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# Dark Matter Production during the Thermalization Era

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**Abstract**

We revisit the non-thermal dark matter (DM) production during the thermalization and reheating era after inflation. The decay of inflaton produces high-energy particles that are thermalized to complete the reheating of the Universe. Before the thermalization is completed, DM can be produced from a collision between the high-energy particles and/or the ambient plasma. We calculate the DM abundance produced from these processes for the case where the cross section of the DM production is proportional to the $n$-th power of the center of mass energy. We find that the collision between the high-energy particles is almost always dominant for $n \gtrsim 4$ while it is subdominant for $n \lesssim 2$. The production from the ambient plasma is dominant when $n \lesssim 3$ and the reheating temperature is of the order of or larger than the DM mass. The production from a collision between the high-energy particle and the ambient plasma is important for $n \lesssim 2$ and the reheating temperature is much lower than the DM mass.

## 1 Introduction

Weakly interacting dark matter (DM) with its abundance determined by the freeze-out process [1] has been intensively studied, especially in connection with solutions to the hierarchy problem. The Large Hadron Collider so far has not found any particles beyond the Standard Model (SM), which encourages us to also consider broader classes of dark matter candidates and production mechanisms. In this paper, we focus on the non-thermal DM production during the thermalization of inflaton decay products, which has been pointed out in Ref. [2]. We will not attempt to apply the production mechanism to a specific model, but the computation presented in this paper should be useful for many models.

In most models of inflation and dark matter, the mass of inflaton is many orders of magnitude larger than the electroweak scale. For example, the inflaton mass in the Starobinsky inflation model [3] is of order $10^{13}$ GeV. The heavy inflaton decays into high-energy particles in the SM so that the Universe is reheated after the end of inflation. The decay products, whose energy is of the same order as the inflaton mass, lose their energy to be thermalized to form thermal plasma.

In the very early stage of reheating, the parametric resonance may take place [4–6]. There high-energy particles are produced non-perturbatively owing to the high occupation number of the inflaton. Particle production in this stage has been investigated in the literature [7]. However, typically, if the reheating proceeds via a very tiny coupling of the inflaton with radiation, *e.g.*, Planck-suppressed operators, the resonance is soon shut off by the cosmic expansion. Afterwards, the reheating process is well described perturbatively. Even in this regime, there is a non-trivial way to create DM which we will investigate in this paper.

The thermalization process via a perturvative reheating has been studied in detail in Refs. [8,9] (see also Refs. [10, 11] for earlier works and Refs. [12–16] in a different context). The high-energy particles emit soft particles, which have relatively small energy, via inelastic scattering processes. The soft particles interact among them to form thermalized ambient plasma. As the temperature of the ambient plasma increases, the interaction between the high-energy particles and the soft particles becomes efficient. Eventually, the high-energy particles become completely thermalized. This is the time when the temperature is maximum after inflation. Still, the inflaton does not decay completely, so that the high-energy decay products are continuously produced from the inflaton decay. When the Hubble parameter becomes comparable to the inflaton decay rate, the reheating process is completed and the Universe becomes dominated by the thermal plasma.

The most important process in the thermalization era is the inelastic scattering process. This process results in the splitting of a high-energy particle, whose energy is larger than the temperature of the ambient plasma, into two high-energy particles. Here we assume $m_\phi > T$. A high-energy particle with an energy as large as the inflaton mass $m_\phi$ splits into two particles whose typical energy is about the half of the initial one. Continuing this splitting process, the original high-energy particle splits into $N$ particles with a typical energy of order $m_\phi/N$. This process continues until the energy drops to the temperature of the ambient plasma. This means that numerous high-energy particles are produced during the thermalization process from the decay of a single inflaton. The DM may be efficiently produced from those high-energy particles before they are completely thermalized [2].

The thermalization via the splitting does not occur instantaneously. There are two important consequences of the finiteness of thermalization time scale. Frist, the maximal temperature of the Universe is overestimated if one assumes the *instantaneous thermalization* [8, 9]. Second, the inflaton decay products, whose initial energy is of the same order as the inflaton mass, survive for a finite time until they lose their energy. These high-energy particles may be relevant for the DM production [2], particularly in the case where the cross section depends positively on the center of mass energy.

Non-thermal production of DM is recently discussed in the literature [17–22]. These works except for Ref. [20] do not take into account the finiteness of the thermalization time scale and estimate the DM production from the scattering of particles in the thermal plasma before the reheating is completed. Ref. [20] takes into account the correct maximal temperature, which is much lower than the one obtained in the *instantaneous thermalization* approximation. Still, they omit the second effect we discussed above and may underestimate the DM production rate.

In this paper, we take into account the finiteness of thermalization time scale. There are three processes to produce DM during the thermalization and reheating era: (A) a collision between high-energy particles, (B) a collision between a high-energy particle and a particle in the ambient plasma, and (C) a collision between particles in the ambient plasma. We calculate DM abundance from these contributions and show the conditions to figure out the dominant process. We will see that the contribution (A) is dominant when the cross section of DM production depends positively and strongly on the center of mass energy as considered in Ref. [20]. The contribution (B) is important when the cross section is mildly dependent on the center of mass energy and the reheating temperature is much lower than the DM mass. This includes the case considered in Ref. [2].

## 2 Thermalization era

In this section, we briefly review the thermalization process of inflaton decay products, which is relevant to the computation of the non-thermal production rate of dark matter in the post-reheating era. Throughout this paper, we assume that the reheating proceeds via a perturbative decay of inflaton. This is the case where the coupling between inflaton and radiation is tiny, *e.g.*, Planck-suppressed operators, because the resonance tends to be shut off immediately due to the cosmic expansion.

As we explained in the introduction, the thermalization of high-energy particles proceeds via the splitting into lower energy particles. This process is the bottleneck of the thermalization if the energy of the high-energy particles, $\sim m_\phi$, is larger than the would-be temperature of radiation after thermalization, *i.e.*, $m_\phi > T$. The rate of the splitting process is suppressed by the destructive interference effect between the parent particle and the daughter particle, which is known as the LPM effect [23–29]. Once the splitting occurs, the subsequent process does not occur until a phase factor, $kx$, varies significantly ($\sim 1$), where $k$ is the 4-momentum of a daughter particle and $x$ is the position of the parent particle. Denoting the emission angle of the daughter particle as $\theta$ ($= k_\perp / k$, where $k_\perp$ is the perpendicular momentum), we obtain $1 \lesssim kx \sim tk\theta^2 \sim tk_\perp^2/k$. Here we have assumed that all the particles are relativistic,

$k^0 \simeq k$. This inequality puts a minimum time so as to emit particles, known as a formation time, $t_{\rm form}$.

Now we need to estimate how the squared perpendicular momentum of the daughter particle, $k_\perp^2$, evolves in the medium.[1] When the daughter particle is scattered by elastic scattering processes, it evolves as a random walk as [13]

$$(k_\perp)^2 \sim \hat{q}_{\rm el}\, t, \tag{1}$$

$$\hat{q}_{\rm el} \sim \int \mathrm{d}^2 q_\perp \frac{\partial \Gamma_{\rm el}}{\partial q_\perp^2} q_\perp^2 \sim \alpha^2 \int \mathrm{d}^3 k\, f(k) \sim \alpha^2 T^3, \tag{2}$$

where we used the elastic scattering rate

$$\frac{\partial \Gamma_{\rm el}}{\partial q_\perp^2} \sim \frac{\alpha^2}{q_\perp^2 (q_\perp^2 + \alpha T^2)} \int \mathrm{d}^3 k\, f(k). \tag{3}$$

Here, $f(k)$ denotes a phase space distribution and $\alpha$ generically represents fine-structure constants for the SM gauge interactions. In the last similarity in Eq. (2), we assume that the number density is dominated by the thermalized sector.[2] Inserting these Eqs. (1) and (2) into $1 \lesssim t k_\perp^2/k$, one may estimate the formation time

$$t_{\rm form}(k) := \left(\frac{k}{\hat{q}_{\rm el}}\right)^{1/2}, \tag{4}$$

which is the minimum time to emit particles. If this is longer than the time scale of elastic scatterings, $1/\Gamma_{\rm el}$, the splitting rate is suppressed. As a result, we obtain

$$\Gamma_{\rm split}(k) \sim \alpha \min\left[\Gamma_{\rm el}, \frac{1}{t_{\rm form}(k)}\right]. \tag{5}$$

In the following we focus on the latter rate *i.e.*, the LPM suppressed rate, dominates the splitting, which implies $k \gtrsim k_{\rm LPM} \equiv \hat{q}_{\rm el}/\Gamma_{\rm el}^2 \sim T$. Note that the splitting rate via the LPM effect does not depend on the energy of the parent particle in non-Abelian gauge theories. The energy loss rate for the parent particle is dominated by the largest possible energy for the daughter particle $k$, which is equal to half of the parent energy. This means that the high-energy particles continuously lose their individual energy via the splitting into high-energy particles.

When $\Gamma_{\rm split}(m_\phi) \gtrsim H$ is satisfied, the inflaton decay products lose their energy within a Hubble time and are thermalized soon after they are produced. The threshold of this regime, which we denote as $t_{\rm max}$, is the time when the temperature reaches its maximal value after inflation. The corresponding temperature turns out to be [8, 9]

$$T_{\rm max} \sim \alpha^{4/5} m_\phi \left(\frac{\Gamma_\phi M_{\rm pl}^2}{m_\phi^3}\right)^{2/5}, \tag{6}$$

[1] If there is no ambient plasma, $k_\perp$ is kept intact. Thus the emitting occurs only for $\theta > 1/(kt)^{1/2}$ where we expect DGLAP vacuum shower dominates the splittings. See Ref. [16] for the classification of splittings processes.

[2] In fact one may show that this is the case for $T < T_{\rm max}$ which we will focus on throughout this paper. See Refs. [8, 9].

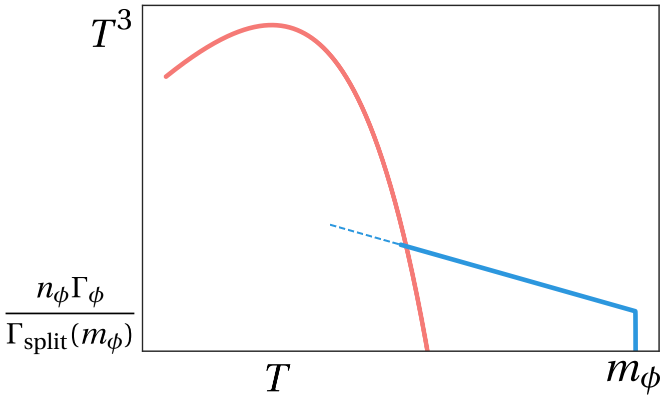


Figure 1: A schematic plot of $n(E)$ as a function of $E$. While the red curve shows the thermal distribution given in Eq. (11), the blue curve shows the high-energy tail given in Eq. (10).

where $\Gamma_\phi$ is the decay rate of the inflaton. After this happens $t \gtrsim t_{\rm max}$, the energy and number density of radiation is dominated by the thermalized sector. The estimation of the maximum temperature is valid only when $T_{\rm max} < m_\phi$ which we assume in the following. Our estimation of the DM abundance is valid without this assumption if the production is dominated at the temperature much below $T_{\rm max}$.

High-energy particles are continuously produced from the inflaton decay until the Hubble parameter decreases to the decay rate: $\Gamma_\phi \simeq H$. This is the time when the reheating is completed and the Universe becomes to be dominated by thermal radiation. We denote the time as $t_{\rm RH}$. From $t_{\rm max}$ to $t_{\rm RH}$, the temperature of the Universe decreases according to

$$\rho_r \sim T^4 \sim H\Gamma_\phi M_{\rm pl}^2, \tag{7}$$

The temperature at $t = t_{\rm RH}$, which is commonly called as the reheating temperature, is given by

$$T_{\rm RH} \sim \sqrt{\Gamma_\phi M_{\rm pl}}. \tag{8}$$

Now we are in a position to discuss a tail of cascading particles which can play an essential role for the non-thermal DM production. Let us emphasize that the high-energy particles are continuously produced from the inflaton decay during the reheating. The time scale of the thermalization of these high-energy particles is given by the splitting rate $1/\Gamma_{\rm split}(m_\phi)$, which is finite. As a result, although this process is completed within a Hubble time for $T < T_{\rm max}$, there remains a tail of cascading particles because the inflaton continuously sources the high-energy particles. The spectrum of these cascading particles can be estimated as follows. For later convenience, we define the number density with energy $E$ denoted by $n(E)$ as follows:

$$n = \int d^3k\, f(k) =: \int d\log E\, n(E). \tag{9}$$

The number density of high-energy particles right after the inflaton decay is $n_\phi \Gamma_\phi / H$. These primary particles with energy $\sim m_\phi$ cascade down to some energy $E$ by splittings. The energy conservation naively implies $n(E) \sim (m_\phi/E) n_\phi \Gamma_\phi / H$. However this population never stays the same energy $E$ for the time interval $H^{-1}$, rather they further cascade within $1/\Gamma_{\rm split}(E)$. By taking this effect into account, we can estimate the spectrum [2] (see also [16]):

$$n(E) \sim \frac{m_\phi}{E} n_\phi \frac{\Gamma_\phi}{H} \frac{H}{\Gamma_{\rm split}(E)} \sim n_\phi \frac{\Gamma_\phi}{\Gamma_{\rm split}(m_\phi)} \left(\frac{m_\phi}{E}\right)^{1/2} \tag{10}$$

for $E > T$ (for a detailed calculation, see Appendix in Ref. [2]). Here $n_\phi$ ($\simeq 3H^2 M_{\rm pl}^2/m_\phi$) is the number density of the inflaton. The characteristic power-law of $E^{-1/2}$ shows the LPM suppression. After cascading down to $T$, the daughter particles get thermalized by their own interactions which results in

$$n(E) \sim n_{\rm eq}(E) \tag{11}$$

for $E < T$. Here $n_{\rm eq}(E)$ denotes the thermal distribution with the temperature $T$.

The whole spectrum, $n(E)$, is shown in Fig. 1. Note again that the high-energy tail in the spectrum never dominates the energy and number density of radiation. Nevertheless this tail can dominate the production of heavy particles such as DM because its energy is much larger than the temperature for $m_\phi \gg T$. As we will see, the scattering involving particles in the high-energy tail is actually the dominant process when the cross section of DM production process positively depends on the center of mass energy and/or the reheating temperature is smaller than the DM mass.

## 3 DM production during the thermalization era

Now we shall take into account the production of DM from those particles. We assume that DM is produced from a pair of SM particles. We also assume that the direct decay of the inflaton into DM is negligible. (This process tends to be non-negligible when DM is charged under the SM gauge group, as DM is necessarily produced by the showering process [30, 31].)

We parametrize the cross section for the DM production as

$$\sigma_{\rm DM}(E_{\rm CM}) = \frac{E_{\rm CM}^n}{M^{n+2}}, \tag{12}$$

where $E_{\rm CM}$ is the center of mass energy and $M$ is a parameter with a mass dimension 1. This kind of cross section comes from a higher dimensional operator with a cutoff of order $M$, where we absorb a coupling constant in the definition of $M$ without loss of generality. The operator comes from an interaction mediated by a mediator of a mass of order $M$. Note that the dependence of the cross section on $E_{\rm CM}$ is modified for $E_{\rm CM} \gtrsim M$ to recover the unitarity. We should replace it by $1/E_{\rm DM}^2$ for $E_{\rm CM} \gtrsim M$. This overestimates the production rate for $E_{\rm CM} > M$, but practically the production is dominated at $E_{\rm CM} \sim M$. In the following discussion, we assume $E_{\rm CM} \lesssim M$ for simplicity. The estimation in the opposite case is obtained by taking $n = 0$ in the following calculation.

Suppose that the DM is produced from a collision of a particle with energy $E_1$ and a particle with energy $E_2$. The rate of DM production for the former particle is then given by

$$\Gamma_{\rm DM}(E_1, E_2) \sim \frac{(E_1 E_2)^{n/2}}{M^{n+2}} n(E_2). \tag{13}$$

The total number density of DM generated per a Hubble time is obtained by performing integrals over the energies $E_{1,2}$,

$$n_{\rm DM} = \int {\rm d}\log E_1 {\rm d}\log E_2 \, n_{\rm DM}(E_1, E_2), \tag{14}$$

where

$$n_{\rm DM}(E_1, E_2) \sim \frac{\Gamma_{\rm DM}(E_1, E_2)}{H} n(E_1). \tag{15}$$

The rest of this paper is devoted to the determination of the dominant contribution of $n_{\rm DM}(E_1, E_2)$ in the integral (14) by inserting $n(E)$ in Eqs. (10) and (11). In the following calculation, we assume $m_{\rm DM} < m_\phi, T_{\rm max}$. We also assume that the annihilation of produced DM is negligible.

### 3.1 (A) Collisions of particles with $E_1$ and $E_2$ ($> T$)

In this subsection, we calculate the DM abundance produced from the collision between the high-energy particles with $T < E_1, E_2 \lesssim m_\phi$. The DM number density produced during one Hubble time in the thermalization era at the temperature of $T$ ($T_{\rm RH} < T < T_{\rm max}$) is given by

$$\left.\frac{n_{\rm DM}^{\rm (NT)}(E_1, E_2)}{s}\right|_T \sim \frac{\Gamma_{\rm DM}(E_1, E_2)}{H(T)} \frac{n(E_1)}{s}, \tag{16}$$

$$\sim \frac{m_\phi^{n-1} T_{\rm RH}^3 T}{\alpha^4 M_{\rm pl} M^{n+2}} \left(\frac{E_1 E_2}{m_\phi^2}\right)^{(n-1)/2}. \tag{17}$$

The subscript (NT) implies that the production involves the high-energy tail, *i.e.*, $T < E_1$ or $T < E_2$. Here we normalize the number density by the effective "entropy density" $1/s$. We define $s$ by $4\rho_\phi / 3 T_{\rm RH}$ during the reheating era so that $s$ is proportional to $1/a^3$ and is equal to the entropy density at $T = T_{\rm RH}$. In other words, the entropy production via the inflaton decay is already factored out in its definition. By using this definition of $s$, the combination of $n_{\rm DM}/s$ is constant in time if there is no additional source of DM nor the entropy.

Equation (16) is the formula derived in Ref. [2]. This contribution is missed if one takes a limit of $\Gamma_{\rm split} \to \infty$, which corresponds to the limit of *instantaneous thermalization*. From Eq. (17), we can see that the maximal values, $E_1 \sim m_\phi$ and $E_2 \sim m_\phi$, dominate the integral (14) for $n > 1$. For $n < 1$ the lowest possible values, $E_1 = E_2 = m_{\rm DM}$, are dominant. If we set $E_1 = E_2 \sim m_\phi$ or $m_{\rm DM}$, the resulting DM abundance is proportional to $T$. Therefore, DM with energy of order $m_\phi$ is produced dominantly at the time when the temperature reaches its maximal value $T_{\rm max}$. Then we obtain

$$\frac{n_{\rm DM}^{\rm (NT)}(E_1, E_2)}{s} \sim \frac{m_\phi^{n-6/5} T_{\rm RH}^{19/5}}{\alpha^{16/5} M_{\rm pl}^{3/5} M^{n+2}} \left(\frac{E_1 E_2}{m_\phi^2}\right)^{(n-1)/2}, \tag{18}$$

with the dominant contribution coming from

$$E_1 E_2 \to \begin{cases} m_{\rm DM}^2 & \text{for} \quad n < 1 \\ m_\phi^2 & \text{for} \quad n > 1. \end{cases} \tag{19}$$

Here, $E_1 E_2$ must be larger than $T_{\max}^2$ by assumption, which requires $T_{\max} \lesssim m_{\rm DM}$ for $n < 1$. Otherwise the contribution B or C is dominant. The result in the latter case ($n > 1$) is obtained in Ref. [20], although the contributions from $T < T_{\max}$ is not discussed and it is not clear if they are subdominant. For $n = 1$, where the dependence of the DM abundance on $E_{1,2}$ vanishes, the integration over $E_{1,2}$ yields logarithmic factors. This is also true for the other contributions B and C when the dependence on $E_{1,2}$ and/or $T$ vanishes. In this paper, we neglect these factors for simplicity.

Finally, we comment that the contribution coming from the time before the temperature reaches the maximal value (*i.e.*, the time when $H > \Gamma_{\rm split}$) is always subdominant. In this case, the number of high-energy particles is given by Eq. (10) with the replacement of $1/\Gamma_{\rm split}(E) \to 1/H$. The factor of $1/H^2$ from $n(E_1)n(E_2)$ is larger for a later epoch, so that the contribution is maximized when the temperature reaches the maximal value.

### 3.2 (B) Collisions of particles with $E_1$ ($> T$) and $T$

In this subsection, we calculate the DM abundance produced from the interaction between the high-energy particle with energy of $m_\phi \gtrsim E_1 > T$ and the thermal plasma $T > E_2$. We assume that $T_{\max} m_\phi > m_{\rm DM}$. Otherwise the produced DM abundance is exponentially suppressed. The result is given by

$$\left.\frac{n_{\rm DM}^{\rm (NT)}(E_1, T)}{s}\right|_T \sim \frac{m_\phi^{n-3} T_{\rm RH}^5}{\alpha^2 M^{n+2}} \left(\frac{E_1}{m_\phi}\right)^{(n-1)/2} \left(\frac{T}{m_\phi}\right)^{(n-5)/2}. \tag{20}$$

Here we have already utilized the fact that the thermal population is dominated by $E_2 \sim T$. The dominant contribution comes from

$$E_1 \to \begin{cases} {\rm Min}\left[m_{\rm DM}^2/T_{\rm RH},\ m_\phi\right] & \text{for} \quad n<1 \\ m_\phi & \text{for} \quad n>1, \end{cases} \tag{21}$$

and

$$T \to \begin{cases} {\rm Max}\left[T_{\rm RH},\ m_{\rm DM}^2/m_\phi\right] & \text{for} \quad n<5 \\ T_{\max} & \text{for} \quad n>5, \end{cases} \tag{22}$$

Here, we should note that $E_1$ must be larger than $T$ by assumption, which may be violated for $m_{\rm DM} \lesssim T_{\rm RH}$ and $n < 1$. Otherwise the contribution C is dominant.

The ratio between the contribution A and B is given by

$$\frac{n_{\rm DM}^{\rm (NT)}(m_\phi, m_\phi)}{n_{\rm DM}^{\rm (NT)}(m_\phi, T)} \sim \begin{cases} \left(\frac{T_{\rm RH}}{T_{\max}}\right)^{3/2} \left(\frac{T_{\rm RH}}{m_\phi}\right)^{1/2} & \text{for} \quad n<1 \\ \left(\frac{T_{\rm RH}}{T_{\max}}\right)^{3/2} \left(\frac{T_{\rm RH}}{m_\phi}\right)^{(2-n)/2} & \text{for} \quad 1<n<5, \\ \left(\frac{m_\phi}{T_{\max}}\right)^{(n-2)/2} & \text{for} \quad 5<n, \end{cases} \tag{23}$$

where we assumed $m_{\rm DM}^2/T_{\rm RH} < m_\phi$. Under the assumption of $T_{\rm max} \lesssim m_\phi$, which is the case for $\Gamma_\phi \lesssim m_\phi^3/M_{\rm pl}^2$, the contribution A is dominant for $n > 5$. One can check that the contribution B is dominant for $n < 2$ because $T_{\rm RH} < T_{\rm max}$.

### 3.3 (C) Collisions of particles in the thermal plasma

Finally, we calculate the DM abundance produced from the thermal plasma. We assume that $T_{\rm max} > m_{\rm DM}$. Otherwise the contribution is exponentially suppressed.

The DM number density produced from the thermal plasma at $T$ ($\in (T_{\rm RH}, T_{\rm max})$) is calculated in Ref. [17]. The result is given by

$$\left.\frac{n_{\rm DM}^{\rm (T)}(T,T)}{s}\right|_T \sim \frac{T^{n-6} T_{\rm RH}^7 M_{\rm pl}}{M^{n+2}}, \tag{24}$$

where we used Eqs. (8) and (11). Here we implicitly assumed that $m_{\rm DM} < T$ so that DM can be produced from the thermal plasma. The main contribution comes from

$$T \to \begin{cases} {\rm Max}\,[T_{\rm RH},\ m_{\rm DM}] & \text{for} \quad n < 6 \\ T_{\rm max} & \text{for} \quad n > 6. \end{cases} \tag{25}$$

When $T_{\rm RH} > m_{\rm DM}$ and $n < -1$, the production dominantly occurs after reheating at $T \sim m_{\rm DM}$, with the abundance given by

$$\left.\frac{n_{\rm DM}^{\rm (T)}(T,T)}{s}\right|_{T\sim m_{\rm DM}} \sim \frac{m_{\rm DM}^{n+1} M_{\rm pl}}{M^{n+2}}. \tag{26}$$

The ratio between the contributions A and C is given by

$$\frac{n_{\rm DM}^{\rm (NT)}(m_\phi, m_\phi)}{n_{\rm DM}^{\rm (T)}} \sim \left(\frac{m_\phi}{\alpha^2 M_{\rm pl}}\right)^{8/5} \left(\frac{m_\phi}{T_{\rm RH}}\right)^{n-14/5}, \tag{27}$$

for $1 < n < 6$ and $m_{\rm DM} < T_{\rm RH}$. This implies that the contribution C is less important for $n \gtrsim 14/5 \simeq 3$.

We compare the contributions B and C for $n = 0, 2$ and find that the contribution C dominates when

$$\alpha\sqrt{M_{\rm pl} m_{\rm DM}} \lesssim T_{\rm RH} \qquad \text{for} \quad n = 0, \tag{28}$$

$$\left(\frac{m_{\rm DM}^8 m_\phi}{\alpha^4 M_{\rm pl}^2}\right)^{1/7} \lesssim T_{\rm RH}, \qquad \text{for} \quad n = 2, \tag{29}$$

where we assume a large $m_\phi$.

Fig. 2 shows which contribution is dominant for $n = 0$ and 2. Here we take $\alpha = 0.01$ and $m_\phi = 10^{13}$ GeV. We can see that the contribution $B$ dominates over $A$ for a low-reheating temperature, $T_{\rm RH} \ll m_{\rm DM}$, as we discussed in Ref. [2]. In the figure, we show the parameter region which explains the observed amount of DM ($m_{\rm DM} n_{\rm DM}/s \simeq 0.4$ eV) with $M = 10^{-3} M_{\rm pl}$ or $M_{\rm pl}$ for $n = 0$ and $M = 10^{-5} M_{\rm pl}$ or $10^{-3} M_{\rm pl}$ for $n = 2$. For a guide to the eye, we show a dashed line representing $T_{\rm RH} = m_{\rm DM}$, which is the threshold determining the dominant contribution for $T$ in Eq. (25).

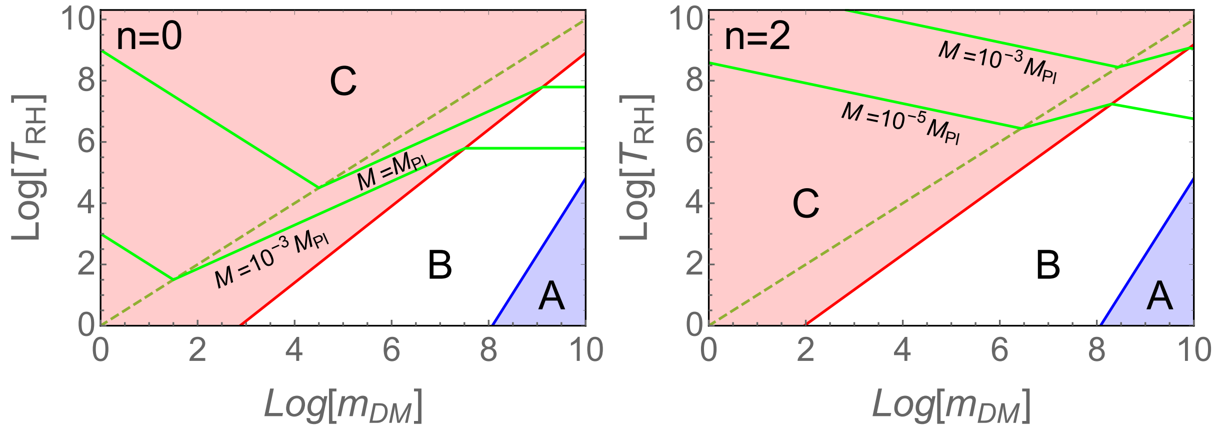


Figure 2: Dominant contribution for the production of DM during the reheating epoch in $T_{\rm RH} - m_{\rm DM}$ space. The blue, white, red regions represent parameter spaces that the contribution A, B, and C is dominant, respectively. We set $n = 0$ (left panel) or $n = 2$ (right panel). Each green line represents parameters where we can explain the observed amount of DM with $M$ being the value shown in the figure. The dashed line represents $T_{\rm RH} = m_{\rm DM}$.

We have checked that the contribution A is almost always dominant for $n \gtrsim 4$. The contribution C can be dominant for $n \simeq 3$, where the threshold is given by Eq. (27). For $n \lesssim 2$, the contributions B or C are dominant depending on $m_{\rm DM}$ and $T_{\rm RH}$, where the threshold is given by Eq. (28) or Eq. (29). The contribution A is dominant for $n \lesssim 2$ only in the case where $m_{\rm DM}^2 / T_{\rm RH} \gg m_\phi$ as one can see at the right-bottom corner in Fig. 2.

## 4 Conclusions and discussion

We have calculated the abundance of DM produced during the thermalization and reheating era after inflation, taking into account the finiteness of thermalization time for high-energy particles from the inflaton decay. The thermalization proceeds via splitting into high-energy particles, whose rate is suppressed by the LPM effect. There are two important effects that are missed in the *instantaneous thermalization* approximation. First, the temperature of the Universe does not reach its maximal value until the thermalization is completed. This means that the maximal temperature is overestimated in the *instantaneous thermalization* approximation. Second, the high-energy particles that are continuously produced from inflaton decay can produce DM before they lose their energy to be thermalized. This process is important when the DM production cross section is positively dependent on the center of mass energy and/or when the DM mass is larger than the reheating temperature.

There are three processes to produce DM during the reheating era: A) a collision between high-energy particles, B) a collision between a high-energy particle and a particle in the ambient plasma, and C) collision between particles in the ambient plasma. The first one is discussed in Ref. [20] though the second contribution is omitted. The second one is discussed in Ref. [2] only for the case of $n = 0$. In this paper, we have generalized the discussion in these papers and have determined the conditions for each contribution to be dominant. We have checked that the first contribution is dominant for the case of $n \gtrsim 3$, where $n$ is the power of the dependence of the DM production cross section on the center of mass energy. The second contribution is dominant for the case where the reheating temperature is much lower than the DM mass. These results are consistent with the results in Refs. [2, 20] when overlapped. The resulting DM abundance is many orders of magnitude larger than the one calculated in the *instantaneous thermalization* approximation in most cases.

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