Chapter B

Theory status of Z-boson physics

Ievgen Dubovyk, II. Institut für Theoretische Physik, Universität Hamburg, 22761 Hamburg, Germany and Deutsches Elektronen–Synchrotron, DESY, 15738 Zeuthen, Germany

Ayres Freitas, Pittsburgh Particle Physics, Astrophysics & Cosmology Center (PITT PACC) and Department of Physics & Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA

Janusz Gluza, Institute of Physics, University of Silesia, 40-007 Katowice, Poland

Krzysztof Grzanka, Institute of Physics, University of Silesia, 40-007 Katowice, Poland

Stanisław Jadach, Institute of Nuclear Physics, PAN, 31-342 Krakow, Poland

Tord Riemann, Institute of Physics, University of Silesia, 40-007 Katowice, Poland and Deutsches Elektronen–Synchrotron, DESY, 15738 Zeuthen, Germany

Johann Usovitsch, Trinity College Dublin - School of Mathematics, Dublin 2, Ireland

Corresponding author: Ayres M. Freitas [afreitas@pitt.edu]

The number of Z bosons collected at LEP, approximately 17 millions in total, made it possible to determine a large amount of electroweak observables with very high precision through measurements of the Z lineshape and of cross section asymmetries, joined by high-precision parity-violating asymmetries measured at the SLC

These measurements are typically expressed through the following set of quantities: The cross-section $e^+e^- \to f\bar{f}$ at the Z pole, $\sigma_f^0 \equiv \sigma_f(s = M_Z^2)$, for different final states $f\bar{f}$, the total width of the Z boson, Γ_Z, determined from the shape of $\sigma_f(s)$, and branching ratios of various final states:

$$\sigma^0_{\rm had} = \sigma[e^+e^- \to \text{hadrons}]_{s=M_Z^2}, \tag{B.1}$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}], \tag{B.2}$$

$$R_\ell = \frac{\Gamma[Z \to \text{hadrons}]}{\Gamma[Z \to \ell^+\ell^-]}, \quad \ell = e, \mu, \tau, \tag{B.3}$$

$$R_q = \frac{\Gamma[Z \to q\bar{q}]}{\Gamma[Z \to \text{hadrons}]}, \quad q = u, d, s, c, b. \tag{B.4}$$

In the definition of these quantities, contributions from s-channel photon exchange, virtual box contributions and initial-state as well as initial–final state interference QED radiation are understood to be already subtracted; see $e.\,g.$ Refs. [1,2].

The precise calculation of the terms to be subtracted, at variable cms energy $\sqrt{s}$ around the Z peak, will be a substantial part of the theoretical analysis for the FCC-ee-Z. Further, for a determination of M_Z and Γ_Z we will have to confront cross section data and predictions around the Z peak position as part of the analysis. Correspondingly, section C of this report contains an updated discussion of QED unfolding in the context of the demanding FCC-ee needs is given. To clarify this fact, the parameters (B.1)–(B.4) have become known as so-called electroweak pseudoobservables (EWPOs), rather than true observables. However, (B.1)–(B.4) still include the effect of final-state QED and QCD radiation. Fortunately, the final-state radiation effects factorize from the massive electroweak corrections almost perfectly; see $e.\,g.$ Refs. [3, 4]. Therefore it is possible to compute the latter, as well as potential contributions from new physics, without worrying about effects from soft and collinear real radiation.

The remaining basic pseudoobservables are cross section asymmetries, measured at the Z pole. The forward-backward asymmetry is defined as

$$A_{\rm FB}^f \;=\; \frac{\sigma_f\left[\theta<\frac{\pi}{2}\right]-\sigma_f\left[\theta>\frac{\pi}{2}\right]}{\sigma_f\left[\theta<\frac{\pi}{2}\right]+\sigma_f\left[\theta>\frac{\pi}{2}\right]}, \tag{B.5}$$

where θ is the scattering angle between the incoming e^- and the outgoing f. It can be approximately written as a product of two terms (for more precise discussion, see C.2.4)

$$A_{\rm FB}^f \;=\; \frac{3}{4}\mathcal{A}_e\mathcal{A}_f, \tag{B.6}$$

with

$$\mathcal{A}_f \;=\; \frac{1-4|Q_f|\sin^2\theta_{\rm eff}^{\rm f}}{1-4|Q_f|\sin^2\theta_{\rm eff}^{\rm f}+8(Q_f\sin^2\theta_{\rm eff}^{\rm f})^2}. \tag{B.7}$$

The $\sin^2\theta_{\rm eff}^{\rm f}$ is called the effective weak mixing angle, which contains the net contributions from all the radiative corrections. The most precise measurements of $A_{\rm FB}^f$ have been obtained for leptonic and bottom-quark final states (f $=\ell$, b). In the presence of polarized electron beams, one can also measure the parity-violating left-right asymmetry

$$A_{\rm LR}^f \;=\; \frac{\sigma_f\left[P_e<0\right]-\sigma_f\left[P_e>0\right]}{\sigma_f\left[P_e<0\right]+\sigma_f\left[P_e>0\right]}=\mathcal{A}_e|P_e|. \tag{B.8}$$

Here P_e denotes the polarization degree of the incident electrons where $P_e<0$ ($P_e>0$) refers to left-handed (right-handed) polarizations, respectively. Since $A_{\rm FB}^f$ and $A_{\rm LR}^f$ are defined as normalized asymmetries, they do not depend on (parity conserving) initial- and final-state QED and QCD radiation effects[1].

The present and predicted future experimental values for the most relevant EWPOs are given in the Wishlist Tab. 1 in the Foreword. In the following, we will compare these numbers with the present theoretical situation and with estimates for future precision calculations. In this context, a discussion of theoretical errors connected with these calculations is crucial.

	$\delta\Gamma_Z$ [MeV]	δR_l $[10^{-4}]$	δR_b $[10^{-5}]$	$\delta\sin^2\theta_{\rm eff}^{\rm l}$ $[10^{-6}]$	$\delta\sin^2\theta_{\rm eff}^{\rm b}$ $[10^{-5}]$
Present EWPO errors					
EXP1 [1]	2.3	250	66	160	1600
TH1 [5–7]	0.5	50	15	45	5
FCC-ee-Z EWPO error estimations					
EXP2 [8] & Tab. 2	0.1	10	$2\div6$	6	70

Table B.1: Present total experimental errors EXP1 and, estimated in 2014 [5–7], theoretical intrinsic errors TH1 for selected EW observables. EXP2 gives corresponding error estimations for the FCC-ee Z-resonance mode, see Foreword.

Tab. B.1 shows the FCC-ee experimental goals for the basic EWPOs. As is evident from the table, the theoretical intrinsic uncertainties of the current results TH1 are safely below the current experimental errors EXP1. However, they are not sufficiently small in view of the FCC-ee experimental precision targets EXP2.

[1] Here it is assumed that any issues related to the determination of the experimental acceptance have been evaluated and unfolded using Monte-Carlo methods.

This situation, as seen from the perspective of 2014, underlines the goals and strategic plan for improvements in the theoretical calculation of radiative SM corrections defined here. Historically, the complete one-loop corrections to the Z-pole EWPOs were calculated for the first time in Ref. [9]. Over the next 32 years many groups with many methods determined partial 2- and 3-loop corrections to EWPOs. A more detailed list of the relevant types of radiative corrections will be given below.

In the last two years, as discussed in Ref. [10], substantial progress in numerical calculations of multiloop and multiscale Feynman integrals was made and the calculation of the last piece of 2-loop corrections, of order $\mathcal{O}(\alpha_{bos}^2)$, to all Z-pole EWPOs [11, 12] became possible. Here "bos" denotes diagrams without closed fermion looops.

All the numerical results discussed below are based on the input parameters gathered in Tab. B.2.

Parameter	Value
M_Z	91.1876 GeV
Γ_Z	2.4952 GeV
M_W	80.385 GeV
Γ_W	2.085 GeV
M_H	125.1 GeV
m_t	173.2 GeV
$m_\mathrm{b}^{\overline{\mathrm{MS}}}$	4.20 GeV
$m_\mathrm{c}^{\overline{\mathrm{MS}}}$	1.275 GeV
m_τ	1.777 GeV
$m_e, m_\mu, m_u, m_d, m_s$	0
$\Delta\alpha$	0.05900
$\alpha_\mathrm{s}(M_\mathrm{Z})$	0.1184
G_μ	1.16638×10^{-5} GeV^{-2}

Table B.2: Input parameters used in the numerical analysis, from Refs. [13–15].

As a concrete example, let us discuss the different higher-order contributions to the Standard Model prediction for the bottom-quark effective weak mixing in more detail. It can be written as

$$\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}} = \Big(1 - \frac{{M_\mathrm{W}}^2}{{M_\mathrm{Z}}^2}\Big)(1 + \Delta\kappa_\mathrm{b}), \tag{B.9}$$

where $\Delta\kappa_\mathrm{b}$ contains the contributions from radiative corrections. Numerical results from loop corrections of different order are shown in Table B.3. Altogether, the corrections included in the table are: electroweak $\mathcal{O}(\alpha)$ [9] and $\mathcal{O}(\alpha^2)$ – fermionic α_{ferm}^2 [6, 16–19] and bosonic α_{bos}^2 [11] EW contributions; $\mathcal{O}(\alpha\alpha_\mathrm{s})$ corrections to internal gauge-boson self-energies [20–24]; leading three- and four-loop corrections in the large-m_t limit, of order $\mathcal{O}(\alpha_\mathrm{t}\alpha_\mathrm{s}^2)$ [25, 26], $\mathcal{O}(\alpha_\mathrm{t}^2\alpha_\mathrm{s})$, $\mathcal{O}(\alpha_\mathrm{t}^3)$ [27, 28], and $\mathcal{O}(\alpha_\mathrm{t}\alpha_\mathrm{s}^3)$ [29–31], where $\alpha_\mathrm{t} \equiv \alpha(m_\mathrm{t}^2)$; and non-factorizable vertex contributions $\mathcal{O}(\alpha\alpha_\mathrm{s})$ [32–37] which account for the fact that the factorization between virtual EW corrections and final-state radiation effects is not exact.

The most recently determined piece, the $\mathcal{O}(\alpha_{\mathrm{bos}}^2)$ electroweak two-loop corrections amount to $\Delta\kappa_\mathrm{b}^{(\alpha^2,\mathrm{bos})} = -0.9855 \times 10^{-4}$, which is comparable in magnitude to the fermionic corrections. Taking into account this new result, an updated error estimation due to missing higher order terms will be discussed later on, see Tab. B.3.

Table B.4 summarizes the known contributions to Z boson production and decay vertices, order by order. The technically challenging *bosonic* two-loop calculation was completed very recently [12]. This result has

Order	Value $[10^{-4}]$	Order	Value $[10^{-4}]$
α	468.945	$\alpha_t^2\alpha_s$	1.362
$\alpha\alpha_s$	-42.655	α_t^3	0.123
$\alpha_t\alpha_s^2$	-7.074	$\alpha_{\rm ferm}^2$	3.866
$\alpha_t\alpha_s^3$	-1.196	$\alpha_{\rm bos}^2$	-0.986

Table B.3: Comparison of different kinds of radiative corrections to $\Delta\kappa_b$ [11], using the input parameters in Tab. B.2. Here $\alpha_t = y_t^2/(4\pi)$, where y_t is the top Yukawa coupling.

been achieved through a combination of different methods: (a) numerical integration of Mellin-Barnes (`MB`) representations with contour rotations and contour shifts for a substantial improvement of the convergence; (b) sector decomposition (`SD`) with numerical integration over Feynman parameters; (c) dispersion relations for sub-loop insertions. The `MB` and `SD` methods were discussed intensively at the workshop [38, 39]; see Chapter E for details.

As is evident from Tab. B.4, the two-loop electroweak corrections to the $Z-$boson partial decay widths are sizeable, of the same order as the $\mathcal{O}(\alpha\alpha_s)$ terms. The bosonic corrections $\mathcal{O}(\alpha_{\rm bos}^2)$ are smaller than the fermionic ones, but larger than previously estimated in Ref. [7]. This underlines that theory error evaluations are always to be taken with a grain of salt.

For the total width Γ_Z, the corrections are also significantly larger than the projected future experimental error EXP2 given in Tab. B.1.

Γ_i [MeV]	Γ_e	Γ_ν	Γ_d	Γ_u	Γ_b	Γ_Z
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$	0.038	0.059	0.191	0.170	0.190	1.20
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha_{\rm bos}^2)$	0.017	0.019	0.059	0.058	0.167	0.51

Table B.4: Loop contributions, in units of MeV, to the partial and total Z widths with fixed M_W as input parameter. Here N_f and N_f^2 refer to corrections with one and two closed fermion loops, respectively, whereas $\alpha_{\rm bos}^2$ denotes contributions without closed fermion loops. Furthermore, $\alpha_t = y_t^2/(4\pi)$, where y_t is the top Yukawa coupling. Table taken from Ref. [12].

These numerical examples demonstrate that radiative electroweak corrections beyond the two-loop level must be calculated for future high-luminosity e^+e^- experiments. In B.4 corrections are calculated using M_W as an input. By calculating M_W obtained from G_μ, we get for $\mathcal{O}(\alpha_{\rm bos}^2)$ instead 0.51 MeV a value 0.34 MeV [12].

Let us discuss impact of radiative corrections in more detail by estimating their potential values.

On one hand, a source of uncertainty for the Standard Model prediction for any EWPO is their dependence on input parameters, as listed in Tab. B.2. The impact of input parameters is best evaluated through a global fit, as shown *e. g.* in Refs. [13, 40]. On the other hand, a separate source of uncertainty is the missing knowledge of theoretical higher-order corrections.

To estimate the latter, one can take different approaches, each of which has its own advantages and disadvantages [41]:

1. Determination of relevant prefactors of a class of higher-order corrections such as couplings, group factors, particle multiplicities, mass ratios, *etc.* and assuming the remainder of the loop amplitude to be order $\mathcal{O}(1)$.
2. Extrapolation under the assumption that higher order radiative corrections can be approximated by a geometric series.
3. Testing the scale dependence of a given fixed-order result in the $\overline{\text{MS}}$ renormalization scheme in order to estimate the size of the missing higher orders; used more often in QCD.
4. Comparing results in the on-shell and $\overline{\text{MS}}$ schemes, where the differences are of the next order in the perturbative expansion.

In Tab. B.5, the intrinsic errors are shown for the Z-boson decay width. TH1 gives numerical estimates that are mainly based on the geometric series extrapolation, but corroborated by some of the other methods. In [12] the α_{bos}^2 contribution has been calculated to be +0.505 MeV with a net numerical precision of about four digits, which eliminates the uncertainty associated with that term completely. It also shifts some of the geometric series extrapolations, such as

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_\mathrm{t}^3) \quad \sim \quad \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_\mathrm{t}^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha^2) \sim 0.2\ \text{MeV}, \tag{B.10}$$

where the full $\mathcal{O}(\alpha^2)$ term was previously not available. The new error estimate TH-new is $\pm$0.4 MeV. As we can see, the estimated theoretical error is still much larger than what is needed for the projected EXP2 goals in Tab. B.1, which is for the Z-boson decay width $\lesssim \pm 0.1$ MeV. The dominant remaining uncertainty stems from unknown three-loop contributions with either QCD loops, $\mathcal{O}(\alpha\alpha_\mathrm{s}^2)$ and $\mathcal{O}(\alpha^2\alpha_\mathrm{s})$, or from electroweak fermionic loops, $\mathcal{O}(N_f^2\alpha^3)$, where N_f^2 refers to diagrams with at least two closed fermion loops.

Once these corrections become available, with a robust intrinsic numerical precision of at least two digits, the remaining theory error will become dominated by missing four-loop terms. Estimating these future errors is rather unreliable at this time using geometric series of perturbation, since it requires two orders of extrapolation. Nevertheless, a rough guess can be obtained by using the following experience-based scaling relations: each order of $N_f\alpha$ and α_bos generate corrections of about 0.1 and 0.01, respectively, and n orders of α_s produce a correction of roughly $n! \times (0.1)^n$, where the $n!$ factor accounts for the combinatorics of the SU(3) algebra. In this fashion one arrives at the TH2 scenario in Tab. B.5[2].

For a safe interpretation of Fcc-ee-Z measurements, the theory error must be subdominant relative to the experimental uncertainties. Comparing the TH2 scenario with the EXP2 numbers, one can see that it does not yet fit this bill. This implies that calculation of 4-loop corrections, or at least the leading parts thereof, will be necessary to fully match the planned precision of the FCC-ee experiments. Since estimates of future theory errors are highly uncertain, and 4-loop contributions are two orders beyond the current state of the art, we do not attempt to make a quantitative estimate of the achievable precision, but it seems plausible that the remaining uncertainty will be well below the EXP2 targets.

Let us now come back to the prospects for computing the missing three-loop contributions. There are two basic factors which play a role: the number of Feynman diagrams (or, correspondingly, the number of Feynman integrals) and the precision with which single Feynman integrals can be calculated. Some basic bookkeeping concerning the number of diagram topologies and different types of diagrams is given in Tab. B.6. First, let us compare the known number of diagram topologies and individual diagrams at two loops and three loops. Comparing the genuine three-loop fermionic diagrams, which are simpler than the bosonic ones, to the already known two-loop bosonic diagrams, there is about an order of magnitude difference in their number: 17580 diagrams for $Z \to bb$ (and 13104 diagrams for $Z \to e^+e^-$) at $\mathcal{O}(\alpha_\mathrm{ferm}^3)$ versus 964 (766) diagrams at

[2] Accounting for "everything else" besides the specific orders listed in Tab. B.5, one may assign a more conservative future theory error estimate of $\delta\Gamma_Z \sim 0.2$ MeV, see also Ref. [41].

δ_1 :	δ_2 :	δ_3 :	δ_4 :	δ_5 :	$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(\alpha^3)$	$\mathcal{O}(\alpha^2\alpha_\mathrm{s})$	$\mathcal{O}(\alpha\alpha_\mathrm{s}^2)$	$\mathcal{O}(\alpha\alpha_\mathrm{s}^3)$	$\mathcal{O}(\alpha_{bos}^2)$	$=\sqrt{\sum_{i=1}^5 \delta_i^2}$
TH1 (estimated error limits from geometric series of perturbation)					
0.26	0.3	0.23	0.035	0.1	0.5
TH1-new (estimated error limits from geometric series of perturbation)					
0.2	0.21	0.23	0.035	$< 10^{-4}$	0.4

δ_1' :	δ_2' :	δ_3' :	δ_4 :		$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(N_f^{\leq 1}\alpha^3)$	$\mathcal{O}(\alpha^3\alpha_\mathrm{s})$	$\mathcal{O}(\alpha^2\alpha_\mathrm{s}^2)$	$\mathcal{O}(\alpha\alpha_\mathrm{s}^3)$		$\sqrt{\delta_1'^2+\delta_2'^2+\delta_2'^3+\delta_4^2}$
TH2 (extrapolation through prefactor scaling)					
0.04	0.1	0.1	0.035	10^{-4}	0.15

Table B.5: The intrinsic theoretical error estimates TH1 for Γ_Z, as given in [7,41], and updates taking into account the newly completed $\mathcal{O}(\alpha_\mathrm{bos}^2))$ corrections TH1-new [12]. TH2 is a projection into the future, assuming $\delta_{2,3}$ and the fermionic parts of δ_1 to be known.

$\mathcal{O}(\alpha_\mathrm{bos}^2)$. In general, however, the number of diagrams is of course not equivalent to the number of integrals to be calculated. At $\mathcal{O}(\alpha_\mathrm{ferm}^3)$ we expect $\mathcal{O}(10^3) - \mathcal{O}(10^4)$ distinct three-loop Feynman integrals before a reduction to a basis, because different classes of diagrams often share parts of their integral basis.

Second, the accuracy with which three-loop diagrams can be calculated must be estimated. For the two-loop bosonic vertex integrals, results have been obtained with a high level of accuracy, which was 8 digits in most cases and at least 6 digits for the few worst integrals; with some room for improvements. The final accuracy of the complete results for the bosonic two-loop corrections to the EWPOs was at the level of at least four digits [11, 12]. To achieve this goal, the Feynman integrals have been calculated numerically, directly in the Minkowskian region, with two main approaches: (i) `SD` as implemented in the packages `FIESTA 3` [42] and `SecDec 3` [43], and (ii) `MB` integrals as implemented in the package `MBsuite` [44–49]. Because fermionic three-loop diagrams are technically not much more complicated than two-loop bosonic integrals ($e.\,g.$ in the case of self-energy insertions the dimensionality of MB integrals increases by only one), an overall two-digit precision for the final phenomenological results appears within reach. This estimate is based on present knowledge and available methods and tools.

Two further remarks are in order. First, the previously estimated value of the bosonic two-loop correction to Γ_Z based on the geometric series (TH1) was at the level of 0.1 MeV, which is much smaller than its actual calculated value [12, 41]. This is partly based on the fact that all final-state flavors sum up because they contribute to $\Gamma_Z(\alpha_\mathrm{bos}^2)$ with the same sign, which was not forseen in the previous estimate. Thus care should be taken in interpreting any theory error estimates. Nonetheless, due to lack of a better strategy, we *assume* that the values TH1-new in Tab. B.5 are representative of the actual size of the currently unknown three-loop corrections. Second, the achievement of at least 2 digits intrinsic net numerical precision for the three-loop electroweak corrections will likely require the evaluation of single Feynman integrals with much higher precision than in two-loop case, since the larger number of diagrams leads to more numerical cancellations, and each new diagram topology poses new challenges for the numerical convergence.

Thus, besides straightforward improvements in numerical calculations based on `SD` and `MB` methods, work on new innovative numerical and analytical techniques (and combinations thereof) should continue and

$Z \to b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	$14 \overset{(A)}{\to} 7 \overset{(B)}{\to} 5$	$211 \overset{(A)}{\to} 84 \overset{(B)}{\to} 51$
Number of diagrams	15	2383 $\overset{(A,B)}{\to}$ 1074	490387 $\overset{(A,B)}{\to}$ 120472
Fermionic loops	0	150	17580
Bosonic loops	15	924	102892
Planar / Non-planar	15 / 0	981/133	84059/36413
QCD / EW	1 / 14	98 / 1016	10386/110086

$Z \to e^+e^-, \ldots$			
Number of topologies	1 loop	2 loops	3 loops
	1	$14 \overset{(A)}{\to} 7 \overset{(B)}{\to} 5$	$211 \overset{(A)}{\to} 84 \overset{(B)}{\to} 51$
Number of diagrams	14	2012 $\overset{(A,B)}{\to}$ 880	397690 $\overset{(A,B)}{\to}$ 91472
Fermionic loops	0	114	13104
Bosonic loops	14	766	78368
Planar / Non-planar	14 / 0	782/98	65487/25985
QCD / EW	0 / 14	0 / 880	144/91328

Table B.6: Number of topologies and diagrams for $Z \to f\bar{f}$ decays in the Feynman gauge. Statistics for planarity, QCD and EW type diagrams is also given. Label (A) denotes statistics after elimination of tadpoles and wavefunction corrections, and label (B) denotes statistics after elimination of topological symmetries of diagrams.

may lead to accelerated progress. There are many other places for future improvements, e.g. optimizations at the 3-loop and 4-loop level of the minimal number of MB-integral dimensions (see Section 3.6 in this report), IBP reductions to master integrals, reliable practical prescriptions for the γ_5 issue at 3 loops and beyond. The numerical methods will certainly be complemented by progress in analytical and semi-analytical approaches (both in methods and tools), to which Chapter E is devoted. Similarly, other EWPOs can be discussed. Table B.7 gathers all present and expected theoretical intrinsic error estimations (see $e.\,g.$ Ref. [41]).

To summarize, FCC-ee-Z imposes very strong demands on future theoretical calculations of currently unknown higher-order quantum EW and QCD corrections. As shown here, different estimations lead to predictions for EWPO error bands which are at the level or of the order of future experimental demands. Then, actual calculations may shift the values and diminish the errors of EWPOs substantially, as it has been shown recently in the case of the Z-boson decay width [12]. Here the result of the bosonic two-loop corrections was found to be larger than the previous estimate by a factor 3–5, depending on the chosen input parametrization. One of the most promising avenues for addressing the challenges of these future calculations are numerical integration methods. These are more flexible than analytical techniques, but are limited by the achievable numerical precision. Our estimations bring us to the conclusion that at least two digits accuracy in future 3- and 4-loop calculations of EWPOs is needed. Therefore, dedicated and increased efforts by the theory community will be important to meet the experimental demands of the FCC-ee-Z or other lepton collider projects in the Z line shape mode and not limit the physical interpretation of the corresponding precision measurements.

FCC-ee-Z EWPO error estimations				
	$\delta\Gamma_Z$ [MeV]	δR_l $[10^{-4}]$	δR_b $[10^{-5}]$	$\delta \sin^2\theta^{\rm l}_{\rm eff}$ $[10^{-5}]$
EXP2 [8]	0.1	10	$2 \div 6$	6
TH1-new	0.4	60	10	45
TH2	0.15	15	5	15
TH3	< 0.07	< 7	< 3	< 7

Table B.7: Comparison of experimental FCC-ee precision goals for selected EWPOs (EXP2, from Table B.1) to various scenarios for theory error estimations. TH1-new is the current theory error based on extrapolations through geometric series. TH2 is an estimate of the theory error (using prefactor scalings), assuming that electroweak 3-loop corrections are known. TH3 denotes a scenario where also the dominant 4-loop corrections are available. Since reliable quantative estimates of TH3 are not possible at this point, only conservative upper bounds on the theory error are given.

Let us stress that apart from the problems mentioned here, there is also the issue of extracting EWPOs from real processes, including the QED unfolding. This is the subject of Chapter C, see also [2] and [50].

Bibliography

[1] ALEPH collab., DELPHI collab., L3 collab., OPAL collab., SLD collab., LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group, S. Schael, et al., Precision electroweak measurements on the Z resonance, Phys. Rept. 427 (2006) 257–454. arXiv:hep-ex/0509008, doi:10.1016/j.physrep.2005.12.006.

[2] J. Gluza, A. Freitas, T. Riemann, et al., The Z boson line shape - from 1.5 loops at LEP to 2.5 loops in future. Talk held by T. Riemann at [51], https://indico.cern.ch/event/669224/contributions/2805418/attachments/1581648/2504620/riemann-FCCeeminiCERN-short.pdf.

[3] A. Arbuzov, M. Awramik, M. Czakon, A. Freitas, M. Grünewald, K. Mönig, S. Riemann, T. Riemann, ZFITTER: A Semi-analytical program for fermion pair production in e^+e^- annihilation, from version 6.21 to version 6.42, Comput. Phys. Commun. 174 (2006) 728–758. arXiv:hep-ph/0507146, doi:10.1016/j.cpc.2005.12.009.

[4] K. G. Chetyrkin, J. H. Kuhn, A. Kwiatkowski, QCD corrections to the e^+e^- cross-section and the Z boson decay rate, CERN 95-03. See also: QCD corrections to the e^+e^- cross-section and the Z boson decay rate: Concepts and results, LBL-36678-REV and Phys. Rept. 277 (1996) 189-281. arXiv:hep-ph/9503396, doi:10.1016/S0370-1573(96)00012-9.

[5] M. Awramik, M. Czakon, A. Freitas, Bosonic corrections to the effective weak mixing angle at $O(\alpha^2)$, Phys. Lett. B642 (2006) 563–566. arXiv:hep-ph/0605339, doi:10.1016/j.physletb.2006.07.035.

[6] M. Awramik, M. Czakon, A. Freitas, B. Kniehl, Two-loop electroweak fermionic corrections to $\sin^2\theta_{\rm eff}^{\rm b\bar{b}}$, Nucl. Phys. B813 (2009) 174–187. arXiv:0811.1364, doi:10.1016/j.nuclphysb.2008.12.031.

[7] A. Freitas, Higher-order electroweak corrections to the partial widths and branching ratios of the Z boson, JHEP 1404 (2014) 070. arXiv:1401.2447, doi:10.1007/JHEP04(2014)070.

[8] "Motivation: Experimental capabilities and requirements", talk held by A. Blondel at [51], https://indico.cern.ch/event/669224/contributions/2805398/attachments/1581811/2499870/Blondel-FCC-ee-physics_case_and_theory_errors-v2.pdf.

[9] A. Akhundov, D. Bardin, T. Riemann, Electroweak one loop corrections to the decay of the neutral vector boson, Nucl. Phys. B276 (1986) 1, Dubna preprint JINR-E2-85-617 (Aug 1985). doi:10.1016/0550-3213(86)90014-3.

[10] I. Dubovyk, J. Gluza, A. Freitas, T. Riemann, J. Usovitsch, "Theory - status", talk held by J. Gluza at [51], https://indico.cern.ch/event/669224/contributions/2805413/attachments/1581532/2499590/FCC_gluza_TheoryStatus.pdf.

[11] I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, The two-loop electroweak bosonic corrections to $\sin^2\theta_{\rm eff}^{\rm b}$, Phys. Lett. B762 (2016) 184–189. arXiv:1607.08375, doi:10.1016/j.physletb.2016.09.012.

[12] I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, Complete electroweak two-loop corrections to Z boson production and decay, Phys. Lett. B783 (2018) 86–94. arXiv:1804.10236, doi:10.1016/j.physletb.2018.06.037.

[13] C. Patrignani, et al., Review of Particle Physics, Chin. Phys. C40 (10) (2016) 100001. doi:10.1088/1674-1137/40/10/100001.

[14] M. Steinhauser, Leptonic contribution to the effective electromagnetic coupling constant up to three loops, Phys. Lett. B429 (1998) 158–161. arXiv:hep-ph/9803313, doi:10.1016/S0370-2693(98)00503-6.

[15] M. Davier, A. Hoecker, B. Malaescu, Z. Zhang, Reevaluation of the Hadronic Contributions to the Muon g-2 and to alpha(MZ), Eur. Phys. J. C71 (2011) 1515, [Erratum: Eur. Phys. J.C72,1874(2012)]. arXiv:1010.4180, doi:10.1140/epjc/s10052-012-1874-8,10.1140/epjc/s10052-010-1515-z.

[16] R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere, Radiative correction effects of a very heavy top, Phys. Lett. B288 (1992) 95–98, [Erratum: Phys. Lett.B312,511(1993)]. arXiv:hep-ph/9205238, doi:10.1016/0370-2693(93)90990-Y,10.1016/0370-2693(92)91960-H.

[17] R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere, Two loop heavy top effects in the Standard Model, Nucl. Phys. B409 (1993) 105–127, doi:10.1016/0550–3213(93)90448–X. doi:10.1016/0550-3213(93)90448-X.

[18] J. Fleischer, O. V. Tarasov, F. Jegerlehner, Two loop heavy top corrections to the rho parameter: A Simple formula valid for arbitrary Higgs mass, Phys. Lett. B319 (1993) 249–256. doi:10.1016/0370-2693(93)90810-5.

[19] J. Fleischer, O. V. Tarasov, F. Jegerlehner, Two loop large top mass corrections to electroweak parameters: Analytic results valid for arbitrary Higgs mass, Phys. Rev. D51 (1995) 3820–3837. doi:10.1103/PhysRevD.51.3820.

[20] A. Djouadi, C. Verzegnassi, Virtual very heavy top effects in LEP/SLC precision measurements, Phys. Lett. B195 (1987) 265.

[21] A. Djouadi, O(alpha alpha-s) Vacuum Polarization Functions of the Standard Model Gauge Bosons, Nuovo Cim. A100 (1988) 357. doi:10.1007/BF02812964.

[22] B. A. Kniehl, Two loop corrections to the vacuum polarizations in perturbative QCD, Nucl. Phys. B347 (1990) 86–104. doi:10.1016/0550-3213(90)90552-O.

[23] B. A. Kniehl, A. Sirlin, Dispersion relations for vacuum polarization functions in electroweak physics, Nucl. Phys. B371 (1992) 141–148. doi:10.1016/0550-3213(92)90232-Z.

[24] A. Djouadi, P. Gambino, Electroweak gauge bosons selfenergies: Complete QCD corrections, Phys. Rev. D49 (1994) 3499–3511, Erratum: Phys. Rev. D53 (1996) 4111. arXiv:hep-ph/9309298, doi:10.1103/PhysRevD.49.3499,10.1103/PhysRevD.53.4111.

[25] L. Avdeev, J. Fleischer, S. Mikhailov, O. Tarasov, $O(\alpha\alpha_s^2)$ correction to the electroweak ρ parameter, Phys. Lett. B336 (1994) 560–566, doi:10.1016/0370–2693(94)90573–8, Erratum–ibid. B349 (1995) 597–598, hep–ph/9406363v2. doi:10.1016/0370-2693(94)90573-8.

[26] K. Chetyrkin, J. H. Kühn, M. Steinhauser, Corrections of order $O(G_F M_t^2 \alpha_s^2)$ to the ρ parameter, Phys. Lett. B351 (1995) 331–338, doi:10.1016/0370–2693(95)00380–4. arXiv:hep-ph/9502291, doi:10.1016/0370-2693(95)00380-4.

[27] J. J. van der Bij, K. G. Chetyrkin, M. Faisst, G. Jikia, T. Seidensticker, Three loop leading top mass contributions to the ρ parameter, Phys. Lett. B498 (2001) 156–162. arXiv:hep-ph/0011373, doi:10.1016/S0370-2693(01)00002-8.

[28] M. Faisst, J. H. Kühn, T. Seidensticker, O. Veretin, Three loop top quark contributions to the ρ parameter, Nucl. Phys. B665 (2003) 649–662. arXiv:hep-ph/0302275, doi:10.1016/S0550-3213(03)00450-4.

[29] Y. Schröder, M. Steinhauser, Four-loop singlet contribution to the ρ parameter, Phys. Lett. B622 (2005) 124–130. arXiv:hep-ph/0504055, doi:10.1016/j.physletb.2005.06.085.

[30] K. G. Chetyrkin, M. Faisst, J. H. Kühn, P. Maierhofer, C. Sturm, Four-loop QCD corrections to the rho parameter, Phys. Rev. Lett. 97 (2006) 102003. arXiv:hep-ph/0605201, doi:10.1103/PhysRevLett.97.102003.

[31] R. Boughezal, M. Czakon, Single scale tadpoles and $O(G_F m_t^2 \alpha(s)^3)$ corrections to the ρ parameter, Nucl. Phys. B755 (2006) 221–238. arXiv:hep-ph/0606232, doi:10.1016/j.nuclphysb.2006.08.007.

[32] A. Czarnecki, J. H. Kühn, Nonfactorizable QCD and electroweak corrections to the hadronic Z boson decay rate, Phys. Rev. Lett. 77 (1996) 3955–3958. arXiv:hep-ph/9608366.

[33] R. Harlander, T. Seidensticker, M. Steinhauser, Complete corrections of order $O(\alpha\alpha_s)$ to the decay of the Z boson into bottom quarks, Phys. Lett. B426 (1998) 125–132. arXiv:hep-ph/9712228, doi:10.1016/S0370-2693(98)00220-2.

[34] J. Fleischer, O. V. Tarasov, F. Jegerlehner, P. Raczka, Two loop $O(\alpha_s G_\mu m_t^2)$ corrections to the partial decay width of the Z^0 into $b\bar{b}$ final states in the large top mass limit, Phys. Lett. B293 (1992) 437–444. doi:10.1016/0370-2693(92)90909-N.

[35] G. Buchalla, A. J. Buras, QCD corrections to the $\bar{s}dZ$ vertex for arbitrary top quark mass, Nucl. Phys. B398 (1993) 285–300. doi:10.1016/0550-3213(93)90110-B.

[36] G. Degrassi, Current algebra approach to heavy top effects in $Z \to b + \bar{b}$, Nucl. Phys. B407 (1993) 271–289. arXiv:hep-ph/9302288, doi:10.1016/0550-3213(93)90058-W.

[37] K. Chetyrkin, A. Kwiatkowski, M. Steinhauser, Leading top mass corrections of order $O(\alpha\alpha_s m_t^2/M_W^2)$ to partial decay rate $\Gamma(Z \to b\bar{b})$, Mod. Phys. Lett. A8 (1993) 2785–2792, doi:10.1142/S0217732393003172. doi:10.1142/S0217732393003172.

[38] I. Dubovyk, J. Gluza, T. Riemann, J. Usovitsch, "MB-suite 1: AMBRE news: non-planar 3-loop vertices", talk held by I. Dubovyk at [51], https://indico.cern.ch/event/669224/overview.

[39] S. Borowka, "pySecDec for phenomenological predictions", talk held at [51], https://indico.cern.ch/event/669224/contributions/2805456/attachments/1582115/2500426/miniWS2018_sophia.pdf.

[40] J. de Blas, M. Ciuchini, E. Franco, S. Mishima, M. Pierini, L. Reina, L. Silvestrini, The Global Electroweak and Higgs Fits in the LHC era, PoS EPS-HEP2017 (2017) 467. arXiv:1710.05402, doi:10.22323/1.314.0467.

[41] A. Freitas, Numerical multi-loop integrals and applications, Prog. Part. Nucl. Phys. 90 (2016) 201–240. arXiv:1604.00406, doi:10.1016/j.ppnp.2016.06.004.

[42] A. V. Smirnov, FIESTA 3: cluster-parallelizable multiloop numerical calculations in physical regions, Comput. Phys. Commun. 185 (2014) 2090–2100. arXiv:1312.3186, doi:10.1016/j.cpc.2014.03.015.

[43] S. Borowka, G. Heinrich, S. P. Jones, M. Kerner, J. Schlenk, T. Zirke, SecDec-3.0: numerical evaluation of multi-scale integrals beyond one loop, Comput. Phys. Commun. 196 (2015) 470–491. arXiv:1502.06595, doi:10.1016/j.cpc.2015.05.022.

[44] J. Gluza, K. Kajda, T. Riemann, AMBRE - a Mathematica package for the construction of Mellin-Barnes representations for Feynman integrals, Comput. Phys. Commun. 177 (2007) 879–893. arXiv:0704.2423, doi:10.1016/j.cpc.2007.07.001.

[45] J. Gluza, K. Kajda, T. Riemann, V. Yundin, Numerical Evaluation of Tensor Feynman Integrals in Euclidean Kinematics, Eur. Phys. J. C71 (2011) 1516. arXiv:1010.1667, doi:10.1140/epjc/s10052-010-1516-y.

[46] K. Bielas, I. Dubovyk, J. Gluza, T. Riemann, Some Remarks on Non-planar Feynman Diagrams, Acta Phys. Polon. B44 (11) (2013) 2249–2255. arXiv:1312.5603, doi:10.5506/APhysPolB.44.2249.

[47] I. Dubovyk, J. Gluza, T. Riemann, Non-planar Feynman diagrams and Mellin-Barnes representations with $AMBRE$ 3.0, J. Phys. Conf. Ser. 608 (1) (2015) 012070. doi:10.1088/1742-6596/608/1/012070.

[48] I. Dubovyk, J. Gluza, T. Riemann, J. Usovitsch, Numerical integration of massive two-loop Mellin-Barnes integrals in Minkowskian regions, PoS LL2016 (2016) 034, https://pos.sissa.it/260/034/pdf. arXiv:1607.07538.

[49] I. Dubovyk, J. Gluza, T. Jelinski, T. Riemann, J. Usovitsch, New prospects for the numerical calculation

of Mellin-Barnes integrals in Minkowskian kinematics, Acta Phys. Polon. B48 (2017) 995. arXiv:1704.02288, doi:10.5506/APhysPolB.48.995.

[50] S. Jadach, "How to calculate QED higher orders in a form useful for MC event generators with soft photon and/or collinear resummation?", talk held at [51], https://indico.cern.ch/event/669224/overview.

[51] A. Blondel, J. Gluza, P. Janot (org.), Mini workshop: *Precision EW and QCD calculations for the FCC studies: methods and tools*, 12-13 January 2018, CERN, Geneva, Switzerland, webpage https://indico.cern.ch/event/669224/.

BU-HEPP-18-04, CERN-TH-2018-145, IFJ-PAN-IV-2018-09, KW 18-003
MITP/18-052, MPP-2018-143, SI-HEP-2018-21

Standard Model Theory for the FCC-ee: The Tera-Z

Report on the 1^{st} Mini workshop: Precision EW and QCD calculations for the FCC studies: methods and tools, 12-13 January 2018, CERN, Geneva

https://indico.cern.ch/event/669224/

A. Blondel[1], J. Gluza[*,2], S. Jadach[3], P. Janot[4], T. Riemann[2,5] (editors),
A. Akhundov[6], A. Arbuzov[7], R. Boels[8],
S. Bondarenko[7], S. Borowka[4], C.M. Carloni Calame[9], I. Dubovyk[8,5],
Ya. Dydyshka[10], W. Flieger[2], A. Freitas[11], K. Grzanka[2], T. Hahn[12], T. Huber[13],
L. Kalinovskaya[10], R. Lee[14], P. Marquard[5], G. Montagna[15], O. Nicrosini[9],
C. G. Papadopoulos[16], F. Piccinini[9], R. Pittau[17], W. Płaczek[18], M. Prausa[19],
S. Riemann[5], G. Rodrigo[20], R. Sadykov[10], M. Skrzypek[3], D. Stöckinger[21], J. Usovitsch[22],
B.F.L. Ward[23,12], S. Weinzierl[24], G. Yang[25], S.A. Yost[26]

[1]*DPNC University of Geneva, Switzerland,* [2]*Institute of Physics, University of Silesia, 40-007 Katowice, Poland,* [3]*Institute of Nuclear Physics, PAN, 31-342 Krakow, Poland,* [4]*CERN, CH-1211 Genéve 23, Switzerland,* [5]*Deutsches Elektronen–Synchrotron, DESY, 15738 Zeuthen, Germany,* [6]*Departamento de Física Teorica, Universidad de València, 46100 València, Spain and Azerbaijan National Academy of Sciences, ANAS, Baku, Azerbaijan,* [7]*Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, 141980 Russia,* [8]*II. Institut für Theoretische Physik, Universität Hamburg, 22761 Hamburg, Germany,* [9]*Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy,* [10]*Dzhelepov Laboratory of Nuclear Problems, JINR, Dubna, 141980 Russia,* [11]*Pittsburgh Particle physics, Astrophysics & Cosmology Center (PITT PACC) and Department of Physics & Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA,* [12]*Max-Planck-Institut für Physik, 80805 München, Germany,* [13]*Naturwissenschaftlich-Technische Fakultät, Universität Siegen, 57068 Siegen, Germany,* [14]*The Budker Institute of Nuclear Physics, 630090, Novosibirsk,* [15]*Dipartimento di Fisica, Università di Pavia, Pavia, Italy,* [16]*Institute of Nuclear and Particle Physics, NCSR Demokritos, 15310, Greece,* [17]*Dep. de Física Teórica y del Cosmos and CAFPE, Universidad de Granada, E-18071 Granada, Spain,* [18]*Marian Smoluchowski Institute of Physics, Jagiellonian University, 30-348 Kraków, Poland,* [19]*Albert-Ludwigs-Universität, Physikalisches Institut, Freiburg, Germany,* [20]*Instituto de Física Corpuscular, Universitat de València - CSIC, 46980 Paterna, València, Spain,* [21]*Institut für Kern- und Teilchenphysik, TU Dresden, 01069 Dresden, Germany,* [22]*Trinity College Dublin - School of Mathematics, Dublin 2, Ireland,* [23]*Baylor University, Waco, TX, USA,* [24]*PRISMA Cluster of Excellence, Inst. für Physik, Johannes Gutenberg-Universität, 55099 Mainz, Germany,* [25]*CAS Key Laboratory of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China,* [26]*The Citadel, Charleston, SC, USA*

[*] Corresponding editor, E-mail: janusz.gluza@cern.ch.

Contents