Prompt η_c meson production at the LHC in the NRQCD with k_T-factorization

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In the framework of the k_T-factorization approach, the prompt production of η_c mesons at the LHC conditions is studied. Our consideration is based on the off-shell amplitudes for hard partonic subprocesses and on the nonrelativistic QCD (NRQCD) formalism for the formation of bound states. We try two latest parametrizations for noncollinear, or transverse momentum dependent (TMD) gluon densities derived from the Catani-Ciafaloni-Fiorani-Marchesini (CCFM) equation. We use the values of the nonperturbative matrix elements obtained from a combined fit of the η_c and J/ψ differential cross sections. Finally, we show an universal set of parameters that provides a reasonable simultaneous description for all of the available data on the prompt J/ψ and η_c production at the LHC.

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I. MOTIVATION

Since long ago, the production of quarkonium states in high energy hadronic collisions remains an area of intense attention from both theoretical and experimental sides. Our present work continues the line started in the previous publications [1–3]. We have already considered there the production of ψ', χ_c, and J/ψ mesons and now come to η_c mesons. As usual, we work in the k_T-factorization approach.

It is worth mentioning that the case of η_c mesons turned out to be rather puzzling for conventional NRQCD calculations at next-to-leading order (NLO) [4, 5]. This time, the theory was very unlucky to have too few free adjustable parameters. Having the nonperturbative matrix elements (NMEs) fixed from fitting all other production data, the theory lost its flexibility and made a prediction for η_c by a huge factor off the measured cross section. The overall situation was even called 'challenging' [4]. The aim of the present note is to show that the approach used consistently in [1–3] meets no troubles with the η_c data.

II. THEORETICAL FRAMEWORK

As it was done previously for ψ', χ_c and J/ψ production [1–3], the present calculations are based on perturbative QCD and nonrelativistic bound state formalism (NRQCD). The production of η_c mesons is dominated by the color singlet (CS) contribution that refers to the partonic subprocess

$$g^*(k_1) + g^*(k_2) \to \eta_c(p) \qquad (1)$$

with the respective cross section

$$\sigma(pp \to \eta_c + X) = \int \frac{2\pi}{x_1 x_2 s\, F}\, \mathcal{F}_g(x_1, \mathbf{k}_{1T}^2, \mu^2)\, \mathcal{F}_g(x_2, \mathbf{k}_{2T}^2, \mu^2) \times \left|\mathcal{M}(g^*g^* \to \eta_c)\right|^2 d\mathbf{k}_{1T}^2\, d\mathbf{k}_{2T}^2\, dy_\eta\, \frac{d\phi_1}{2\pi}\, \frac{d\phi_2}{2\pi}, \qquad (2)$$

where k_1 and k_2 denote the initial gluon 4-momenta, ϕ_1 and ϕ_2 are the respective azimuthal angles, y_η is the rapidity of η_c meson, x_1 and x_2 are the gluon longitudinal momentum fractions, $\mathcal{M}(g^*g^* \to \eta_c)$ is the hard scattering amplitude, and $\mathcal{F}_g(x_i, \mathbf{k}_{iT}^2, \mu^2)$ is the transverse momentum dependent (TMD, or unintegrated) gluon density in a proton. In accordance with the general definition [6], the off-shell gluon flux factor in (2) is taken as $F = 2\lambda^{1/2}(\hat{s}, k_1^2, k_2^2)$, where $\hat{s} = (k_1 + k_2)^2$.

In addition to the above, we have considered a number of color octet (CO) contributions and contribution from the feed-down $h_c \to \eta_c X$ process. The CO terms refer to the perturbative production of a color-octet $c\bar{c}^{[8]}$ pair followed by nonperturbative gluon radiation bringing the intermediate $c\bar{c}^{[8]}$ state to a real (colorless) meson:

$$g^*(k_1) + g^*(k_2) \to c\bar{c}^{[8]} \to \eta_c(p) + \text{soft gluons}. \qquad (3)$$

The intermediate color octet $c\bar{c}^{[8]}$ state can be either of ${}^1S_0$, ${}^3S_1$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2$, or ${}^1P_1$, where we use standard spectroscopic notation. The probabilities of the subsequent nonperturbative soft transitions are not calculable within the theory and are usually accepted as free model parameters. There are, however, certain restrictions coming from some general principles. Whenever calculable or not, the nonperturbative amplitudes must

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be identical for transitions in both directions (i.e., from vectors to scalars and vice versa), as it is motivated by the heavy quark spin symmetry (HQSS). The amplitudes can only differ by an overall normalizing factor representing the averaging over spin degrees of freedom. Thus, we strictly have from this property [7]:

$$\begin{aligned}
\left\langle \mathcal{O}^{\eta_c}\left[{}^1S_0^{[1]}\right]\right\rangle &= \frac{1}{3}\left\langle \mathcal{O}^{J/\psi}\left[{}^3S_1^{[1]}\right]\right\rangle \\
\left\langle \mathcal{O}^{\eta_c}\left[{}^1S_0^{[8]}\right]\right\rangle &= \frac{1}{3}\left\langle \mathcal{O}^{J/\psi}\left[{}^3S_1^{[8]}\right]\right\rangle \\
\left\langle \mathcal{O}^{\eta_c}\left[{}^3S_1^{[8]}\right]\right\rangle &= \left\langle \mathcal{O}^{J/\psi}\left[{}^1S_0^{[8]}\right]\right\rangle \\
\left\langle \mathcal{O}^{\eta_c}\left[{}^1P_1^{[8]}\right]\right\rangle &= 3\left\langle \mathcal{O}^{J/\psi}\left[{}^3P_0^{[8]}\right]\right\rangle \\
\left\langle \mathcal{O}^{h_c}\left[{}^1P_1^{[1]}\right]\right\rangle &= 3\left\langle \mathcal{O}^{\chi_{c0}}\left[{}^3P_0^{[1]}\right]\right\rangle \\
\left\langle \mathcal{O}^{h_c}\left[{}^1S_0^{[8]}\right]\right\rangle &= 3\left\langle \mathcal{O}^{\chi_{c0}}\left[{}^3S_1^{[8]}\right]\right\rangle \qquad (4)
\end{aligned}$$

The above relations require a simultaneous fit for the η_c and J/ψ production data. This fit turned out to be impossible in the traditional NRQCD scheme. The calculated cross sections were either found to be at odds with the measurements [4] or at odds with theoretical principles [5].

The crucial point in the above papers is the presence of a large unwanted contribution to the η_c production cross section from the intermediate ${}^3S_1^{[8]}$ state (unwanted, as the η_c production cross section is saturated by the color singlet channel alone; a fact, already pointed out in [8]). The corresponding nonperturbative matrix element is an HQSS counterpart of the ${}^1S_0^{[8]}$ matrix element engaged in the production of J/ψ mesons, where it is needed to make the outgoing J/ψ meson unpolarised: this spinless state is employed to dilute strong J/ψ polarization in other channels. Note by the way that the size of $\langle \mathcal{O}^{J/\psi}[{}^1S_0^{[8]}]\rangle$ matrix element used in [4] is in conflict with the NRQCD quark relative velocity counting rules.

In our present approach, we follow the interpretation of nonperturbative color octet transitions in terms of multipole radiation theory. Then, the final state J/ψ mesons come nearly unpolarized [9], either because of the cancellation between the ${}^3P_1^{[8]}$ and ${}^3P_2^{[8]}$ contributions, or as a result of two successive color-electric (E1) dipole transitions in the chain ${}^3S_1^{[8]} \to {}^3P_J^{[8]} \to J/\psi$ with $J = 0, 1, 2$. Thus, we can avoid the ${}^1S_0^{[8]}$ contribution to J/ψ and, as a consequence, get rid of the ${}^3S_1^{[8]}$ contribution to η_c production.

In the numerical analysis shown below, we tried two latest sets of TMD gluon densities in a proton, referred to as JH'2013 set 1 and JH'2013 set 2 [10]. These gluon densities were obtained from CCFM evolution equation where the input parametrization (used as boundary conditions) was fitted to the proton structure function $F_2(x, Q^2)$. Following [11], we take the charmonia masses $m(\eta_c) = 2.9839$ GeV, $m(h_c) = 3.52538$ GeV, $m(J/\psi) = 3.0969$ GeV and the branching fractions

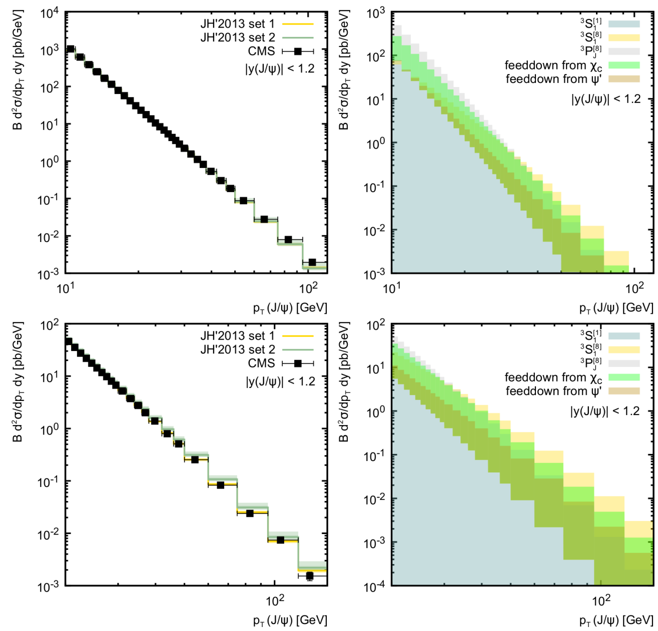

FIG. 1: Transverse momentum distribution of prompt J/ψ mesons produced in pp collisions at $\sqrt{s} = 7$ TeV (upper plots) and $\sqrt{s} = 13$ TeV (lower plots). The shaded bands on the left panels represent the total uncertainties of our calculations (i.e. scale uncertainties and the uncertainties coming from NMEs fit, summed in quadrature), as estimated for JH'2013 set 2 gluon density. The relative contributions from the different production mechanisms are shown on the right panels. The experimental data are from CMS [14].

$B(J/\psi \to \mu^+\mu^-) = 0.05961$ and $B(h_c \to \eta_c\gamma) = 0.51$. The renormalization and factorization scales were set to $\mu_R^2 = m^2 + \mathbf{p}_T^2$ and $\mu_F^2 = \hat{s} + \mathbf{Q}_T^2$, where m and $\mathbf{p}_T$ are the mass and transverse momentum of the produced charmonium, and $\mathbf{Q}_T$ is the transverse momentum of the initial off-shell gluon pair. The choice of μ_R is rather standard for charmonia production, while the unusual choice of μ_F is connected with the CCFM evolution (see [10] for details). The analytic expressions for the hard scattering amplitudes in (1) and (3) were otained using the algebraic manipulation system FORM [12]. The multidimensional phase space integration has been performed by means of the Monte-Carlo technique using the routine VEGAS [13].

III. NUMERICAL RESULTS

To determine the NMEs of J/ψ mesons (as well as their η_c counterparts) we performed a combined fit of J/ψ and η_c transverse momentum distributions using the latest CMS [14], ATLAS [15] and LHCb data [16] collected at 7, 8 and 13 TeV. Here, the factorization principle seems to be on solid theoretical grounds because of not too low p_T values for both J/ψ and η_c mesons. We do not impose

TABLE I: Sets of NME's for J/ψ production as determined from the different fits

	JH set 1	JH set 2	Kniehl *et al.* [17]	Gong *et al.* [18]
$\left\langle \mathcal{O}^{J/\psi}\left[{}^3S_1^{[1]}\right]\right\rangle$ /GeV^3	1.16	1.16	1.32	1.16
$\left\langle \mathcal{O}^{J/\psi}\left[{}^1S_0^{[8]}\right]\right\rangle$ /GeV^3	0.0	0.0	0.304	0.097
$\left\langle \mathcal{O}^{J/\psi}\left[{}^3S_1^{[8]}\right]\right\rangle$ /GeV^3	$(4.2 \pm 0.9) \cdot 10^{-4}$	$(1.6 \pm 0.2) \cdot 10^{-3}$	0.00168	-0.0046
$\left\langle \mathcal{O}^{J/\psi}\left[{}^3P_0^{[8]}\right]\right\rangle$ /GeV^5	0.023 ± 0.002	0.024 ± 0.002	-0.00908	-0.0214

any kinematic restrictions but the experimental acceptance. The fitting procedure was separately done in each of the rapidity subdivisions under the requirement that the NMEs be strictly positive, and then the mean-square average of the fitted values was taken. Note that we used the results of a global fit for the entire charmonium family (including, in particular, χ_{cJ} and ψ' states) [19] to properly calculate the feed-down contributions from h_c, χ_{cJ}, and ψ' decays.

For some (yet unrecognized) reasons, our ${}^1P_1^{[1]}$ production amplitude (needed to calculate the feed-down $h_c \to \eta_c X$) disagrees with the one found in the literature. Our calculation is off-shell, but has continuous on-shell limit that can be promptly compared with [20, 21]. The contribution is anyway small and unimportant numerically; but the discrepancy is still of interest from the academic point of view. For the lack of details presented in [20, 21], we cannot repeat their calculation. The details of our calculation are explained in the Appendix.

The numerical values of our NMEs for J/ψ and h_c mesons are written out in Tables I and II. For comparison, we also present here several sets of NMEs [17, 18, 22, 23], obtained in the NLO NRQCD by other authors. The NMEs shown for h_c mesons are translated from χ_c NMEs using HQSS formulas. The fits differ from one another by somehow differently selected data sets. The corresponding values of NMEs for η_c meson are collected in Table III. They can be easily obtained from Table I using the HQSS relations (4).

A comparison of our predictions with the experimental results is displayed in Figs. 1 and 2. The theoretical uncertainty bands include both scale uncertainties and the uncertainties coming from the NMEs fitting procedure. First of them were obtained by varying the μ_R scale around its default value by a factor of 2. This was accompanied with using the JH'2013 set 2+ and JH'2013 set 2- in place of the JH'2013 set 2, in accordance with [10]. One can see that we have achieved a reasonably good agreement between our calculations and LHCb measurements (with both of the considered TMD gluons), simultaneously for the prompt η_c and J/ψ production data collected at different energies and in the whole p_T range. The presented results can give a significant impact on the understanding of charmonia production within NRQCD.

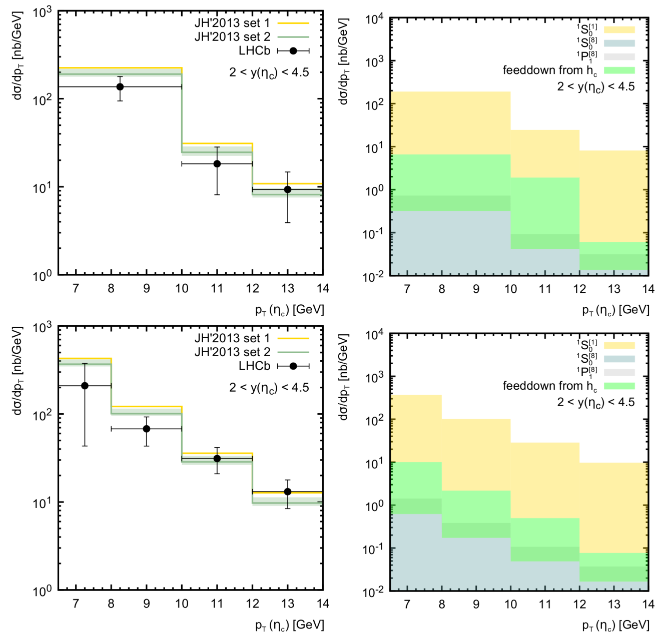

FIG. 2: Transverse momentum distribution of prompt η_c mesons produced in pp collisions at $\sqrt{s} = 7$ TeV (upper plots) and $\sqrt{s} = 8$ TeV (lower plots). Shaded bands on the left panels represent the total uncertainties of our calculations (i.e. scale uncertainties and the uncertainties coming from NMEs fit, summed in quadrature), as estimated for JH'2013 set 2 gluon density. The relative contributions from the different production mechanisms are shown on the right panels. The experimental data are from LHCb [16].

IV. CONCLUSIONS

We have considered the production of charmonium states at the LHC and found a consistent simultaneous description for the J/ψ and η_c data. Our nonperturbative matrix elements strictly obey the heavy quark spin symmetry rules.

The fundamental difference with the traditional NRQCD scheme (which was unable to accommodate the whole data set) is in a different treatement of the nonperturbative color-octet transitions. The latter are interpreted in our approach in terms of multipole radiation theory. Then the J/ψ mesons are produced unpolarized, thus making no need in a diluting ${}^1S_0^{[8]}$ contribution to

TABLE II: Sets of NME's for h_c production as determined from the different fits

	JH set 1	JH set 2	Zhang *et al.* [22]	Likhoded *et al.* [23]
$\left\langle \mathcal{O}^{h_c}\left[{}^1P_1^{[1]}\right]\right\rangle/\text{GeV}^5$	3.1 ± 0.4	3.2 ± 0.5	0.96	4.51
$\left\langle \mathcal{O}^{h_c}\left[{}^1S_0^{[8]}\right]\right\rangle/\text{GeV}^3$	$(6.0 \pm 3.0)\cdot 10^{-4}$	$(1.5 \pm 0.9)\cdot 10^{-3}$	0.00603	0.00132

J/ψ production and, as a consequence, requiring no ${}^3S_1^{[8]}$ contribution to η_c production. In the forthcoming paper [19] we are going to present a global fit for the entire charmonium family, including J/ψ, χ_{cJ}, $\psi(2S)$ and η_c mesons.

V. APPENDIX. OFF-SHELL PRODUCTION AMPLITUDE FOR ${}^1P_1^{[1]}$ STATE

In this section, we consider the gluon-gluon fusion subprocess

$$g(k_1,\epsilon_1,a) + g(k_2,\epsilon_2,b) \to g(k_3,\epsilon_3,c) + c\bar{c}(p,\epsilon^\alpha), \quad (5)$$

where the symbols in the parentheses indicate the momentum, the polarization, and the color of the interacting quanta. The calculation of this subprocess at $\mathcal{O}(\alpha_s^3)$ relates to six Feynman diagrams:

$$\begin{aligned}
\mathcal{M}_1 &= \text{tr}\{\not\epsilon_1(\not p_c - \not k_1 + m_c)\,\not\epsilon_2(-\not p_{\bar c} - \not k_3 + m_c)\,\not\epsilon_3\,\mathcal{P}_S\} \\
&\times \left[k_1^2 - 2(p_c k_1)\right]^{-1}\left[k_3^2 + 2(p_{\bar c} k_3)\right]^{-1}, \quad (6)\\
\mathcal{M}_2 &= \text{tr}\{\not\epsilon_1(\not p_c - \not k_1 + m_c)\,\not\epsilon_3(-\not p_{\bar c} + \not k_2 + m_c)\,\not\epsilon_2\,\mathcal{P}_S\} \\
&\times \left[k_1^2 - 2(p_c k_1)\right]^{-1}\left[k_2^2 - 2(p_{\bar c} k_2)\right]^{-1}, \quad (7)\\
\mathcal{M}_3 &= \text{tr}\{\not\epsilon_3(\not p_c + \not k_3 + m_c)\,\not\epsilon_1(-\not p_{\bar c} + \not k_2 + m_c)\,\not\epsilon_2\,\mathcal{P}_S\} \\
&\times \left[k_3^2 + 2(p_c k_3)\right]^{-1}\left[k_2^2 - 2(p_{\bar c} k_2)\right]^{-1}, \quad (8)\\
\mathcal{M}_4 &= \text{tr}\{\not\epsilon_2(\not p_c - \not k_2 + m_c)\,\not\epsilon_1(-\not p_{\bar c} - \not k_3 + m_c)\,\not\epsilon_3\,\mathcal{P}_S\} \\
&\times \left[k_2^2 - 2(p_c k_2)\right]^{-1}\left[k_3^2 + 2(p_{\bar c} k_3)\right]^{-1}, \quad (9)\\
\mathcal{M}_5 &= \text{tr}\{\not\epsilon_2(\not p_c - \not k_2 + m_c)\,\not\epsilon_3(-\not p_{\bar c} + \not k_1 + m_c)\,\not\epsilon_1\,\mathcal{P}_S\} \\
&\times \left[k_2^2 - 2(p_c k_2)\right]^{-1}\left[k_1^2 - 2(p_{\bar c} k_1)\right]^{-1}, \quad (10)\\
\mathcal{M}_6 &= \text{tr}\{\not\epsilon_3(\not p_c + \not k_3 + m_c)\,\not\epsilon_2(-\not p_{\bar c} + \not k_1 + m_c)\,\not\epsilon_1\,\mathcal{P}_S\} \\
&\times \left[k_3^2 + 2(p_c k_3)\right]^{-1}\left[k_1^2 - 2(p_{\bar c} k_1)\right]^{-1}, \quad (11)\\
\mathcal{M} &= \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4 + \mathcal{M}_5 + \mathcal{M}_6, \quad (12)
\end{aligned}$$

with the property $\mathcal{M}_1 = \mathcal{M}_6$, $\mathcal{M}_2 = \mathcal{M}_5$, $\mathcal{M}_3 = \mathcal{M}_4$. The color factor is universal and is equal to $d^{abc}/4\sqrt{3}$. This set of diagrams is complete; no other diagrams can contribute at the order $\mathcal{O}(\alpha_s^3)$ to the production of a meson with the given quantum numbers $J^{PC} = 1^{+-}$.

The amplitudes $\mathcal{M}_i$ contain spin projection operators which discriminate the spin-singlet and spin-triplet $c\bar{c}$ states:

$$\begin{aligned}
\mathcal{P}_{S=0} &= (\not p_{\bar c} - m_c)\gamma_5(\not p_c + m_c)\cdot(2m_c)^{-3/2}, \quad (13)\\
\mathcal{P}_{S=1} &= (\not p_{\bar c} - m_c)\,\not\epsilon_\psi(\not p_c + m_c)\cdot(2m_c)^{-3/2}, \quad (14)
\end{aligned}$$

where m_c is the charmed quark mass. These projectors are orthogonal to each other, as they should be: $tr\{\mathcal{P}_0\overline{\mathcal{P}_1}\} = 0$. For the ${}^1P_1^{[1]}$ state we evidently have to use the projector $\mathcal{P}_0$.

The orbital angular momentum L is associated with the relative momentum q of the quarks in a bound state. The relative momentum q is defined as

$$p_c = \frac{1}{2}p + q, \quad p_{\bar c} = \frac{1}{2}p - q. \quad (15)$$

According to a general formalism developed in [24, 25], the terms showing no dependence on q are identified with the contributions to the $L = 0$ state; the terms linear in q^α are related to the $L = 1$ state with the proper polarization vector ϵ^α (see below); the quadratic terms $q^\alpha q^\beta$ refer to the $L = 2$ state with the polarization tensor $\epsilon^{\alpha\beta}$; and so on. The decomposition of $\mathcal{M}$ in powers of q is carried out by expanding the subprocess amplitude as

$$\mathcal{M}(q) = \mathcal{M}|_{q=0} + q^\alpha(\partial\mathcal{M}/\partial q^\alpha)|_{q=0} + ..., \quad (16)$$

where q is assumed to be a small quantity. The amplitude $\mathcal{M}(q)$ has to be multiplied by the bound state wave finction $\Psi(q)$ and integrated over q. A term-by-term integration of Eq.(16) is performed using the relations

$$\begin{aligned}
\int \frac{d^3q}{(2\pi)^3}\Psi(q) &= \frac{1}{\sqrt{4\pi}}\mathcal{R}(x{=}0), \quad (17)\\
\int \frac{d^3q}{(2\pi)^3}q^\alpha\Psi(q) &= -i\epsilon^\alpha\frac{\sqrt{3}}{\sqrt{4\pi}}\mathcal{R}'(x{=}0), \quad (18)
\end{aligned}$$

etc., where $\mathcal{R}(x)$ is the radial wave function in the coordinate representation (the Fourier transform of $\Psi(q)$). This formula completes our derivation of the production matrix element. The resulting expression has been explicitly tested for gauge invariance by substituting the gluon momentum k_i for the polarization vector ϵ_i. We have observed gauge invariance even with off-shell initial gluons.

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TABLE III: Sets of NME's for η_c production as determined from the different fits

	JH set 1	JH set 2	Kniehl *et al.* [17]	Gong *et al.* [18]
$\left\langle \mathcal{O}^{\eta_c}\left[{}^1S_0^{[1]}\right]\right\rangle/\mathrm{GeV}^3$	0.39	0.39	0.44	0.39
$\left\langle \mathcal{O}^{\eta_c}\left[{}^3S_1^{[8]}\right]\right\rangle/\mathrm{GeV}^3$	0.0	0.0	0.304	0.097
$\left\langle \mathcal{O}^{\eta_c}\left[{}^1S_0^{[8]}\right]\right\rangle/\mathrm{GeV}^3$	$(1.4 \pm 0.3) \cdot 10^{-4}$	$(5.3 \pm 0.7) \cdot 10^{-4}$	0.00056	-0.0015
$\left\langle \mathcal{O}^{\eta_c}\left[{}^1P_1^{[8]}\right]\right\rangle/\mathrm{GeV}^5$	0.069 ± 0.006	0.072 ± 0.006	-0.02724	-0.0642

[1] S.P. Baranov, A.V. Lipatov, N.P. Zotov, Eur. Phys. J. C **75**, 455 (2015).

[2] S.P. Baranov, A.V. Lipatov, N.P. Zotov, Phys. Rev. D **93**, 094012 (2016).

[3] S.P. Baranov, A.V. Lipatov, Phys. Rev. D **96**, 034019 (2017).

[4] M. Butenshoen, Z.-G. He, B.A. Kniehl, Phys. Rev. Lett. **114**, 092004 (2015).

[5] H. Han, Y.-Q. Ma, C. Meng, H.-S. Shao, K.-T. Chao, Phys. Rev. Lett. **114**, 092005 (2015).

[6] E. Bycling, K. Kajantie, *Particle Kinematics* (John Wiley and Sons, New York, 1973).

[7] G.T. Bodwin, E. Braaten, G.P. Lepage, Phys. Rev. D **51**, 1125 (1995); Phys. Rev. D **55**, 5853(E) (1997).

[8] A.K. Likhoded, A.V. Luchinsky, S.V. Poslavsky, Mod. Phys. Lett. A **30**, 1550032 (2015).

[9] S.P. Baranov, Phys. Rev. D **93**, 054037 (2016).

[10] F. Hautmann, H. Jung, Nucl. Phys. B **883**, 1 (2014).

[11] Particle Data Group, Chin. Phys. C **40**, 100001 (2016).

[12] J.A.M. Vermaseren, *Symbolic Manipulations with FORM* (Computer Algebra Nederland, Kruislaan, SJ Amsterdaam, 1991, ISBN 90-74116-01-9).

[13] G.P. Lepage, J. Comp. Phys. **27**, 192 (1978).

[14] CMS Collaboration, Phys. Rev. Lett. **114**, 191802 (2015);
Phys. Lett. B **780**, 251 (2018).

[15] ATLAS Collaboration, Eur. Phys. J. C **76**, 47 (2016).

[16] LHCb Collaboration, Eur. Phys. J. C **75**, 311 (2015).

[17] M. Butenshoen, B.A. Kniehl, Phys. Rev. D **84**, 051501(R) (2011).

[18] B.Gong, L.-P. Wan, J.-X. Wang, H.-F. Zhang, Phys. Rev. Lett. **110**, 042002 (2013).

[19] S.P. Baranov, A.V. Lipatov, in preparation.

[20] M.M. Meijer, J. Smith, and W.L. van Neerven, Phys. Rev. D **77**, 034014, (2008), `arXiv:0710.3090`.

[21] R. Gastmans, W. Troost, and T.T. Wu, Nucl. Phys. B **291**, 731 (1987).

[22] H.-F. Zhang, L. Yu, S.-X. Zhang, L. Jia, Phys. Rev. D **93**, 054033 (2016).

[23] A.K. Likhoded, A.V. Luchinsky, S.V. Poslavsky, Phys. Rev. D **90**, 074021 (2014).

[24] H. Krasemann, Z. Phys. C **1**, 189 (1979.)

[25] G. Guberina, J. Kühn, R. Peccei, and R. Rückl, Nucl. Phys. B **174**, 317 (1980.)