Accidental SO(10) Axion from Gauged Flavour

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An accidental U(1) Peccei-Quinn (PQ) symmetry automatically arises in a class of SO(10) unified theories upon gauging the $SU(3)_f$ flavour group. The PQ symmetry is protected by the $\mathbb{Z}_4 \times \mathbb{Z}_3$ center of $SO(10) \times SU(3)_f$ up to effective operators of canonical dimension six. However, high-scale contributions to the axion potential posing a PQ quality problem arise only at $d = 9$. In the pre-inflationary PQ breaking scenario the axion mass window is predicted to be $m_a \in [7 \times 10^{-8}, 10^{-3}]$ eV, where the lower end is bounded by the seesaw scale and the upper end by iso-curvature fluctuations. A high-quality axion, that is immune to the PQ quality problem, is obtained for $m_a \gtrsim 0.02$ eV. We finally offer a general perspective on the PQ quality problem in grand unified theories.

Contents

1   Introduction

The Peccei-Quinn (PQ) [1, 2] mechanism relies on a global $U(1)_{PQ}$ symmetry that is broken in the infrared (IR) by the QCD anomaly. Global symmetries have no fundamental meaning, but they are rather understood to arise as accidental symmetries e.g. in quantum field theories. Well-known examples are baryon and lepton number in the Standard Model (SM). Although an effective $U(1)_{PQ}$ is sometimes imposed "by hand", a proper *PQ theory* should achieve that in an *automatic* way. This was the term used in the early days [3], when first attempts were put forth to get an accidental $U(1)_{PQ}$ in grand unified theories (GUTs). The quick decline of the electroweak Weinberg-Wilczek [4, 5] axion and the rise of the so-called *invisible* axion [6–9], brought in another related puzzle, known as the *PQ quality problem* [3, 10–14]: why is $U(1)_{PQ}$ an extremely good symmetry of ultraviolet (UV) physics? In fact, there is no reason to expect global symmetries to be exact, but even a tiny explicit breaking of $U(1)_{PQ}$ in the UV would spoil the PQ solution to the strong CP problem. Although the PQ quality problem is eventually a matter of UV physics which cannot be definitively assessed without a calculable theory of quantum gravity, the requirement of having the $U(1)_{PQ}$ to arise accidentally is a bare minimum that a sensible PQ theory in should structurally achieve.

It is the purpose of this work to revisit this question in the context of SO(10) GUTs. Since the SO(10) symmetry, by itself, is not sufficient to provide an automatic $U(1)_{PQ}$, some ingredient must be clearly added. An interesting possibility, put forth by Chang and Senjanovič [15], is to employ the $SU(3)_f$ flavour symmetry of SO(10) (for related approaches without GUTs, see [16, 17]). Remarkably, if the SO(10) representations are properly selected, the simultaneous presence of the horizontal ($SU(3)_f$) and vertical (SO(10)) symmetry leads to an automatic PQ symmetry. While Ref. [15] focussed on the case of *global* $SU(3)_f$, motivated by the possibility of testing the $SU(3)_f$ breaking dynamics at low-energy via the associated Goldstone bosons (familons), from a modern perspective it would be more satisfactory to get a $U(1)_{PQ}$ symmetry to arise from *local* $SU(3)_f$. The main obstacle for such a program is the cancellation of the $SU(3)_f^3$ gauge anomaly, which requires to extend the fermion content of SO(10). In this work, we show how this can be consistently done, and we extend the analysis of [15] in several respects: *i*) we point out that the same approach can be used beyond the renormalizable level, thus providing also a way to tackle the PQ quality problem; *ii*) we identify the physical axion field and compute its low-energy couplings to SM matter fields. In this step, we realized that the charge assignment of [15] must be slightly modified, in order to avoid an alignment between two SO(10) Higgs representations which would otherwise lead to a Weinberg-Wilczek axion; *iii*) we address $SU(3)_f$ breaking dynamics.

A relevant phenomenological feature of the accidental SO(10) axion is that its decay constant is bounded from above by the seesaw scale, which can be translated into a lower bound on the axion mass, $m_a \gtrsim 7 \times 10^{-8}$ eV. In fact, as we are going to show, in order to get an automatic PQ symmetry the SO(10) representations responsible for $U(1)_{PQ}$ breaking need to have a non-trivial SO(10) center. This immediately rules out the GUT-scale axion ($m_a \lesssim 10^{-9}$ eV), in which the PQ breaking is connected to the first stage of SO(10) breaking, and selects instead SO(10) representations with $B - L$ breaking

vacuum expectation values (VEVs) at intermediate mass scales well below GUT. Other phenomenological constraints on the axion mass depend on whether the PQ is broken before or after inflation. In the former case, an upper bound of about $m_a \lesssim 10^{-3}$ eV originates from iso-curvature fluctuations generated by the massless axion field during inflation. On the other hand, a high-quality PQ symmetries requires a relatively heavy axion, $m_a \gtrsim 0.02$ eV, that is viable only if the PQ symmetry is broken after inflation and never restored afterwards. This scenario is threatened by a genuine domain wall problem, which however can be overcome by a small explicit breaking of the $U(1)_{PQ}$ (in terms of Planck-suppressed operators), compatibly with the PQ solution of the strong CP problem. We also discuss the relevance of astrophysical bounds in such case and the possibility that DM is wholly comprised by axions in the high-quality axion mass window.

The paper is structured as follows. In Sect. 2 we show how to obtain an accidental $U(1)_{PQ}$ symmetry upon gauging the flavour group of SO(10) and specify a minimal realistic model. In Sect. 3 we extend the analysis beyond the renormalizable level, in order to identify at which operator level the $U(1)_{PQ}$ gets broken. Here, we also comment on the physical relevance of the PQ quality problem and on the possibility to address it within the present framework. Sect. 4 is devoted to the identification of the physical axion field, which requires some care due to the fact that the axion gets kinetically mixed with neutral massive vectors arising from SO(10) breaking. Bringing the axion in a canonical form is actually a necessary step for computing its low-energy couplings to SM matter fields (as detailed in App. A). Next, in Sect. 5 we address the cancellation of the $SU(3)_f^3$ gauge anomaly as well as $SU(3)_f$ breaking dynamics. In Sect. 6 we discuss the phenomenological profile of the accidental SO(10) axion and conclude in Sect. 7, with a general perspective on the question of an automatic $U(1)_{PQ}$ in GUTs.

2 Gauging the way to the SO(10) axion

A family of SM fermions plus a right-handed neutrino reside into a spinorial SO(10) representation, ψ_{16}, which gets triplicated in order to account for three SM chiral families. Schematically, a typical SO(10) Yukawa Lagrangian reads

$$\mathscr{L}_Y = y_{10}\,\psi_{16}\psi_{16}\phi_{10} + \tilde{y}_{10}\,\psi_{16}\psi_{16}\phi_{10}^\star + y_{\overline{126}}\,\psi_{16}\psi_{16}\phi_{\overline{126}} + \text{h.c.}\,, \qquad (2.1)$$

where we restricted for simplicity to a $10 + \overline{126}$ reducible Higgs representation, and we have taken a complex ϕ_{10} (as required by realistic fermion masses and mixings [18–21]), while the anti self-dual $\phi_{\overline{126}}$ is by construction complex.[1]

In the $y_{10}, \tilde{y}_{10}, y_{\overline{126}} \to 0$ limit the global symmetry group of the SO(10) fermion sector comprising three copies of ψ_{16} is

$$U(3) = U(1)_{PQ} \times SU(3)_f\,, \qquad (2.2)$$

where the abelian factor is a PQ symmetry, since it is anomalous under QCD. In fact, the chiral embedding of SM matter into the spinorial of SO(10) implies a non-zero $U(1)_{PQ}$-$SU(3)_c^2$ anomaly (cf. Sect. 4).

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[1]The reader not familiar with SO(10) properties can find a basic introduction e.g. in Sect. 2 of [22].

In the following, we wish to argue that the *gauging* of $SU(3)_f$ leads to an accidental $U(1)_{PQ}$ in the full Lagrangian, if the $SO(10)$ Higgs representations are properly chosen along the lines of [15]. Since the $SO(10)$ spinors transform in the fundamental of $SU(3)_f$, $\psi_{16} \sim \mathbf{3}$, in order to make the Yukawa Lagrangian $SU(3)_f$ invariant we need to assign both $\phi_{10} \sim \overline{\mathbf{6}}$ and $\phi_{\overline{126}} \sim \overline{\mathbf{6}}$, since $SO(10)$ contractions are symmetric.[2] Then the $\tilde{y}_{10}$ term in Eq. (2.1) is forbidden by $SU(3)_f$ gauge invariance and a $U(1)_{PQ}$ automatically arises in the Yukawa sector:

$$\psi_{16} \to e^{i\alpha}\psi_{16}\,, \quad \phi_{10,\overline{126}} \to e^{-2i\alpha}\phi_{10,\overline{126}}\,, \tag{2.3}$$

where α indicates the parameter of the $U(1)_{PQ}$ transformation and PQ charges are explicitly denoted as $PQ(\psi_{16}) = 1$, $PQ(\phi_{10,\overline{126}}) = -2$, etc.

Other Higgs representations need to be introduced for the spontaneous symmetry breaking of $SO(10) \times U(1)_{PQ}$ ($SU(3)_f$ breaking will be discussed separately in Sect. 5.2). To this end we consider a (real) adjoint ϕ_{45} and a ϕ_{16}. Their role is the following:

- ϕ_{45}: this is the smallest representation which can break $SO(10)$ down to a rank-5 sub-group featuring an unbroken $U(1)_R \times U(1)_{B-L}$ Cartan subalgebra, although it requires to go beyond the tree approximation for the minimization of the scalar potential [23–26]. As far as concerns the transformation properties of ϕ_{45} under $SU(3)_f$, since it does not couple to ψ_{16} at the renormalizable level, it can be taken to be a singlet. In fact, even if we were to assign ϕ_{45} to a non-trivial $SU(3)_f$ representation (say a fundamental), a PQ breaking operator of the type $\phi_{45}^3 \equiv \epsilon_{(3)}^{abc}(\phi_{45})_a^{ij}(\phi_{45})_b^{jk}(\phi_{45})_c^{ki}$ would be always allowed by $SO(10) \times SU(3)_f$ invariance. Similar considerations apply to other $SO(10)$ representations with a trivial $SO(10)$ center, like 54 and 210, which could be similarly employed in place of ϕ_{45}.

- ϕ_{16}: the need for this extra representation [27] is due to the fact that $\phi_{\overline{126}}$ breaks $U(1)_R \times U(1)_{B-L} \times U(1)_{PQ} \to U(1)'_{PQ} \times U(1)_Y$, where $U(1)'_{PQ}$ is a remnant PQ symmetry (a linear combination of the original PQ and the broken gauge generators) that would be eventually broken at the electroweak scale, leading to a phenomenologically untenable Weinberg-Wilczek axion. Hence, the ϕ_{16} (together with $\phi_{\overline{126}}$) is needed to ensure a proper rank reduction down to the SM group. In order for the ϕ_{16} to participate to $U(1)_{PQ}$ breaking, it needs to couple in a non-trivial way to ϕ_{10} and/or $\phi_{\overline{126}}$ so that it can get charged under the PQ. The simplest, viable option is that $\phi_{16} \sim \overline{\mathbf{3}}$, thus allowing for the gauge invariant operator $\phi_{16}^2\phi_{10}^\star(\phi_{45})$ (the parenthesis meaning both the invariant with and without ϕ_{45}), so that $PQ(\phi_{16}) = -1$.[3]

Summarizing, the Higgs sector includes $\phi_{10} \sim \overline{\mathbf{6}}$, $\phi_{\overline{126}} \sim \overline{\mathbf{6}}$, $\phi_{45} \sim \mathbf{1}$ and $\phi_{16} \sim \overline{\mathbf{3}}$. Then the scalar potential, $\mathcal{V} = \mathcal{V}_2 + \mathcal{V}_3 + \mathcal{V}_4$ (with the subscript denoting the dimensionality of

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[2]On the contrary, a ϕ_{120} would have to be assigned to a $\mathbf{3}$, being the $\psi_{16}\psi_{16}\phi_{120}$ $SO(10)$ contractions antisymmetric.

[3]Refs. [15, 27] considered instead the case in which the following operators $\phi_{16}^2\phi_{10}(\phi_{45})$ and $\phi_{16}^2\phi_{\overline{126}}(\phi_{45})$ are present, and hence $PQ(\phi_{16}) = 1$. As we are going to show in Sect. 4 (cf. footnote (6)), the latter choice implies an alignment between $\langle\phi_{16}\rangle$ and $\langle\phi_{\overline{126}}\rangle$, ending up in a Weinberg-Wilczek axion.

Field	Lorentz	SO(10)	$\mathbb{Z}_4$	SU(3)$_f$	$\mathbb{Z}_3$	U(1)$_{\text{PQ}}$
ψ_{16}	$(1/2, 0)$	16	i	$\mathbf{3}$	$e^{i2\pi/3}$	1
$\psi_1^{1,\ldots,16}$	$(1/2, 0)$	1	1	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	0
ϕ_{10}	$(0, 0)$	10	-1	$\overline{\mathbf{6}}$	$e^{i2\pi/3}$	-2
ϕ_{16}	$(0, 0)$	16	i	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	-1
$\phi_{\overline{126}}$	$(0, 0)$	$\overline{126}$	-1	$\overline{\mathbf{6}}$	$e^{i2\pi/3}$	-2
ϕ_{45}	$(0, 0)$	45	1	$\mathbf{1}$	1	0

Table 1: Field content of the model and relative transformation properties under SO(10) $\times$ SU(3)$_f$, its $\mathbb{Z}_4 \times \mathbb{Z}_3$ center and the accidental U(1)$_{\text{PQ}}$. In light gray, exotic fermions which ensure SU(3)$_f^3$ anomaly cancellation (cf. Sect. 5.1).

the operators), features the following SO(10) $\times$ SU(3)$_f$ invariant terms

$$\mathcal{V}_2 = |\phi_{10}|^2 + |\phi_{\overline{126}}|^2 + \phi_{45}^2 + |\phi_{16}|^2 \,, \tag{2.4}$$

$$\mathcal{V}_3 = \phi_{16}^2 \phi_{10}^\star + \text{h.c.} \,, \tag{2.5}$$

$$\mathcal{V}_4 = \mathcal{V}_2^2\text{-terms} + \phi_{10}^2 \phi_{\overline{126}}^{\star 2} + \phi_{10} \phi_{\overline{126}} \phi_{\overline{126}}^{\star 2} + \phi_{16}^2 \phi_{10}^\star \phi_{45} + \text{h.c.} \,, \tag{2.6}$$

where $\mathcal{V}_2^2$-terms stands for quartics obtained by "squaring" $\mathcal{V}_2$ (including all possible linearly independent invariants made by the same amount of fields). Note that the operator $\phi_{16}^2 \phi_{\overline{126}}^\star (\phi_{45})$ is not allowed by SO(10) invariance, but due to the interplay with the ϕ_{10} there is a sufficient amount of scalar potential terms so that a single abelian global symmetry survives accidentally. This can be identified with the U(1)$_{\text{PQ}}$, with transformation properties

$$\phi_{16} \to e^{-i\alpha} \phi_{16} \,, \quad \phi_{45} \to \phi_{45} \,, \tag{2.7}$$

while those of ϕ_{10} and $\phi_{\overline{126}}$ are given in Eq. (2.3). The SU(3)$_f$ symmetry forbids the following terms which are allowed by SO(10) invariance:

$$\phi_{10}^2 \,, \ \phi_{10}^4 \,, \ \phi_{16}^4 \,, \ \phi_{\overline{126}}^4 \,, \ \phi_{10}^2 \phi_{45}^2 \,, \ \phi_{10}^2 \phi_{\overline{126}}^2 \,, \ \phi_{10} \phi_{\overline{126}}^3 \,, \ \phi_{45}^2 \phi_{\overline{126}}^2 \,, \ \phi_{16}^2 \phi_{10}(\phi_{45}) \,, \ \phi_{16}^2 \phi_{\overline{126}}(\phi_{45}) \,. \tag{2.8}$$

The origin of the accidental U(1)$_{\text{PQ}}$ can be neatly understood in terms of the action of the centers of SO(10) and SU(3)$_f$, which are respectively $\mathbb{Z}_4$ and $\mathbb{Z}_3$.[4] The transformation properties of the model fields under the gauge and the U(1)$_{\text{PQ}}$ accidental symmetries are summarized in Table 1.

Note that for consistency we have also introduced 16 (SO(10)-singlet) exotic fermions in the $\overline{\mathbf{3}}$ of SU(3)$_f$, to ensure SU(3)$_f^3$ anomaly cancellation. Their spectrum will be discussed in Sect. 5, together with SU(3)$_f$ breaking.

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[4]The center $Z(G)$ of a group G is the set of elements that commute with every element of G. $Z(\text{SU}(3)) = \mathbb{Z}_3$, which is generated by $e^{2\pi i/3}\mathbb{1}_3$; $Z(\text{SO}(10)) = \mathbb{Z}_4$, which is generated by $i\Gamma_0$, with Γ_0 denoting the "chirality" operator of the SO(10) Clifford algebra (see e.g. [37]).

3 Peccei-Quinn quality

After having obtained the $U(1)_{PQ}$ to arise accidentally in the renormalizable Lagrangian, one should worry about possible sources of PQ breaking in the UV, which are often parametrized via effective operators suppressed by a cut-off scale Λ_{UV}. A simple estimate shows that $U(1)_{PQ}$ should be preserved by operators up to dimension $d \gtrsim 9$, assuming for instance $\Lambda_{UV} \sim M_{Pl}$ and an axion decay constant $f_a \sim 10^9$ GeV. This is obtained by requiring that the energy density from UV sources of PQ breaking is about 10^{-10} times smaller than the energy density of the QCD axion potential

$$\left(\frac{f_a}{\Lambda_{UV}}\right)^{d-4} f_a^4 \lesssim 10^{-10} \Lambda_{QCD}^4 \,, \tag{3.1}$$

so that the induced axion VEV displacement from zero is $\langle a \rangle / f_a \lesssim 10^{-10}$, within the bound from the neutron electric dipole moment (nEDM).

Interestingly, the gauging of $SU(3)_f$ provides some protection also beyond the renormalizable level. Given the field content in Table 1, we proceed to identify at which operator level the $U(1)_{PQ}$ gets broken in the scalar potential. The lowest-dimensional PQ-breaking operators are found to be:

$$\phi_{10}^6 \qquad\qquad (d=6)\,, \tag{3.2}$$
$$\phi_{\overline{126}}^6 \qquad\qquad (d=6)\,, \tag{3.3}$$
$$\phi_{16}^6 \phi_{10}^3 \qquad\qquad (d=9)\,, \tag{3.4}$$
$$\phi_{16}^6 \phi_{\overline{126}}^3 \qquad\qquad (d=9)\,, \tag{3.5}$$
$$\phi_{16}^{12} \qquad\qquad (d=12)\,. \tag{3.6}$$

This classification can be easily understood in terms of the action of the $\mathbb{Z}_4 \times \mathbb{Z}_3$ center as displayed in Table 1: invariance under $\mathbb{Z}_3$ requires a number of fields that is a multiple of 3 and after that one has to compensate powers in order to get a $\mathbb{Z}_4$ singlet. Note that the $d=9$ operators $\phi_{16}^6 \phi_{10}^{\star 3}$ and $\phi_{16}^6 \phi_{\overline{126}}^{\star 3}$ are also allowed by gauge invariance, but they preserve $U(1)_{PQ}$.

Some comments are in order, regarding the impact of those operators on the PQ quality problem:

- ϕ_{10} can only develop electroweak scale VEVs, hence the Planck-suppressed operators ϕ_{10}^6 and $\phi_{16}^6 \phi_{10}^3$ do not pose a problem for the PQ quality issue. Then, we only need to worry about operators developing "large" VEVs compared to the electroweak scale, i.e. ϕ_{16} and $\phi_{\overline{126}}$.

- Remarkably, the operators $\phi_{\overline{126}}^6$ and ϕ_{16}^{12} do not yield a large contribution to the axion potential, since (once projected on the SM vacuum) extra electroweak VEV insertions are needed. To see this, recall the decompositions under $SO(10) \to SU(5) \times U(1)_Z$ [28]: $16 \to 1(-5) + \dots$ and $\overline{126} \to 1(+10) + \dots$, along the $SU(5)$ singlet components. Clearly, $\phi_{\overline{126}}^6$ and ϕ_{16}^{12} have zero projection on $V_{\overline{126}}^6$ and V_{16}^{12} (the latter denoting the $SU(5)$-singlet VEVs), since it is not possible to compensate the $U(1)_Z$ charge.

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- The leading contribution to the axion potential is then given by the operator $\phi_{16}^6 \phi_{\overline{126}}^3$, with a non-zero projection on the high-scale VEVs $V_{16}^6 V_{\overline{126}}^3$. In order to assess the impact of that PQ-breaking operator on the axion potential, one has still to identify the physical axion field. This step will be done in detail Sect. 4, but we anticipate here the results for the assessment of the PQ quality problem. Using Eq. (4.16) we find the following contribution to the axion potential:

$$\mathcal{V}_{\text{PQ-break}} = k \frac{\phi_{16}^6 \phi_{\overline{126}}^3}{M_{\text{Pl}}^5} + \text{h.c.} \supset \frac{|k|}{2^{7/2}} \frac{V_{16}^6 V_{\overline{126}}^3}{M_{\text{Pl}}^5} \cos\left(\frac{a}{V/12} + \delta\right), \qquad (3.7)$$

with $\delta = \text{Arg}\, k$ denoting a generic $\mathcal{O}(1)$ phase and V a function of $V_{16,\overline{126}}$ defined in Eq. (4.18) (for $c_1 = 1$). Considering the total axion potential $\mathcal{V} = \mathcal{V}_{\text{PQ-break}} + \mathcal{V}_{\text{QCD}}$, with $\mathcal{V}_{\text{QCD}} = -\Lambda_{\text{QCD}}^4 \cos(a/f_a)$ and f_a given in terms of $V_{16,\overline{126}}$ via Eq. (4.20), one finds the induced QCD theta term (valid for $\langle a \rangle / f_a \ll 1$):

$$\theta_{\text{eff}} \equiv \frac{\langle a \rangle}{f_a} \approx \frac{|k| V_{16}^6 V_{\overline{126}}^3 \sin\delta}{8\sqrt{2} M_{\text{Pl}}^5 \Lambda_{\text{QCD}}^4}. \qquad (3.8)$$

Requiring the neutron electric dipole moment bound, $|\theta_{\text{eff}}| \lesssim 2.0 \times 10^{-10}$, implies $(V_{16}^3 V_{126}^6)^{1/9} \lesssim 2.0 \times 10^9$ GeV (for e.g. $|k| = 1$ and $\sin\delta = 1$). A more useful estimate in terms of the axion decay constant will be given in Sect. 6.

- Last, but not least, assuming no new physics above the GUT scale apart for gravity, it is unclear whether quantum gravity would generate power-like PQ-breaking operators, such as those in Eq. (3.7). Nonetheless, it is believed that gravity does violate global symmetries, based on semi-classical arguments related to black holes and Hawking radiation. In scenarios in which Einstein gravity is minimally coupled to the axion field, non-conservation of the PQ global charge arises from non-perturbative effects described by Euclidean wormholes. Those are calculable in the semi-classical limit [29–33] and give a correction to the axion potential of the order of $M_{\text{Pl}}^4 e^{-S_{\text{wh}}}$, where $S_{\text{wh}} \sim M_{\text{Pl}}/f_a$ is the wormhole action. Being exponentially suppressed, such contribution poses a problem for the PQ solution only for $f_a \gtrsim 10^{17}$ GeV, which is however much above the intermediate-scale values of f_a that will be considered here.

All in all, we conclude that not only the gauging of $\text{SU}(3)_f$ leads to an accidental $\text{U}(1)_{\text{PQ}}$, but it also structurally protects it against PQ breaking effective operators. To solve the PQ quality issue in its standard formulation in terms of Planck-suppressed, power-like effective operators one needs PQ breaking VEVs smaller than about 10^9 GeV.

4 Axion's guts (physical axion field)

A crucial quantity that qualifies the U(1) accidental symmetry as a PQ symmetry is the QCD anomaly of the classically conserved PQ current, J_μ^{PQ}. At the SO(10) level, that is

(employing the compact notation $G\tilde{G} \equiv G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu}$, etc.)

$$\partial^\mu J^{\text{PQ}}_\mu = \frac{\alpha_{\text{SO(10)}}N_{\text{SO(10)}}}{4\pi}F_{\text{SO(10)}}\tilde{F}_{\text{SO(10)}}$$

$$\supset \frac{\alpha_s N_{\text{SO(10)}}}{4\pi}G\tilde{G} + \frac{\alpha_W N_{\text{SO(10)}}}{4\pi}W\tilde{W} + \frac{5}{3}\frac{\alpha_Y N_{\text{SO(10)}}}{4\pi}B\tilde{B}$$

$$\supset \frac{\alpha_s N_{\text{SO(10)}}}{4\pi}G\tilde{G} + \frac{8}{3}\frac{\alpha N_{\text{SO(10)}}}{4\pi}F\tilde{F} \equiv \frac{\alpha_s N}{4\pi}G\tilde{G} + \frac{\alpha E}{4\pi}F\tilde{F}\,, \qquad (4.1)$$

where the matching with the SM gauge bosons is understood at the GUT scale. The matching with QED is obtained by projecting on the photon component ($W^3 \to \sin\theta_W A$ and $B \to \cos\theta_W A$) and using $\alpha = \alpha_W \sin^2\theta_W = \alpha_Y \cos^2\theta_W$. In the last step we made contact with the standard notation for the QCD (N) and QED (E) anomaly coefficients, which read respectively $N = N_{\text{SO(10)}}$ and $E = 8/3\,N_{\text{SO(10)}}$, in terms of the SO(10) anomaly factor

$$N_{\text{SO(10)}} = n_g T(\psi_{16})\text{PQ}(\psi_{16}) = 6\,, \qquad (4.2)$$

with $n_g = 3$ (number of generations), $T(\psi_{16}) = 2$ (Dynkin index of the spinorial of SO(10)) and $\text{PQ}(\psi_{16}) = 1$. Upon the anomalous PQ rotation $\psi_{16} \to e^{i\alpha}\psi_{16}$, the QCD theta term in $\mathscr{L}_{\text{QCD}} \supset \frac{\alpha_s\theta}{8\pi}G\tilde{G}$ is shifted as $\theta \to \theta + 2N\alpha$. Since different values of θ correspond to different physics modulo 2π, the U(1)$_{\text{PQ}}$ symmetry is explicitly broken down to a $\mathbb{Z}_{2N}$, corresponding to $\alpha = 2\pi n/(2N)$ with $n = 0, 1, \ldots, 2N - 1$.

The anomaly does not depend on the mass of the fermions running in the triangle loop, and hence it must be preserved through the various stages of symmetry breaking. This feature will be useful for identifying the physical axion field and its low-energy couplings. In the following, we provide the anatomy of the axion field in SO(10) at three different levels of increasing complexity: *i*) a single, dominant SM-singlet and PQ-breaking VEV, *ii*) two SM-singlet and PQ-breaking VEVs of comparable size and *iii*) the full-fledged case including also electroweak VEVs, a necessary step to compute low-energy axion couplings to SM matter fields. The last, self-contained derivation is deferred to App. A.

4.1 A dominant SM-singlet and PQ-breaking VEV

Let us first consider a phenomenologically relevant limit where physics should be clearer, and focus on the breaking pattern

$$\text{SO(10)} \times \text{U(1)}_{\text{PQ}} \xrightarrow{\langle\phi_{45}\rangle_{B-L}} \text{SU(3)}_c \times \text{SU(2)}_L \times \text{SU(2)}_R \times \text{U(1)}_{B-L} \times \text{U(1)}_{\text{PQ}}$$

$$\xrightarrow{V_{\overline{126}}} \text{SU(3)}_c \times \text{SU(2)}_L \times \text{U(1)}_Y \times \text{U(1)}'_{\text{PQ}}$$

$$\xrightarrow{V_{16}} \text{SU(3)}_c \times \text{SU(2)}_L \times \text{U(1)}_Y\,, \qquad (4.3)$$

where we assumed the hierarchy of VEVs: $\langle\phi_{45}\rangle_{B-L} \gg V_{\overline{126}} \gg V_{16}$, as motivated by gauge coupling unification [34–36]. Here, $\langle\phi_{45}\rangle_{B-L}$ denotes a specific orientation of the adjoint VEV, which can be achieved via a renormalizable scalar potential after including one-loop

corrections [23–26]; while the other two intermediate-scale VEVs are defined in terms of the following polar field decompositions (along the SM-singlet components)

$$\phi_{16} = \frac{1}{\sqrt{2}}(V_{16} + \ldots)e^{i\frac{a_{16}}{V_{16}}}, \qquad \phi_{\overline{126}} = \frac{1}{\sqrt{2}}(V_{\overline{126}} + \ldots)e^{i\frac{a_{\overline{126}}}{V_{\overline{126}}}}, \tag{4.4}$$

with the projections of the angular modes a_{16} and $a_{\overline{126}}$ on the axion field yet to be identified. After the second breaking stage, a remnant $U(1)'_{PQ}$ global symmetry is left invariant by $\langle\phi_{\overline{126}}\rangle$ (which cannot reduce the rank by more than one unit). The latter symmetry can be expressed as a linear combination of the original $U(1)_{PQ}$ and the broken Cartan gauge generators as[5]

$$PQ' = c_1 PQ + c_2(B - L). \tag{4.5}$$

Given $\{PQ, B - L\}(\langle\phi_{\overline{126}}\rangle) = \{-2, -2\}$, the defining property, $PQ'(\langle\phi_{\overline{126}}\rangle) = 0$, implies $c_2 = -c_1$. The coefficient c_1 can be fixed by matching the anomaly in Eq. (4.2) with the one computed in terms of PQ' charges, that is

$$N = n_g T(3)(2\,PQ'(q) + 2\,PQ'(q^c)) = 6c_1, \tag{4.6}$$

where we used $n_g = 3$, $T(3) = 1/2$ (the Dynkin index of the fundamental of $SU(3)_c$) and the PQ' charges $PQ'(q) = 2c_1/3$ (for the $SU(2)_L$ quark doublet, with $B - L = 1/3$) and $PQ'(q^c) = 4c_1/3$ (for the $SU(2)_R$ quark doublet, with $B - L = -1/3$). The matching with the $SO(10)$ anomaly in Eq. (4.2) requires then $c_1 = 1$, and hence $PQ' = PQ - (B - L)$. Given $\{PQ, B - L\}(\langle\phi_{16}\rangle) = \{-1, 1\}$, it is readily verified that the action of PQ' on $\langle\phi_{16}\rangle$ is non-trivial ($PQ'(\langle\phi_{16}\rangle) = -2$) and hence it allows us to consistently break $U(1)'_{PQ}$ at the scale V_{16}.[6] Neglecting $(V_{16}/V_{\overline{126}})^2$ corrections as well as subleading electroweak VEVs, the axion field is identified with the angular component of the ϕ_{16} field, $a = a_{16}$.

In the other relevant limit, $V_{16} \gg V_{\overline{126}}$, one obtains that another linear combination $U(1)''_{PQ}$, defined as $PQ'' = PQ + (B - L)$, is left invariant by $\langle\phi_{16}\rangle$, which is further broken by $\langle\phi_{\overline{126}}\rangle$ ($PQ''(\langle\phi_{\overline{126}}\rangle) = -4$), so that the axion field is $a = a_{\overline{126}}$.

4.2 Two SM-singlet and PQ-breaking VEVs

The more general case with $V_{16} \sim V_{\overline{126}}$ must interpolate between the two limits above and it can be obtained as follows. Consider the classically conserved currents

$$J_\mu^{PQ} = q_{16}V_{16}\partial_\mu a_{16} + q_{\overline{126}}V_{\overline{126}}\partial_\mu a_{\overline{126}}, \tag{4.7}$$

$$J_\mu^{B-L} = (B - L)_{16}V_{16}\partial_\mu a_{16} + (B - L)_{\overline{126}}V_{\overline{126}}\partial_\mu a_{\overline{126}}, \tag{4.8}$$

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[5]Since at this point we only consider VEVs with zero hypercharge, $Y = T_R^3 + (B - L)/2$, it is not necessary to include the T_R^3 generator.

[6]Choosing instead the PQ charge of ϕ_{16} equal to 1, leads to $PQ'\langle\phi_{16}\rangle = 0$. Namely $\langle\phi_{16}\rangle$ and $\langle\phi_{\overline{126}}\rangle$ are aligned, thus not providing a proper breaking pattern for $U(1)'_{PQ}$, that gets eventually broken at the electroweak scale.

with gauge charges $(B - L)_{16} = 1$ and $(B - L)_{\overline{126}} = -2$, and the *physical* PQ charges $q_{16,\overline{126}}$. The latter are linear combination of the PQ charges in Table 1 and broken gauge generators, in general

$$q = c_1 \mathrm{PQ} + c_2(B - L) + c_3 Y \,, \tag{4.9}$$

(with $c_3 = 0$ as long as we consider only the SM-singlet VEVs $V_{16,\overline{126}}$). The q charges can be fixed by requiring that the PQ and $B - L$ currents are orthogonal:

$$q_{16} V_{16}^2 - 2 q_{\overline{126}} V_{\overline{126}}^2 = 0 \,, \tag{4.10}$$

which ensures no kinetic mixings between the axion field and the $B - L$ massive gauge boson, thus providing a canonical axion field. The latter is defined as

$$a = \frac{1}{V}(q_{16} V_{16} a_{16} + q_{\overline{126}} V_{\overline{126}} a_{\overline{126}}) \,, \tag{4.11}$$

with

$$V^2 = (q_{16})^2 V_{16}^2 + (q_{\overline{126}})^2 V_{\overline{126}}^2 \,, \tag{4.12}$$

so that $J_\mu^{\mathrm{PQ}} = V \partial_\mu a$ and, compatibly with the Goldstone theorem, $\langle 0 | J_\mu^{\mathrm{PQ}} | a \rangle = i V p_\mu$. Inverting the orthogonal transformation in Eq. (4.11), one readily obtains the projection of the angular modes on the axion field:

$$a_{16} \to q_{16} V_{16} \frac{a}{V} \,, \qquad a_{\overline{126}} \to q_{\overline{126}} V_{\overline{126}} \frac{a}{V} \,, \tag{4.13}$$

whose weight factors are extracted from Eq. (4.10) and (4.12), and can be conveniently written as

$$q_{16} V_{16} = V \cos \omega \,, \qquad q_{\overline{126}} V_{\overline{126}} = V \sin \omega \,, \tag{4.14}$$

in terms of the vacuum parameter

$$\tan \omega = \frac{V_{16}}{2 V_{\overline{126}}} \,. \tag{4.15}$$

So we can finally read the axion composition of the complex fields in Eq. (4.4), which is

$$\phi_{16} = \frac{1}{\sqrt{2}}(V_{16} + \dots)e^{i \cos \omega \frac{a}{V_{16}}} \,, \qquad \phi_{\overline{126}} = \frac{1}{\sqrt{2}}(V_{\overline{126}} + \dots)e^{i \sin \omega \frac{a}{V_{\overline{126}}}} \,, \tag{4.16}$$

For instance, taking the limit $V_{\overline{126}} \gg V_{16}$ ($\omega \to 0$) one reproduces the previous result that the axion dominantly correspond to the angular mode of ϕ_{16}.

The orthogonality condition in Eq. (4.10), together with the matching of the UV and IR charges of ϕ_{16} and $\phi_{\overline{126}}$ in Eq. (4.9), allows us to fix

$$c_2 = -c_1 \cos 2\omega \,, \qquad q_{16} = -2c_1 \cos^2 \omega \,, \qquad q_{\overline{126}} = -4c_1 \sin^2 \omega \,, \tag{4.17}$$

and hence, using Eq. (4.12) (and Eq. (4.14) regarding the absolute sign)

$$V = -\frac{4c_1 V_{\overline{126}} V_{16}}{\sqrt{V_{16}^2 + 4 V_{\overline{126}}^2}} \,. \tag{4.18}$$

10

Again, the value of c_1 can be determined by matching the anomaly between the UV and the broken theory, that is

$$N = n_g T(3)(2\,q(q) + 2\,q(q^c)) = 6c_1\,, \tag{4.19}$$

where $q(q) = 2c_1/3(\cos^2\omega + 2\sin^2\omega)$ and $q(q^c) = 2c_1/3(2\cos^2\omega + \sin^2\omega)$. Hence, $c_1 = 1$ in order to match the UV anomaly. The correct physical limits $V = -2V_{16}$ (for $V_{\overline{126}} \to \infty$) and $V = -4V_{\overline{126}}$ (for $V_{16} \to \infty$) are then recovered.[7] Finally, the axion decay constant is

$$f_a = \frac{V}{2N} = -\frac{V_{\overline{126}}V_{16}}{3\sqrt{V_{16}^2 + 4V_{\overline{126}}^2}}\,. \tag{4.20}$$

In the following, to ease the notation when discussing phenomenological bounds, we will refer to f_a meaning its absolute value.

The most general case including also electroweak VEVs that participate to $U(1)_{PQ}$ breaking is discussed in App. A, together with the derivation of low-energy axion couplings.

4.3 Axion domain wall problem?

The discrete $\mathbb{Z}_{2N}$ symmetry left invariant by the QCD anomaly (cf. discussion below Eq. (4.2)), implies that the axion potential has $2N = 12$ degenerate minima. At the QCD phase transition, this leads to the formation of domain-wall-like structures at the boundaries between regions of different vacua, which quickly dominate the energy density of the Universe. However, in the presence of extra global/local symmetries it could happen that some of those minima are connected. In particular, if the $\mathbb{Z}_{2N}$ discrete symmetry can be fully embedded in the center of a non-abelian symmetry group, then the axion domain walls will quickly disappear through the emission of Goldstone bosons, thus providing an elegant solution to the axion domain wall problem [37]. Solutions of this type have been often considered in the context of SO(10) [37–39], whose $\mathbb{Z}_4$ center provides a starting point for embedding the $\mathbb{Z}_{2N}$ discrete symmetry. This is possible in models where $2N = 4$ or if extra global/local symmetries beyond SO(10) are invoked (which is the case considered here).

While such a solution was claimed to be at play also in the $SO(10) \times SU(3)_f^{global}$ model of Ref. [15], sharing several similarities with the present work, it turns out that this conclusion is flawed by the fact that in the model of [15] the PQ symmetries of ϕ_{16} and $\phi_{\overline{126}}$ were aligned, eventually leading to a phenomenologically untenable Weinberg-Wilczek axion (cf. the discussion in footnote (3)). After assigning the ϕ_{16} to a proper $SU(3)_f$ representation (cf. Table 1), so that its PQ charge is not aligned to the one of $\phi_{\overline{126}}$, it turns out that the remnant $\mathbb{Z}_{2N}$ symmetry cannot be embedded in the center of $SO(10) \times SU(3)_f$ and hence the axion domain wall problem persists. To show this, let us

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[7]To see that, take for instance the $V_{\overline{126}} \to \infty$ limit, corresponding to $a \to a_{16}$. Then, given that under a PQ transformation $a_{16,\overline{126}} \to a_{16,\overline{126}} + \alpha q_{16,\overline{126}} V_{16,\overline{126}}$ the axion field transforms as $a \to a + \alpha V$ (cf. Eq. (4.11)), we can make the identification $V = q_{16}V_{16} \to -2V_{16}$.

define the action of the centers as $r_P = \exp\left(i2\pi/P\right)$. Denoting as r_{12} the action of a PQ transformation with discrete parameter $\alpha = 2\pi/12$, r_4 and r_3 the actions of the SO(10) and SU(3)$_f$ centers (as displayed in Table 1), one would like to have that $r_{12} = r_4^{-1}r_3$ on all the model fields, so that the two sets of symmetries can be identified. It is easy to check that while such identification works for the fields ψ_{16} ($r_{12} = r_4^{-1}r_3 = \exp\left(i\pi/6\right)$), ϕ_{10}, $\phi_{\overline{126}}$ ($r_{12} = r_4^{-1}r_3 = \exp\left(-i\pi/3\right)$) and ϕ_{45} ($r_{12} = r_4^{-1}r_3 = 1$), it fails for ϕ_{16} ($r_{12} = \exp\left(-i\pi/6\right) \neq r_4^{-1}r_3 = \exp\left(i5\pi/6\right)$) and $\psi_1^{1,\ldots,16}$ ($r_{12} = 1 \neq r_4^{-1}r_3 = \exp\left(i2\pi/3\right)$).

We hence conclude that the accidental SO(10) axion has a genuine domain wall problem, with domain wall number $N_{\mathrm{DW}} \equiv 2N = 12$. A straightforward solution is given by the pre-inflationary PQ breaking scenario (in which domain walls are inflated away). Instead, in the post-inflationary PQ breaking scenario the Planck-suppressed sources of explicit PQ breaking discussed in Sect. 3 could potentially lift the vacuum degeneracy and lead to a fast decay of the domain walls [40, 41], compatibly with the PQ solution of the strong CP problem. This possibility will be explored in more detail in Sect. 6.

5 SU(3)$_f$ sector

We next discuss two issues related to the SU(3)$_f$ sector, namely the cancellation gauge anomalies and the spontaneous symmetry breaking of SU(3)$_f$.

5.1 SU(3)$_f^3$ anomaly

The only relevant gauge anomaly to be cancelled is the SU(3)$_f^3$ one (while SO(10) is anomaly free). The simplest way to cancel the latter is to introduce 16 exotic fermions, ψ_1^α ($\alpha = 1, \ldots, 16$) which are SO(10) singlets and transform in the $\overline{\mathbf{3}}$ of SU(3)$_f$.[8] Due to their quantum numbers they have no renormalizable interactions with the other fields of Table 1, so that they also acquire no PQ charge. Their spectrum is eventually controlled by SU(3)$_f$ breaking, as discussed in the following. Let us also note that there is a remnant U(1)$_{\mathrm{PQ}}\times$SU(3)$_f^2$ anomaly (since only ψ_{16} are charged under the PQ). So, in order for the axion to relax to zero the θ term of QCD (and not the one of SU(3)$_f$), SU(3)$_f$ should be spontaneously broken (effectively suppressing the contribution to the axion potential due to the Higgsing of SU(3)$_f$ instantons). This is anyway compatible with the requirement that the flavour symmetry must be completely broken in order to give mass to SM fermions.

5.2 SU(3)$_f$ breaking

We assume that SU(3)$_f$ breaks completely, in two steps

$$\mathrm{SU}(3)_f \xrightarrow{M_1} \mathrm{SU}(2)_f \xrightarrow{M_2} \mathbf{1}\,, \tag{5.1}$$

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[8]Other options to cancel the SU(3)$_f^3$ anomaly include the following copies of SO(10)-singlet representations: $2 \times \overline{\mathbf{6}} + 2 \times \overline{\mathbf{3}}$, $3 \times \overline{\mathbf{6}} + 5 \times \mathbf{3}$, etc.

where **1** denotes the trivial group (SU(2)$_f$ completely broken). As suggested by SM charged fermion masses, we also assume a hierarchy of scales $M_1 \gg M_2$. This could be achieved via two (misaligned) scalars transforming as $\phi^{1,2} \sim \overline{\mathbf{3}}$ or $\mathbf{6}$ of SU(3)$_f$. At the same time, Yukawa operators $\psi_1^\alpha \psi_1^\alpha \phi^{1,2}$, $\psi_1^\alpha \psi_1^\alpha (\phi^{1,2})^2/M_{\mathrm{Pl}}$, etc. (in the diagonal basis), would give a mass of order $M_{1,2}$, $M_{1,2}^2/M_{\mathrm{Pl}}$, etc. to the exotic fermions after complete SU(3)$_f$ breaking. However, if $\phi^{1,2}$ were to transform under an SO(10) representation with trivial center, i.e. **1**, **45**, **54** or **210**, the following operators would be also allowed: $\phi_{10}^2 (\phi^{1,2})^\star$, $\phi_{10}^2 (\phi^{1,2})^{\star 2}$ and $(\phi^{1,2})^3$, whose simultaneous presence would break explicitly the U(1)$_{\mathrm{PQ}}$ at the renormalizable level. This can be avoided if we assigned instead $\phi^{1,2}$ e.g. to a **16** or $\overline{\mathbf{126}}$.

A remarkably economical possibility (that is also unavoidable in absence of extra sources of SU(3)$_f$ breaking at the GUT scale) is actually that of using the representations ϕ_{16} and $\phi_{\overline{126}}$ in Table 1 to break simultaneously SO(10) $\times$ U(1)$_{\mathrm{PQ}}$ *and* SU(3)$_f$. There are no obvious obstructions to this program, which ties together the "horizontal" and "vertical" breaking, since the representations involved have the group-theoretical power to properly reduce the symmetry,[9] while their high-scale VEVs would allow us to decouple the associated flavour breaking dynamics at safely large scales.

On the other hand, within such approach the 16 exotic fermions ψ_1^α remain massless at the renormalizable level. The following effective operators, involving a pair of exotic fermions are allowed by SO(10) $\times$ SU(3)$_f$ invariance (omitting the $\alpha = 1, \dots, 16$ index)

$$\frac{1}{M_{\mathrm{Pl}}} \psi_1 \psi_1 \phi_{10}^2 \,, \qquad \frac{1}{M_{\mathrm{Pl}}^2} \psi_1 \psi_1 \phi_{16}^2 \phi_{\overline{126}} \,, \qquad \frac{1}{M_{\mathrm{Pl}}^3} \psi_1 \psi_1 \phi_{16}^4 \,, \qquad \frac{1}{M_{\mathrm{Pl}}^3} \psi_1 \psi_1 (\phi_{\overline{126}}^\star)^4 \,. \qquad (5.2)$$

Only the second operator gives a sizeable contribution to the mass of the exotic fermions, that are lifted up to $(V_{16}^2 V_{\overline{126}})/M_{\mathrm{Pl}}^2 \lesssim 10$ TeV (for $V_{16,\overline{126}} \lesssim 10^{14}$ GeV), while the other operators have only projections on electroweak VEVs, thus providing a tiny contribution to the mass of the exotic fermions.[10] For lower values of $V_{16,\overline{126}}$ the exotic fermions are lighter and they might eventually contribute to dark radiation if once in thermal contact with the SM via SU(3)$_f$ interactions.

Of course, it would be interesting to study whether the above flavour dynamics could be predictive for reproducing SM fermion masses and mixings. Here, we stress that the present approach to flavour differs from more standard ones (such as e.g. the one of Ref. [42]), in which the Yukawas transform under SU(3)$_f$, being themselves flavon fields interacting with SM fields via effective operators. In our case instead it is crucial, in order to obtain an automatic U(1)$_{\mathrm{PQ}}$, that the flavour dynamics acts at the renormalizable level

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[9]For instance, it should be clear that the VEV orientation $\langle \phi_{16} \rangle_{a=3}^{\alpha=16}$ breaks SO(10) $\times$ SU(3)$_f \to$ SU(5) $\times$ SU(2)$_f$, etc.

[10]One might worry that the operators in Eq. (5.2) break the PQ symmetry and hence do contribute to the axion potential upon closing fermion loops. Note, however, that it is possible to make at least one of those operators formally PQ invariant, by a proper assignment of the PQ charge of the exotic fermions, respectively PQ(ψ_1) = $\{-2, 2, 2, -4\}$, so that the simultaneous presence of at least two of them is required in order to genuinely break the PQ symmetry. The largest contribution comes by combining the second operator, involving a SM-singlet VEV, with another operator that necessarily involves an electroweak scale VEV. The latter suppression makes the contribution to the axion potential safely negligible.
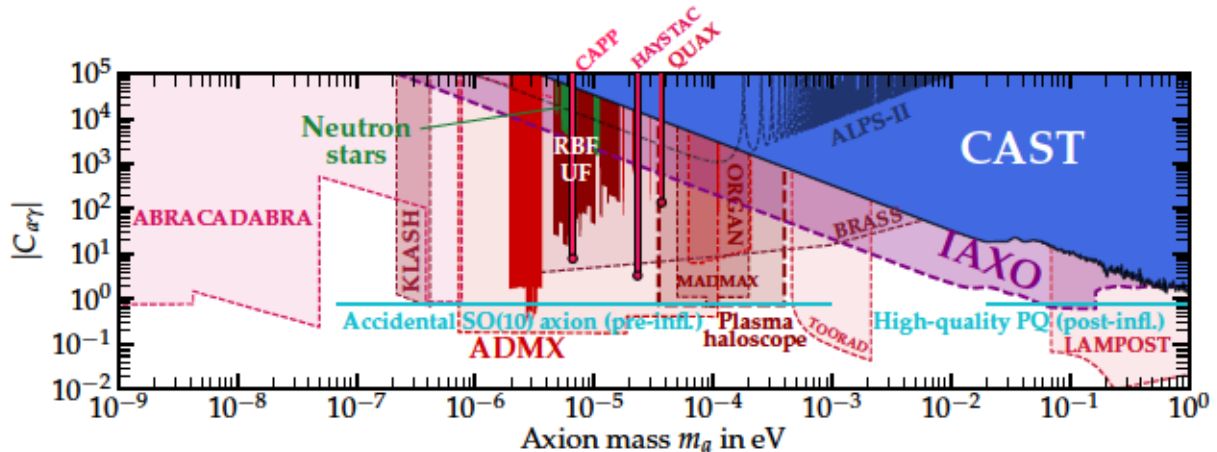
Figure 1: Axion-photon coupling (Eq. (A.26) with $E/N = 8/3$) and sensitivity of present (full lines) and future (dashed lines) axion experiments. The accidental SO(10) axion mass window (in the pre-inflationary PQ breaking scenario) corresponds to $m_a \in [7 \times 10^{-8}, 10^{-3}]$ eV. A high-quality PQ symmetry is obtained for $m_a \gtrsim 0.02$ eV (in the post-inflationary PQ breaking scenario). Axion limits from [44].

and that a single Higgs representation breaks both SO(10) and $SU(3)_f$, which is closer in spirit to the approach of Ref. [43]. A quantitative analyses of fermion masses and mixings following this path is left for future studies.

6 Axion phenomenology

In this Section we describe the phenomenological profile of the accidental SO(10) axion. Axion couplings to photons and SM matter fields have been computed in App. A (respectively Eq. (A.26) and Eqs. (A.30)–(A.31)). For the present scenario, the axion coupling to photons represents the main experimental probe (as shown in Fig. 1). The mass range of the accidental SO(10) axion is constrained by various considerations, depending on whether the PQ symmetry is broken before or after inflation.

6.1 Pre-inflationary PQ breaking

If the PQ symmetry is broken before (and during) inflation and not restored afterwards, the dark matter (DM) relic density via the vacuum misalignment mechanism is given by [45, 46]

$$\Omega_a h^2 = 0.12 \left(\frac{f_a}{9 \times 10^{11} \text{ GeV}} \right)^{1.165} \theta_{\text{in}}^2, \qquad (6.1)$$

which is valid for $\theta_{\text{in}} \lesssim 1$. In the non-linear regime, $\theta_{\text{in}} \to \pi$, DM axions are allowed to have masses as high as $m_a \lesssim 1$ meV, but no higher because quantum fluctuations during inflation would imply too large iso-curvature fluctuations [47].

On the other hand, a lower bound on m_a follows as well within the model, since f_a is bounded from above by the seesaw scale. Rewriting Eq. (4.20) as $|f_a| = |\sin 2\omega| V_{B-L}/12$, with $V_{B-L} \equiv \sqrt{V_{16}^2 + 4V_{\overline{126}}^2}$, we find the upper limit $f_a \leq V_{B-L}/12$. Note that the mass of the $B-L$ gauge boson will be proportional to V_{B-L} (because of the relative $B-L$ charges of the two VEVs). Hence, V_{B-L} assumes the meaning of $B-L$ breaking scale, subject to constraints both from gauge coupling unification and neutrino masses. In particular, right-handed neutrino masses are controlled at tree-level by $V_{\overline{126}}$, via $M_{\nu_R} = y_{\overline{126}} V_{\overline{126}}/\sqrt{2}$, and at two loops [48, 49] by V_{16}. Very conservatively, we take $V_{B-L} \lesssim 10^{15}$ GeV, corresponding to $m_a \gtrsim 7 \times 10^{-8}$ eV.

Fig. 1 shows the predictions of the model in the axion-photon vs. axion-mass plane. Barring a hole in sensitivity around $m_a \sim 10^{-7}$ eV, the parameter space of the accidental SO(10) axion in the pre-inflationary PQ breaking scenario will be mostly explored in the coming decades, under the crucial assumption that the axion comprises the whole DM (otherwise the sensitivity of axion DM experiments on the axion-photon coupling is diluted as $(\Omega_a/\Omega_{\rm DM})^{1/2}$).

It should be noted that although the allowed axion mass window spans over several orders of magnitude, further refinements of the present analysis will likely narrow down the axion mass range. This is due to extra constraints on the $B-L$ and PQ breaking scales originating from the requirement of reproducing SM fermion masses and mixings as well as from gauge coupling unification. However, we refrain from applying standard unification constraints to the present model, since those are affected by the replication of the SO(10) Higgs fields under SU(3)$_f$, which could drastically change the running [50], depending on the pattern of SU(3)$_f$ breaking.

6.2 Post-inflationary PQ breaking

If the PQ is broken after inflation or restored afterwards, one has also topological defects (axion strings and domain walls) that contribute to the axion DM relic density, on top of the usual misalignment mechanism. Such a scenario is theoretically motivated by the possibility of addressing the PQ quality problem in the present framework. In fact, using the estimate in Eq. (3.8) for the induced QCD theta term and the expression of f_a in terms of $V_{16,\overline{126}}$ (cf. Eq. (4.20)), we find that in order to satisfy the nEDM bound, $|\theta_{\rm eff}| \lesssim 10^{-10}$, one needs $f_a \lesssim 3.0 \times 10^8$ GeV (correspondingly, $m_a \gtrsim 0.02$ eV). The more general parameter space in terms of $V_{\overline{126}}$ and V_{16} is displayed in Fig. 2.

In the post-inflationary PQ breaking scenario there is a main phenomenological issue given by the axion domain wall problem (recall that the model has $N_{\rm DW} = 12$, see Sect. 4.3). A small breaking of the U(1)$_{\rm PQ}$ via the Planck suppressed operator in Eq. (3.7) could remove the degeneracy among the vacua of the axion potential, effectively leading to the domain walls to decay before they dominate the energy density of the universe [40]. Parametrizing the PQ breaking via a bias term in the scalar potential [41, 51]

$$\mathcal{V}_{\rm bias} = -2\Xi V^4 \cos\left(\frac{a}{V} + \delta\right), \tag{6.2}$$

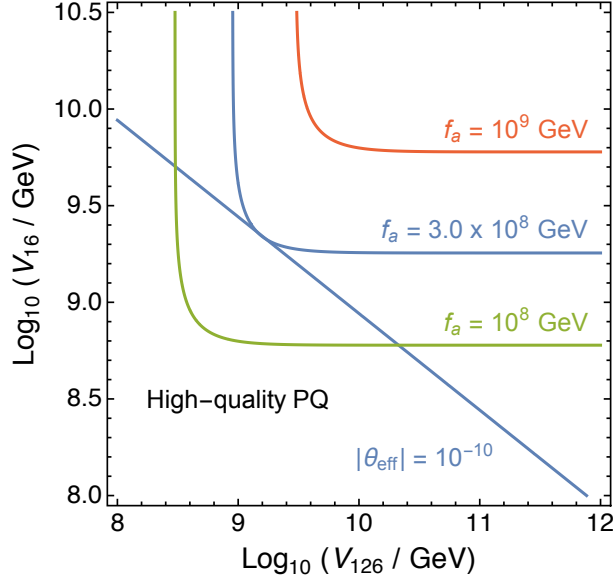with $V = N_{\rm DW} f_a$, the energy density difference between two neighbouring minima, e.g. in

Figure 2: Parameter space of the accidental SO(10) axion leading to a high-quality $U(1)_{PQ}$. For the estimate of the induced QCD theta term we used Eq. (3.8) with $|k| = 1$ and $\sin\delta = 1$.

$a = 0$ and $a = 2\pi/N_{DW}$, is (for $\sin\delta \approx 1$) $\Delta\mathcal{V} = \mathcal{V}_{bias}(2\pi/N_{DW}) - \mathcal{V}_{bias}(0) \approx \Xi V^4(1 - \cos(2\pi/N_{DW}))$. A simple estimate of the decay time of the domain wall is obtained by equating the volume pressure $p_V \sim \Delta\mathcal{V}$ and the tension force $p_T \sim \sigma_{DW}/t$ (with $\sigma_{DW} \approx 9m_a f_a^2$ denoting the wall tension [52, 53]). Hence,

$$t_{decay} \approx \frac{\sigma_{DW}}{\Xi V^4(1 - \cos(2\pi/N_{DW}))} \approx 5 \times 10^{-5} \text{ s } \left(\frac{10^{-50}}{\Xi}\right)\left(\frac{12}{N_{DW}}\right)^4\left(\frac{m_a}{0.02 \text{ eV}}\right)^3. \quad (6.3)$$

Matching the bias parameter with the potential in Eq. (3.7) due to the Planck-suppressed operator $\phi_{16}^6\phi_{\overline{126}}^3$, we obtain $\Xi \sim (V/M_{Pl})^5 \approx 5 \times 10^{-48}(0.02 \text{ eV}/m_a)^5(N_{DW}/12)^5$. Hence, for m_a in the ballpark of 0.02 eV the domain walls safely decay before Big Bang nucleosynthesis (and also well before they start to dominate the energy density of the Universe). Note, however, that due to the $t_{decay} \propto m_a^8$ dependence the bound saturates fast for larger m_a (e.g. $t_{decay} \sim 10$ s for $m_a = 0.2$ eV).

For $m_a \gtrsim 0.02$ eV, astrophysical constraints are also at play (see Sect. 4.7 in [54] for an updated summary). In particular, limits from red giants and white dwarfs cooling on the axion-electron coupling and the SN1987A bound on the axion-nucleon couplings are in the ballpark of $m_a \sim 0.02$ eV, depending however on electroweak vacuum parameters (cf. Eqs. (A.30)–(A.31)). Their perturbativity domain will differ from those of the standard DFSZ [8, 9] axion model (as it does happen in PQ-extended left-right symmetric models [55]). Hence, it is possible that astrophysical constraints could be relaxed to some extent compared to the DFSZ axion case, thus entering the region probed by IAXO in Fig. 1. A detailed analysis of astrophysical constraints is beyond the scopes of the present work, but we note in passing that the axion-electron coupling $g_{ae} = c_e m_e/f_a$ (with c_e given

in Eq. (A.31)), is of the size required to explain the so-called "stellar cooling anomalies" [56] for $m_a \approx 0.02$ eV and $c_e \approx 0.1$.

Finally, it is interesting to note that recent simulations of the axion-string network suggest a much larger contribution to the total axion relic density than what traditionally thought [57, 58]. Rescaling the results of Ref. [58] for a generic domain wall number N_{DW}, $m_a(N_{\mathrm{DW}}) = (Q(N_{\mathrm{DW}})/Q(N_{\mathrm{DW}} = 1))^{6/7} m_a(N_{\mathrm{DW}} = 1)$ (with the function Q given in Eq. (36) of [58], after replacing $\xi_\star \log_\star \to N_{\mathrm{DW}}^2 \xi_\star \log_\star$), one obtains a lower bound on the axion DM mass for $N_{\mathrm{DW}} = 12$ of about $m_a \gtrsim 7.5$ meV, which still neglects extra strings-domain walls contribution after the onset of axion oscillations. Hence, the requirement of the axion comprising the whole DM could potentially be compatible also with the high-quality axion mass window in Fig. 1.

7   Conclusions

Obtaining an automatic $\mathrm{U}(1)_{\mathrm{PQ}}$ in GUTs is a longstanding problem, with remarkably few successful attempts.[11] Any progress in that direction would certainly make the connection between two motivated frameworks (the axion and grand unification) a more convincing one. Here, we have shown that the gauging the $\mathrm{SU}(3)_f$ flavour group in $\mathrm{SO}(10)$ leads to an automatic $\mathrm{U}(1)_{\mathrm{PQ}}$ if the $\mathrm{SO}(10)$ Higgs representations are properly chosen. Moreover, the PQ symmetry is protected also beyond the renormalizable level, thus providing a possible way to tackle the PQ quality problem. In particular, the leading contributions to the axion potential arise from $d = 9$ operator which, if Planck-suppressed, imply that the axion must be relatively heavy, $m_a \gtrsim 0.02$ eV. Disregarding instead the PQ quality problem in its standard formulation in terms of power-like effective operators, the axion can be as light as about 7×10^{-8} eV, as implied by the upper bound on f_a given by the seesaw scale. What emerges is an *intermediate-scale* axion, that is quite different from the usual *GUT-scale* axion, $m_a \lesssim 10^{-9}$ eV, obtained by dominantly breaking the PQ symmetry at the GUT scale. In the latter case, the $\mathrm{U}(1)_{\mathrm{PQ}}$ breaking $\mathrm{SO}(10)$ representations, i.e. 45, 54 or 210, have a trivial $\mathrm{SO}(10)$ center and thus the PQ symmetry does not arise automatically. From this perspective, the GUT-scale axion and in general $\mathrm{SO}(10) \times \mathrm{U}(1)_{\mathrm{PQ}}$ models in which the PQ breaking is related to the VEV of a complex 45, 54 or 210 (even at intermediate mass scales, see e.g. [61–66]) appear to be theoretically less motivated, since it is more difficult to obtain an automatic $\mathrm{U}(1)_{\mathrm{PQ}}$ (compared to models in which the $\mathrm{U}(1)_{\mathrm{PQ}}$ is broken by $\mathrm{SO}(10)$ representations with non-trivial $B - L$ [15, 21, 27]).

Some of the considerations above extend as well to other GUT groups. Recent non-GUT constructions based on $\mathrm{SU}(\mathcal{N})$ gauge dynamics [67, 68], showed that it is possible to protect the PQ symmetry via the $\mathbb{Z}_{\mathcal{N}}$ center of $\mathrm{SU}(\mathcal{N})$ up to operators of dimension $\mathcal{N}$. One could try to follow a similar path for GUTs. $\mathrm{SU}(5)$ is broken to the SM in

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[11]The original work of Georgi et al. [3] based on $\mathrm{SU}(9)$ stands out as possibly the only successful one, that relies only on GUT dynamics. Ref. [13] proposed a SUSY $E_6 \times \mathrm{U}(1)'$ gauge model, where the $\mathrm{U}(1)_{\mathrm{PQ}}$ arises accidentally and it is protected against higher-dimensional operators. However, as observed in [59], the solution to the PQ quality problem is spoiled by soft SUSY breaking effects. Other approaches to the PQ quality problem based on composite dynamics such as [60], can be made compatible with GUTs.

one step, either via a 24 or a 75, both of which have however a trivial center (in fact, all SU(5) representations containing a SM-singlet direction have also zero quintality), thus making structurally difficult to get an automatic $U(1)_{PQ}$ in SU(5) where the PQ symmetry is broken via a complex adjoint [69–72]. Non-minimal GUTs such as SU(6) or E_6 are more promising in this respect, since they feature representations with SM-singlet directions (which can achieve large VEVs compared to the electroweak scale) and at the same time transform non-trivially under their respective $\mathbb{Z}_6$ and $\mathbb{Z}_3$ centers. Pati-Salam instead looks much alike SO(10). For instance, the models discussed in Refs. [73, 74] feature a PQ breaking representation that transforms trivially under the $\mathbb{Z}_4$ center of $SU(4)_{PS}$, and hence the $U(1)_{PQ}$ is not protected. Following a similar path as in the present work, an accidental Pati-Salam axion could arise by breaking the PQ symmetry with representations that have non-trivial $B-L$ quantum numbers (e.g. a 4 and a 10) and transform non-trivially under flavour.

Finally, it is remarkable that if flavour has to play a role for the $U(1)_{PQ}$ to arise accidentally, three generations are indeed quite special, since for example $n_g = 1, 2, 4$ would have not worked [15, 16]. So, ironically, $n_g = 3$ could have been responsible for making the CKM phase physical and at the same time washing out CP violation from strong interactions.

Acknowledgments

A   Low-energy axion couplings

In this Appendix we provide a derivation of low-energy SO(10) axion couplings to SM matter fields,[12] that requires the inclusion of electroweak VEVs for the identification of the physical axion field. Table 2 summarizes the global (PQ) and gauge ($B-L$ and Y) charges of the SO(10) sub-multiplets which host the axion as an angular component (this requires a complex scalar with a $Q = T_L^3 + Y = 0$ neutral component), i.e.

$$\Phi_i \supset \frac{V_i}{\sqrt{2}} e^{i\frac{a_i}{V_i}} \,, \tag{A.1}$$

with $\Phi = \{\Delta^{\overline{126}}, \delta^{\overline{126}}, H_d^{16}, H_u^{10}, H_d^{10}, H_u^{\overline{126}}, H_d^{\overline{126}}, \Delta_L^{\overline{126}}\}$ spanning over the fields in Table 2.

The low-energy PQ symmetry, with charge q, is a linear combination of the UV PQ charges and two broken Cartan generators, which can be chosen as $B-L$ and Y, namely

$$q = c_1 PQ + c_2(B - L) + c_3 Y \,. \tag{A.2}$$

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[12]See also Ref. [65] for a similar approach to SO(10) axion couplings. Note that the latter paper did not consider the specific SO(10) model studied here.

	$\Delta^{\overline{126}}$	δ^{16}	H_d^{16}	H_u^{10}	H_d^{10}	$H_u^{\overline{126}}$	$H_d^{\overline{126}}$	$\Delta_L^{\overline{126}}$
PQ	-2	-1	-1	-2	-2	-2	-2	-2
$B-L$	-2	1	-1	0	0	0	0	2
Y	0	0	$-1/2$	$1/2$	$-1/2$	$1/2$	$-1/2$	1

Table 2: Global (PQ) and local ($B-L$ and Y) charges of the SO(10) sub-multiplets hosting the physical axion field.

It can be shown, analogously to the cases discussed in Sect. 4, that in order to match $\mathrm{U}(1)_{\mathrm{PQ}}$ anomalies in terms of UV and IR charges, $c_1 = 1$, which we henceforth assume in the following. Given the PQ current $J_\mu^{\mathrm{PQ}} = \sum_i q_i V_i \partial_\mu a_i$ The canonical axion field is defined as

$$a = \frac{1}{V} \sum_i q_i V_i a_i, \qquad V^2 = \sum_i q_i^2 V_i^2, \tag{A.3}$$

so that $J_\mu^{\mathrm{PQ}} = V \partial_\mu a$ and, compatibly with the Goldstone theorem, $\langle 0 | J_\mu^{\mathrm{PQ}} | a \rangle = i V p_\mu$. Under a PQ transformation $a_i \to a_i + \alpha q_i V_i$ the axion field transforms as $a \to a + \alpha V$. Inverting the orthogonal transformation in Eq. (A.3), one readily obtains the projection of the angular modes on the axion field:

$$a_i \to q_i V_i \frac{a}{V}. \tag{A.4}$$

To determine the q charges we proceed as follows. First, we require the orthogonality between the axion current and the gauge currents $J_{B-L} = \sum_i (B-L)_i V_i \partial_\mu a_i$ and $J_Y = \sum_i Y_i V_i \partial_\mu a_i$ (to avoid kinetic mixings of the axion field with massive gauge bosons). This yields, respectively:

$$-2 q_{\overline{126}} V_{\overline{126}}^2 + q_{16} V_{16}^2 - q_{H_d^{16}} (v_{16}^d)^2 = 0, \tag{A.5}$$

$$-\frac{1}{2} q_{H_d^{16}} (v_{16}^d)^2 + \frac{1}{2} q_{H_u^{10}} (v_{10}^u)^2 - \frac{1}{2} q_{H_d^{10}} (v_{10}^d)^2 + \frac{1}{2} q_{H_u^{\overline{126}}} (v_{\overline{126}}^u)^2 - \frac{1}{2} q_{H_d^{\overline{126}}} (v_{\overline{126}}^d)^2 = 0. \tag{A.6}$$

Second, by decomposing the invariants with non-trivial global re-phasings in the SO(10) scalar potential (cf. Eqs. (2.5)–(2.5)):[13]

$$\phi_{16}^2 \phi_{10}^\star \supset H_d^{16} \delta^{16} (H_d^{10})^\star, \tag{A.7}$$

$$\phi_{10}^2 \phi_{\overline{126}}^{\star 2} \supset (H_u^{10})^2 (H_u^{\overline{126}})^{2\star}, (H_d^{10})^2 (H_d^{\overline{126}})^{2\star}, \tag{A.8}$$

one obtains the following extra constraints on q charges (recall Eq. (A.1) and Eq. (A.4)):

$$q_{H_d^{16}} + q_{16} - q_{H_d^{10}} = 0, \tag{A.9}$$

$$q_{H_u^{10}} - q_{H_u^{\overline{126}}} = q_{H_d^{10}} - q_{H_d^{\overline{126}}} = 0, \tag{A.10}$$

from which we see that the (electroweak) q charges of ϕ_{10} and $\phi_{\overline{126}}$ are aligned.

To close the system of linear equations, in order to extract the q charges, it is necessary to include also the matching between UV and IR PQ charges in Eq. (A.2) for all the scalar fields.

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[13]In the following, we set to zero the $SU(2)_L$ triplet VEV, $v_{\Delta_L^{\overline{126}}}$. Keeping the latter, axion couplings to SM charged fermions receive safely negligible corrections of the order of $v_{\Delta_L^{\overline{126}}}^2 / v^2 \ll 1$ (with v denoting the electroweak scale) [75].

Here, we present the result in the simplifying limit $v_{H_d^{16}} \to 0$ (although for the final expressions of axion couplings we will keep $v_{H_d^{16}} \neq 0$), that is

$$c_2 = \frac{V_{16}^2 - 4V_{\overline{126}}^2}{V_{16}^2 + 4V_{\overline{126}}^2}, \tag{A.11}$$

$$c_3 = 4\frac{v_u^2 - v_d^2}{v^2}, \tag{A.12}$$

$$q_{\overline{126}} = -\frac{4V_{16}^2}{V_{16}^2 + 4V_{\overline{126}}^2}, \tag{A.13}$$

$$q_{16} = -\frac{8V_{\overline{126}}^2}{V_{16}^2 + 4V_{\overline{126}}^2}, \tag{A.14}$$

$$q_{H_d^{16}} = \frac{8V_{\overline{126}}^2(v_d^2 - v_u^2) - 4V_{16}^2 v_u^2}{(V_{16}^2 + 4V_{\overline{126}}^2)v^2}, \tag{A.15}$$

$$q_{H_u^{10}} = q_{H_u^{\overline{126}}} = -4\frac{v_d^2}{v^2}, \tag{A.16}$$

$$q_{H_d^{10}} = q_{H_d^{\overline{126}}} = -4\frac{v_u^2}{v^2}, \tag{A.17}$$

$$V^2 = 16\left(\frac{V_{16}^2 V_{\overline{126}}^2}{V_{16}^2 + 4V_{\overline{126}}^2} + \frac{v_u^2 v_d^2}{v^2}\right). \tag{A.18}$$

where we defined $v_u^2 = v_{H_u^{10}}^2 + v_{H_u^{\overline{126}}}^2$, $v_d^2 = v_{H_d^{10}}^2 + v_{H_d^{\overline{126}}}^2$ and $v^2 = v_u^2 + v_d^2$.

To compute low-energy axion couplings to SM charged fermions, we decompose $\psi_{16}\psi_{16}\phi_{10} \supset qu^c H_u^{10} + qd^c H_d^{10} + \dots$. The axion is removed from the Yukawa interaction via a family universal axion-dependent transformation (in Weyl notation and suppressing generation indices)

$$u \to e^{-iq_{H_u^{10}}\frac{a}{2V}}u, \qquad\qquad u^c \to e^{-iq_{H_u^{10}}\frac{a}{2V}}u^c, \tag{A.19}$$

$$d \to e^{-iq_{H_d^{10}}\frac{a}{2V}}d, \qquad\qquad d^c \to e^{-iq_{H_d^{10}}\frac{a}{2V}}d^c, \tag{A.20}$$

(equivalently, in Dirac notation, $u \to e^{iq_{H_u^{10}}\gamma_5\frac{a}{2V}}u$, etc.), and analogously for charged leptons. The same rotation also removes the axion from the other Yukawa interaction, $\psi_{16}\psi_{16}\phi_{\overline{126}}^\star$, due to the alignment of the (electroweak) PQ charges of ϕ_{10} and $\phi_{\overline{126}}$.

The SM chiral fermion transformation above generate the anomalous terms

$$\delta\mathscr{L} = \frac{\alpha_s N}{4\pi}\frac{a}{V}G\tilde{G} + \frac{\alpha E}{4\pi}\frac{a}{V}F\tilde{F}, \tag{A.21}$$

with the anomaly factors

$$N = -n_g T(3)\left(2\frac{q_{H_u^{10}}}{2} + 2\frac{q_{H_d^{10}}}{2}\right) = 6, \tag{A.22}$$

$$E = -n_g\left(q_{H_u^{10}}3(2/3)^2 + q_{H_d^{10}}3(-1/3)^2 + q_{H_d^{10}}(-1)^2\right) = 16, \tag{A.23}$$

using $n_g = 3$, $T(3) = 1/2$, etc. So, in particular, upon choosing the standard normalization of the axion-gluon coupling in terms of the Lagrangian term

$$\frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G}, \tag{A.24}$$

the axion decay constant reads (using Eq. (A.18) with the sign fixed as in Eq. (4.20) for the case of no electroweak VEVs)

$$f_a = \frac{V}{2N} = -\frac{1}{3}\sqrt{\frac{V_{16}^2 V_{\overline{126}}^2}{V_{16}^2 + 4V_{\overline{126}}^2} + \frac{v_u^2 v_d^2}{v^2}}\,, \tag{A.25}$$

while $E/N = 8/3$, which enters the axion-photon coupling [76]

$$C_{a\gamma} = E/N - 1.92(4)\,, \tag{A.26}$$

defined in terms of the Lagrangian term $\mathscr{L} \supset \frac{\alpha}{8\pi f_a} C_{a\gamma} a F\tilde{F}$.

On the other hand, the variation of the fermion kinetic terms yields (in Dirac notation)

$$\delta(\overline{u}i\slashed{\partial}u) = -q_{H_u^{10}}\frac{\partial_\mu a}{2V}\overline{u}\gamma^\mu\gamma_5 u\,, \tag{A.27}$$

$$\delta(\overline{d}i\slashed{\partial}d) = -q_{H_d^{10}}\frac{\partial_\mu a}{2V}\overline{d}\gamma^\mu\gamma_5 d\,, \tag{A.28}$$

$$\delta(\overline{e}i\slashed{\partial}e) = -q_{H_d^{10}}\frac{\partial_\mu a}{2V}\overline{e}\gamma^\mu\gamma_5 e\,. \tag{A.29}$$

Hence, defining the axion coupling to SM fermions via $\mathscr{L} \supset \frac{\partial_\mu a}{2f_a} c_f \overline{f}\gamma_\mu\gamma_5 f$, and replacing $f_a = V/(2N)$, we find the (family universal) couplings

$$c_u = -\frac{q_{H_u^{10}}}{2N} = \frac{1}{3}\frac{V_{16}^2(v_d^2 + v_{H_d^{16}}^2) + 2V_{\overline{126}}^2(2v_d^2 + v_{H_d^{16}}^2) + v_d^2 v_{H_d^{16}}^2}{(V_{16}^2 + 4V_{\overline{126}}^2)(v^2 + v_{H_d^{16}}^2) + v^2 v_{H_d^{16}}^2} \approx \frac{v_d^2}{3v^2}\,, \tag{A.30}$$

$$c_d = c_e = -\frac{q_{H_u^{10}}}{2N} = \frac{1}{3}\frac{V_{16}^2 v_u^2 + 2V_{\overline{126}}^2(2v_u^2 + v_{H_d^{16}}^2) + v_u^2 v_{H_d^{16}}^2}{(V_{16}^2 + 4V_{\overline{126}}^2)(v^2 + v_{H_d^{16}}^2) + v^2 v_{H_d^{16}}^2} \approx \frac{v_u^2}{3v^2}\,, \tag{A.31}$$

where in the last step we provided simplified expressions in the $v_{H_d^{16}} \to 0$ limit.

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