Kinetic Simulation of Nonrelativistic Perpendicular Shocks of Young Supernova Remnants. IV. Electron Heating

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Kinetic simulation of nonrelativistic perpendicular shocks of young supernova remnants. IV. Electron heating.

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ABSTRACT

High Mach number collisionless shocks are found in planetary systems and supernova remnants (SNRs). Electrons are heated at these shocks to the temperature well above the Rankine-Hugoniot prediction. However processes responsible for electron heating are still not well understood. We use a set of large-scale Particle-In-Cell simulations of non-relativistic shocks in high Mach number regime to clarify the electron heating processes. The physics of these shocks is defined by ion reflection at the shock ramp. Further interaction of the reflected ions and the upstream plasma excites electrostatic Buneman and two-stream ion-ion Weibel instabilities. Electrons are heated via shock surfing acceleration, the shock potential, magnetic reconnection, stochastic Fermi scattering and the shock compression. The main contributor is the shock potential. Magnetic field lines are tangled due to the Weibel instability, which allows the parallel electron heating by the shock potential. The constrained model of the electron heating predicts the ion-to-electron temperature ratio within observed values at SNR shocks and in Saturn's bow shock.

Keywords: acceleration of particles, instabilities, ISM – supernova remnants, methods – numerical, plasmas, shock waves

1. INTRODUCTION

Collisionless shocks are commonly observed in such places as planetary systems, supernova remnants (SNRs) and jets of active galactic nuclei when plasmas move with super-sonic velocities. In such flows, collective particle interactions form a shock layer on kinetic plasma scales much smaller than the collisional mean free path. In the shock transition part of the bulk kinetic energy is converted into energies of thermal particles and electromagnetic fields through wave-particle interactions. The microphysics of these processes is still not fully understood, especially in high Mach number regime.

High Mach number shocks are supercritical, which means that the upstream kinetic energy can not be entirely dissipated via Joule heating (Ohmic dissipation). If Mach number is above a certain limit ($M_s \gtrsim 2.76$, Marshall 1955) part of the kinetic energy is dissipated via ion reflection by the shock potential. In quasi-perpendicular shocks the interaction of reflected ions with the upstream plasma leads to excitation of two-stream instabilities. The electrostatic Buneman instability (Buneman 1958) is excited at the leading edge of the shock foot and it results from interaction of the hot reflected ions and the cold upstream electrons. Deeper in the shock foot the interaction of reflected and upstream ions results in the Weibel instability (Fried 1959). These instabilities shape the shock structure forming the shock foot with electrostatic Buneman waves and Weibel filamentary structures, strongly turbulent ramp and overshoot regions, and the shock downstream filled with thermalized plasma.

This paper is the fourth in a series of works focusing on the analysis of high-Mach-number perpendicular shock physics by dint of Particle-in-Cell (PIC) simulations. Previously (Bohdan et al. 2019a,b, 2020, hereafter Papers I, II and III) we discussed processes responsible for a production of nonthermal electrons, namely

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shock surfing acceleration (SSA, Paper I), its influence on the nonthermal downstream electron population (Paper II) and magnetic reconnection (Paper III). In these papers we have defined how acceleration efficiencies depend on the sonic and Alfvénic Mach numbers and ion-to-electron mass ratio (hereafter referred to as the mass ratio). We found that regardless of the Mach number SSA negligibly contributes to the nonthermal electron population in perpendicular shocks and realistic mass ratio. Magnetic reconnection appears to be very active and operates as efficient electron accelerator in shocks with higher Mach numbers, however the nonthermal electron production efficiency does not depend on the mass ratio used in simulations.

Here we study electron heating processes and magnetic field amplification at high Mach number shocks with $M_A \gtrsim 20$. It was already reported that PIC simulations of low Mach number shocks (Tran & Sironi 2020) demonstrate good consistency of simulations and in-situ measurements of the Earth's (Schwartz et al. 1988) and Saturn's (Masters et al. 2011) bow shocks. The super-adiabatic electron heating (above the limit predicted by the Rankine-Hugoniot condition) is associated with the cross-shock potential and interaction with ion-scale waves in the shock transition. However there is a lack of understanding how electrons are heated in the high Mach number regime.

If no energy exchange between electrons and ions is expected in the shock transition region, then the downstream electron temperature is described by the Rankine-Hugoniot condition. Observations of Balmer-dominated shocks SNRs (Rakowski 2005; van Adelsberg et al. 2008; Ghavamian et al. 2013) and in-situ measurements of Saturn's bow shock (Masters et al. 2011) reveal the downstream electron-to-ion temperature ratio (hereafter referred to as the temperature ratio) to be in a range of $T_e/T_i \approx 0.05 - 0.5$, which is well above the Rankine-Hugoniot prediction of $T_e/T_i \approx m_e/m_i$. In our previous studies (Bohdan et al. 2017, Paper II) we reported that the electron temperature observed in PIC simulations is considerably higher than predicted by the Rankine-Hugoniot conditions, but at the same time an energy equipartition between ions and electrons is not reached and electrons are colder than ions. Similar results were demonstrated in the PIC simulations of Kato & Takabe (2010), in which $T_e/T_i \approx 0.33$. These results suggest that super-adiabatic electron heating occurs in high Mach number shocks.

We have previously discussed that electrons can be accelerated to relativistic energies via a number of mechanisms: SSA (Shimada & Hoshino 2000; Hoshino & Shimada 2002), magnetic reconnection (Matsumoto

et al. 2015), stochastic Fermi-like acceleration (SFA, Bohdan et al. 2017), and stochastic shock drift acceleration (Matsumoto et al. 2017). In addition to accelerating particles, these processes cause some amount of electron heating, however the relative contribution from each is yet to be determined. Electrons also can be heated via the shock potential which is widely discussed in low Mach number regime (Thomsen et al. 1987; Chen et al. 2018; Tran & Sironi 2020).

Electron heating processes are mediated by electromagnetic effects over electron temporal and spatial scales that are much shorter than the ion gyroradius or the ion skin depth. Magnetohydrodynamic (MHD) and hybrid simulations can not describe physical processes on such small scales. Thus, fully kinetic simulations are needed for a proper description of the electron physics. In this paper we constrain the heating model which aims to predict the temperature ratio at high Mach number shocks. The paper is organized as follows. We present a short description of simulation setup in Section 2. The results are presented in Section 3. The discussion and summary are in Section 4.

2. SIMULATION SETUP

Simulations are performed with the modified version of the relativistic electromagnetic TRISTAN code (Buneman 1993) with MPI-based parallelization (Niemiec et al. 2008; Wieland et al. 2016) and the particle sorting optimization (Dorobisz et al. 2018). The Vay solver (Vay 2008) is used to update particle positions. The triangular-shape-cloud particle shapes (the second-order approximation) and Friedman (1990) filter for electric and magnetic fields are used to suppress the numerical grid-Cherenkov short-wave radiation. To perform shock simulations we use a 2D3V code configuration which follows two spatial coordinates and all three components of particle velocities and electromagnetic fields.

The flow-flow simulation setup is used to initialize shocks. Note that the same setup was used in our previous works (Bohdan et al. (2017), Papers I, II and III) and more detailed description can be found in Paper I. The simulation setup assumes collision of two counter-streaming electron-ion plasma flows which leads to the formation of two shocks separated by a contact discontinuity. Here we refer to shocks as the *left* and the *right* shocks. The absolute values of the beam velocities are equal, $v_L = v_R = v_0 = 0.2c$. Plasma beams are equal in density but their temperatures differ by a factor of 1000. Thus the *electron* plasma beta (the ratio of the electron plasma pressure to the magnetic pressure) for

Table 1. Simulation Parameters

Runs	m_i/m_e	M_A	M_s		β_e	
			*1	*2	*1	*2
A1, A2	50	22.6	1104	35	$5 \cdot 10^{-4}$	0.5
B1, B2	100	31.8	1550	49	$5 \cdot 10^{-4}$	0.5
C1, C2	100	46	2242	71	$5 \cdot 10^{-4}$	0.5
D1, D2	200	32	1550	49	$5 \cdot 10^{-4}$	0.5
E1, E2	200	44.9	2191	69	$5 \cdot 10^{-4}$	0.5
F1, F2	400	68.7	3353	106	$5 \cdot 10^{-4}$	0.5

NOTE—Parameters of simulation runs described in this paper. Listed are: the ion-to-electron mass ratio m_i/m_e, and Alfvénic, M_A, and sonic Mach numbers, M_s, the electron plasma beta, β_e. The last two values are shown separately for the *left* (runs *1) and the *right* (runs *2) shock. All runs use the in-plane magnetic field configuration $\varphi = 0^o$.

the left beam is $\beta_{e,L} = 5 \cdot 10^{-4}$ and $\beta_{e,R} = 0.5$ for the right beam.

The large scale upstream magnetic field makes an angle $\varphi = 0^o$ with the simulation plane (the so-called *in-plane* configuration). Note, that such simulations give us a good representation of the 3D shock physics (Bohdan et al. 2017; Matsumoto et al. 2017). The adiabatic index is thus $\Gamma = 5/3$. The resulting shock speed equals $v_{sh} = 0.263c$ in the *upstream* reference frame. The Alfvén velocity is defined as $v_A = B_0/\sqrt{\mu_0(N_e m_e + N_i m_i)}$, where μ_0 is the vacuum permeability, N_i and N_e are the ion and the electron number densities, and B_0 is the far-upstream magnetic-field strength. The sound speed reads $c_s = (\Gamma k_B T_i/m_i)^{1/2}$, where k_B is the Boltzmann constant and T_i is the ion temperature defined as $k_B T_i = m_i v_{th}^2/2$, where v_{th} is defined as the most probable speed of the upstream plasma particles in the upstream reference frame. The Alfvénic, $M_A = v_{sh}/v_A$, and sonic, $M_s = v_{sh}/c_s$, Mach numbers of the shocks are defined in the conventional *upstream* reference frame. Note that the *sonic* Mach number, M_s, of the two shocks differ by a factor of $\sqrt{1000} \simeq 30$ because of the difference in β_e.

The ratio of the electron plasma frequency, $\omega_{pe} = \sqrt{e^2 N_e/\epsilon_0 m_e}$, to the electron gyrofrequency, $\Omega_e = eB_0/m_e$, is in the range of $\omega_{pe}/\Omega_e = 8.5 - 17.3$. Here, e is the electron charge, and ϵ_0 is the vacuum permittivity. The electron skin depth in the upstream plasma is common for all runs and equals $\lambda_{se} = 20\Delta$, where Δ is the size of grid cells. As the unit of length we use the ion skin depth, $\lambda_{si} = \sqrt{m_i/m_e}\lambda_{se}$. As the time-step $\delta t = 1/40\,\omega_{pe}^{-1}$ is used. The time scales are given in terms of the upstream ion Larmor frequency, Ω_i, where $\Omega_i = eB_0/m_i$. The number density in the far-upstream region is 20 particles-per-cell for each species.

In the following sections, we discuss the results of six large-scale numerical experiments (runs A–F), featuring

in total twelve physically distinct simulated shocks. We therefore refer to each of these shock cases as to a separate simulation run, and label the shocks in the left plasma (cold shock, $\beta_{e,L} = 5 \cdot 10^{-4}$) with *1, and the right shocks with *2 (warm shock, $\beta_{e,R} = 0.5$). The derived parameters of the simulation runs described in this paper are listed in Table 1. If the result is valid for both cold and warm shocks we use only a letter without a digit.

These simulations cover a wide range of mass ratios ($m_i/m_e = 50 - 400$) and Alfvénic Mach numbers ($M_A = 22.6 - 68.7$), thus permitting a thorough investigation of the influence of these parameters on the electron heating processes. The goal of this paper is to clarify these dependencies in order to give a prediction for realistic high Mach number shocks.

3. RESULTS

In Paper II we showed that the downstream electron temperature is approximately proportional to the mass ratio, therefore a similar fraction of the upstream ion kinetic energy is transferred to thermal electrons in the simulations discussed here. It also results in similar temperature ratios in all runs (see Table 2).

Here we want to constrain the electron heating model based on simulation data. First, we identify processes responsible for the electron heating. Next, using known properties of mechanisms and particle data, we determine how individual heating efficiencies depend on the mass ratio (m_i/m_e), Alfvénic Mach number (M_A) and the shock velocity (v_{sh}). Finally, using energy gains from individual heating processes, we calculate numerical coefficients for them in order to confirm the heating model and simulation results.

3.1. *Electron energy evolution*
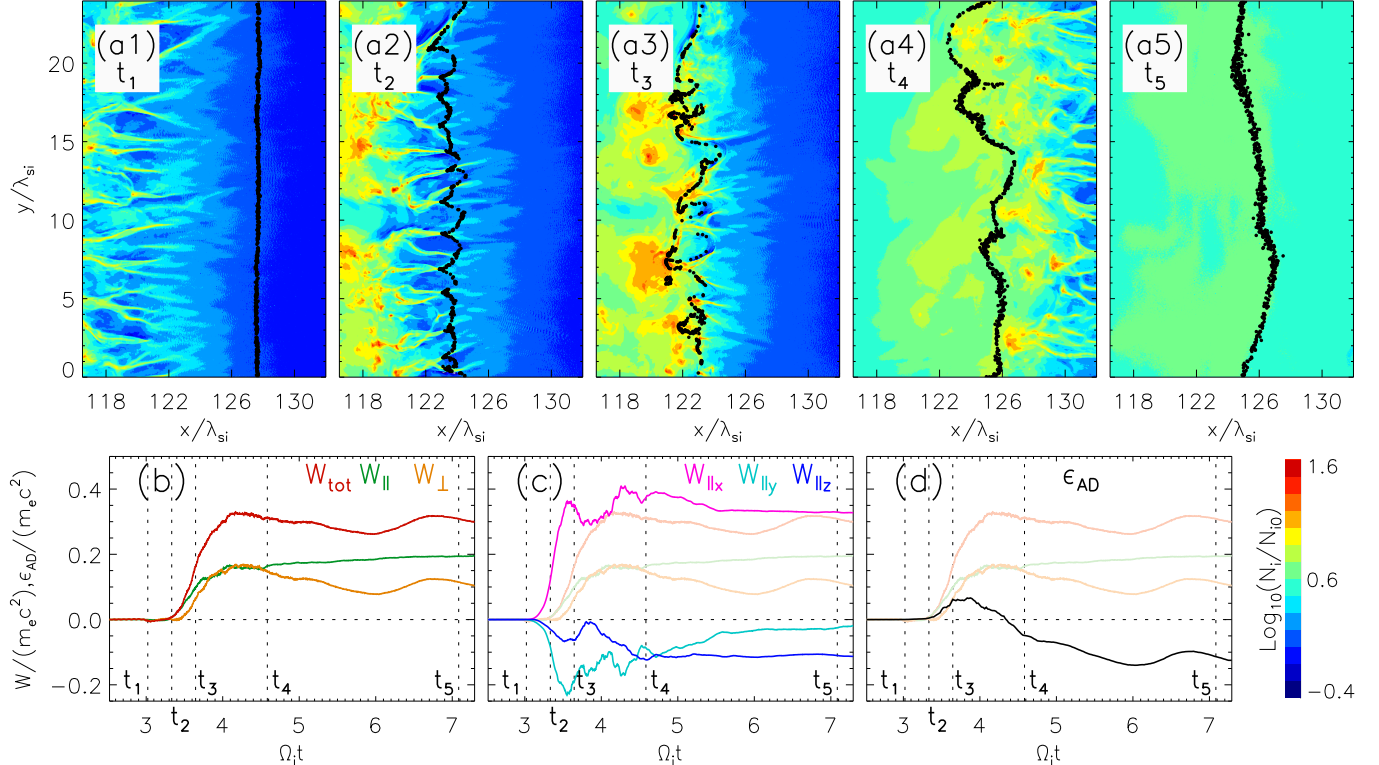
Figure 1. Evolution of the work components of traced electrons for run B2. Panels (a1)(a5) present the positions of traced electrons (black dots) at the time t1t5 on density maps of the shock region. Panel (b): the temporal evolution of the total work done by electric field (W_{tot}, red line), the parallel work component ($W_\parallel$, green line) and the perpendicular work component ($W_\perp$, orange line). Panel (c): magenta, light blue and dark blue lines are x- ($W_{\parallel x}$), y- ($W_{\parallel y}$) and z-components ($W_{\parallel z}$) of the parallel work. Panel (d): evolution of the adiabatic component (W_{AD}, see Section 3.6). Faded curves in panel (c) and (d) represent lines from panel (a). The vertical dashed lines in panels (b), (c) and (d) are time markers for $t_1 - t_5$, the horizontal dashed line is the zero level.

Particle energies change due to work, W_{tot}, done by the electric field. For identification of heating process we split this work into two parts, namely, the work done in the directions parallel, $W_\parallel$, and perpendicular, $W_\perp$, to the local magnetic field. Therefore the electron kinetic energy is calculated as

$$\varepsilon = W_{\text{tot}} + \varepsilon_0 = W_\parallel + W_\perp + \varepsilon_0 \ , \qquad (1)$$

where ε_0 is an initial kinetic energy, $W_\parallel = \int_{t'=0}^{t} eE_\parallel v_\parallel dt'$ and $W_\perp = \int_{t'=0}^{t} eE_\perp v_\perp dt'$. All variables denoted with $\perp$ or $\parallel$ are vector components perpendicular and parallel to the local magnetic field, respectively. Taking into account that the electron downstream temperature is approximately proportional to the mass ratio, we derive energy incomes of individual processes in terms of the ion upstream kinetic energy, $m_i v_{\text{sh}}^2$. Note, that energy incomes strongly vary during shock self-reformation cycles, thus in our calculation we use values averaged over one reformation cycle.

To demonstrate the main stages of the electron heating we use the energy history of electrons selected in the shock upstream of run B2 (Fig. 1). Electrons reside at the leading edge of the shock foot with electrostatic Buneman waves at t_1 (panel (a1)), the Weibel instability region of the foot at t_2 (panel (a2)), the ramp region where magnetic reconnection occurs at t_3 (panel (a3)), the turbulent overshoot at t_4 (panel (a4)) and the downstream at t_5 (panel (a5)).

The total energy change of the electron population is represented by W_{tot} in Figure 1(b). The energy of the electrons starts to grow when they enter the shock foot at t_2 (panel (a2)). Figure 1(b) shows that the energy reaches the maximal value at $\Omega_i t \approx 3.7$ when particles reside in the shock overshoot, and then it gradually decreases when electrons are advected to the shock downstream. At the final step about 70% of the total energy is produced by parallel heating and the remaining part has a perpendicular origin.

We have identified five processes responsible for the electron heating: SSA via interaction with Buneman waves at the leading edge of the shock foot, a heating by the shock potential at the shock foot and the ramp, mag-

Table 2. Thermal properties of the downstream plasma

Run	$\dfrac{k_\mathrm{B}T_\mathrm{e}}{m_\mathrm{e}c^2}$ (Paper II)	$\dfrac{k_\mathrm{B}T_\mathrm{e}}{m_\mathrm{e}c^2}$ (eq. 12)	$T_\mathrm{e}/T_\mathrm{i}$	$T_\mathrm{e}/T_\mathrm{i}$ (eq. 15)
A1	0.107 ± 0.004		0.17 ± 0.04	
A2	0.091 ± 0.004	0.1 ± 0.03	0.14 ± 0.03	0.16 ± 0.05
B1	0.216 ± 0.004		0.18 ± 0.03	
B2	0.183 ± 0.004	0.19 ± 0.05	0.15 ± 0.03	0.16 ± 0.05
C1	0.253 ± 0.002		0.19 ± 0.03	
C2	0.217 ± 0.003	0.22 ± 0.07	0.17 ± 0.02	0.19 ± 0.06
D1	0.332 ± 0.024		0.15 ± 0.03	
D2	0.28 ± 0.003	0.34 ± 0.08	0.11 ± 0.01	0.15 ± 0.04
E1	0.394 ± 0.005		0.16 ± 0.02	
E2	0.368 ± 0.009	0.38 ± 0.1	0.16 ± 0.02	0.17 ± 0.05
F1	0.765 ± 0.035		0.18 ± 0.04	
F2	0.732 ± 0.02	0.8 ± 0.2	0.18 ± 0.04	0.18 ± 0.06

Note—Electron temperature and electron-to-ion temperature ratio in the shock downstream. Second column - the downstream electron temperatures listed in Paper II. Third column - the downstream electron temperature calculated via equation 12. Fourth column - the electron-to-ion temperature ratio observed in simulations. Fifth column - the electron-to-ion temperature ratio calculated via equation 15.

netic reconnection in the shock ramp, SFA in the shock ramp and the overshoot, and an adiabatic heating which follow plasma compression and decompression.

3.2. *Heating via SSA*

The Buneman instability is excited because of the relative motion of the upstream electrons to the reflected ions, with the energy budget available from this process approximately given by $E_\mathrm{SSA,avail} \approx 2m_\mathrm{e}v_\mathrm{sh}^2 \simeq 0.15m_\mathrm{e}c^2$ (Amano & Hoshino 2009). Later this energy is converted into electron heating, therefore it can be used as an upper-limit for the energy provided to electrons via SSA. In Paper II we discussed that only about 1% of the available energy is transferred to the *nonthermal* high energy electrons. However, in this case we are interested in the *total* energy gain, which is about $0.1E_\mathrm{SSA,avail}$ for the selected population of electrons. Thus we estimate the energy gain via SSA as

$$\varepsilon_\mathrm{SSA} \approx \alpha_\mathrm{SSA}m_\mathrm{e}v_\mathrm{sh}^2 \; = \alpha_\mathrm{SSA}\frac{m_\mathrm{e}}{m_\mathrm{i}}m_\mathrm{i}v_\mathrm{sh}^2 \;, \qquad (2)$$

where α_SSA depends on the magnetic field configuration of the simulation and it equals $\alpha_\mathrm{SSA,0} = 0.2 \pm 0.1$ for the in-plane case , where the subscript '0' denotes the angle of the initial magnetic field to the simulation plane. For

the selected electron set the heating via SSA takes place at $\Omega_\mathrm{i}t \approx 3$, it is not well visible in Figure 1 because of the very small efficiency compared to other processes. Note that SSA contributes to the perpendicular work component. It is also easy to see from equation 2 that the heating due to SSA may contribute a substantial amount of energy only in case of small mass ratios.

Previously we demonstrated in Bohdan et al. (2017); Matsumoto et al. (2017) that the realistic 3D SSA efficiency is reproduced in simulations with an out-of-plane magnetic field configuration ($\varphi = 90^o$) where Buneman waves are well captured in the simulation plane. In runs with such a field configuration, the heating efficiency is higher, with around 50% of the available energy transferred and $\alpha_\mathrm{SSA,90} = 1 \pm 0.5$. This value we put in the heating model of realistic shocks.

3.3. *Heating by the shock potential*

In our simulations electrons are magnetized, thus they follow its initial magnetic flux tube and can freely move only along magnetic field lines. Therefore if any plasma disturbance creates regions with aligned magnetic and electric fields (which initially are perpendicular to each other) electrons can be energized via parallel heating. At a shock transition of high Mach number shocks the Weibel instability and magnetic reconnection are responsible for generation of turbulent magnetic fields

In Figure 1(a2), the electron positions show that magnetic field lines are deformed because of additional B_x and B_z components generated by the Weibel instability. In addition to that, the shock potential (E_x) starts to operate in this region. The magnetic field structure is represented by non-propagating linearly polarized magnetic field waves generated by the Weibel instability (Fig. 2). This field is regular with the spatial scale of λ_si. Deeper in the shock, compressions and magnetic reconnection make the magnetic field structure more turbulent and chaotic (Fig. 1(a3)). Therefore an initial upstream field configuration is destroyed in the shock transition, creating suitable conditions for the parallel heating of electrons.

Parallel acceleration for selected portion of electrons occurs in two stages. During the first stage, $\Omega_\mathrm{i}t = 3.1 - 3.6$, electrons reside in the shock foot with regular filamentary magnetic field produced by the Weibel instability. The electron energy increases because of a rapid growth of $W_\parallel$. During the second stage, $\Omega_\mathrm{i}t = 3.6 - 5$, electrons are at the shock ramp-overshoot, where the magnetic field is turbulent, and $W_\parallel$ grows slower than in the first stage. Here we call these parts of the parallel work as the regular component, $\varepsilon_\mathrm{SP,r}$, and the turbulent component, $\varepsilon_\mathrm{SP,t}$, because of the magnetic field struc-
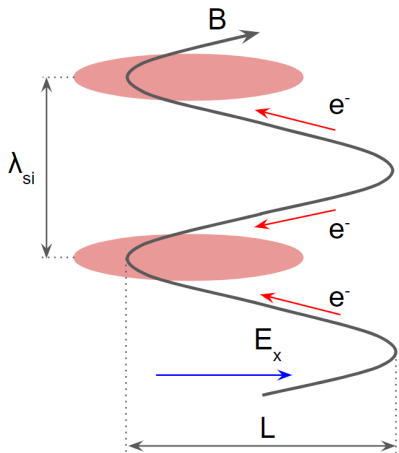
Figure 2. Scheme of electron heating by the shock potential in the shock foot. Red ellipses indicate dense Weibel filaments. Electron motion is designated by red arrows. Blue arrow is the shock potential, E_x, in the shock foot. $\lambda_{\rm si}$ represents the Weibel instability wavelength and L is a measure of a magnetic field line widening because of B_x amplification.

ture at the corresponding regions. Note that 'SP' refers to the shock potential.

While the Weibel instability develops, electrons are moving to the left and forming dense filaments (Fig. 2). During this motion they gain some energy because of the work done by E_x-field (see $W_{\parallel,x}$ in Figure 1(c)). We found that in this region $W_{\parallel,x}$ is always the main energy contributor and for further discussions we ignore $W_{\parallel,y}$ and $W_{\parallel,z}$. Thus the energy income is

$$\varepsilon_{\rm SP,r} \propto eE_x L = eE_x \frac{B_x}{B_y}\frac{\lambda_{\rm si}}{2} \propto eE_x M_{\rm A}\frac{\lambda_{\rm si}}{2} \propto$$
$$\propto \frac{1}{2}m_{\rm i}v_{\rm sh}^2 \ . \tag{3}$$

Here we use two relations derived from simulation data. The saturation level of the normalized B_x generated by the Weibel instability is roughly proportional to the Alfvénic Mach number, so the ratio of $\frac{1}{M_{\rm A}}\frac{B_x}{B_y}$ is constant. We also can express E_x in terms of the upstream fields. The shock potential, $e\phi$, is proportional to the ion upstream kinetic energy. We can therefore estimate the electric field as $E_x \propto \phi/L_{\rm ramp} \propto r_{\rm g} = v_{\rm sh}/\Omega_{\rm i}$, then $E_x = \eta v_{\rm sh}B_0$. In Paper III we discussed that all shocks host very similar conditions at the shock ramp, namely the same fraction of reflected ions, the same relative velocities etc. Therefore the coefficient η appears to be the same in our simulations.

Using the observed parallel energy income during this stage, we can derive the regular heating component:

$$\varepsilon_{\rm SP,r} = \alpha_{\rm SP,r}m_{\rm i}v_{\rm sh}^2 \ , \tag{4}$$

where $\alpha_{\rm SP,r} = 0.012 \pm 0.002$.

The ramp-overshoot region is strongly turbulent thus the scheme presented in Figure 2 is no longer valid. However the thorough investigation of $W_{\parallel}$ at this region demonstrates that the main energy contributor is still the x-component of the electric field. Note that a part of $W_{\parallel}$ during the second stage is explained by magnetic reconnection (see section 3.4) but we account for it in our calculations.

As soon as the magnetic field is turbulent electrons have a possibility to move some distance along the x-axis, which is proportional to the length of the turbulent region, $L \approx L_{\rm ramp} \propto r_{\rm gi} = M_{\rm A}\lambda_{\rm si}$. As we discussed above the average electric field is proportional to the upstream electric field, $E'_x \propto v_{\rm sh}B_0$. The only difference is that this field is opposite (negative) to the shock potential which heats electrons during the first stage. Therefore the chaotic component is also proportional to the upstream ion kinetic energy, $\varepsilon_{\rm SP,t} \propto m_{\rm i}v_{\rm sh}^2$. Using the simulation data we can write that

$$\varepsilon_{\rm SP,t} = \alpha_{\rm SP,t}m_{\rm i}v_{\rm sh}^2 \ , \tag{5}$$

where $\alpha_{\rm SP,t} = 0.013 \pm 0.002$ which is consistent (as error bars overlap) with what we estimated for the regular component. Such a result is not surprising because during this process magnetic field lines are straightening and electrons are redistributed back evenly along magnetic field lines in the presence of a negatively directed E_x. It is essentially the opposite to what happens in the Weibel instability region, albeit in more turbulent and chaotic way.

3.4. *Heating by magnetic reconnection*

In Paper III we discussed how magnetic reconnection accelerates electrons to nonthermal energies. We also found that magnetic reconnection becomes more active in shocks with high Alfvén Mach numbers and the fraction of electrons involved in magnetic reconnection correlates with $M_{\rm A}$. It makes the heating via magnetic reconnection strongly dependent on Alfvén Mach number.

For the selected electron population magnetic reconnection takes place at $\Omega_{\rm i}t \approx 3.5 - 3.6$. In Paper II we discussed that magnetic reconnection can contribute both to $W_{\perp}$ (e.g., Fermi-like processes) and $W_{\parallel}$ (e.g., parallel heating at x-points). For run B2 the average energy of the electrons that have undergone reconnection is $0.21m_{\rm e}c^2$. Heating via SSA and the regular part of the shock potential occurs before magnetic reconnection, thus energy gained by electrons due to magnetic reconnection is $\varepsilon_{\rm MR,sim} = 0.21m_{\rm e}c^2 - \varepsilon_{\rm SSA} - \varepsilon_{\rm SP,r} = 0.115m_{\rm e}c^2$. Taking into account that about 26% (see

Paper III) of electrons are involved in magnetic reconnection, the fraction of the total heating delivered by magnetic reconnection is about 11% in run B2.

The number of vortices generated during magnetic reconnection differs by a factor of 60 between runs A and F (see Table 2. in Paper III). However the heating efficiency grows not so quickly. Magnetic reconnection gives 6% of the total heating in runs A, 11% in runs B, 18% in runs C, 15% in runs D, 17% in runs E, 20% in runs F. It happens because the electron temperature at reconnection sites, T_{MR}, becomes smaller compared to the downstream electron temperature, T_e, in high M_{A} cases. T_{MR}/T_e is about 1 in runs A and 0.5 in runs F.

To approximate the heating via magnetic reconnection we also need to determine boundary conditions. We assume that magnetic reconnection is switched-off when $M_{\mathrm{A}} < 20$, in Paper III we have already demonstrated that in run A the number of reconnection sites is very small and it drops quickly. In the high Mach number limit ($M_{\mathrm{A}} \gtrsim 100$) we expect saturation of magnetic reconnection efficiency at a level which does not exceed 25%, because already in run F almost all electrons are involved in the process and the ratio T_{MR}/T_e will likely stabilize around 0.5. Therefore the heating due to magnetic reconnection we estimate as

$$\varepsilon_{\mathrm{MR}} = \alpha_{\mathrm{MR}} m_{\mathrm{i}} v_{\mathrm{sh}}^2 \ ,$$

$$\alpha_{\mathrm{MR}} = \begin{cases} 0 & , M_{\mathrm{A}} < 20 \\ \alpha_{\mathrm{MR}}^* (M_{\mathrm{A}} - 20)^{\frac{1}{3}} & , 20 < M_{\mathrm{A}} < 100 \\ \alpha_{\mathrm{MR}}^* \, 80^{\frac{1}{3}} & , M_{\mathrm{A}} > 100 \end{cases} \qquad (6)$$

where $\alpha_{\mathrm{MR}}^* = (22 \pm 4) \cdot 10^{-4}$. For the discussion of the heating model in Section 3.7 we assume that magnetic reconnection contributes equally to $W_\perp$ and $W_\parallel$.

3.5. *Heating via SFA*

In Bohdan et al. (2017) we have discussed that high-energy electrons are accelerated via SFA. However this process works regardless of the particle energy and also contribute to the electron heating.

During the time interval of $\Omega_{\mathrm{i}} t \approx 3.6 - 4.5$ electrons are in the shock ramp-overshoot region (Fig. 1(a3),(a4)) and the perpendicular work component grows (Fig. 1(b)). This region is characterized by a turbulent magnetic field hosting appropriate conditions for SFA. When a particle interacts with moving magnetic field structures it is accelerated by a motional electric field, $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$, thus SFA contributes to $W_\perp$.

To estimate energy income in SFA we need to know a number of collisions of electron with scattering centers, N_{coll}, and the average velocity of these centers, v_{mag}.

The energy income via SFA is

$$\frac{\varepsilon_{\mathrm{SFA}}}{\varepsilon_e} \propto \left(\frac{v_{\mathrm{mag}}}{v_e}\right)^2 N_{\mathrm{coll}} = \left(\frac{v_{\mathrm{mag}}}{v_e}\right)^2 \frac{T}{\Delta t} \propto$$
$$\propto \left(\frac{v_{\mathrm{mag}}}{v_e}\right)^2 \frac{r_{\mathrm{gi}} v_e}{v_{\mathrm{sh}} \lambda_{\mathrm{si}}} = M_{\mathrm{A}} \frac{v_{\mathrm{mag}}^2}{v_e v_{\mathrm{sh}}} \ , \qquad (7)$$

where ε_e and v_e is the electron energy and velocity before SFA, $N_{\mathrm{coll}} = T/\Delta t$, where $T \propto r_{\mathrm{gi}}/v_{\mathrm{sh}}$ is the total time an electron spends in the shock transition, $\Delta t \propto \lambda_{\mathrm{si}}/v_e$ is the average time between collisions, where the average distance between scattering centers are defined by Weibel instability with the spatial scale of λ_{si}. In our simulations we found that v_{mag} does not depend on the mass ratio or Mach number and it can be represented as $v_{\mathrm{mag}} = \chi v_{\mathrm{sh}}$, where $\chi \approx 0.25$ was derived from simulations.

Heating by SFA occurs after energization by SSA, the regular part of the shock potential and magnetic reconnection, thus we can assume that initial energy for SFA is $\varepsilon_e = (m_{\mathrm{e}} v_e^2)/2 = \varepsilon_{\mathrm{SSA}} + \varepsilon_{\mathrm{SP,r}} + \varepsilon_{\mathrm{MR}}$. Therefore the energy income from SFA is:

$$\varepsilon_{\mathrm{SFA}} = \alpha_{\mathrm{SFA}} m_{\mathrm{i}} v_{\mathrm{sh}}^2 \ ,$$

$$\alpha_{\mathrm{SFA}} = \alpha_{\mathrm{SFA}}^* M_{\mathrm{A}} \chi^2 \sqrt{\frac{(\alpha_{\mathrm{SSA}} + \alpha_{\mathrm{SP,r}} + \alpha_{\mathrm{MR}}) m_{\mathrm{e}}}{2 m_{\mathrm{i}}}} \qquad (8)$$

where $\alpha_{\mathrm{SFA}}^* = 0.7 \pm 0.2$.

Here we used nonrelativistic formulas because it has a minor effect for cases discussed here and it fits well for nonrelativistic SNR shocks or planetary bow shocks.

3.6. *Adiabatic heating*

In the shock transition plasma is compressed, thus we need to account for a heating because of adiabatic compression. During compression the plasma temperature can be approximated by

$$TV^{\Gamma-1} = const \qquad (9)$$

where V is the volume of the flux tube and $\Gamma = 5/3$ is the adiabatic index. Using the total derivative of equation 9 we estimate the energy income from adiabatic compression as

$$\varepsilon_{\mathrm{AD}} = \sum_n \varepsilon_{\mathrm{AD},n} \ ,$$

$$\varepsilon_{\mathrm{AD},n} = k_{\mathrm{B}} T_n \left(1 - \frac{V_{n+1}}{V_n}\right) \ . \qquad (10)$$

and sum is done over time steps while electrons travel from the upstream to the downstream. Here we use B-field maps to derive the volume of the flux tube and the

traced particle data to estimate electron temperature inside the selected flux tube.

Overall impact of a compression-decompression cycle appears to be negative (Fig. 1(d)). Such result is slightly counterintuitive, however it can happen if an additional heating process operates during a compression-decompression cycle. Initially the electron temperature is low thus the adiabatic heating during compression is also low. However decompression happens when electrons cross the shock overshoot and they are already hot. Therefore the adiabatic cooling is high and in our case it overcomes the adiabatic heating during the compression. It results in overall negative impact of the compression-decompression cycle.

As expected the behaviour of the adiabatic energy income is consistent with $W_\perp$ and $W_{\rm tot}$ in the shock downstream ($\Omega_i t > 5$) where only adiabatic compression and decompression is responsible for the electron energy change.

Compression operates on top of all discussed above processes and adiabatic influence can be represented as a modulation of energy incomes from processes contributing to $W_\perp$. Thus

$$W_\perp = \alpha_{\rm AD}\varepsilon_\perp$$
$$\alpha_{\rm AD} = 0.6 \pm 0.1 \tag{11}$$

where $\varepsilon_\perp$ is the sum of energy incomes from processes contributing to $W_\perp$.

3.7. Model of the electron heating.

To compile the heating model we combine all individual heating efficiencies (eq. 2, 4, 5, 6, 8, 11). Recall that both shock potential components contribute to the parallel work, magnetic reconnection contribute evenly to the parallel and perpendicular parts, and SSA, SFA and adiabatic heating affect on the perpendicular component. Therefore expanding equation 11 we can write

$$W_{\rm tot} = \varepsilon_{\rm SP,r} + \varepsilon_{\rm SP,t} + \varepsilon_{\rm MR}/2 +$$
$$+ \alpha_{\rm AD}(\varepsilon_{\rm SSA} + \varepsilon_{\rm SFA} + \varepsilon_{\rm MR}/2) , \tag{12}$$

and the downstream electron temperature reads

$$k_{\rm B}T_e = \frac{2}{3}\left(W_{\rm tot} + \varepsilon_0\right) . \tag{13}$$

The amount of available energy for the particle heating is defined by Rankine-Hugoniot relations, namely by the ion temperature jump condition. Here we neglect electron kinetic energy and the ion thermal energy in the shock upstream. The available energy is

$$E_{\rm avail} = \frac{3}{2}T_i = \frac{3}{2} \cdot \frac{5}{16}M_s^2 T_{up,i} = \frac{1}{2}m_i v_0^2. \tag{14}$$

Thus in the downstream reference frame, where our simulations are performed, the upstream ion kinetic energy is the source for the downstream thermal energies of ions and electrons. Therefore the temperature ratio can be calculated as

$$\frac{T_{\rm e}}{T_{\rm i}} = \frac{W_{\rm tot}}{E_{\rm avail} - W_{\rm tot}} . \tag{15}$$

Note that here we also neglect energies of electric and magnetic fields which are small compared to other components.

The modeled downstream temperatures and temperature ratios are listed in Table 2 and prediction of the heating model is consistent with the simulations results.

4. SUMMARY AND DISCUSSION

This work is the fourth paper in a series investigating physics of nonrelativistic perpendicular high Mach number shocks. In our previous studies we discussed production of nonthermal electrons via SSA operating at the shock foot (Papers I and II) and via magnetic reconnection that results from the nonlinear decay of ion Weibel filaments at the shock ramp (Paper III). In this paper we investigate super-adiabatic electron heating. We constrain the heating model for the downstream electron-to-ion temperature ratio in realistic high Mach number shocks. 2D simulations with an in-plane magnetic field configuration are capable of reproducing the main shock structures found in 3D simulations (Bohdan et al. 2017; Matsumoto et al. 2017) and therefore electron heating is also well captured at a fraction of the computational expense. Our simulations are performed for a wide range of physical parameters ($m_{\rm i}/m_{\rm e} = 50 - 400$ and $M_{\rm A} = 22.6 - 68.7$) which permits predictions for realistic shocks.

In Paper II we discussed that all shocks demonstrate super-adiabatic electron heating, and that the downstream electron temperature is well above that predicted by the Rankine-Hugoniot condition. Here we calculate the downstream temperature ratio which is in range of $T_{\rm e}/T_{\rm i} = 0.11 - 0.19$.

Electrons are heated via SSA, parallel acceleration due to the shock potential, magnetic reconnection, and SFA. Electrons energized via SSA, magnetic reconnection and SFA have been already extensively studied, and here heating via the shock potential at the shock foot is discussed for the first time. At perpendicular shocks the upstream magnetic field and the shock potential are perpendicular to each other. Nevertheless, operating at the shock foot, the Weibel instability generates a B_x magnetic field component which permits parallel heating by the shock potential. It consist of a regular part which deals with ordered Weibel filaments at the shock foot

and a turbulent part which operates deeper in the turbulent ramp-overshoot region.

In addition to these processes, compression at the shock transition influences the plasma temperature, but its total effect is surprisingly negative. During passage from the upstream to the downstream plasma is compressed to the overshoot density ($N_{max}/N_0 \approx 8$) and then relaxes and reaches a compression ratio of 4 expected for high Mach numbers. The adiabatic heating at the compression stage is smaller than cooling after the shock overshoot, on account of nonadiabatic heating in-between. Hence the net adiabatic cooling.

Among these processes only SSA is limited by the *electron* upstream kinetic energy as the instability is driven by inbound electrons. All other processes are the ion-related phenomena, and the incoming energy is proportional to the *ion* upstream kinetic energy. They are actually responsible for the super-adiabatic electron heating. Combining all detected heating processes we constrain the heating model. It accurately predicts the downstream electron temperature of perpendicular nonrelativistic high Mach number shocks performed by means of PIC simulations.
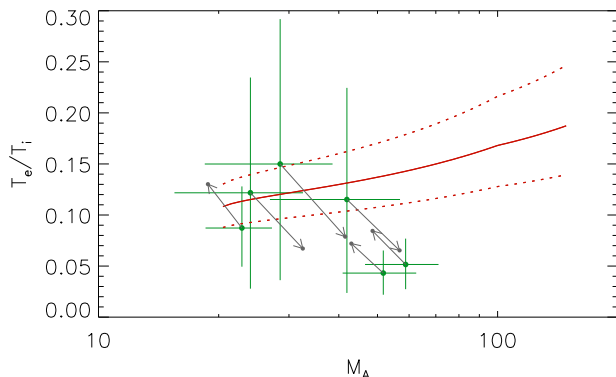
Figure 3. Comparison of the model temperature ratio with Cassini measurements during Saturn's bow shock crossings with $M_A > 20$ and $\theta_{B_n} > 45^o$ (Masters et al. 2011). Red solid line represents the heating model (eq. 15) and red dotted lines are errorbars. Green dots are Cassini measurements, with grey arrows pointing to the uncorrected values.

We apply the heating model to realistic shocks using the real proton-to-electron mass ratio and assuming that heating processes are not significantly different between our 2D in-plane shock simulations and real 3D shocks. In SNR shocks with propagation velocity above 1000 km/s, which are more than likely to be high Mach number shocks, the observed temperature ratio is $T_e/T_i = 0.05 - 0.2$ (Rakowski 2005; van Adelsberg et al. 2008; Ghavamian et al. 2013), based on X-ray spectra for T_e and Balmer-line width for T_i. We do not precisely

know the Alfvénic Mach number of SNR shocks, and so we simply assume that it is above 20. The predicted temperature ratio is $T_e/T_i = 0.09 - 0.25$ for $M_A > 20$, which indicates a good match between observations and PIC simulations.

At Alfvénic Mach numbers $M_A > 100$ the Weibel instability is heavily saturated which limits parallel heating by the shock potential and magnetic reconnection, and the slow growth of the electron/ion temperature ratio in our model would not necessarily continue. For $20 < M_A < 100$ our model predicts that the temperature ratio is independent of the shock velocity. We do not reproduce in our simulations the the empirical relation $T_e/T_i \propto v_{sh}^{-2}$ (van Adelsberg et al. 2008), which for the standard jump conditions for T_i is equivalent to stating that T_e is independent of v_{sh} and hence electron heating not governed by the bulk-kinetic energy supply of inbound particles. Instead, we see that the electron temperature is roughly proportional to the ion upstream kinetic energy. Caution is advised when comparing the temperature ratio immediately downstream of the shock, that is measured in our simulations, with that found in observations which reflects the state of plasma a few months or years after passage through the shock.

The Solar system also hosts high Mach number shocks. One of the best studied examples is Saturn's bow shock whose properties have been measured in-situ by the Cassini spacecraft. The plasma beta upstream of Saturn's bow shock is about 0.1 (Jackman & Arridge 2011), which is within the parameter range of our simulations. The postshock temperature ratio for the quasi-perpendicular Saturn's bow shock was published by Masters et al. (2011). Figure 3 shows a comparison of the heating model with these data. For comparison, we consider cases when the shock has a high Mach number ($M_A > 20$) and is quasi-perpendicular ($\theta_{B_n} > 45^o$), which severely limits the number of data points. Cassini can only measure the electron temperature. The ion temperature is inferred from the shock speed that itself is an estimate requiring transformation from the spacecraft frame to the shock frame. To indicate the level of correction that Masters et al. (2011) had to impose, we add to Fig. 3 grey arrows that undo thoses changes and point to the original measurement. Some data points are consistent within the errorbars provided by our model, but two values are below the prediction. This may happen for few reasons. Our simulations consider only perpendicular shocks, but the inferred shock obliquity for the shock crossings shown in Fig. 3 is in the range $55^o < \theta_{B_n} < 80^o$. Second, in our simulations shocks propagate in homogeneous media, and we can not

account for density or field fluctuations, both of which are present upstream of real shocks, that may impact on the temperature ratio. These assumptions therefore lead to additional uncertainties in our calculations of the temperature ratio. Nevertheless, we can state here that the heating model yields a reasonably good estimate of the temperature ratio at quasi-perpendicular high Mach number shocks.

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