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## Resummation of large logarithms in the VFN scheme for DIS heavy-quark production

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We consider the impact of the resummation of large logarithms, which appear in the QCD evolution of the heavy-quark distributions, on the phenomenology of deep-inelastic heavy-quark production. The heavy-quark PDFs are derived using the fixed-order matching conditions as a boundary for the QCD evolution and the result obtained is compared to the distributions defined by the matching conditions at all scales. With such an approach, the effect of heavy-quark PDF evolution is found to be sizable at LO and dramatically reduces at NLO. The NNLO evolved distributions are not very different from the NLO ones at large scales, however, show substantial differences at low virtualities, i.e. where the additional large logarithms are numerically not important, while a mismatch between the NLO accuracy of the matching conditions and the NNLO accuracy in the evolution kernels causes a substantial excess in the heavy-quark distributions. This excess propagates into the variable flavor number (VFN) scheme predictions for the deep-inelastic structure functions and has to be compensated by a decrease in the small- $x$  gluon distribution determined from PDF fits based on the VFN scheme, which should be considered as a theoretical uncertainty in VFN PDF fits and reaches  $\sim 30\%$  for the small- $x$  gluon distribution extracted from the data on deep-inelastic charm-quark production.

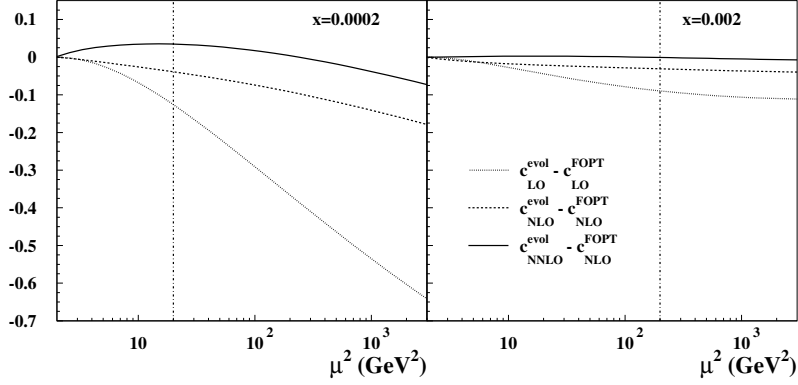
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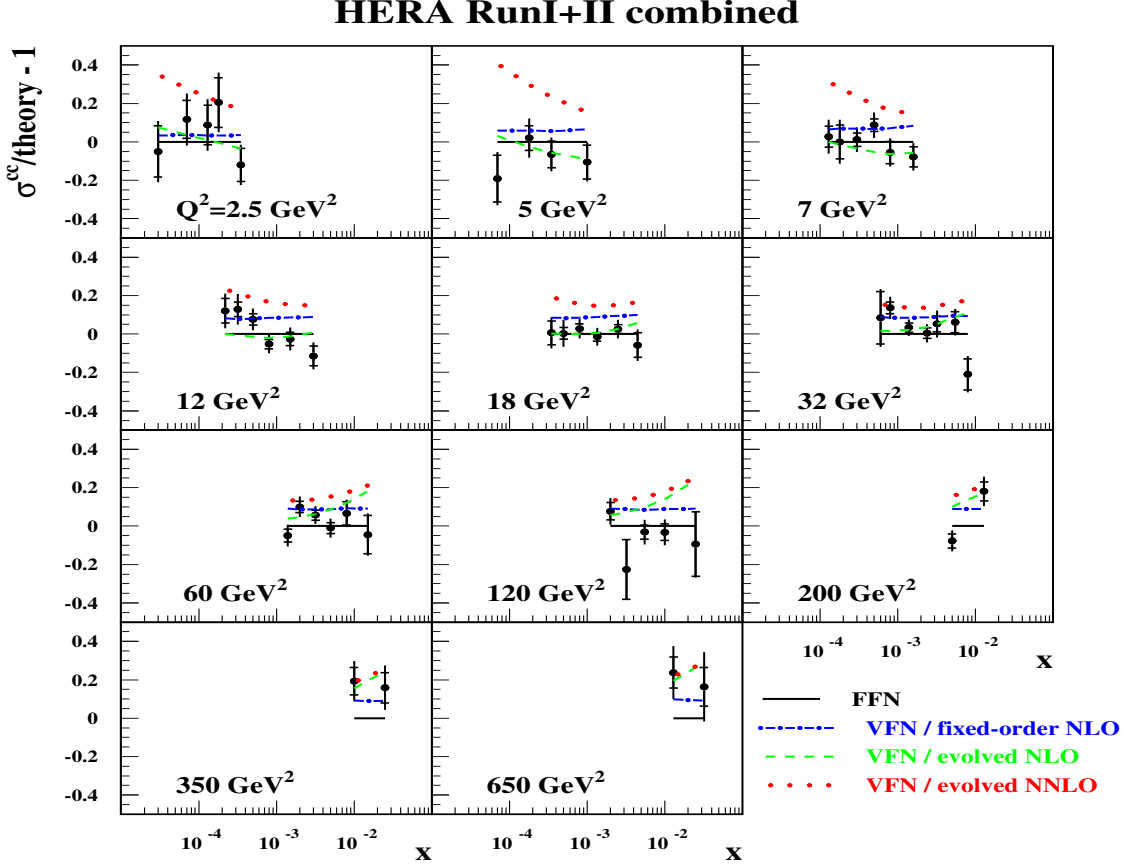
The heavy-quark production in deep-inelastic-scattering (DIS) of leptons off nucleons provides a valuable tool for the study of the QCD dynamics. This process can be described using the QCD-improved parton model with the higher-order perturbative corrections taken into account. In particular, a good description of the available high-precision data on heavy-flavor DIS production, collected in a wide range of momentum transfer  $Q^2$  and Bjorken  $x$ , can be achieved by employing a factorization scheme with three light flavors in the initial state in combination with the QCD perturbative correction up to the next-to-next-to-leading-order (NNLO) [1, 2]. The fixed-order calculations still might be insufficient at extremely large  $Q^2$ , when the terms  $\sim \ln(Q^2/m_h)$ , where  $m_h$  is the heavy-quark mass, give important contributions [3]. To circumvent this problem, a scheme with massless heavy-flavor distributions was suggested [4]. These distributions are evolved similarly to the massless ones and in this way automatically incorporate the large logarithmic contributions through resummation provided by the evolution equations. In the present study we check the impact of such a resummation on the QCD analysis of the available DIS data. The analysis framework is provided by the ABM PDF fit, which now includes the most recent HERA data on the DIS  $c$ - and  $b$ -quark production [5]. These data can be well described within the 3-flavor scheme. However, for



**Figure 1:** Difference between evolved  $c$ -quark distribution and the ones obtained within fixed-order perturbative theory (FOPT) conditions in various orders of QCD (LO, NLO and NNLO) versus the factorization scale  $\mu$  and at representative values of the parton momentum fraction  $x$  (left:  $x = 0.0002$ , right:  $x = 0.002$ ) taking the pole mass of  $c$ -quark,  $m_c = 1.4$  GeV. The vertical dashed-dotted line displays upper margin for the HERA collider kinematics.

the purposes of the present analysis we also consider a variant of the ABM fit based on the massless treatment of the heavy-quark DIS production. There are many variants of the massless heavy-quark schemes available in the literature: ACOT, FONLL, RT and their numerous modifications used in various PDF fits, cf. e.g. [6]. However, they all employ a common conceptual framework: The heavy-quark production cross sections are constructed as a combination of the terms corresponding to the 3-flavor and massless schemes in order to provide a smooth transition from the former to the latter as  $Q^2$  rises. Such a transition is also ensured by the Buza-Matiounine-Smith-van Neerven (BMSN) prescription of the VFN scheme [7, 9]. However, the original BMSN approach is based on the heavy-quark PDFs derived using the fixed-order matching conditions. Therefore resummation effects are missing in this case. In the present study we consider the standard BMSN prescription and its variant with the heavy-quark PDF evolution taken into account. First, we compare the PDF

shapes obtained within these two approaches and then check the impact of such a variation on the results of the global PDF fit including the most recent HERA data on semi-inclusive  $c$ -quark DIS production [5]. This allows us to get a deeper insight into the crucial ingredients of the VFN approach and to better quantify the theoretical uncertainties of this formalism.



**Figure 2:** The pulls obtained for the combined HERA data on DIS  $c$ -quark production [5] in the FFN version of the present analysis (solid lines) versus  $x$  in bins on  $Q^2$ . The predictions obtained using the BMSN version of the VFN scheme with various versions of the heavy-quark PDFs with respect to the FFN fit are displayed for comparison (dotted-dashes: fixed order NLO, dashes: evolving from the NLO matching conditions with the NLO splitting functions, dots: the same for the NLO matching conditions combined with the NNLO splitting functions). The 3-flavor PDFs obtained in the FFN fit are used throughout.

The comparisons are based on the ABMP16 PDF fit [1], which employs the fixed-flavor-number (FFN) scheme with three light quarks in the initial state for the description of heavy-flavor DIS production. An optional VFN framework is provided by the  $c$ - and  $b$ -quark distributions, which are generated from the 3-flavor PDFs through matching conditions of Ref. [7] presently up to the next-to-leading-order (NLO) of perturbative QCD.<sup>1</sup>

In the original BMSN approach these heavy-flavor PDFs are used for all factorization scales. In the modified BMSN scheme we produce the QCD-evolved PDFs using the fixed-order (FO)

<sup>1</sup>For the two-mass effects a generalized VFN scheme at NLO has been given in [8].

heavy-flavor PDFs as boundary condition at the initial scale, which is conventionally selected at the value of heavy-quark mass  $m_h$ , where  $h = c, b$ . Since the NLO matching is used for the boundary conditions, a consistent evolution implies using the NLO splitting functions. However, in order to cope with the recent precision of DIS data the NNLO PDFs are needed. Therefore, commonly the NNLO evolution is used despite of the NLO accuracy of the boundary conditions. The evolved leading-order (LO) heavy-quark PDFs are substantially smaller than the fixed-order ones, cf. Fig. 1. For the NLO case this difference is dramatically reduced due to the logarithmic terms emerging in the evolution. They are partially taken into account in the NLO terms of the FO matching conditions, although the evolution still pushes the PDFs to smaller values. For the combination of the NNLO evolution with the NLO boundary conditions the trend changes and the evolved PDFs become larger than the FO ones. The numerical impact of the resummation is still limited since even at very large scales the NNLO PDFs are not very different from the FO and evolved ones obtained at NLO. Instead, the effect manifests itself more significantly at small scales. Therefore it is related rather to the mismatch between the theoretical accuracy in the boundary conditions and the evolution kernel. This discrepancy should disappear with account of the upcoming NNLO corrections to OMEs [10]. Meanwhile, the difference between the FO NLO and evolved NNLO PDFs provides an estimate of the theoretical uncertainty in the NNLO implementation of VFN schemes.

To check the impact of this uncertainty on the PDFs obtained from analysis of the DIS data we perform several variants of the PDF fit with different treatments of the heavy-quark contributions. For a better discrimination of the schemes we employ in the fit a recent combination of the H1 and ZEUS data on the DIS  $c$ - and  $b$ -quark production with reduced uncertainties [5]. Furthermore, we drop the inclusive HERA data in order to show the sensitivity of the PDFs to the scheme choice in greater detail. For the same reason, the collider data on  $W$ - and  $Z$ -boson production, which provide some constraint on the small- $x$  gluon distribution, are also dropped and the data on DIS off deuteron targets are added instead in order to keep disentangling of the  $u$ - and  $d$ -quark distributions. The FFN fit is performed taking the NLO massive Wilson coefficients with the pole-mass definition <sup>2</sup> and the value of  $m_c^{pole} = 1.4$  GeV, which ensures a good description of the HERA data, cf. Fig. 2. This choice is also needed for a consistent comparison with the version of the VFN scheme based on the NLO heavy-quark evolution, given available theoretical accuracy of the massive OMEs.

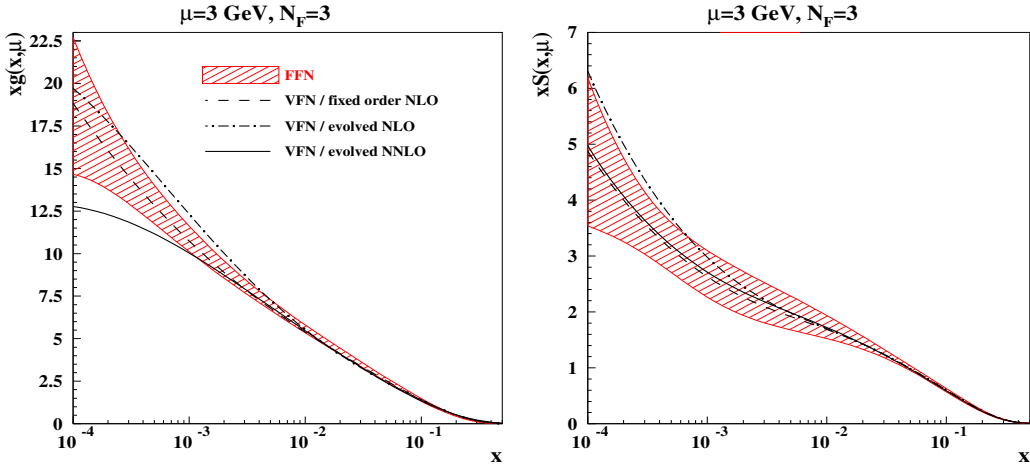
The present analysis employs a BMSN prescription of the VFN scheme, which reads for the  $c$ -quark production structure function  $F_{2,c}$  as follows:

$$F_{2,c}^{\text{BMSN}}(x, Q^2) = F_{2,c}^{\text{FFN}}(x, Q^2) + F_{2,c}^{\text{ZMVFN}}(x, Q^2) - F_{2,c}^{\text{ASYM}}(x, Q^2), \quad (1)$$

where  $F_{2,c}^{\text{FFN}}$  is the expression for the 3-flavor scheme with massive Wilson coefficients,  $F_{2,c}^{\text{ASYM}}$  describes the limit of  $F_{2,c}^{\text{FFN}}$  for asymptotic values of  $Q^2 \gg m_c^2$  and  $F_{2,c}^{\text{ZMVFN}}$  is computed using the 4-flavor PDFs in combination with the massless Wilson coefficients. At large values of  $Q^2$   $F_{2,c}^{\text{FFN}} \approx F_{2,c}^{\text{ASYM}}$ . Therefore  $F_{2,c}^{\text{BMSN}}(x, Q^2)$  reproduces 4-flavor VFN scheme expression. At low values of  $Q^2$   $F_{2,c}^{\text{ZMVFN}} \approx F_{2,c}^{\text{ASYM}}$  and  $F_{2,c}^{\text{BMSN}} \approx F_{2,c}^{\text{FFN}}$ . Furthermore, a smooth transition between FFN and VFN schemes is provided when the FO  $c$ -quark distributions are employed in  $F_{2,c}^{\text{ZMVFN}}$  [9]. For

<sup>2</sup>The transition to the heavy-quark mass definition in the  $\overline{\text{MS}}$  scheme for the corresponding heavy-flavor Wilson coefficients is known, cf. [10].

this reason the BMSN predictions obtained with the FO heavy-flavor PDFs are similar to the FFN ones for the kinematics of existing data, cf. Fig. 2. The BMSN predictions made with the NLO-evolved PDFs are close to the FO results, in line with the comparison of Fig. 1. Meanwhile, the use of NNLO-evolved PDFs leads to much larger values, particularly at low  $Q^2$ . Due to this excess the results obtained with the VFN scheme based on the heavy-quark NNLO evolution are substantially different from the ones of other fit variants. The small- $x$  gluon distribution is most sensitive to details of the VFN scheme settings, cf. Fig. 3, and the spread observed gives an estimate of the theoretical scheme uncertainties due to missing higher orders of perturbative QCD. In particular, the difference which appears due to switching between the NLO- and NNLO-evolution ansatz, related to the yet incomplete NNLO corrections in the massive OMEs, is more sizable for the small- $x$  gluon distribution and amounts to  $\sim 30\%$  at  $x \sim 10^{-4}$ .



**Figure 3:** The central values of 3-flavor gluon  $xg(x, \mu)$  (left) and the total light-flavor sea  $xS(x, \mu)$  (right) distributions at the factorization scale  $\mu = 3$  GeV versus  $x$  obtained in the various versions of the VFN scheme (solid: evolved NNLO, dashes: fixed-order NLO, dashed-dots: evolved NLO). The  $1\sigma$  error band for the FFN results is given for comparison (hatched area).

It is worth to mention that this uncertainty is relevant for all versions of the VFN schemes employed in PDF fits. Meanwhile, the mismatch between perturbative accuracy of the massive OMEs and the massless evolution kernels also leads to the nonphysical kink in  $Q^2$ -dependence of the DIS structure functions at  $Q^2 \sim m_h^2$ . In many VFN scheme implementations this kink is smoothed out by introducing various damping coefficients, which also formally reduce corresponding theoretical uncertainty. Since the shape of such coefficients is not based on solid theoretical arguments, they in turn, introduce additional uncertainties [11], which effectively reflect the original one. For the FFN description of the DIS data this uncertainty is irrelevant, but it appears in the PDF fits including collider data on the production of massive final states (e.g.,  $W^-$ ,  $Z^-$ , Higgs-bosons), which are routinely described using the 5-flavor PDFs. Its impact is reduced as compared to the VFN fits due to the fact that it is localized at small factorization scales, cf. Fig. 1. However, the complete NNLO OMEs are still required in order to achieve ultimate PDF precision [10].

In summary, we considered the impact of the resummation of large logarithms, which appear in the QCD evolution of the heavy-quark distributions, on the phenomenology of the DIS heavy-

quark production. The heavy-quark PDFs are derived using the fixed-order matching conditions as a boundary for the QCD evolution and the result obtained has been compared to the distribution defined by the matching conditions at all scales. With such an approach the effect of heavy-quark PDF evolution is found to be sizable at LO and to be dramatically reduced at NLO, since the large logarithmic terms, which are resummed by the evolution, are partially taken into account in the NLO corrections to the fixed-order distributions. The NNLO evolved distributions are not very different from the NLO ones at large scales, however, demonstrate a substantial difference at low values of  $Q^2$ . This implies, that the additional large logarithms are numerically unimportant, while a mismatch between the NLO accuracy of the matching conditions and NNLO accuracy in the evolution kernels results into a substantial excess of the heavy-quark distributions at small scales. This excess propagates into the variable flavor number (VFN) scheme predictions for the deep-inelastic structure functions and has to be compensated by a decrease in the small- $x$  gluon distribution determined from PDF fits based on the VFN scheme, which should be considered as a theoretical uncertainty in VFN PDF fits and reaches  $\sim 30\%$  for the small- $x$  gluon distribution extracted from the data on deep-inelastic charm-quark production.

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