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Quantum Corrections for D-Brane Models with Broken Supersymmetry

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Abstract

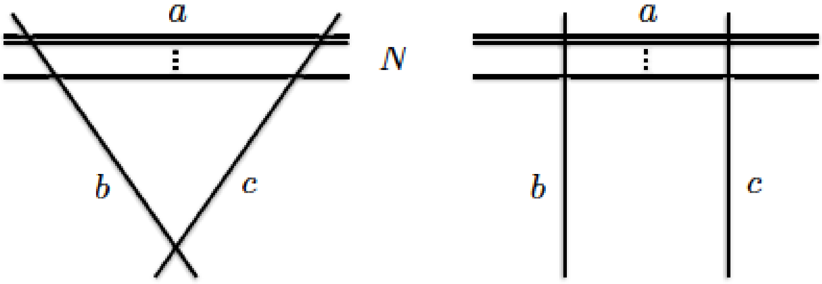


Figure 1: Left: intersections of brane stack a with branes b and c, and brane b with c in the second torus T_2^2; right: intersections of brane stack a with branes b and c in torus T_1^2 where branes b and c are parallel.

In this case, the tadpole conditions (4) read explicitly,

$$\begin{aligned} N + 2 &= 16 \ , \\ N m_a^2 m_a^3 + m_b^2 m_b^3 + m_c^2 m_c^3 &= 0 \ , \\ m_b^1 m_b^3 + m_c^1 m_c^3 &= 0 \ , \\ m_b^1 m_b^2 + m_c^1 m_c^2 &= 0 \ . \end{aligned} \tag{5}$$

One easily verifies that these equations are solved by the ansatz in Table 1, with $N = 14$, $l = 7$. The chosen wrapping numbers imply that not all branes intersect in all tori: a and a', and b and c are parallel in the first torus, whereas b and c' are parallel in the second and in the third torus. This situation is illustrated in Figure 1.

On each brane an $\mathcal{N} = 4$ supermultiplet of zero-modes in the adjoint representation of the gauge group is localized. The branes intersect at angles determined by the wrapping numbers. At these intersections fermions and scalars in bi-fundamental representations $(N_e, \bar{N}_f)$ are localized. For non-zero intersection numbers

$$I_{ef} = \otimes_i \left(n_e^i m_f^i - m_e^i n_f^i \right) \tag{6}$$

the fermion spectrum is chiral. The fermions are left-handed for $I_{ef} > 0$ and right-handed for $I_{ef} < 0$, corresponding to left-handed fermions in the complex conjugate representation $(\bar{N}_e, N_f)$. At the intersections of the brane system defined in Table 1 one obtains the left-handed fermions listed in Table 2. There are matter fields that carry 'colour', transforming as N or $\bar{N}$ under $SU(N)$. They form a chiral representation of the full gauge group, whereas colour singlet 'Higgs fields' form vector-like representations. The quantum numbers of the chiral fermions allow Yukawa couplings that are most conveniently expressed in terms of the associated chiral superfields,

$$\mathcal{L}_Y \supset \sum_{r,s}^{3(l-2)} y_{rs}^{(1)} \bar{\boldsymbol{N}}_{1,0}^r \boldsymbol{N}_{0,1}^s \boldsymbol{1}_{-1,-1} + \sum_{r,s}^{l+2} y_{rs}^{(2)} \bar{\boldsymbol{N}}_{0,1}^r \boldsymbol{N}_{1,0}^s \boldsymbol{1}_{-1,-1} \ . \tag{7}$$

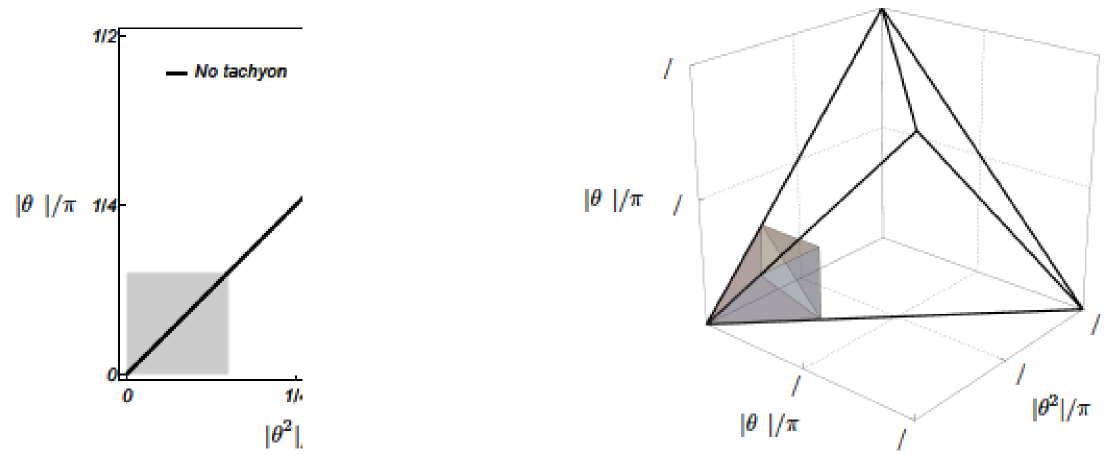


Figure 2: Domain of angles for which no tachyons appear. Left: line in $\theta^2 - \theta^3$-plane in case of no flux in first torus, i.e. $\theta^1 = 0$. Right: tetrahedron in case of fluxes in all tori. The gray areas indicate small-angle domains with $|\theta^i|/\pi < 0.15$.

and for the ab'- and ac-sector one obtains

$$l\rho_2 < \rho_1 < (l+4)\rho_2 \ . \tag{17}$$

With $l = 7$, the last condition (17) is the stronger one and implies condition (16). Hence, once the conditions (14) and (17) are satisfied, all scalars in the sectors ab, ac', ab' and ac are massive and supersymmetry is completely broken. The angles satisfying these conditions form a tetrahedron [16]. It is illustrated in Figure 2, together with a domain of small angles.

The appearance of tachyons is a generic feature of non-supersymmetric intersecting D-brane models. However, it is argued that such tachyons can be removed by couplings to moduli fields that parametrize the distance between branes in tori where they are parallel. In the T-dual picture discussed in the following section these moduli correspond to Wilson-lines ξ, ξ' that acquire vacuum expectation values (see, for example, [16, 21]). In the present model the corresponding superpotential terms would have the form (in superfield notation, see Table 2),

$$W_{\xi,\xi'} = \lambda_1\ \xi\ \mathbf{1}_{1,-1}\mathbf{1}_{-1,1} + \lambda_2\ \xi'\ \mathbf{1}_{1,1}\mathbf{1}_{-1,-1} \ . \tag{18}$$

Clearly, existence and stability of a ground state require an appropriate potential for ξ, ξ'. At tree-level the potential is flat. To compute the one-loop quantum correction to the potential is an essential goal of this paper. To achieve this we first construct the T-dual type I string compactification on a magnetized torus, which allows a straightforward computation of the full mass spectrum of the model as well as Yukawa couplings.

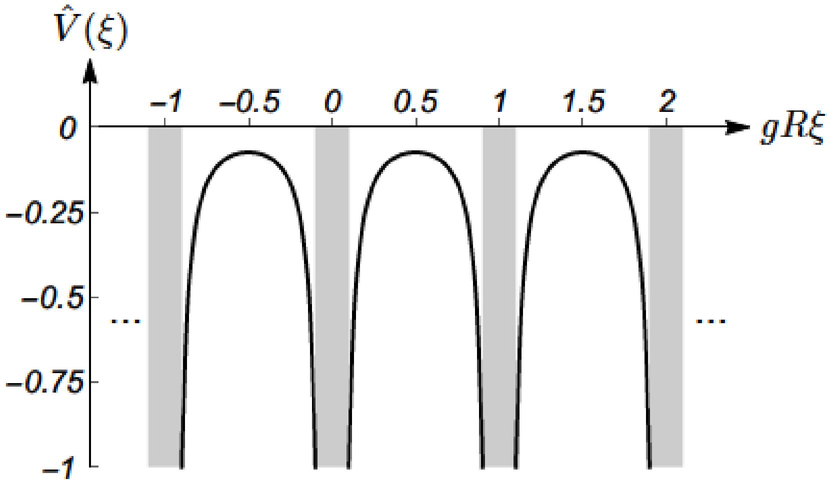


Figure 3: Wilson-line potential, normalized to its value at the border to the tachyonic region, which is chosen to have the width $\Delta = 2\sqrt{gfR^2} = 0.2$ (see text).

The ultraviolet divergence is now encoded in the term $l_i = 0$. Adding to the sum a counter term

$$-c_1 e^{-\mu_1^2 t} - c_2 e^{-\mu_2^2 t} , \tag{134}$$

with

$$1 - c_1 - c_2 = 0 , \quad c_1\mu_1^2 + c_2\mu_2^2 = 0 , \tag{135}$$

implies that

$$\sum_{m_i} \exp\Big(-t\Big(\frac{m_i}{R_i} + g\xi_i\Big)^2\Big) - \frac{\pi^2 \prod_i R_i}{t^2}\Big(c_1 e^{-\mu_1^2 t} + c_2 e^{-\mu_2^2 t}\Big) \tag{136}$$

is finite as $t \to 0$, yielding a finite integral $V(\xi) = \int dt V(t,\xi)$. Note, that the terms

$$\int \frac{dt}{t^5} e^{-\mu_i^2 t} \propto \int \frac{dt}{t} \int \frac{d^8k}{(2\pi)^8} e^{-(k^2+\mu_i^2)t} \propto \int \frac{d^8k}{(2\pi)^8} \ln(k^2 + \mu_i^2) \tag{137}$$

correspond to Pauli-Villars regulators in 8d field theories.

Stationary points of the potential have to satisfy

$$\frac{\partial V(t,\xi)}{\partial \xi_i} \propto \sum_{m_j} \Big(\frac{m_i}{R_i} + g\xi_i\Big) \exp\Big(-t\Big(\frac{m_j}{R_j} + g\xi_j\Big)^2\Big) = 0 . \tag{138}$$

The solutions are given by

$$\hat{\xi}_i = \frac{n_i}{2gR_i} , \quad n_i \in \mathbb{Z} , \tag{139}$$

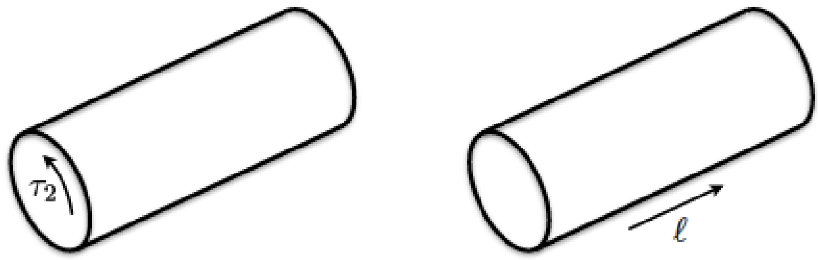


Figure 4: Left: loop-diagram for open string; right: tree-diagram for closed string.

The analogous expression for the amplitude A_{bc} is easily found to be

$$A_{bc} = \frac{I_{bc}^2 I_{bc}^3}{2(4\pi^2\alpha')^2} \int_0^\infty \frac{d\tau_2}{\tau_2^3} \frac{(V_8 - S_8)(\epsilon_2\tau;\epsilon_3\tau|\tau)}{\eta^4} \frac{2i\eta}{\theta_1(\epsilon_2\tau|\tau)} \frac{2i\eta}{\theta_1(\epsilon_3\tau|\tau)} P_{\mathbf{m_1}+\xi_1} \ , \tag{152}$$

where one can now use the Jacobi identity (228),

$$\begin{aligned}(V_8 - S_8)(\epsilon_2\tau;\epsilon_3\tau|\tau) \equiv& \frac{1}{2\eta^4}\big(\theta_3^2\theta_3(\epsilon_2\tau|\tau)\theta_3(\epsilon_3\tau|\tau) \\ &- \theta_4^2\theta_4(\epsilon_2\tau|\tau)\theta_4(\epsilon_2\tau|\tau) - \theta_2^2\theta_2(\epsilon_2\tau|\tau)\theta_2(\epsilon_3\tau|\tau)\big) \\ =& \frac{1}{\eta^4}\theta_1^2\Big(\frac{(\epsilon_2+\epsilon_3)\tau}{2}\Big|\tau\Big)\theta_1^2\Big(\frac{(\epsilon_2-\epsilon_3)\tau}{2}\Big|\tau\Big) \ .\end{aligned} \tag{153}$$

The stacks in the ab-sector intersect in all three tori. In this case, there are no standard Kaluza-Klein sums, but Landau levels in the three tori. The cylinder partition function reads

$$A_{ab} = \frac{I_{ab}^1 I_{ab}^2 I_{ab}^3}{2(4\pi^2\alpha')^2} \int_0^\infty \frac{d\tau_2}{\tau_2^3} \frac{(V_8 - S_8)(\epsilon_1\tau;\epsilon_2\tau;\epsilon_3\tau|\tau)}{\eta^2} \prod_{i=1}^3 \frac{2i\eta}{\theta_1(\epsilon_i\tau|\tau)} \ , \tag{154}$$

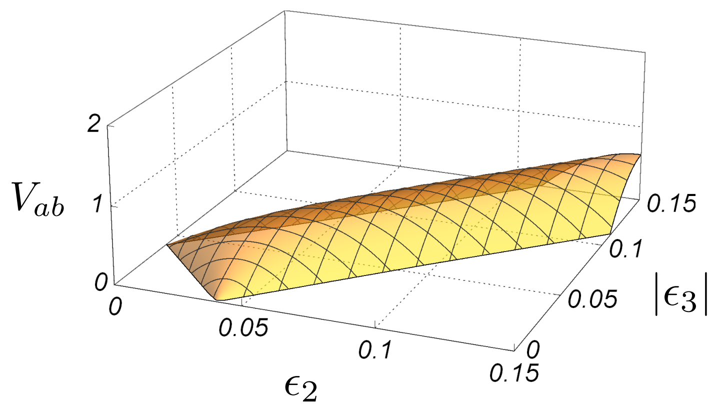
which can again be simplified with the help of the Jacobi identity (228),

$$\begin{aligned}(V_8 - S_8)(\epsilon_1\tau;\epsilon_2\tau;\epsilon_3\tau|\tau) \equiv& \frac{\theta_3\prod_{i=1}^3\theta_3(\epsilon_i\tau|\tau) - \theta_4\prod_{i=1}^3\theta_4(\epsilon_i\tau|\tau) - \theta_2\prod_{i=1}^3\theta_2(\epsilon_i\tau|\tau)}{2\eta^4} \\ =& -\frac{1}{\eta^4}\theta_1\Big(\frac{(\epsilon_1+\epsilon_2+\epsilon_3)\tau}{2}\Big|\tau\Big)\theta_1\Big(\frac{(-\epsilon_1+\epsilon_2+\epsilon_3)\tau}{2}\Big|\tau\Big) \\ &\times\theta_1\Big(\frac{(\epsilon_1-\epsilon_2+\epsilon_3)\tau}{2}\Big|\tau\Big)\theta_1\Big(\frac{(\epsilon_1+\epsilon_2-\epsilon_3)\tau}{2}\Big|\tau\Big) \ .\end{aligned} \tag{155}$$

Notice that the potential vanishes whenever

$$\epsilon_1 \pm \epsilon_2 \pm \epsilon_3 = 0 \ , \tag{156}$$

which encode the standard condition for supersymmetry restoration (see Eq. (12)), $\theta^1 \pm \theta^2 \pm \theta^3 = 0$, as explained in [13].



V_{ab}
2
1
0
0
0.05
0.1
0.15
ϵ_2
0
0.05
0.1
0.15
$|\epsilon_3|$

$\boldsymbol{N}$ $\boldsymbol{N}$ $\boldsymbol{N}$ $\boldsymbol{N}$

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