

DEUTSCHES ELEKTRONEN-SYNCHROTRON
Ein Forschungszentrum der Helmholtz-Gemeinschaft



DESY 20-018
arXiv:2002.06122
February 2020

Status of the WHIZARD Generator for Linear Colliders

J. Reuter, S. Brass, P. Bredt, V. Rothe, P. Stienemeier

Deutsches Elektronen-Synchrotron DESY, Hamburg

W. Kilian

Department Physik, Universität Siegen

T. Ohl

Fakultät für Physik und Astronomie, Universität Würzburg

ISSN 0418-9833

NOTKESTRASSE 85 - 22607 HAMBURG

DESY behält sich alle Rechte für den Fall der Schutzrechtserteilung und für die wirtschaftliche Verwertung der in diesem Bericht enthaltenen Informationen vor.

DESY reserves all rights for commercial use of information included in this report, especially in case of filing application for or grant of patents.

To be sure that your reports and preprints are promptly included in the
HEP literature database
send them to (if possible by air mail):

DESY Zentralbibliothek Notkestraße 85 22607 Hamburg Germany	DESY Bibliothek Platanenallee 6 15738 Zeuthen Germany
---	---

Status of the WHIZARD generator for linear colliders

JÜRGEN REUTER^{1 2a}, SIMON BRASS^{3a}, PIA BREDT^{4a}, WOLFGANG KILIAN^{5b}, THORSTEN OHL^{6c},
VINCENT ROTHE^{7a}, PASCAL STIENEMEIER^{8a}

^a*DESY Theory Group,
Notkestr. 85, 22607 Hamburg, Germany*

^b*University of Siegen, Physics Department, Walter-Flex-Straße 3, 57068 Siegen, Germany*

^c*University of Würzburg, Faculty of Physics and Astronomy, Campus Hubland Nord, Emil-Hilb-Weg
22, 97074 Würzburg, Germany*

ABSTRACT

This summarizes the talk given at the LCWS 2019 conference in Sendai, Japan, on the progress of the WHIZARD event generator in terms of new physics features and technical improvements relevant for the physics programme of future lepton and especially linear colliders. It takes as a reference the version 2.8.2 released in October 2019, and also takes into account the development until version 2.8.3 to be released in February 2020.

¹Talk presented at the International Workshop on Future Linear Colliders (LCWS2019), Sendai, Japan, 28 October - 1 November, 2019. C19-10-28.

²juergen.reuter@desy.de

³simon.brass@desy.de

⁴pia.bredt@desy.de

⁵kilian@physik.uni-siegen.de

⁶ohl@physik.uni-wuerzburg.de

⁷vincent.rothe@desy.de

⁸pascal.stienemeier@desy.de

1 Introduction

WHIZARD is a multi-purpose event generator for collider physics [1]. It is a very general framework for all types of colliders, but with a special emphasis on the physics program at lepton colliders, and has been used for many studies and design reports for e.g. ILC, CLIC and FCC-ee [2,3,4,5,6]. Hard scattering process matrix elements are generated with WHIZARD’s intrinsic (tree-level) matrix element generator `0’Mega` [7], using the color-flow formalism for QCD [8]. It supports all particles up to spin 2, and also fermion-number violating vertices [9,10,11,12]. `0’Mega` can write matrix-element code as compiled process code (libraries) or as byte-code instructions in the form of a virtual machine [13]. The latter produces very small and efficient matrix element instructions. The NLO automation will be discussed in Sec. 2.3. WHIZARD comes with two different final- and initial-state parton shower implementations, a k_T -ordered shower as well as an analytic parton shower [14]. For LC simulations, WHIZARD ships with the final `Pythia6` version [15] for shower and hadronization; it also has a full-fledged interface to `Pythia8` [16]. This is very handy as it directly transfers data between the two event records of the generators and allows WHIZARD to use all of `Pythia8`’s machinery for matching and merging. WHIZARD also automatically assigns underlying resonances to full off-shell processes and gives the correct information of resonant shower systems to the parton shower.

One of the special features of WHIZARD is its framework for the support of lepton collider physics, including electron PDFs with resummation of soft photons to all orders and hard-collinear photons up to third order in α , the generation of ISR photon p_T spectra, sampling of lepton collider beam spectra [17], proper simulation of polarized beams, crossing angles and photon-induced background processes.

WHIZARD has a large number of hard-coded Beyond the Standard Model (BSM) models. The newest development for new physics, especially regarding completely general Lorentz tensor structures, will be described in Sec. 2.2.

2 New physics and technical features

2.1 Performance and integration, technical features

WHIZARD has a very modular infrastructure that allows to easily exchange different components: there are several different phase-space algorithms implemented, as well as several different Monte Carlo integration options. Besides the traditional `VAMP` integrator [18], there is now a conceptually identical implementation generalized to an MPI-based parallelization. In contrast to event generation which can always be trivially parallelized, adaptive phase space integration cannot so easily parallelized, and is a major bottleneck for high-multiplicity tree- and especially loop-level processes. This `VAMP2` integrator [19] will now be further improved with a dynamic load balancer that allows for non-blocking communication between the different workers. The new setup will be released in version 3.0 α , cf. below. Even without the load balancer speed-ups between 10 and 100 are observed, depending on the complexity of processes.

Further technical improvements are the finalization of the proper event headers for the LCIO event interface for the LC software framework, as well as the completion of the interface to `HepMC3`. Rescanning of event files in order to recalculate hard matrix elements without recalculating the phase space, now also work with beam spectra and structure functions. Alternative weights (squared matrix elements) can now be written out not only in LHE and `HepMC` formats, but also to LCIO.

Process	σ^{LO} [pb]	σ^{NLO} [pb]	K	Process	σ^{LO} [fb]	σ^{NLO} [fb]	K
$pp \rightarrow jj$	$1.157(2) \cdot 10^6$	$1.604(7) \cdot 10^6$	1.39	$e^+e^- \rightarrow jj$	622.73(4)	639.41(9)	1.03
$pp \rightarrow Z$	$4.2536(3) \cdot 10^4$	$5.4067(2) \cdot 10^4$	1.27	$e^+e^- \rightarrow jjj$	342.4(5)	318.6(7)	0.93
$pp \rightarrow Zj$	$7.207(2) \cdot 10^3$	$9.720(17) \cdot 10^3$	1.35	$e^+e^- \rightarrow jjjj$	105.1(4)	103.0(6)	0.98
$pp \rightarrow Zjj$	$2.352(8) \cdot 10^3$	$2.735(9) \cdot 10^3$	1.16	$e^+e^- \rightarrow jjjjj$	22.80(2)	24.35(15)	1.07
$pp \rightarrow W^\pm$	$1.3750(5) \cdot 10^5$	$1.7696(9) \cdot 10^5$	1.29	$e^+e^- \rightarrow b\bar{b}$	92.32(1)	94.78(7)	1.03
$pp \rightarrow W^\pm j$	$2.043(1) \cdot 10^4$	$2.845(6) \cdot 10^4$	1.39	$e^+e^- \rightarrow b\bar{b}b\bar{b}$	$1.64(2) \cdot 10^{-1}$	$3.67(4) \cdot 10^{-1}$	2.24
$pp \rightarrow W^\pm jj$	$6.798(7) \cdot 10^3$	$7.93(3) \cdot 10^3$	1.17	$e^+e^- \rightarrow t\bar{t}$	166.4(1)	174.53(6)	1.05
$pp \rightarrow ZZ$	$1.094(2) \cdot 10^1$	$1.4192(32) \cdot 10^1$	1.3	$e^+e^- \rightarrow t\bar{t}j$	48.3(2)	53.25(6)	1.1
$pp \rightarrow ZZj$	$3.659(2) \cdot 10^0$	$4.820(11) \cdot 10^0$	1.32	$e^+e^- \rightarrow t\bar{t}jj$	8.612(8)	10.46(6)	1.21
$pp \rightarrow ZW^\pm$	$2.775(2) \cdot 10^1$	$4.488(4) \cdot 10^1$	1.62	$e^+e^- \rightarrow t\bar{t}jjj$	1.040(1)	1.414(10)	1.36
$pp \rightarrow ZW^\pm j$	$1.604(6) \cdot 10^1$	$2.103(4) \cdot 10^1$	1.31	$e^+e^- \rightarrow t\bar{t}t\bar{t}$	$6.463(2) \cdot 10^{-4}$	$11.91(2) \cdot 10^{-4}$	1.84
$pp \rightarrow W^+W^-(4f)$	$0.7349(7) \cdot 10^2$	$1.027(1) \cdot 10^2$	1.4	$e^+e^- \rightarrow t\bar{t}t\bar{t}j$	$2.722(1) \cdot 10^{-5}$	$5.250(14) \cdot 10^{-5}$	1.93
$pp \rightarrow W^+W^-j(4f)$	$2.868(1) \cdot 10^1$	$3.733(8) \cdot 10^1$	1.3	$e^+e^- \rightarrow t\bar{t}b\bar{b}$	0.186(1)	0.293(2)	1.58
$pp \rightarrow W^+W^+jj$	$1.483(4) \cdot 10^{-1}$	$2.238(6) \cdot 10^{-1}$	1.51	$e^+e^- \rightarrow t\bar{t}H$	2.022(3)	1.912(3)	0.95
$pp \rightarrow W^-W^-jj$	$6.755(4) \cdot 10^{-1}$	$9.97(3) \cdot 10^{-1}$	1.48	$e^+e^- \rightarrow t\bar{t}Hj$	0.2540(9)	0.2664(5)	1.05
$pp \rightarrow W^+W^-W^\pm(4f)$	$1.309(1) \cdot 10^{-1}$	$2.117(2) \cdot 10^{-1}$	1.62	$e^+e^- \rightarrow t\bar{t}Hjj$	$2.666(4) \cdot 10^{-2}$	$3.144(9) \cdot 10^{-2}$	1.18
$pp \rightarrow ZW^+W^-(4f)$	$0.966(2) \cdot 10^{-1}$	$1.682(2) \cdot 10^{-1}$	1.74	$e^+e^- \rightarrow t\bar{t}\gamma$	12.71(4)	13.78(4)	1.08
$pp \rightarrow W^+W^-W^\pm Z(4f)$	$0.642(2) \cdot 10^{-3}$	$1.240(2) \cdot 10^{-3}$	1.93	$e^+e^- \rightarrow t\bar{t}Z$	4.64(1)	4.94(1)	1.06
$pp \rightarrow W^\pm ZZ$	$0.588(2) \cdot 10^{-5}$	$1.229(2) \cdot 10^{-5}$	2.09	$e^+e^- \rightarrow t\bar{t}Zj$	0.610(4)	0.6927(14)	1.14
$pp \rightarrow t\bar{t}$	$4.588(2) \cdot 10^2$	$6.740(9) \cdot 10^2$	1.47	$e^+e^- \rightarrow t\bar{t}Zjj$	$6.233(8) \cdot 10^{-2}$	$8.201(14) \cdot 10^{-2}$	1.32
$pp \rightarrow t\bar{t}j$	$3.131(3) \cdot 10^2$	$4.194(9) \cdot 10^2$	1.34	$e^+e^- \rightarrow t\bar{t}W^\pm jj$	$2.41(1) \cdot 10^{-4}$	$3.695(9) \cdot 10^{-4}$	1.53
$pp \rightarrow t\bar{t}t\bar{t}$	$4.511(2) \cdot 10^{-3}$	$9.070(9) \cdot 10^{-3}$	2.01	$e^+e^- \rightarrow t\bar{t}\gamma\gamma$	0.382(3)	0.420(3)	1.1
$pp \rightarrow t\bar{t}Z$	$5.281(8) \cdot 10^{-1}$	$7.639(9) \cdot 10^{-1}$	1.45	$e^+e^- \rightarrow t\bar{t}\gamma Z$	0.220(1)	0.240(2)	1.09
				$e^+e^- \rightarrow t\bar{t}\gamma H$	$9.748(6) \cdot 10^{-2}$	$9.58(7) \cdot 10^{-2}$	0.98
				$e^+e^- \rightarrow t\bar{t}ZZ$	$3.756(4) \cdot 10^{-2}$	$4.005(2) \cdot 10^{-2}$	1.07
				$e^+e^- \rightarrow t\bar{t}W^+W^-$	0.1370(4)	0.1538(4)	1.12
				$e^+e^- \rightarrow t\bar{t}HH$	$1.367(1) \cdot 10^{-2}$	$1.218(1) \cdot 10^{-2}$	0.89
				$e^+e^- \rightarrow t\bar{t}HZ$	$3.596(1) \cdot 10^{-2}$	$3.581(2) \cdot 10^{-2}$	1

Table 1: Selection of validated processes at LO and NLO QCD with WHIZARD. e^+e^- processes (left) are for 1 TeV fixed beams, pp processes are for 13 TeV. The scale is the scalar transverse energy, H_T . Jets are clustered with the anti- k_T algorithm and jet radius $\Delta R = 0.5$, with cuts of $p_T > 30\text{GeV}$ for the Born jets.

2.2 Beyond the standard model physics

Besides of the full SM samples for TESLA, ILC, CLIC and CEPC, WHIZARD has been extensively used for BSM simulations where it contains e.g. complete implementations of Little Higgs models [20,21,22,23,24,25]. Another interesting feature is WHIZARD's ability to calculate unitarity constraints for vector boson scattering (VBS) and multi-boson processes and to deliver unitarized amplitudes for SMEFT dim-6/dim-8 operators and simplified models [26,27,28,29,30,31], while precision SM predictions for VBS can be found in [32]. Ongoing work deals with the automatic calculation of unitarity limits for multiple (transversal) vector boson production both for hadron and (high-energy) lepton colliders.

Nowadays, new physics models are almost exclusively included via automated interfaces, e.g. to FeynRules [33,34]. These explicit interfaces have now been superseded by WHIZARD's implementation of its UFO [35] interface. WHIZARD now (with the upcoming versions 2.8.3 and 3.0 α) supports this completely including spins 1/2, 3/2, 0, 1, 2, 3, 4, 5, automatic construction of 5-, 6-ary and even higher vertices, fermion-number violating vertices, four-fermion vertices (and higher), SLHA-type input files for BSM models and customized propagators defined in the UFO files. This makes the old interfaces to FeynRules and SARAH [36] deprecated, however, they will be kept for backwards compatibility.

2.3 Next-to-leading order QCD automation

WHIZARD started first with hard-coded next-to-leading order (NLO) projects regarding QED and electroweak corrections for SUSY production [37,38] and NLO QCD correction for $pp \rightarrow b\bar{b}b\bar{b}$ [39,40]. Now,

WHIZARD is based on an automated implementation of the FKS subtraction algorithm [41]. In this automated implementation only the virtual amplitudes are external from one-loop providers (OLP, there are interfaces to `Openloops` [42,43], `Recola` [44] and `GoSam` [45]), while subtraction terms are automatically generated in WHIZARD. The NLO QCD has been fully validated as can be seen from Table 1. First applications of this automated interface have been devoted to linear collider top physics in the continuum [46] and in the threshold region [47]. These examples also show NLO calculations with factorized processes as well as NLO QCD decays. Recently, the selection of heavy-flavor jets in the jet clustering (bottom and charm) as well as a veto for them has been added, and also the possibility for photon isolation to separate perturbative QCD from nonperturbative effects in photon-jet fragmentation. The final validation is being finished now, there are still a few ongoing issues especially regarding easier usage, but an alpha version of WHIZARD 3.0 officially releasing NLO QCD automation will be done in March 2020. WHIZARD allows for a completely automatized POWHEG-type matching (and damping) [48] to the parton shower (for final state showering). While the corresponding matching for initial-state showering is being implemented, the work on NLO electroweak corrections has been started and first total cross sections for simple processes are already available. Next steps here are the complete validation, as well as the proper matching to the higher-order corrections for incoming electron PDFs. Also, the work for other NLO matching schemes has started.

2.4 Summary and Outlook

This is a status report of the close-to-final release version 2.8.2/2.8.3 of the WHIZARD version 2 series, showing intense work on the complete NLO QCD automation, the completion of automatic generation of arbitrary Lorentz tensor representations and the UFO interface, and many technical and convenience developments driven by the upcoming 250 GeV full SM Monte Carlo mass production for ILC with 2 ab^{-1} integrated luminosity.

Acknowledgments

This work was funded by the Deutsche Forschungsgemeinschaft under Germanys ExcellenceStrategy EXC 2121 Quantum Universe 390833306. JRR wants to thank the organizers for a fruitful and interesting conference in Sendai, which was followed by an intense and very productive LC generator group meeting at University of Tokyo.

References

- [1] W. Kilian, T. Ohl and J. Reuter, Eur. Phys. J. C **71**, 1742 (2011) doi:10.1140/epjc/s10052-011-1742-y [arXiv:0708.4233 [hep-ph]].
- [2] K. Fujii *et al.*, arXiv:1506.05992 [hep-ex].
- [3] J. de Blas *et al.*, CERN Yellow Rep. Monogr. Vol. 3 (2018) doi:10.23731/CYRM-2018-003 [arXiv:1812.02093 [hep-ph]].
- [4] H. Baer *et al.*, arXiv:1306.6352 [hep-ph].
- [5] T. Behnke *et al.*, arXiv:1306.6329 [physics.ins-det].

- [6] A. Abada *et al.* [FCC Collaboration], *Eur. Phys. J. ST* **228**, no. 2, 261 (2019). doi:10.1140/epjst/e2019-900045-4
- [7] M. Moretti, T. Ohl and J. Reuter, hep-ph/0102195.
- [8] W. Kilian, T. Ohl, J. Reuter and C. Speckner, *JHEP* **1210**, 022 (2012) doi:10.1007/JHEP10(2012)022 [arXiv:1206.3700 [hep-ph]].
- [9] T. Ohl and J. Reuter, *Eur. Phys. J. C* **30**, 525 (2003) doi:10.1140/epjc/s2003-01301-7 [hep-th/0212224].
- [10] J. A. Aguilar-Saavedra *et al.*, *Eur. Phys. J. C* **46**, 43 (2006) doi:10.1140/epjc/s2005-02460-1 [hep-ph/0511344].
- [11] K. Hagiwara, W. Kilian, F. Krauss, T. Ohl, T. Plehn, D. Rainwater, J. Reuter and S. Schumann, *Phys. Rev. D* **73**, 055005 (2006) doi:10.1103/PhysRevD.73.055005 [hep-ph/0512260].
- [12] J. Kalinowski, W. Kilian, J. Reuter, T. Robens and K. Rolbiecki, *JHEP* **0810**, 090 (2008) doi:10.1088/1126-6708/2008/10/090 [arXiv:0809.3997 [hep-ph]].
- [13] B. Chokoufe Nejad, T. Ohl and J. Reuter, *Comput. Phys. Commun.* **196**, 58 (2015) doi:10.1016/j.cpc.2015.05.015 [arXiv:1411.3834 [physics.comp-ph]].
- [14] W. Kilian, J. Reuter, S. Schmidt and D. Wiesler, *JHEP* **04**, 013 (2012) doi:10.1007/JHEP04(2012)013 [arXiv:1112.1039 [hep-ph]].
- [15] T. Sjostrand, S. Mrenna and P. Z. Skands, *JHEP* **0605**, 026 (2006) doi:10.1088/1126-6708/2006/05/026 [hep-ph/0603175].
- [16] T. Sjostrand *et al.*, *Comput. Phys. Commun.* **191**, 159 (2015) doi:10.1016/j.cpc.2015.01.024 [arXiv:1410.3012 [hep-ph]].
- [17] T. Ohl, *Comput. Phys. Commun.* **101**, 269 (1997) doi:10.1016/S0010-4655(96)00167-1 [hep-ph/9607454].
- [18] T. Ohl, *Comput. Phys. Commun.* **120**, 13 (1999) doi:10.1016/S0010-4655(99)00209-X [hep-ph/9806432].
- [19] S. Brass, W. Kilian and J. Reuter, *Eur. Phys. J. C* **79**, no. 4, 344 (2019) doi:10.1140/epjc/s10052-019-6840-2 [arXiv:1811.09711 [hep-ph]].
- [20] W. Kilian and J. Reuter, *Phys. Rev. D* **70**, 015004 (2004) doi:10.1103/PhysRevD.70.015004 [hep-ph/0311095].
- [21] W. Kilian, D. Rainwater and J. Reuter, *Phys. Rev. D* **71**, 015008 (2005) doi:10.1103/PhysRevD.71.015008 [hep-ph/0411213].

- [22] W. Kilian, D. Rainwater and J. Reuter, Phys. Rev. D **74**, 095003 (2006) Erratum: [Phys. Rev. D **74**, 099905 (2006)] doi:10.1103/PhysRevD.74.095003, 10.1103/PhysRevD.74.099905 [hep-ph/0609119].
- [23] J. Reuter and M. Tonini, JHEP **1302**, 077 (2013) doi:10.1007/JHEP02(2013)077 [arXiv:1212.5930 [hep-ph]].
- [24] J. Reuter, M. Tonini and M. de Vries, JHEP **1402**, 053 (2014) doi:10.1007/JHEP02(2014)053 [arXiv:1310.2918 [hep-ph]].
- [25] D. Dercks, G. Moortgat-Pick, J. Reuter and S. Y. Shim, JHEP **1805**, 049 (2018) doi:10.1007/JHEP05(2018)049 [arXiv:1801.06499 [hep-ph]].
- [26] M. Beyer, W. Kilian, P. Krstonsic, K. Monig, J. Reuter, E. Schmidt and H. Schroder, Eur. Phys. J. C **48**, 353 (2006) doi:10.1140/epjc/s10052-006-0038-0 [hep-ph/0604048].
- [27] A. Alboteanu, W. Kilian and J. Reuter, JHEP **0811**, 010 (2008) doi:10.1088/1126-6708/2008/11/010 [arXiv:0806.4145 [hep-ph]].
- [28] W. Kilian, T. Ohl, J. Reuter and M. Sekulla, Phys. Rev. D **91**, 096007 (2015) doi:10.1103/PhysRevD.91.096007 [arXiv:1408.6207 [hep-ph]].
- [29]
- [29] W. Kilian, T. Ohl, J. Reuter and M. Sekulla, Phys. Rev. D **93**, no. 3, 036004 (2016) doi:10.1103/PhysRevD.93.036004 [arXiv:1511.00022 [hep-ph]].
- [30] C. Fleper, W. Kilian, J. Reuter and M. Sekulla, Eur. Phys. J. C **77**, no. 2, 120 (2017) doi:10.1140/epjc/s10052-017-4656-5 [arXiv:1607.03030 [hep-ph]].
- [31] S. Brass, C. Fleper, W. Kilian, J. Reuter and M. Sekulla, Eur. Phys. J. C **78**, no. 11, 931 (2018) doi:10.1140/epjc/s10052-018-6398-4 [arXiv:1807.02512 [hep-ph]].
- [32] A. Ballestrero *et al.*, Eur. Phys. J. C **78**, no. 8, 671 (2018) doi:10.1140/epjc/s10052-018-6136-y [arXiv:1803.07943 [hep-ph]].
- [33] N. D. Christensen, C. Duhr, B. Fuks, J. Reuter and C. Speckner, Eur. Phys. J. C **72**, 1990 (2012) doi:10.1140/epjc/s10052-012-1990-5 [arXiv:1010.3251 [hep-ph]].
- [34] N. D. Christensen and C. Duhr, Comput. Phys. Commun. **180**, 1614 (2009) doi:10.1016/j.cpc.2009.02.018 [arXiv:0806.4194 [hep-ph]].
- [35] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer and T. Reiter, Comput. Phys. Commun. **183**, 1201 (2012) doi:10.1016/j.cpc.2012.01.022 [arXiv:1108.2040 [hep-ph]].
- [36] F. Staub, Comput. Phys. Commun. **185**, 1773 (2014) doi:10.1016/j.cpc.2014.02.018 [arXiv:1309.7223 [hep-ph]].

- [37] W. Kilian, J. Reuter and T. Robens, *Eur. Phys. J. C* **48**, 389 (2006) doi:10.1140/epjc/s10052-006-0048-y [hep-ph/0607127].
- [38] T. Robens, J. Kalinowski, K. Rolbiecki, W. Kilian and J. Reuter, *Acta Phys. Polon. B* **39**, 1705 (2008) [arXiv:0803.4161 [hep-ph]].
- [39] T. Binoth, N. Greiner, A. Guffanti, J. Reuter, J.-P. Guillet and T. Reiter, *Phys. Lett. B* **685**, 293 (2010) doi:10.1016/j.physletb.2010.02.010 [arXiv:0910.4379 [hep-ph]].
- [40] N. Greiner, A. Guffanti, T. Reiter and J. Reuter, *Phys. Rev. Lett.* **107**, 102002 (2011) doi:10.1103/PhysRevLett.107.102002 [arXiv:1105.3624 [hep-ph]].
- [41] S. Frixione, Z. Kunszt and A. Signer, *Nucl. Phys. B* **467**, 399 (1996) doi:10.1016/0550-3213(96)00110-1 [hep-ph/9512328].
- [42] F. Cascioli, P. Maierhofer and S. Pozzorini, *Phys. Rev. Lett.* **108**, 111601 (2012) doi:10.1103/PhysRevLett.108.111601 [arXiv:1111.5206 [hep-ph]].
- [43] F. Buccioni, J. N. Lang, J. M. Lindert, P. Maierhofer, S. Pozzorini, H. Zhang and M. F. Zoller, *Eur. Phys. J. C* **79**, no. 10, 866 (2019) doi:10.1140/epjc/s10052-019-7306-2 [arXiv:1907.13071 [hep-ph]].
- [44] S. Actis, A. Denner, L. Hofer, J. N. Lang, A. Scharf and S. Uccirati, *Comput. Phys. Commun.* **214**, 140 (2017) doi:10.1016/j.cpc.2017.01.004 [arXiv:1605.01090 [hep-ph]].
- [45] G. Cullen *et al.*, *Eur. Phys. J. C* **74**, no. 8, 3001 (2014) doi:10.1140/epjc/s10052-014-3001-5 [arXiv:1404.7096 [hep-ph]].
- [46] B. Chokouf Nejad, W. Kilian, J. M. Lindert, S. Pozzorini, J. Reuter and C. Weiss, *JHEP* **1612**, 075 (2016) doi:10.1007/JHEP12(2016)075 [arXiv:1609.03390 [hep-ph]].
- [47] F. Bach, B. C. Nejad, A. Hoang, W. Kilian, J. Reuter, M. Stahlhofen, T. Teubner and C. Weiss, *JHEP* **1803**, 184 (2018) doi:10.1007/JHEP03(2018)184 [arXiv:1712.02220 [hep-ph]].
- [48] J. Reuter, B. Chokoufe, A. Hoang, W. Kilian, M. Stahlhofen, T. Teubner and C. Weiss, *J. Phys. Conf. Ser.* **762**, no. 1, 012059 (2016) doi:10.1088/1742-6596/762/1/012059 [arXiv:1602.06270 [hep-ph]].