

Soft corrections to inclusive DIS at four loops and beyond

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We study the threshold corrections to the structure functions in deep-inelastic scattering (DIS) at the fifth logarithmic (N^4LL) order of the soft-gluon exponentiation in massless perturbative QCD. Using recent results for the splitting functions and the quark form factor, we derive the fourth-order contribution to the coefficient f^q of the form factor and from it the N^4LL part of the exponentiation coefficient B^{DIS} in the limit of a large number of colours. An approximation scheme is shown that leads to sufficiently accurate N^4LL results for full QCD. The N^4LL corrections are small and lead to a further stabilization of the perturbative expansion for the soft-gluon exponent.

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1. Introduction

The Wilson coefficients (coefficient functions) for the structure functions of inclusive DIS have been a subject of research since the early days of QCD. These quantities are not only relevant for determining the parton distribution functions (PDFs) and the strong coupling constant α_s using structure function data, see, e.g., [1], but also to other processes and less inclusive observables in DIS, see, e.g., [2]. The main Wilson coefficients for DIS are presently known up to the third order in α_s in massless perturbative QCD [3]. Their perturbative expansion is well-behaved except close to the kinematic endpoints $x = 0$ and $x = 1$ of the Bjorken variable. The dominant terms $\ln^\ell(1-x)/(1-x)_+$ in the latter (threshold) limit are resummed by the soft-gluon exponentiation, see, e.g., [4–6], which is best formulated in Mellin N -space [4]. So far this resummation has been performed up to the next-to-next-to-next-to-leading logarithmic ($N^3\text{LL}$) accuracy [7].

The threshold resummation coefficients are closely related to the large- x limit of the quark-quark splitting functions for the PDFs and to the quark form factor [8,9] which are both fully known to order α_s^3 [10–13]. Recently the computations of these quantities have been extended to order α_s^4 in the (Ln_c) limit of a large number of colours [14, 15]. Together with approximate results for the n_f -independent [16] and exact expression for the n_f -dependent contributions to the cusp anomalous dimension in full QCD [17] these results facilitate the effective extension of the threshold resummation for the DIS Wilson coefficients to the next ($N^4\text{LL}$) logarithmic order. In the following we recall the theoretical framework, present the $N^4\text{LL}$ resummation coefficient and briefly address the numerical implications of this result for the resummation of DIS in QCD.

2. Theoretical framework and new fourth-order coefficients

The all-order large- N behaviour of the DIS Wilson coefficients for F_1 , F_2 and F_3 can be written as

$$C^N(Q^2) = g_0(Q^2) \cdot \exp[G^N(Q^2)] + \mathcal{O}(N^{-1} \ln^n N), \quad (2.1)$$

where the resummation exponent G^N of the dominant $N^0 \ln^n N$ contributions is given by [18]

$$G^N = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left[\int_{\mu_f^2}^{(1-z)Q^2} \frac{dq^2}{q^2} A^q(\alpha_s(q^2)) + B^{\text{DIS}}(\alpha_s((1-z)q^2)) \right]. \quad (2.2)$$

Here A^q is the (light-like) quark cusp anomalous dimension and B^{DIS} is the resummation coefficient for DIS. Both have perturbative series, $A^q = \sum_i a_s^i A_i^q$ etc, in terms of the strong coupling which we normalize as $a_s \equiv \alpha_s/4\pi$. Performing the integrations one can organize the exponent as

$$G^N = \ln N g^{(1)}(\lambda) + g^{(2)}(\lambda) + a_s g^{(3)}(\lambda) + a_s^2 g^{(4)}(\lambda) + a_s^3 g^{(5)}(\lambda) + \dots, \quad (2.3)$$

where $\lambda = \beta_0 a_s \ln N$ or $\lambda = \beta_0 a_s \ln \tilde{N}$ with $\ln \tilde{N} = \ln N + \gamma_e$. The first $n+1$ terms in (2.3) are required for the resummation at $N^n\text{LL}$ accuracy. The $N^2\text{LL}$ and $N^3\text{LL}$ contributions to G^N have been derived in [7, 19]; explicit expressions can be found in (3.3) – (3.6) of [7]. The lengthy new function $g^{(5)}(\lambda)$ entering at $N^4\text{LL}$ will be presented in [20]. The N -independent prefactor g_0 is presently known to order α_s^3 from the all- N calculation in [3], see (4.6) – (4.8) of [7],

$$g_0 = 1 + a_s g_{01} + a_s^2 g_{02} + a_s^3 g_{03} + \mathcal{O}(a_s^4). \quad (2.4)$$

The resummation to $N^4\text{LL}$ requires the terms up to A_5^q and B_4^{DIS} in their corresponding expansions. The impact of the former quantity, for which a first estimate has been obtained in [21], is very small. B^{DIS} can be calculated from knowledge of the quark form factor or the DIS Wilson coefficients. The form factor satisfies a differential equation which follows from the renormalization group and gauge invariance. Its solution can be found in terms of the cusp anomalous dimension A^q and the function G^q containing the quantity f^q related to a universal eikonal anomalous dimension and the coefficient B^q of $\delta(1-x)$ in the quark-quark splitting function. The four-loop coefficient of G^q (which appears in the $1/\epsilon$ coefficient in the solution of the form factor) can be written as

$$G_4^q = 2B_4^q + f_4^q + \beta_2 f_{01}^q + \beta_1 f_{02}^q + \beta_0 f_{03}^q + \mathcal{O}(\epsilon), \quad (2.5)$$

where the quantities f_{0n}^q are (combinations of) known lower-order coefficients of G^q , see [12] and (20) of [9]. Hence f_4^q can be determined in the large- n_c limit from the results of [14, 15]. We find

$$\begin{aligned} f_4^q \Big|_{L n_c} = & C_F n_c^3 \left(\frac{9364079}{6561} - \frac{1186735}{729} \zeta_2 - \frac{837988}{243} \zeta_3 + \frac{115801}{27} \zeta_4 + \frac{11896}{9} \zeta_2 \zeta_3 + 3952 \zeta_5 \right. \\ & \left. - \frac{4796}{9} \zeta_3^2 - \frac{129547}{54} \zeta_6 - 416 \zeta_2 \zeta_5 - 720 \zeta_3 \zeta_4 - 1700 \zeta_7 \right) + C_F n_c^2 n_f \left(-\frac{247315}{432} \right. \\ & \left. + \frac{412232}{729} \zeta_2 + \frac{102205}{243} \zeta_3 - \frac{7589}{6} \zeta_4 - \frac{824}{9} \zeta_2 \zeta_3 - \frac{740}{9} \zeta_5 + \frac{2816}{9} \zeta_3^2 + \frac{15611}{27} \zeta_6 \right) \\ & + C_F n_c n_f^2 \left(\frac{329069}{17496} - \frac{22447}{729} \zeta_2 + \frac{25300}{243} \zeta_3 + \frac{140}{3} \zeta_4 - \frac{176}{9} \zeta_2 \zeta_3 - \frac{856}{9} \zeta_5 \right) \\ & + C_F n_f^3 \left(-\frac{16160}{6561} - \frac{16}{81} \zeta_2 - \frac{400}{243} \zeta_3 + \frac{128}{27} \zeta_4 \right). \end{aligned} \quad (2.6)$$

The a_s^4 contribution to resummation B^{DIS} reads, in terms of the genuine four-loop contributions f_4^q and B_4^q , which are exactly known only in the $L n_c$ limit for now, and lower-order coefficients,

$$\begin{aligned} B_4^{\text{DIS}} = & -f_4^q - B_4^q - \beta_2 \left(f_{01}^q + g_{01} - \frac{1}{2} \zeta_2 A_1^q \right) + \beta_0^3 \left(3\zeta_2 f_{01}^q + 3\zeta_2 g_{01} + 2\zeta_3 f_1^q + 2\zeta_3 B_1^q \right) \\ & + \frac{3}{2} \zeta_4 A_1^q - \frac{3}{4} \zeta_2^2 A_1^q + \beta_0 \beta_1 \left(\frac{5}{2} \zeta_2 f_1^q + \frac{5}{2} \zeta_2 B_1^q + \frac{5}{3} \zeta_3 A_1^q \right) + \beta_0^2 \left(3\zeta_2 f_2^q + 3\zeta_2 B_2^q + 2\zeta_3 A_2^q \right) \\ & - \beta_1 \left(f_{02}^q + 2g_{02} - (g_{01})^2 - \zeta_2 A_2^q \right) - \beta_0 \left(f_{03}^q + 3g_{03} - 3g_{02}g_{01} - (g_{01})^3 - \frac{3}{2} \zeta_2 A_3^q \right), \end{aligned} \quad (2.7)$$

where g_{0i} are to be taken without the γ_e terms in (4.6) – (4.8) of [7]. Its explicit form is given by

$$\begin{aligned} B_4^{\text{DIS}} \Big|_{L n_c} = & C_F n_c^3 \left(-\frac{2040092429}{139968} + \frac{23011973}{1944} \zeta_2 + \frac{517537}{36} \zeta_3 - \frac{312481}{36} \zeta_4 - \frac{39838}{9} \zeta_2 \zeta_3 \right. \\ & \left. - \frac{50680}{9} \zeta_5 - 988 \zeta_3^2 + \frac{12467}{6} \zeta_6 + 496 \zeta_2 \zeta_5 + 688 \zeta_3 \zeta_4 + 2260 \zeta_7 \right) \\ & + C_F n_c^2 n_f \left(\frac{83655179}{11664} - \frac{5160215}{972} \zeta_2 - \frac{639191}{162} \zeta_3 + \frac{24856}{9} \zeta_4 + \frac{8624}{9} \zeta_2 \zeta_3 \right. \\ & \left. + 200 \zeta_5 - 32 \zeta_3^2 - \frac{1201}{3} \zeta_6 \right) + C_F n_f^3 \left(\frac{50558}{2187} + \frac{80}{81} \zeta_3 - \frac{1880}{81} \zeta_2 + \frac{40}{9} \zeta_4 \right) \\ & + C_F n_c n_f^2 \left(-\frac{5070943}{5832} + \frac{160903}{243} \zeta_2 + \frac{14618}{81} \zeta_3 - \frac{2110}{9} \zeta_4 - \frac{400}{9} \zeta_2 \zeta_3 + \frac{904}{9} \zeta_5 \right). \end{aligned} \quad (2.8)$$

3. Numerical implications

The lower-order coefficients B_l^{DIS} have the same structure as (2.7), i.e., they contain $-f_l^q - B_l^q$ and lower-order coefficients. Therefore, by comparing the exact results to an approximation at $N^l\text{LL}$ in which the Ln_c expression for $-f_l^q - B_l^q$ is used together with the exact lower-order coefficients, we can check whether (2.7) with the Ln_c results for f_4^q and B_4^q can be expected to provide a good approximation for B_4^{DIS} and hence G^N at the $N^4\text{LL}$ accuracy of full QCD.

This comparison is carried out in Fig. 1 for $l = 2$ and $l = 3$ (at $l = 1$ there is no difference between the Ln_c limit and full QCD). The Ln_c curves are off by less than 0.5% at $N^2\text{LL}$ and 0.25% at $N^3\text{LL}$ for G^{DIS} in the N -range shown and, at $x \leq 0.9$, for the convolution of its exponential with a schematic but sufficiently realistic form for a quark PDF. Therefore we can safely expect that the Ln_c numbers will deviate from (presumably exceed) the exact QCD results by well below 1%.

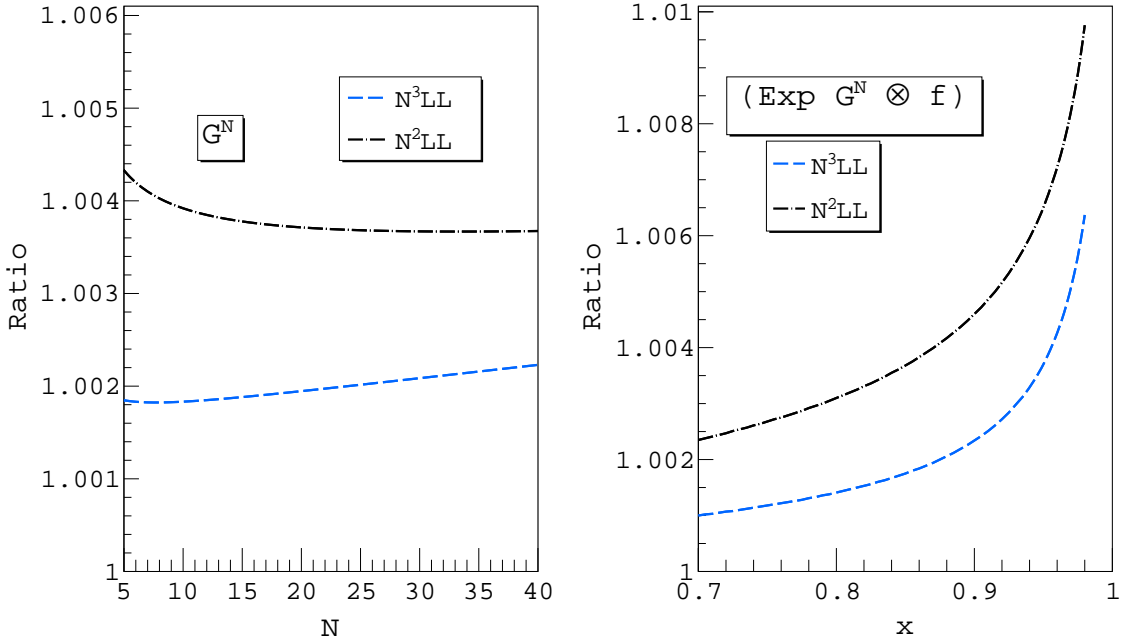


Figure 1: The ratio of the large- n_c approximation, defined as above, and the exact results at $N^2\text{LL}$ and $N^3\text{LL}$ for the DIS resummation exponent G^N (left) and for the convolution of the exponential with the schematic quark PDF shape $xf = x^{0.5}(1-x)^3$ (right) for $\alpha_s = 0.2$ and $n_f = 3$ flavours.

The cumulative effect, relative to the NLL results, of the exact $N^2\text{LL}$ and $N^3\text{LL}$ contributions and our new $N^4\text{LL}$ corrections, as above determined using the Ln_c limit of $-f_l^q - B_l^q$ in (2.7), is illustrated in Fig. 2. Unlike the $N^3\text{LL}$ contribution, the $N^4\text{LL}$ correction is almost negligible at $N \leq 15$ and $x \leq 0.9$. Even at $N = 40$, the functions $g^{(n)}(\lambda)$ add only 6%, 1.6% and 1% to the NLL result, respectively, for $n = 2$, $n = 3$ and $n = 4$, where the latter Ln_c result is presumably a slight overestimate. The corresponding $N^2\text{LL}$, $N^3\text{LL}$ and $N^4\text{LL}$ percentages for the convolution of $\exp G^N$ with $xf = x^{0.5}(1-x)^3$ at $x = 0.95$ read 9.5%, 1.5% and 0.5%, where we have performed the Mellin inversion using a standard contour, see, e.g., [22], which constitutes a ‘minimal prescription’ contour [4] in the context of the present exponentiation. It appears that the expansion of G^N to $N^4\text{LL}$ for the structure functions in inclusive DIS is sufficient for all practical purposes.

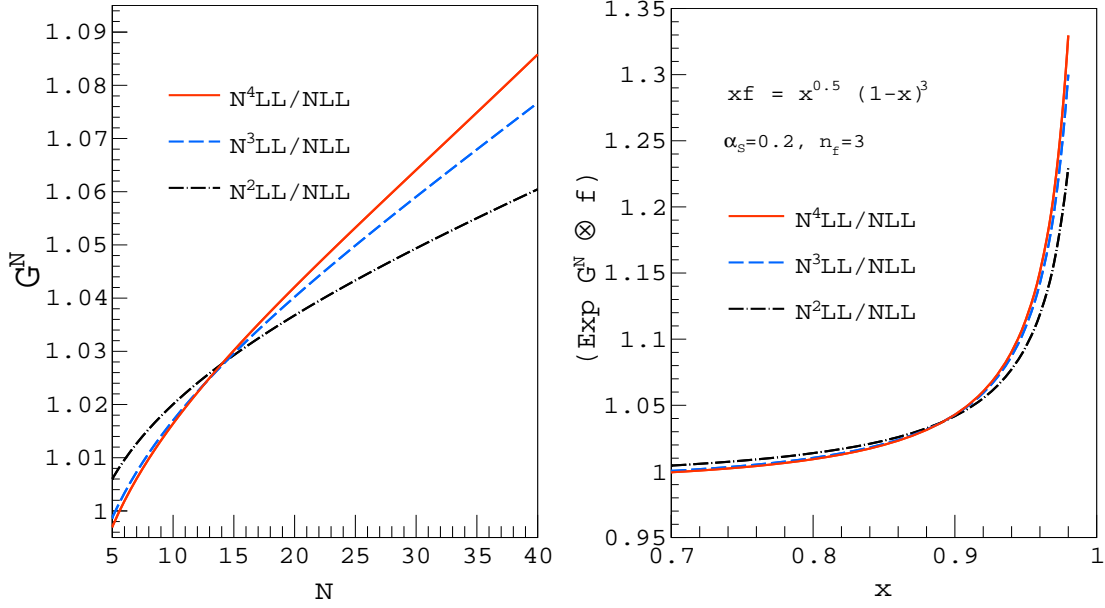


Figure 2: Left: The DIS resummation exponent G^N in (2.3) up to N^4 LL accuracy, normalized to the NLL result at the standard reference point $\alpha_s = 0.2$ for $n_f = 3$. Right: corresponding x -space results for $\exp G^N$ after convoluted with a schematic form of a quark PDF of the proton.

4. Summary and outlook

We have studied the soft-gluon exponentiation (SGE) of inclusive DIS at the fifth logarithmic (N^4 LL) order. Recent four-loop results on splitting functions and the quark form factor [14, 15] facilitate the exact determination of the form-factor coefficient f^q and the SGE coefficient B^{DIS} at order α_s^4 in the large- n_c (Ln_c) limit. Both coefficients are relevant beyond the context of DIS: Like the lightlike quark and gluon cusp anomalous dimensions $A^{q,g}$ [10], the quantities $f^{q,g}$ are maximally non-Abelian and related by a simple Casimir scaling up to three loops. We expect that the generalized Casimir scaling of [16] also applies to $f^{q,g}$, hence our result (2.6) fixes also f^g at large n_c . The coefficient here called B^{DIS} is due to the outgoing unobserved quark; hence it contributes to the SGE for many other processes including, e.g., direct photon production [6].

The Ln_c approximation to the N^4 LL resummation exponent G^N for inclusive DIS, defined as discussed above, is sufficiently accurate to demonstrate that the N^4 LL corrections are small: they contribute well below 1% over a wide range in N and x . As shown in [23], the $1/N \ln^\ell N$ non-SGE contributions are larger; the highest four of these logarithms are currently known to all orders [23,24] – recall the parameter ξ_{DIS_4} unspecified in [23] was fixed in [24]. We have considered the case of $n_f = 3$ light flavours. In electromagnetic and neutral-current DIS, also charm production close to threshold needs to be taken into account beyond the threshold for $c\bar{c}$ production, see [25].

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