

D E U T S C H E S   E L E K T R O N E N - S Y N C H R O T R O N  
(DESY)

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Desy-Notiz A 2.82

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Hamburg, den 14. August 1961  
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STRONG FOCUSING SYNCHROTRON MAGNET  
WITH TILTED FOCUSING PLANES

ABSTRACT

A strong focussing synchrotron magnet design of the type suggested by Sigurgeisson<sup>1)</sup> is considered, in which the two focussing planes are tilted by  $45^\circ$  with respect to the conventional orientation. When used in a FODO-electron synchrotron, this magnet type would eliminate classical radiation anti-damping of betatron oscillations. Some advantages and disadvantages of the magnet are discussed. A magnet of this type may also be used as a strong focussing pair spectrometer.

The magnetic field in a strong focussing synchrotron magnet may be considered as a linear superposition of a homogeneous deflecting field  $B_0$  and a quadrupole field with its axis coinciding with the reference orbit. This quadrupole field has two focussing planes which are orthogonal to the field and to each other. In a coordinate system parallel to these focussing planes, the two components of transverse particle motion are nearly independent of each other within linear approximation.

In a conventional synchrotron magnet, one of the focussing planes is parallel to the deflecting field  $B_0$ , and, for vertical  $B_0$ , the orientation of the focussing planes is vertical and horizontal, respectively.

More generally, as suggested by Sigurgeisson<sup>1, 2)</sup>, synchrotron magnets may be conceived for any given relative orientation between the two superimposed fields.

Here, we shall briefly consider a synchrotron magnet design in which the deflecting field  $B_0$  and the focussing planes of the superimposed quadrupole field are at an angle of  $45^\circ$ . For practical applications, this magnet is the most interesting because of its symmetry properties.

Fig. 1 demonstrates the magnet design. The field in the magnet aperture is a section of a quadrupole field, which has its center of symmetry in the intersection of the two coordinate axes  $\bar{\nu}$  and  $\bar{\tau}$ . The aperture is symmetric to the line  $\bar{\nu} = \bar{\tau}$ .

For a curved magnet sector of this type with vertical entry and exit of the reference orbit, the linearized

equations of motion are given by

$$\left. \begin{aligned} \nu'' + k \cdot \nu + \frac{1}{2\varrho^2} (\nu - \tau) &= \frac{1}{\sqrt{2} \cdot \varrho} \cdot \frac{\Delta p}{p_0} \quad \text{and} \\ \tau'' - k \cdot \tau - \frac{1}{2\varrho^2} (\nu - \tau) &= - \frac{1}{\sqrt{2} \cdot \varrho} \cdot \frac{\Delta p}{p_0} \end{aligned} \right\} (1)$$

with  $\nu, \tau$  = transverse deviations from reference orbit

$$\nu'' = \frac{d^2 \nu}{ds^2}$$

$$k = \frac{e \cdot g}{p}$$

$g$  = field gradient

$p_0$  = momentum of reference particle

$\varrho$  = radius of curvature of reference orbit

Eqs. (1) show that for  $k \gg \frac{1}{2\varrho^2}$  the two components of particle motion are independent of each other in the coordinates  $\nu$  and  $\tau$ .

They show furthermore, that in a FODO-synchrotron structure built of such magnets, there is a complete symmetry between the two normal modes of transverse oscillation. This has an interesting consequence. As shown by Robinson<sup>3)</sup>, in an ordinary AG electron synchrotron the classical radiation anti-damping of betatron oscillations can be eliminated by introducing additional elements for coupling of horizontal and vertical particle motion. An electron

synchrotron of the type considered here provides such coupling inherently, and there is no anti-damping of betatron oscillations caused by classical radiation effects.

Physically, the magnet aperture in Fig. 1 is limited on two sides by iron poles following hyperbolic equipotential lines  $\bar{\nu} \cdot \bar{\tau} = \text{const.}$ , as indicated in Fig. 1. On the other two sides, the aperture is limited by coil faces which are chosen to be approximately orthogonal to the central hyperbola, since in this case one can use a rectangular coil cross section without introducing any noticeable field nonlinearities. This can be seen from the dotted line in Fig. 1, which has been calculated by Dr. Lublow<sup>4)</sup>. It shows the coil contour which would generate an exact quadrupole field within the aperture. In the magnet considered here, a noticeable deviation of the rectangle from this contour occurs only so far away from the aperture that it does not contribute nonlinearities of practical significance.

The magnet shown in Fig. 1 looks like a "window frame" deflecting magnet with a slightly curved aperture. In order to get a feeling for its merits and disadvantages as compared to conventional synchrotron magnets, its relative field gradient and its aperture dimensions have been adapted to the acceptance and structure of the DESY-magnet. Both magnet types are shown to scale in Fig. 2, I and II. They compare as follows:

- 1) The conventional-type synchrotron in general needs two different magnets F and D. This allows maximum economy by adjusting the two gap heights to the two

different beam heights in F and D magnets<sup>5)</sup>. A synchrotron built of magnet type II is less economical because it does not allow any practical adjustment of this kind. In addition, it needs a larger gap height because the closed orbits for particles with momentum deviation oscillate vertically. As compared to type I, for twice the current density in the coils this results in an increase of about 50 % in stored energy. On the other hand, it is an advantage of having only one magnet type II.

The "window frame"-structure of magnet II offers the following advantages:

- 2) Design of the pole contour is very easy because the ideal hyperbola contour produces a precise quadrupole field over the entire aperture at medium field strength.
- 3) The field  $B_0$  can be made about 50 % higher than in magnet I.
- 4) It is easier to control the end fringing fields.

On the other hand, the "window frame"-structure of magnet II has the disadvantage that

- 5) beam injection and ejection is more difficult. However, special long straight sections may be introduced for these purposes. It is also possible to build the considered magnet type in C-shape, as shown in Fig. 2, III.

Because of its symmetry properties, the considered magnet type may also offer some advantages in special experimental applications, for instance as a strong focussing pair spectrometer.

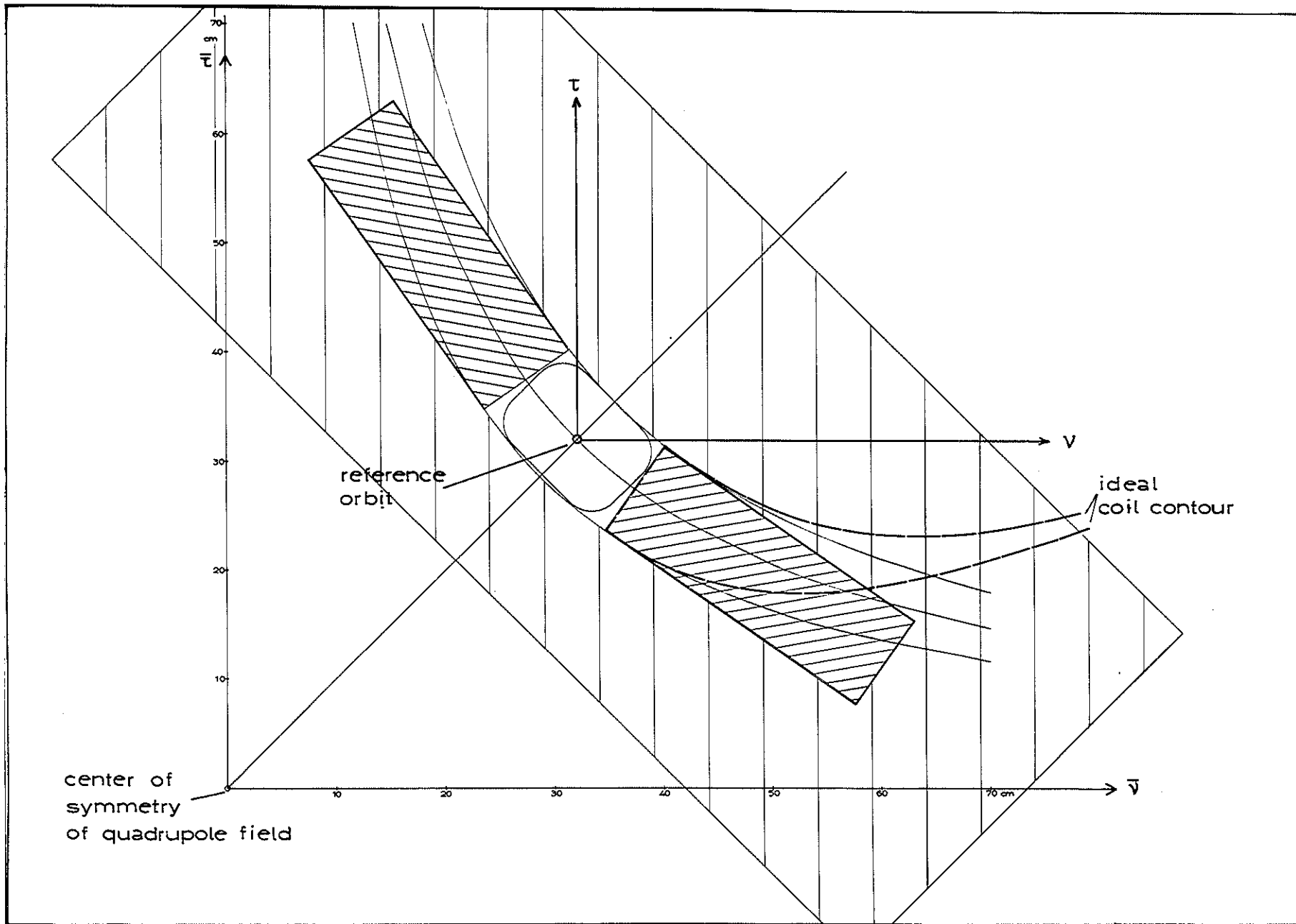
REFERENCES

- 1) T. Sigurgeisson, CERN 55-14 (1955)
- 2) C. Frønsdal, CERN/T/CF-1
- 3) K.W. Robinson, Phys. Rev. 111, 373 (1958)
- 4) D. Lublow, private communication
- 5) W. Hardt, DESY-Bericht A 1.5, 1959

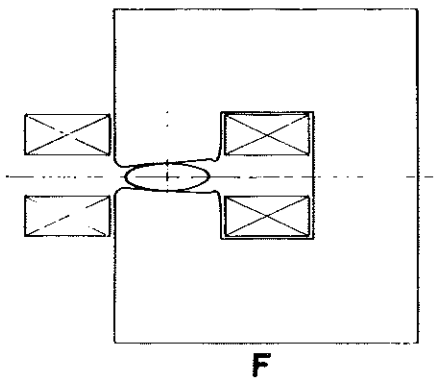
LIST OF FIGURE LEGENDS:

Figure 1: Magnet with tilted focussing planes,  
shown as a section of a quadrupole field  
with its center of symmetry in  $\bar{y} = \bar{z} = 0$ .  
The dotted line indicates an ideal coil cross  
section contour.

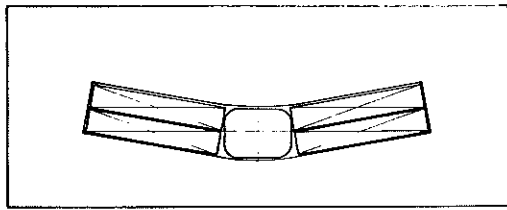
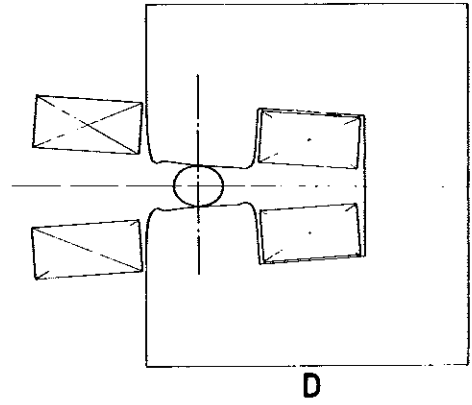
Figure 2: Cross sections of synchrotron magnets  
I: Conventional synchrotron magnet (DESY)  
II: Suggested synchrotron magnet,  
"window frame"-type  
III: Suggested synchrotron magnet, C-type.



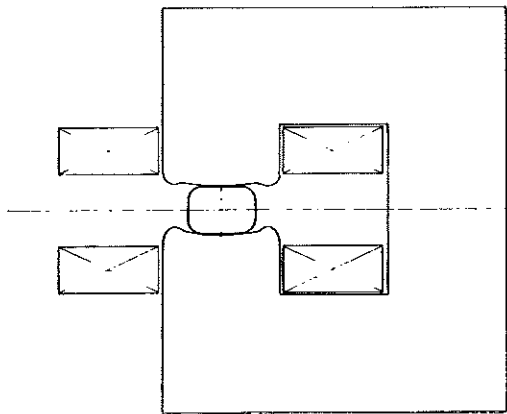
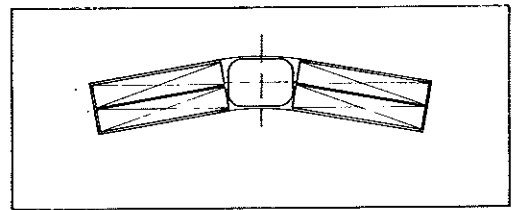




I



II



III

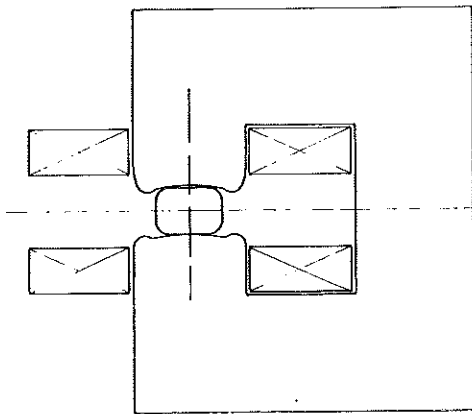


FIG. 2