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Relaxion Dark Matter

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We highlight a new connection between the Standard Model hierarchy problem and the dark matter sector. The key piece is the relaxion field, which besides scanning the Higgs mass and setting the electroweak scale, also constitutes the observed dark matter abundance of the universe. The relaxation mechanism is realized during inflation, and the necessary friction is provided by particle production, with no need for a large number of e-folds and no new physics at the TeV scale. Using this framework we show that the relaxion is a phenomenologically viable dark matter candidate in the keV mass range.

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Introduction. The non-baryonic matter component of the universe, the dark matter (DM), constitutes about a fifth of the total energy density of our universe. Despite the several experimental searches and the impressive effort of the community, the non-gravitational nature of DM is still unknown. In the last decades, a lot of attention was devoted to the class of beyond Standard Model (SM) theories which can provide a weakly interacting massive particle (WIMP) DM candidate, which features an appealing connection between the dark sector and the electroweak scale. In light of no definitive evidence of new physics at the TeV scale and strong exclusion limits from WIMP direct detection experiments, to go beyond the WIMP paradigm became crucial. In this letter we propose another option which can closely connect the Higgs naturalness problem with the DM sector. The link is the relaxion field.

The cosmological relaxation of the electroweak scale is a recent proposal to address the SM hierarchy problem making use of the relaxion, a new axion-like field, which scans the Higgs mass parameter during its cosmological evolution [1]. The relaxion evolution stops due to a back-reaction mechanism which turns on when the Higgs vacuum expectation value (VEV) is at the electroweak scale. This paradigm shift fits in the interface between particle physics and early universe cosmology and gave rise to a varied literature, including studies related to the model building challenges [2–9], concerns about the inflationary and reheating sectors [10–15], alternatives to inflation [16–18], UV completions [19–22], developments on the model building front [23–31], a leptogenesis realization [32], and experimental signatures [33–37].

The constructions discussed in [2, 35] use a Higgs-dependent barrier as back-reaction mechanism to stop the field during inflation. In these models, the relaxion abundance is too small to explain DM. A second field, which scans the barriers' amplitude in [2], can instead meet this requirement being produced via the misalignment mechanism. Oppositely, if relaxation proceeds after inflation, the relaxion is overproduced [18].

In the framework used here, relaxation happens during inflation and the relaxion stopping mechanism is provided by particle production [17]. This construction does not require new physics close to the TeV scale, and it can be realized without the need of large number of e-folds or super-Planckian field excursions. In this note, we point out that the relaxion field, which sets the value of the electroweak scale, can simultaneously account for the observed DM density, when it is produced after reheating by the scattering of SM particles. Similarly to the case where the DM production happens via the freeze-in mechanism (see *e.g.* [38] for a review), the relaxion is never in thermal equilibrium with the SM bath and its comoving number density freezes to a constant value when the number density of the particles dominating its production become Boltzmann-suppressed.

Parameter space for the model. In this section we derive the parameter space for the model proposed in [17] in which relaxation happens during inflation and particle production is the stopping mechanism.[1] The Lagrangian can be written as

$$\mathcal{L} \supset \frac{1}{2}\left(\Lambda^2 - g'\Lambda\phi\right)h^2 + g\Lambda^3\phi - \frac{\lambda}{4}h^4 - \Lambda_b^4\cos\left(\frac{\phi}{f'}\right) - \frac{\phi}{4\mathcal{F}}\left(g_2^2 W^a_{\mu\nu}\widetilde{W}^{a\,\mu\nu} - g_1^2 B_{\mu\nu}\widetilde{B}^{\mu\nu}\right), \qquad (1)$$

where ϕ is an axion-like field with decay constant f', h is the Higgs field, Λ is the cutoff of the theory, the dimensionless parameters g and g' are assumed to be spurions that explicitly break the axion shift symmetry, λ is the Higgs quartic coupling, Λ_b is the scale at which the ϕ cosine potential is generated. The effective scale $\mathcal{F}$ controls the interaction of the relaxion with the SM gauge bosons. We assume that the relaxation dynamics takes place in the broken phase so that the Higgs mass parameter, $\mu_h^2(\phi) \equiv (-\Lambda^2 + g'\Lambda\,\phi)$, is large and negative when the scanning process starts, $\mu_h^2(\phi_{\rm ini}) \sim -\Lambda^2$. The last term in Eq. (1) is the interaction responsible to slow down the relaxion once the particle production is triggered. B and W are the SM gauge bosons with g_1 and

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[1] See [18] for an analysis of the case in which relaxation happens after inflation.

g_2 being the corresponding U(1) and SU(2) gauge couplings. When expanded in the mass eigenstates this term reads

$$-\frac{\phi}{\mathcal{F}}\epsilon^{\mu\nu\rho\sigma}\Big(2g_2^2\partial_\mu W_\nu^-\partial_\rho W_\sigma^+ + (g_2^2-g_1^2)\partial_\mu Z_\nu\partial_\rho Z_\sigma - 2g_1g_2\partial_\mu Z_\nu\partial_\rho A_\sigma\Big)\,. \tag{2}$$

In what follows we will just consider the tachyonic instability from the $Z\widetilde{Z}$ term and absorb the gauge coupling in the definition of the corresponding field such that $1/f = (g_2^2 - g_1^2)/\mathcal{F}$, as in [18].

The non-generic coupling structure of Eq. (2) is designed to avoid a tree-level coupling to photons $\phi F\widetilde{F}$, which would lead to exponential photon production during the relaxion's evolution, thus slowing down the field independently of the Higgs VEV and spoiling the mechanism.[2] Such a term is unavoidably generated through radiative corrections, and we will discuss under which conditions it does not harm the construction. The leading interaction with the SM gauge bosons in Eq. (1) generates a coupling to SM fermions (at one loop) and to photons (at one and two loops) [39, 40]:

$$\frac{\partial_\mu\phi}{f_F}(\bar{\psi}\gamma^\mu\gamma_5\psi) \quad\text{and}\quad \frac{\phi}{4f_\gamma}F\widetilde{F} \tag{3}$$

where

$$\frac{1}{f_F} = \frac{3\alpha_{\rm em}^2}{4\mathcal{F}}\left[\frac{Y_{F_L}^2+Y_{F_R}^2}{\cos^4\theta_W} - \frac{3}{4\sin^4\theta_W}\right]\log\frac{\Lambda^2}{m_W^2}, \tag{4}$$

and

$$\frac{1}{f_\gamma} = \frac{2\alpha_{\rm em}}{\pi\sin^2\theta_W\mathcal{F}}B_2\left(x_W\right) + \sum_F\frac{N_c^FQ_F^2}{2\pi^2f_F}B_1\left(x_F\right), \tag{5}$$

where N_c^F is the color factor, Q_F is the electric charge of the fermion F with mass m_F, and $x_i \equiv 4m_i^2/m_\phi$. The functions $B_{1,2}$ are given by:

$$B_1(x) = 1 - x[f(x)]^2,\;\; B_2(x) = 1-(x-1)[f(x)]^2, \tag{6}$$

$$f(x) = \begin{cases}\arcsin\frac{1}{\sqrt{x}}, & x\geq 1\\ \frac{\pi}{2}+\frac{i}{2}\log\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}, & x<1.\end{cases} \tag{7}$$

When the relaxion is light, which will turn out to be the case of interest for our DM scenario, these functions scale like $B_1(x_F)\to -m_\phi^2/(12m_F^2)$ and $B_2(x_W)\to m_\phi^2/(6m_W^2)$ as $m_\phi^2\to 0$. This implies, for instance, that when the relaxion is lighter than the electron mass, the induced coupling to photons originated from the coupling in Eq. (1) is suppressed.

The relaxation mechanism can be described as follows. The ϕ field rolls down its potential until it reaches a critical velocity $\dot{\phi}_c$ when there is an exponential production of gauge bosons, which makes the relaxion slow down due to the transfer of its energy to the gauge bosons. This back-reaction mechanism becomes clear once we examine the equations of motion for ϕ and for a massive vector V_μ with interaction $\mathcal{L}\supset\phi/(4f)V_{\mu\nu}\widetilde{V}^{\mu\nu}$,

$$\ddot{\phi} - g\Lambda^3 + g'\Lambda h^2 + \frac{\Lambda_b^4}{f'}\sin\frac{\phi}{f'} + \frac{1}{4f}\langle V\widetilde{V}\rangle = 0, \tag{8}$$

$$\ddot{V}_\pm + (k^2+m_V^2\mp k\frac{\dot{\phi}}{f})V_\pm = 0, \tag{9}$$

where the $\langle V\widetilde{V}\rangle$ is the expectation value of the quantum operator and $V_\pm$ refers to the two transverse polarizations of V_μ.[3] Note that V_+ has a tachyonic growing mode when $\omega_{k,+}^2 \equiv k^2+m_V^2-k\frac{\dot{\phi}}{f}<0$. Taking into account that the first mode that becomes tachyonic is $k_c=\dot{\phi}/(2f)$, which is the one for which $\omega_{k,+}$ is minimum, we get that V_+ grows exponentially for

$$|\dot{\phi}| \gtrsim \dot{\phi}_c \sim 2f\,m_V. \tag{10}$$

Consequently, the term $\langle V\widetilde{V}\rangle$ becomes the dominant one in the equation of motion for the relaxion field, making $\dot{\phi}$ decrease. After ϕ has slowed down, the constant cosine potential acting as a barrier can make the relaxion evolution stop. If V is a massive SM gauge boson, then its mass depends on the Higgs field, implying that condition (10) is not satisfied when the Higgs field value is large so there is no relevant particle production in this regime. To obtain the correct value of the electroweak scale, the back-reaction should be triggered when the mass of the EW bosons is m_Z, *i.e.* for $\dot{\phi}_c\sim 2fm_Z$, where m_Z is the SM Z boson mass. The parameters of the model in (1) should be arranged such that this back-reaction turns on when ϕ is close to the critical value Λ/g', generating a parametric hierarchy between the cutoff scale Λ and the electroweak scale. This happens if the classical Higgs field h follows closely the minimum of its potential.

There are several conditions that should be fulfilled in order to get a successful stopping mechanism using particle production during inflation. We refer to the original paper [17] for a discussion of such conditions (see App. A for the summary of the required conditions that should be consistently combined). Besides the requirements presented in [17], following [18] we impose that the loop-induced coupling $\phi F\widetilde{F}$ does not lead to efficient photons production (see App. A for details).

Taking into account all the constraints on the particle production mechanism, we identify the allowed parameter parameter space for the model in Eq. (1) which is

[2] This can, for example, descend from a left-right symmetric UV completion [39].

[3] We neglect the longitudinal mode V_L as it does not have a tachyonic instability.

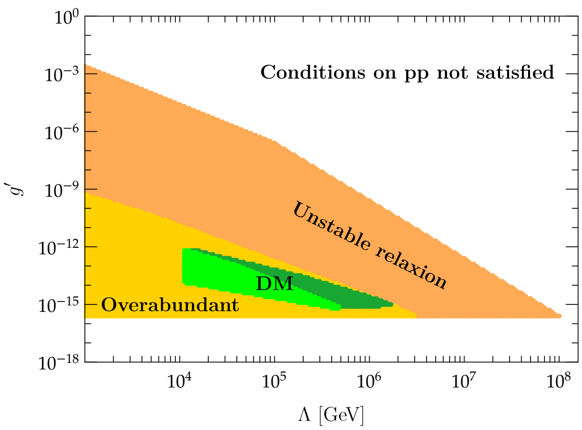

FIG. 1. Parameter space consistent with relaxation during inflation using particle production (pp) as the stopping mechanism. In the orange region ϕ is unstable; in the yellow one the relaxion is overproduced; the green region is compatible with the relaxion being DM; the light green one results after applying indirect detection bounds.

characterized by five parameters: the cut-off Λ, the coupling g', the barriers' height Λ_b, the decay constant f' and the Hubble constant during inflation H_I. The scale f can be fixed in terms of the other parameters using the particle production trigger condition in Eq. (10) and the value of the slow-roll velocity, which gives

$$f = \frac{g'\Lambda^3}{6H_I m_Z}\,. \tag{11}$$

Additionally, we assume $g = g'$ as the terms proportional to g and g' in Eq. (1) may be generated in a similar manner in the UV model.

The coloured region in Fig. 1 shows the values of Λ, g' for which the relaxion mechanism can be realized successfully. To each point it corresponds a range in the other three free parameters.

Relaxion as Dark Matter. The relaxion can be produced via vacuum misalignment and through thermal scattering. In the first case, after the relaxion gets stuck in one of the barriers, it will eventually start to oscillate freely when particle production becomes inefficient, leading to a energy density which red-shifts as non-relativistic matter. Since in our scenario the relaxation dynamics happens during inflation, the energy density stored in the field is diluted away and the misalignment contribution to the relaxion abundance is negligible [17] (see also [2, 35]). The only possibility to produce a significant relaxion abundance is then via scattering.

A population of relaxion particles is produced through $a + b \leftrightarrow \phi + c$ interactions, where the species $a, b,$ and c belong to the SM and are in thermal equilibrium. The relaxion abundance is controlled by the Boltzmann equation [41]

$$\frac{dY_\phi}{dx} = -\frac{\Gamma}{xH}\left(Y_\phi - Y_\phi^{\rm eq}\right), \tag{12}$$

where we define the dimensionless variables $Y_\phi = n_\phi/s$ and $x = m_\phi/T$ with n_ϕ being the relaxion number density, s the entropy density, and m_ϕ the relaxion mass. The equilibrium number density of ϕ is $Y_\phi^{\rm eq} = n_\phi^{\rm eq}/s \approx 0.278/g_*$ where g_* is the number of relativistic degrees of freedom, and H is the Hubble rate. The quantity Γ is given by the sum over the interaction rates, $\Gamma \equiv \sum_i \Gamma_i$ with $\Gamma_i = n_{c_i}\langle\sigma v\rangle_i$, where the sum includes gluon scattering [42], Primakoff scattering (via $\phi\gamma\gamma$, $\phi Z\gamma$, and ϕBB, see *e.g.* [35, 43, 44]), Compton scattering (via $\phi\bar{l}l$ and $\phi\bar{q}q$, where l and q refer to a SM lepton and quark, respectively, see *e.g.* [35, 43]), and Primakoff and Compton processes through the mixing with the Higgs (see *e.g.* [35]). We neglected all the interference terms. We expect this to be a sensible approximation, as we checked that at each temperature the subdominant processes are highly suppressed compared to the main one. As we discuss below, our relevant temperatures are always below the pion mass, so the relaxion production through pion conversion is Boltzmann-suppressed. At such low temperatures, the dominant process is Compton scattering $\gamma + e \leftrightarrow \phi + e$, with a rate

$$\Gamma_C \approx \frac{3\zeta(3)}{\pi^2}\alpha_{\rm em}\frac{m_e^2 T}{f_e^2}\,, \tag{13}$$

where f_e is given in Eq. (4). Assuming that the initial number density of relaxion particles is negligible, $Y_\phi(x_0) = 0$, the solution of Eq. (12) is given by

$$Y_\phi(x) = Y_\phi^{\rm eq}\left[1 - \exp\left(-\int_{x_0}^{x}\frac{\Gamma}{x'H}dx'\right)\right], \tag{14}$$

where $T_0 = m_\phi/x_0$ can be identified with the reheating temperature (we will comment more on this below). If the argument of the integral is large, then $Y_\phi(x) \approx Y_\phi^{\rm eq}$ and the correct DM abundance can only be met for a very light (thus hot) DM component. We must therefore be in the opposite situation, in which the argument of the integral in Eq. (14) is much smaller than one, and we can approximate Y_ϕ by

$$Y_\phi(x) \approx Y_\phi^{\rm eq}\int_{x_0}^{x}\frac{\Gamma}{x'H}dx'. \tag{15}$$

Compton scattering is active until electrons become non-relativistic. This is why we need a rather low T_0 (as discussed in the following), which guarantees that the interactions are out-of-equilibrium ($\Gamma_i/H < 1$) and that the relaxion never enters in thermal equilibrium with the SM bath.

The relaxion decays through the loop-induced couplings to photons and SM fermions as in Eq. (3) (see *e.g.* [39, 40]), by the leading interaction with the electroweak gauge bosons in Eq. (1), and via the mixing with the Higgs as in Eq. (1) for which we used the results in [45]. We only consider 2-body decays in our analysis. As we shall see in the following, relaxion dark matter is typically in the keV range. In this mass ballpark the relaxion can only decay into photons and neutrinos. The decay into photons happens both via the mixing with the Higgs and through the loop-induced coupling of Eq. (3). For simplicity, we will assume in the following that neutrinos are Majorana fermions in which case the decay through this channel is suppressed compared to the one into photons as it proceeds via higher dimensional operators (see *e.g.* [40]). If neutrinos are Dirac fermions, this can be the dominant decay channel. Even in this case, the bounds from indirect detection on the DM decaying into photons (see next section) imply stronger constraints on the relaxion lifetime.

Results and discussion. In order to identify the parameter space that can lead to the correct DM abundance, we performed a scan looking for points in $\{\Lambda, g', \Lambda_b, f'\}$ which can satisfy the DM hypothesis. For each point, the value of H_I is fixed by the requirement of the DM abundance, and we compared this with the range allowed from the conditions on particle production. The result is shown in Fig. 1. The green region is the one where the relaxion is stable, all the bounds on a successful relaxation with particle production can be simultaneously satisfied and the relaxion abundance matches the observed DM one. The light green part is the one in which, additionally, the constraints from indirect detection are satisfied. In the yellow region the relaxion can be made stable, but it is overproduced. Finally, in the orange region the relaxion's lifetime is shorter than the age of the universe. The sharp cut in the green region at $\Lambda \gtrsim 10^4\,\text{GeV}$ descends from the condition that the relaxion slowly rolls with a velocity that does not exceed the cutoff Λ^2, after fixing H_I to get the correct relic abundance (see for details App. A).

On top of the above constraints, we applied a lower bound on the DM mass from structure formation. Observations from Lyman-α forest [46, 47] are in tension with a thermal relic below a few keV. Note that in our case the DM velocity distribution is non-thermal, which may weaken some of these bounds (see [38] and references therein). It would be interesting to explore such feature which we leave for future work. Here we will simply impose that the relaxion should be heavier than $2\,\text{keV}$.

Figure 2 shows the minimal and maximal allowed DM mass for each point, including all the range of T_0. The range of relaxion mass compatible with the DM hypothesis is $\sim 2\,\text{keV} - 70\,\text{keV}$.

The other free parameters of the model take the following values: $f' \sim 10^{11} - 10^{13}\,\text{GeV}$, $\Lambda_b \sim 10^3 - 10^4\,\text{GeV}$, $H_I \sim 10^{-10} - 10^{-6}\,\text{GeV}$. Correspondingly, the scale f lies in the range $\sim 10^6 - 10^7\,\text{GeV}$. The number of e-folds required to realize the relaxation mechanism is at most $\sim 10^6$, but in most of the parameter space it is $\lesssim \mathcal{O}(100)$.

Finally, Fig. 3 shows how the allowed region depends on the value we choose for the reheating temperature. As it was anticipated, a drawback of our scenario is that the reheating temperature is rather low, $T_0 \lesssim 30\,\text{MeV}$, to avoid overabundance. If thermal equilibrium is reached, in this mass range the relaxion would be overabudant by a factor of $10 - 1000$. The reheating temperature can be made larger than T_0 if a dilution mechanism is active above the latter. For example, the decay of additional unstable particles can inject entropy in the plasma, reducing Y_ϕ. Alternatively, in the perturbative reheating scenario, the temperature of the universe during the reheating phase can be much larger than the final reheating temperature [48], while entropy injection keeps the relaxion abundance low. This could therefore allow baryogenesis at high temperature. Here we just assume that the relaxion abundance is negligible at T_0.

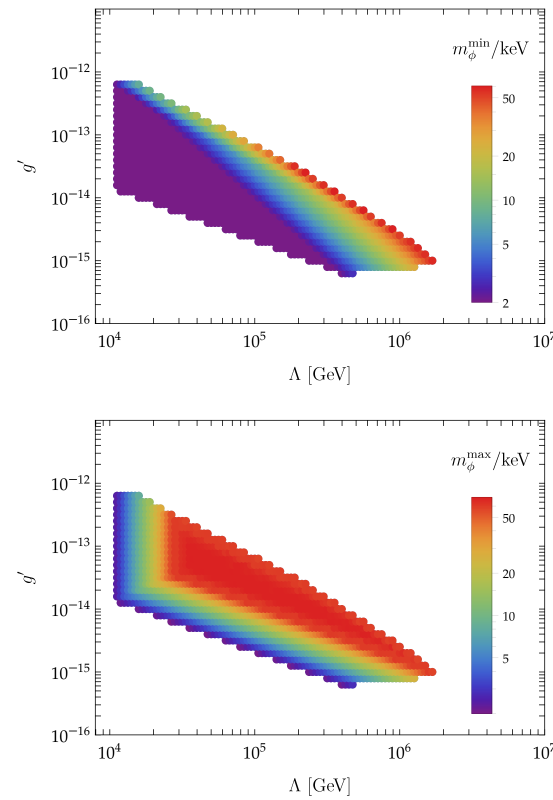

FIG. 2. Minimal and maximal relaxion masses for the allowed parameter space, selecting the green region in Fig. 1.

Strong constraints on the model come from the observations of the galactic and extra-galactic diffuse X-ray and γ-ray background. We consider the constraints on decaying DM from [49] which uses the diffuse photon spectra data from different satellites. For our parameter space, which comprises masses around the keV range, the relevant bounds are given by the satellites HEAO-1 [50] and INTEGRAL [51]. In Fig. 1 we show in light green the region in agreement with the bounds on the lifetime of a scalar DM decaying into two photons, $\tau_\phi \gtrsim 10^{26-28}\,\text{s}$ for $m_\phi > 4\,\text{keV}$ [49]. This constrains the relaxion mass to be below $m_\phi \lesssim 4\,\text{keV}$. Extrapolating the bound from [49] to lower masses further constrains the parameter space, but the result does not change significantly. This places relaxion DM in a knife-edge position: on the one hand, new results from indirect searches in the keV mass range

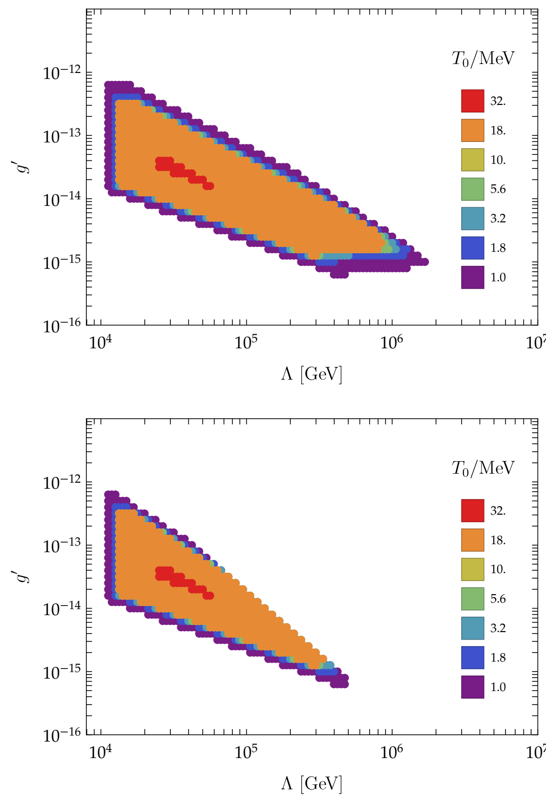

FIG. 3. Allowed dark matter region as a function of the reheating temperature. The region shrinks for higher temperature T_0. The plot on the top corresponds to the whole green region in Fig. 1 while the one on the bottom selects the light green region in Fig. 1.

could rule out this scenario; on the other hand, a numerical solution of the Boltzmann equation could weaken the lower bound on the relaxion mass, thus opening the parameter space for lighter DM.

Another important constraints are given by astrophysical probes. We consider the cooling bounds on photophobic axion-like particles of [39] to constrain our relaxion DM parameter space. The relaxion coupling to electrons (see Eq. (4)) is constrained from red giants observations, which results in a lower bound for the coupling in Eq. (1) of $f \gtrsim 3 \times 10^7\,\text{GeV}$ for $m_\phi \lesssim 10^{-5}$ GeV. Even more stringent is the bound from Supernova 1987A, which for $m_\phi \lesssim 0.1\,\text{GeV}$, disfavors $f \lesssim 10^8\,\text{GeV}$. It should be noticed that the uncertainties associated with the bounds derived from astrophysical sources are typically within an order of magnitude [39]. This implies that our scenario is in a mild tension with such bounds. On the other hand, the parameter space for successful relaxation with particle production is also subject to some variation. Indeed, the consistency conditions applied here are in general conservative. We then expect that by relaxing some of these requirements, one may open the resulting parameter space.

In this work, we showed that the relaxion mechanism can naturally provide a phenomenologically viable warm DM candidate in the keV mass range. We identified the relevant parameter space in the scenario in which relaxation happens during inflation, using particle production as a source of friction. We discussed astrophysical and indirect detection constraints on our model.

Recently, there has been an increasing interest in direct detection experiments that can probe the sub-MeV mass range, in particular, in the development of new techniques which would allow us to explore new regions of the DM parameter space (see *e.g.* [52, 53]). The relaxion would be a well motivated DM candidate in the keV range, which encourages new studies in this mass ballpark.

It would be interesting to further explore the consequences of such a model on structure formation, and perform a dedicated analysis of the indirect detection bounds. We leave these studies for future work.

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Appendix A: Conditions for relaxation during inflation with back-reaction from particle production

In this appendix we discuss the requirements that should be fulfilled in order to have relaxation during inflation with back-reaction provided by particle production [17]. First, the relaxion should not affect the inflationary dynamics, implying that the relaxion potential is subdominant compared to the inflaton one. This gives a lower bound on the inflation scale H_I:

$$V_\phi \sim \Lambda^4 \lesssim H_I^2 M_{\text{Pl}}^2 \,. \tag{A1}$$

In addition, the assumption that ϕ evolves classically is valid only if the classical evolution dominates over the quantum fluctuations during inflation. Therefore we impose that, over a Hubble time, $(\delta\phi)_{\text{class}} \gtrsim (\delta\phi)_{\text{quant}}$ with $(\delta\phi)_{\text{class}} \sim V'_\phi/(3H_I^2)$ and $(\delta\phi)_{\text{quant}} \sim H_I/(2\pi)$. This gives us an upper bound on the inflation scale,

$$H_I \lesssim \left(\frac{2\pi}{3}\right)^{1/3} \left(g'\Lambda^3\right)^{1/3} , \tag{A2}$$

where we used that $V'_\phi \sim g'\Lambda^3$.

Furthermore, inflation should last long enough such that the relaxion has time to scan the Higgs mass parameter. The minimal number of e-folds which is required to scan a field range $\Delta\phi \sim \Lambda/g'$, is given by

$$\mathcal{N}_e \sim (\delta\phi_{\rm class})^{-1}\frac{\Lambda}{g'} \sim \frac{3H_I^2}{g'^2\Lambda^2} \sim \frac{\Lambda^4}{12\, m_Z^2 f^2}\,, \tag{A3}$$

where in the last step we used that the slow roll velocity is $\dot{\phi} \sim 2m_Z f$.

We also need to make sure that the Higgs field is efficiently tracking the minimum of its potential during the scanning process. This ensures that the back-reaction from the exponential production of gauge bosons is triggered when the VEV is at the electroweak scale. Hence, we impose that

$$\frac{\dot{v}}{v^2} \lesssim 1, \tag{A4}$$

where $v = (\Lambda^2 - g'\Lambda\phi)^{1/2}/\sqrt{\lambda}$ is the minimum of the Higgs potential given in Eq. (1). Eq. (A4) needs to be satisfied until the Higgs field value has reached the electroweak scale.

Another necessary condition is that the average slow-roll velocity during the scanning has to be large enough to overcome the barriers generated by the cosine potential in Eq. (1),

$$\dot{\phi}_{\rm roll} \gtrsim \Lambda_b^2, \tag{A5}$$

where $\dot{\phi}_{\rm roll} \sim V'_\phi/(3H_I) + \delta(t)$ with $V'_\phi = g\,\Lambda^3$ and $\delta(t)$ being a contribution due to the cosine potential.

Additionally, once the back-reaction has turned on, the barriers must be high enough to stop the relaxion evolution, requiring that

$$\Lambda_b^4 \gtrsim g\,\Lambda^3 f'. \tag{A6}$$

We should also ensure that once relaxion is slowing down, the Higgs mass does not change by an amount larger than the correct value, i.e.

$$\Delta m_h \sim \frac{\Delta m_h^2}{m_h} \sim \frac{1}{m_h} g'\Lambda\,\dot{\phi}\,\Delta t_{\rm pp} \lesssim m_h\,, \tag{A7}$$

where $\Delta t_{\rm pp}$ is the characteristic time scale for particle creation which can be estimate by taking into account that the particle production is affected by the presence of the thermal plasma, and is given by $\Delta t_{\rm pp} \sim T^2 f^3/\dot{\phi}^3$ [17]. Here T is obtained by assuming that the relaxion kinetic energy is transferred to radiation:

$$\frac{1}{2}\dot{\phi}^2 \sim \frac{1}{2}\left(\frac{V'_\phi}{3H_I}\right)^2 \sim \frac{\pi^2}{30} g_* T^4\,. \tag{A8}$$

To be conservative, we evaluate Eq. (A7) with $\dot{\phi} \sim \Lambda_b^2$.

We impose that the kinetic energy lost by ϕ due to particle production is larger than the one gained by rolling down the potential,

$$\Delta K_{\rm rolling} \lesssim \Delta K_{\rm pp}. \tag{A9}$$

We estimate the two terms as $\Delta K_{\rm pp} \sim \dot{\phi}^2/2$ and $\Delta K_{\rm rolling} \sim \frac{dK}{dt}\Delta t_{\rm pp}$, where $dK/dt = -dV/dt \sim g\Lambda^3\dot{\phi}$. Again, to be conservative, we take $\dot{\phi}^2/2 \sim \Lambda_b^4$.

On top of that, one should guarantee that the particle production is faster than the expansion rate,

$$\Delta t_{\rm pp} < H_I^{-1}, \tag{A10}$$

so that the energy dissipation efficently slows down the relaxion field.

Furthermore, the scanning must have enough precision to resolve the electroweak scale. The mass parameter μ_h^2 cannot vary more than the Higgs mass over one period of the cosine potential,

$$g'\Lambda\,\delta\phi = g'\Lambda\,(2\pi f') \lesssim m_h^2\,. \tag{A11}$$

It is crucial that the induced coupling to photons in Eq. (3) is suppressed enough, otherwise the dissipation from particle production would be relevant independently of the value of the Higgs mass. Then we have to impose that the produced photons are efficiently diluted by the cosmic expansion

$$\Delta t_\gamma > H_I^{-1}, \tag{A12}$$

where $\Delta t_\gamma \sim T^2 f_\gamma^3/\dot{\phi}^3$ with f_γ given in Eq. (5). The relaxion induced coupling to photons through the Higgs mixing (see Eq. (1)) is very suppressed and the dilution requirement in (A12) for this contribution is trivially satisfied.

A last condition concerns the restoration of the shift symmetry. After the relaxion has been trapped into one of the wiggles, the temperature cannot be larger than the confinement scale, $T < \Lambda_b$. This condition is only relevant if the sector which generates the cosine potential gets in thermal equilibrium with the SM model. This can be estimated as follows. We assume the barriers are generated by some QCD-like gauge group, which is coupled to the relaxion via a term $\phi G'\widetilde{G}'/f'$. Then, we naively estimate the rate for $g'g' \leftrightarrow ZZ$ interactions as $\Gamma \sim T^5/(f^2 f'^2)$, which must be larger than the Hubble rate H_I to achieve thermalization.

All in all, the conditions that apply to the parameters of our model are the following:

$$H_I \gtrsim \frac{g'\Lambda}{3} \qquad \text{slow-roll velocity} \tag{A13}$$

$$H_I \gtrsim \frac{g'^2\Lambda^4}{3v_{\rm EW}^3\lambda^{3/2}} \qquad \text{Higgs tracking the minimum} \tag{A14}$$

$$H_I \lesssim \frac{g'\Lambda^3}{3\Lambda_b^2} \qquad \text{overcome the wiggles} \tag{A15}$$

$$H_I \gtrsim \left(\frac{10^{-4}g'^5\Lambda^{15}}{\sqrt{g_*}m_Z^3\Lambda_b^8}\right)^{1/4} \qquad \text{efficient dissipation} \tag{A16}$$

$$H_I \gtrsim \left(\frac{10^{-4}g'^5\Lambda^{13}}{\sqrt{g_*}m_h^2 m_Z^3\Lambda_b^4}\right)^{1/4} \qquad \text{small Higgs mass variation} \tag{A17}$$

$$H_I \gtrsim \text{Min}\left[\left(\frac{5}{3}\frac{g'^2\Lambda^6}{g_*\pi^2\Lambda_b^4}\right)^{1/2}, \left(\frac{230 m_Z^8 g'^2\Lambda^6}{g_*^5 f'^8}\right)^{1/6}\right] \qquad \text{no symmetry restoration} \tag{A18}$$

$$H_I \gtrsim \frac{\Lambda^2}{M_{\rm Pl}} \qquad \text{inflaton potential dominates} \tag{A19}$$

$$H_I \lesssim \left(\frac{2\pi}{3}\right)^{1/3} g'^{1/3}\Lambda \qquad \text{classical rolling dominates} \tag{A20}$$

$$H_I \gtrsim \frac{16}{9\pi^2 g_{\rm EW}^2}\frac{\dot\phi^3}{T^2 f_\gamma^3} \qquad \text{photon dilution} \tag{A21}$$

$$H_I \lesssim \frac{16}{9\pi^2 g_{\rm EW}^2}\frac{\dot\phi^3}{T^2 f^3} \qquad \text{particle production fast} \tag{A22}$$

$$g' \lesssim \frac{\Lambda_b^4}{\Lambda^3 f'} \qquad \text{stopping condition} \tag{A23}$$

$$g' \lesssim \frac{m_h^2}{2\pi f'\Lambda} \qquad \text{scanning with enough precision} \tag{A24}$$

$$f' \gtrsim \Lambda_b, \Lambda \quad \text{and} \quad f \gtrsim \Lambda \qquad \text{consistency of the EFT} \tag{A25}$$

[1] P. W. Graham, D. E. Kaplan, and S. Rajendran, Phys. Rev. Lett. **115**, 221801 (2015), arXiv:1504.07551 [hep-ph].

[2] J. R. Espinosa, C. Grojean, G. Panico, A. Pomarol, O. Pujolàs, and G. Servant, Phys. Rev. Lett. **115**, 251803 (2015), arXiv:1506.09217 [hep-ph].

[3] R. S. Gupta, Z. Komargodski, G. Perez, and L. Ubaldi, JHEP **02**, 166 (2016), arXiv:1509.00047 [hep-ph].

[4] S. Abel and R. J. Stewart, JHEP **02**, 182 (2016), arXiv:1511.02880 [hep-th].

[5] K. Choi and H. Kim, Phys. Lett. **B759**, 520 (2016), arXiv:1511.07201 [hep-th].

[6] L. E. Ibanez, M. Montero, A. Uranga, and I. Valenzuela, JHEP **04**, 020 (2016), arXiv:1512.00025 [hep-th].

[7] A. Hebecker, F. Rompineve, and A. Westphal, JHEP **04**, 157 (2016), arXiv:1512.03768 [hep-th].

[8] L. McAllister, P. Schwaller, G. Servant, J. Stout, and A. Westphal, JHEP **02**, 124 (2018), arXiv:1610.05320 [hep-th].

[9] N. Fonseca, L. de Lima, C. S. Machado, and R. D. Matheus, Phys. Rev. **D94**, 015010 (2016), arXiv:1601.07183 [hep-ph].

[10] S. P. Patil and P. Schwaller, JHEP **02**, 077 (2016), arXiv:1507.08649 [hep-ph].

[11] J. Jaeckel, V. M. Mehta, and L. T. Witkowski, Phys. Rev. **D93**, 063522 (2016), arXiv:1508.03321 [hep-ph].

[12] L. Marzola and M. Raidal, Mod. Phys. Lett. **A31**, 1650215 (2016), arXiv:1510.00710 [hep-ph].

[13] S. Di Chiara, K. Kannike, L. Marzola, A. Racioppi, M. Raidal, and C. Spethmann, Phys. Rev. **D93**, 103527 (2016), arXiv:1511.02858 [hep-ph].

[14] W. Tangarife, K. Tobioka, L. Ubaldi, and T. Volansky, JHEP **02**, 084 (2018), arXiv:1706.03072 [hep-ph].

[15] K. Choi, H. Kim, and T. Sekiguchi, Phys. Rev. **D95**, 075008 (2017), arXiv:1611.08569 [hep-ph].

[16] E. Hardy, JHEP **11**, 077 (2015), arXiv:1507.07525 [hep-ph].

[17] A. Hook and G. Marques-Tavares, JHEP **12**, 101 (2016), arXiv:1607.01786 [hep-ph].

[18] N. Fonseca, E. Morgante, and G. Servant, (2018), arXiv:1805.04543 [hep-ph].

[19] B. Batell, G. F. Giudice, and M. McCullough, JHEP **12**, 162 (2015), arXiv:1509.00834 [hep-ph].

[20] J. L. Evans, T. Gherghetta, N. Nagata, and Z. Thomas,

JHEP **09**, 150 (2016), arXiv:1602.04812 [hep-ph].
[21] J. L. Evans, T. Gherghetta, N. Nagata, and M. Peloso, Phys. Rev. **D95**, 115027 (2017), arXiv:1704.03695 [hep-ph].
[22] N. Fonseca, B. Von Harling, L. De Lima, and C. S. Machado, JHEP **07**, 033 (2018), arXiv:1712.07635 [hep-ph].
[23] B. Batell, M. A. Fedderke, and L.-T. Wang, JHEP **12**, 139 (2017), arXiv:1705.09666 [hep-ph].
[24] O. Antipin and M. Redi, JHEP **12**, 031 (2015), arXiv:1508.01112 [hep-ph].
[25] A. Agugliaro, O. Antipin, D. Becciolini, S. De Curtis, and M. Redi, Phys. Rev. **D95**, 035019 (2017), arXiv:1609.07122 [hep-ph].
[26] Z. Lalak and A. Markiewicz, J. Phys. **G45**, 035002 (2018), arXiv:1612.09128 [hep-ph].
[27] O. Matsedonskyi, JHEP **01**, 063 (2016), arXiv:1509.03583 [hep-ph].
[28] O. Davidi, R. S. Gupta, G. Perez, D. Redigolo, and A. Shalit, (2017), arXiv:1711.00858 [hep-ph].
[29] F. P. Huang, Y. Cai, H. Li, and X. Zhang, Chin. Phys. **C40**, 113103 (2016), arXiv:1605.03120 [hep-ph].
[30] O. Matsedonskyi and M. Montull, Phys. Rev. **D98**, 015026 (2018), arXiv:1709.09090 [hep-ph].
[31] O. Davidi, R. S. Gupta, G. Perez, D. Redigolo, and A. Shalit, (2018), arXiv:1806.08791 [hep-ph].
[32] M. Son, F. Ye, and T. You, (2018), arXiv:1804.06599 [hep-ph].
[33] T. Kobayashi, O. Seto, T. Shimomura, and Y. Urakawa, Mod. Phys. Lett. **A32**, 1750142 (2017), arXiv:1605.06908 [astro-ph.CO].
[34] K. Choi and S. H. Im, JHEP **12**, 093 (2016), arXiv:1610.00680 [hep-ph].
[35] T. Flacke, C. Frugiuele, E. Fuchs, R. S. Gupta, and G. Perez, JHEP **06**, 050 (2017), arXiv:1610.02025 [hep-ph].
[36] H. Beauchesne, E. Bertuzzo, and G. Grilli di Cortona, JHEP **08**, 093 (2017), arXiv:1705.06325 [hep-ph].
[37] C. Frugiuele, E. Fuchs, G. Perez, and M. Schlaffer, (2018), arXiv:1807.10842 [hep-ph].
[38] N. Bernal, M. Heikinheimo, T. Tenkanen, K. Tuominen, and V. Vaskonen, Int. J. Mod. Phys. **A32**, 1730023 (2017), arXiv:1706.07442 [hep-ph].
[39] N. Craig, A. Hook, and S. Kasko, (2018), arXiv:1805.06538 [hep-ph].
[40] M. Bauer, M. Neubert, and A. Thamm, JHEP **12**, 044 (2017), arXiv:1708.00443 [hep-ph].
[41] E. W. Kolb and M. S. Turner, Front. Phys. **69**, 1 (1990).
[42] E. Masso, F. Rota, and G. Zsembinszki, Phys. Rev. **D66**, 023004 (2002), arXiv:hep-ph/0203221 [hep-ph].
[43] M. S. Turner, Phys. Rev. Lett. **59**, 2489 (1987).
[44] D. Cadamuro and J. Redondo, JCAP **1202**, 032 (2012), arXiv:1110.2895 [hep-ph].
[45] F. Bezrukov and D. Gorbunov, JHEP **05**, 010 (2010), arXiv:0912.0390 [hep-ph].
[46] M. Viel, G. D. Becker, J. S. Bolton, and M. G. Haehnelt, Phys. Rev. **D88**, 043502 (2013), arXiv:1306.2314 [astro-ph.CO].
[47] V. Iršič *et al.*, Phys. Rev. **D96**, 023522 (2017), arXiv:1702.01764 [astro-ph.CO].
[48] G. F. Giudice, E. W. Kolb, and A. Riotto, Phys. Rev. **D64**, 023508 (2001), arXiv:hep-ph/0005123 [hep-ph].
[49] R. Essig, E. Kuflik, S. D. McDermott, T. Volansky, and K. M. Zurek, JHEP **1311**, 193 (2013), arXiv:1309.4091 [hep-ph].
[50] D. E. Gruber, J. L. Matteson, L. E. Peterson, and G. V. Jung, Astrophys. J. **520**, 124 (1999), arXiv:astro-ph/9903492 [astro-ph].
[51] L. Bouchet, E. Jourdain, J. P. Roques, A. Strong, R. Diehl, F. Lebrun, and R. Terrier, Astrophys. J. **679**, 1315 (2008), arXiv:0801.2086 [astro-ph].
[52] Y. Hochberg, Y. Zhao, and K. M. Zurek, Phys. Rev. Lett. **116**, 011301 (2016), arXiv:1504.07237 [hep-ph].
[53] K. Schutz and K. M. Zurek, Phys. Rev. Lett. **117**, 121302 (2016), arXiv:1604.08206 [hep-ph].