle\tau_2\rangle\left(1-\frac{R^2}{2\langle\tau_2\rangle}\right)^2}\right](\partial R)^2 = -\frac{R^2+\langle\tau_2\rangle}{\langle\tau_2\rangle^2\left(1-\frac{R^2}{2\langle\tau_2\rangle}\right)^2}(\partial R)^2\,. \tag{5.17}$$

In order to absorb the $\langle\tau_2\rangle$ dependence, we define $\tilde{R}\equiv R/\sqrt{2\langle\tau_2\rangle}$ such that the kinetic term becomes

$$-\frac{2(1+2\tilde{R}^2)}{(1-\tilde{R}^2)^2}\,(\partial\tilde{R})^2\,. \tag{5.18}$$

This allows us to define the canonically normalized field φ corresponding to the radial field R as

$$d\varphi \equiv 2\frac{\sqrt{1+2\tilde{R}^2}}{1-\tilde{R}^2}d\tilde{R} \quad . \tag{5.19}$$

We see that $\tilde{R}\to 1$ corresponds to $\varphi\to\infty$, which as done in §3.2 we can use to express φ in terms of $\epsilon = 1-\tilde{R}$

$$d\varphi = -\frac{\sqrt{3}}{1-\tilde{R}}+\mathcal{O}(1) \tag{5.20}$$

such that

$$1-\tilde{R} = \epsilon = e^{-\frac{\varphi}{\sqrt{3}}} \quad . \tag{5.21}$$

This expression is precisely eq. (3.10) for $\alpha = 2$. Using analogous arguments to those in §3, we make an expansion of the scalar potential as eq. (3.7). This generic structure of the scalar potential of Φ_i as a power law series around its minimum often arises for open string moduli in setups with controlled moduli stabilization and supersymmetry breaking. For example, refs. [97, 98] argue explicitly that mobile D3-branes at the IR end of the warped throat of a KKLT or LVS compactification acquire a scalar potential of the general form of eq. (3.7). This results into a computable spectrum of discrete values of $p\geq 1$ while the coefficients $a_{i,p}$, $b_{ij,p}$ are tunable Wilson coefficients except the one arising from the conformal curvature coupling of the D3-brane moduli.

Finally, in analogy with §3, if the motion is purely radial, we see that for an arbitrary number N of open string moduli Φ_i driving exponential plateau inflation, we arrive at

$$\alpha = 2 \quad\Rightarrow\quad n_s = 1-\frac{2}{N_e}\simeq 0.97\,,\; r=\frac{12\alpha}{N_e^2}\simeq 0.007 \tag{5.22}$$

as universal observable predictions. Similar to the simplest fibred inflation models, if the shift by the D3-brane Kähler potential was made on τ_1 rather than τ_2, the effective $\alpha = 1/2$. The predictions for inflation happening along the radial direction would therefore be

$$\alpha = 1/2 \quad \Rightarrow \quad n_s = 1 - \frac{2}{N_e} \simeq 0.97\,, \; r = \frac{12\alpha}{N_e^2} \simeq 0.002\,. \tag{5.23}$$

These predictions can be altered if the angular directions are active during inflation and truly multifield dynamics takes place (see e.g. [99]). In addition, effects of the WGC precluding semi-infinite plateaus often include steepening from growing corrections [95, 100–102]. We expect these to change the above predictions as well.

In closing the discussion, we wish to note the following: embedding pole N-flation into string theory so far seems to require realizing it in the context of a $K3$ or T^4-fibred CY compactification. These models are known to contain another sector capable of driving α-attractor inflation [96] using the two Kähler moduli of the fibration, leading to what is known as 'fibre inflation' [90]. The Kähler moduli of fibre inflation constitute examples of half-plane fields and contain their own axion partners as the imaginary parts. Applying a WGC based bound in terms of the these half-plane field axions to the field range of fibre inflation itself is a natural question arising from our analysis of pole N-flation, which however falls outside the scope of pole N-flation. Consequently, we leave this issue for future work.

6 Conclusions

Pole inflation/α-attractors is an intriguing class of models that suggests that the observed primordial power spectrum may be a universal consequence of a pole in the field space metric. That is to say, regardless of a broad range of microphysical considerations, ultimately observables are determined by just a few key parameters characterising the pole. This property is two-sided. On one hand, such a mechanism seriously limits the potential for learning about fundamental physics from cosmology, given there are many fundamental parameters one simply cannot hope to infer from cosmological data. On the other hand, such robust predictions provide an especially appealing target for future observational surveys and in principle would enable a small number of exceptionally sharp statements about the underlying theory. For example, the model predicts that primordial gravitational waves may be detectable. In the context of this model, such a detection would imply the existence of a hyperbolic moduli space [12, 103], which in turn may be viewed as indirect evidence for extra dimensions.[6] To make such statements, however, it is crucial to understand the robustness of the mechanism both from a phenomenological viewpoint and from the perspective of its possible embedding in string theory or another theory of quantum gravity. Considerable progress in this direction has been done by showing that the so-called 'fibre inflation' model [90–92] is a string realization of α-attractors with $\alpha = 1/2\,,\,2$ [96]. Furthermore,

[6] Hyperbolic moduli spaces arise generically in Kaluza-Klein compactification of higher-dimensional Einstein gravity, and hence also in string theory.

investigations on the effects of string moduli backreaction [104] and Kähler corrections [46] have given strong evidences of the special resilience of this attractor mechanism.

In this paper, we have taken a step forward towards a consistent realization of the pole inflation dynamics in string theory, by exploring the possibility of assistance of many fields in the inflaton sector. The proposed *pole N-flation* model consists of several open string moduli, such as D3-branes, whose collective motion reduces the distance each brane should traverse in order to yield the inflationary attractor phase. Allowing each individual brane to be sufficiently far from the moduli boundary improves the radiative stability of this model.

In §4, we focus on the limitations that UV physics imposes on the effective description of pole inflation when this is embedded into supergravity as a low-energy limit of string theory. We find the existence of axionic partners with decay constants which explicitly depend on the distance to the boundary. This fact has direct consequences for inflation. The bounds which the weak gravity conjecture (WGC) imposes on the periodicity of the axions ($f \lesssim M_{\rm P}$) automatically result in a net constraint on the available length of the exponentially flat plateau typical of pole inflation. We show that in the original single superfield pole-inflation, with a single brane, the inflaton is not even allowed to reach the plateau region of the scalar potential. Moreover, we find that when inflation is driven by the assistance of N branes, these constraints do *not* weaken — we find that the *upper bound* on the canonical radial field range set by the strong forms of the WGC as e.g. the lattice WGC scales like $\Delta\varphi \lesssim \mathcal{O}(1)$, exactly as in the single field case $N = 1$. Rendering the plateau region of the potential available for slow-roll inflation requires relaxing the WGC to milder forms. We interpret these findings as an important WGC bound on the range of validity of the effective field theory of this cosmological scenario.

The universality of the pole inflation/α-attractor mechanism also emerges in our analysis. Despite the presence of N fields, the form of the exponential plateau remains unaltered from the single field case. This implies that when inflation occurs along the collective radial direction, we recover the single field predictions. This may be contrasted with other many-field inflationary constructions, where the predictions at large N are typically distinct from the single field limit [5, 28, 105–110] (however, see Ref. [31] for a counter example). That said, a full analysis of the large-N dynamics of this model remains to be explored, as a subset of the angular field directions may also be sufficiently light to play a role in the inflationary dynamics. This may give rise to richer phenomenology through multifield effects which have the capacity to modify the original predictions of the model. While studying the complete dynamics will be a computationally heavy task, the necessary tools have recently been made publicly available [6, 111–113]; we leave this for future work.

Regarding the implementation of pole N-flation in type IIB string theory, while we have made first steps in §5 by embedding the model in fibred geometries, developing a consistent program for moduli stabilization within this scenario remains an important step to be addressed. We see this as an exciting avenue to be explored.

Acknowledgements

We are grateful to Shamit Kachru, Renata Kallosh, Andrei Linde, Eva Silverstein, Yvette Welling and Yusuke Yamada for helpful comments and discussions. We are particularly indebted to Miguel Montero, Jakob Moritz and Irene Valenzuela for numerous illuminating conversations and explanations. AR and AW are grateful to the SITP in Stanford for warm hospitality while developing this work. JF, AR and AW are supported by the ERC Consolidator Grant STRINGFLATION under the HORIZON 2020 grant agreement no. 647995. MD is supported by the German Science Foundation (DFG) within the Collaborative Research Centre 676 *Particles, Strings and the Early Universe.* MS is supported by the Research Foundation - Flanders (FWO) and the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 665501. MS acknowledges also financial support by 'The Foundation Blanceflor Boncompagni Ludovisi nèe Bildt', by the 'German Academic Exchange Service' (DAAD) and by the foundation 'Angelo Della Riccia' for his research stay at DESY.

Appendix

A Properties of angular element

The angular element $\Omega_i(\psi_\beta)$ as defined by eq. (3.3) can be expressed as

$$\Omega_i = \begin{cases} \cos(\psi_1), & \text{if } i = 1 \\ \prod_{\mu=1}^{i-1} \sin(\psi_\mu)\cos(\psi_i), & \text{if } 1 < i < N \\ \prod_{\mu=1}^{N-1} \sin(\psi_\mu), & \text{if } i = N \end{cases} \tag{A.1}$$

and therefore

$$\partial_\beta \Omega_i = \begin{cases} 0, & \text{if } i < \beta \\ -\Omega_i \frac{\sin(\psi_\beta)}{\cos(\psi_\beta)}, & \text{if } i = \beta\ . \\ \Omega_i \frac{\cos(\psi_\beta)}{\sin(\psi_\beta)}, & \text{if } i > \beta \end{cases} \tag{A.2}$$

Using these expressions it is easy to see that, after take without loss of generality $\beta \geq \gamma$,

$$\sum_{i=1}^{N} \partial_\beta \Omega_i \partial_\gamma \Omega_i = \sum_{i=\beta}^{N} \partial_\beta \Omega_i \partial_\gamma \Omega_i. \tag{A.3}$$

In the case $\beta \neq \gamma$ it further simplifies

$$\sum_{i=1}^{N} \partial_\beta \Omega_i \partial_\gamma \Omega_i = \frac{\cos(\psi_\gamma)}{\sin(\psi_\gamma)} \sum_{i=\beta}^{N} \Omega_i \partial_\beta \Omega_i = 0. \tag{A.4}$$

For the case of $\beta = \gamma$, note that

$$\sum_{i=1}^{N}(\partial_\beta \Omega_i)^2 = \sum_{i=\beta}^{N}(\partial_\beta \Omega_i)^2 = \Omega_\beta^2 \frac{\sin^2(\psi_\beta)}{\cos^2(\psi_\beta)} + \sum_{i=\beta+1}^{N} \Omega_i^2 \frac{\cos^2(\psi_\beta)}{\sin^2(\psi_\beta)} \tag{A.5}$$

which is in general not zero. For example, when $\beta = 1$ this reduces to 1:

$$\sum_{i=1}^{N}(\partial_\beta \Omega_i)^2 = \frac{1}{2}\partial^2_{\beta\beta} \sum_{i=1}^{N} \Omega_i^2 - \sum_{i=1}^{N} \Omega_i \partial^2_{\beta\beta} \Omega_i = -\sum_{i=1}^{N} \Omega_i \partial_{\beta\beta} \Omega_i = 1. \tag{A.6}$$

Throughout the paper it is of special interest the configuration where all Ω_i are the same and therefore $\Omega_i^2 = 1/N$. In order to derive eq. (3.12), note that

$$\begin{aligned}
&\Omega_1^2 = \cos^2(\psi_1) = \frac{1}{N} \quad \rightarrow \quad \frac{\cos^2(\psi_1)}{\sin^2(\psi_1)} = \frac{1}{N-1} \\
&\Omega_2^2 = \sin^2(\psi_1)\cos^2(\psi_2) = \frac{1}{N} \quad \rightarrow \quad \cos^2(\psi_2) = \frac{1}{N-1} \quad \rightarrow \quad \frac{\cos^2(\psi_2)}{\sin^2(\psi_2)} = \frac{1}{N-2} \\
&\vdots \\
&\frac{\cos^2(\psi_\beta)}{\sin^2(\psi_\beta)} = \frac{1}{N-\beta}\ .
\end{aligned} \tag{A.7}$$

This relation, together with eq. (A.5), implies that in the configuration of interest

$$\sum_{i=1}^{N}(\partial_\beta \Omega_i)^2 = \frac{N-\beta+1}{N}\ . \tag{A.8}$$

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