Internal Report
DESY F15-80/04
May 1980

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We review, for the use by experimentalists, the present status of elementary theory of the B-mesons decays. The review emphasizes only a few points and is not intended to be complete in any respect.

1. Definition and Bounds on the b-quark Coupling Constants

In the model with three quark doublets the weak charged current has the form:

$$j_\mu = \bar{q}_A V_{AC} \gamma_\mu (1+\gamma_5) q_C$$

where q_A and q_C stand for the charge + 2/3 and for the charge - 1/3 quarks respectively, q_A = u, c, t , q_C = d, s, b and V_{AC} is a unitary matrix which can be parametrized by four observable angles $\theta_{1,2,3}$ and δ :

$$(\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} c_1 & s_1 c_2 & s_1 s_3 \\ -c_2 s_1 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -s_2 c_1 c_3 - s_3 c_2 e^{i\delta} & c_2 c_3 e^{i\delta} - c_1 s_2 s_3 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (1)$$

where $c_i = \cos\theta_i$; $s_i = \sin\theta_i$ $\theta_1 = \theta_{\text{Cabibbo}}$. Eq.(1) is the famous Kobayashi-Maskawa[1] matrix. One should keep in mind, however, that a too naive use of it can be misleading. In particular, there can be "accidental" cancellations and the phase of the matrix element is not immediately translated into observable CP violation (see below). It might be useful therefore to give bounds directly on the couplings of the b-quark to u,c,t-quarks.

It is known to a good accuracy that the b-quark couples to the u-quark rather weakly[2]:

$$|V_{ub}|^2 \lesssim 8 \times 10^{-3}$$

where the bound is due to the fact that the known couplings of the u-quark practically saturate the universality bound:

$$1 - |V_{ud}|^2 - |V_{us}|^2 = (4 \pm 4) \cdot 10^{-3}$$

It is known also that b-quark has quite a sizable coupling to the heavier t-quark[3,4]:

$$|V_{tb}|^2 \gtrsim 0.5 - 0.7$$

This bound depends actually on the mass of the t-quark and we quote the result for m_t = 15 - 30 GeV. Unitarity then bounds the V_{cb} in the following way:

$$|V_{cb}|^2 \lesssim 0.3 - 0.5$$

The present bounds allow, in particular, for any ratio of the couplings V_{ub}/V_c There is a trend in many theoretical papers to assume that there is a sort of "kinship" between heavy quarks (see, e.g., Ref.5) so that b-quark couples stronger to the c-quark. Actually, all these speculations arise from the phenomenologically successfull relation $tg^2\theta_c \simeq m_d/m_s$. However, there exist models[6] which incorporate the latter relation but predict $|V_{cb}| \ll |V_{ub}|$. So, at present one should be open-minded.

It should be also mentioned that if the only source of CP violation in the K system is the KM phase none of the angles in the standard parameterization can vanish. The reason for this is that for all the CP-odd amplitudes the KM-mixing results in the factor

$$\xi = c_1 c_2 c_3 s_1^2 s_2 s_3 \sin\delta$$

which vanishes when any of the angles is zero. Moreover, from the known CP violation in the K-mesons one finds[7]

$$\xi \simeq \sin^2\theta_c\, \varepsilon_K \simeq 10^{-4}$$

so that some of the angles $\theta_{2,3}$ or the phase δ should be rather small.

2. B-meson Decays and KM Angles

The ratio of the coupling constants $|V_{ub}|/|V_{cb}|$ can be determined in the most direct way from the ratio of the corresponding inclusive leptonic decays:

$$\frac{\Gamma(B \to l\nu X_c)}{\Gamma(B \to l\nu X)} = \frac{|V_{cb}|^2 f\left(\frac{m_c^2}{m_b^2}\right)}{|V_{cb}|^2 f\left(\frac{m_c^2}{m_b^2}\right) + |V_{ub}|^2}$$

where X_c includes only charm states, $f(m_c^2/m_b^2)$ is the kinematical factor:

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

$$f(m_c^2/m_b^2) \simeq 0.6$$

It is worth emphasizing that the theory predicts only the ratio of the total semileptonic decay widths, while the ratio of exclusive channels, say, $\Gamma(B \to \ell\nu D)/\Gamma(B \to \ell\nu\rho)$ depends heavily on unknown hadron dynamics. In particular, there should be present rather strong form-factor-like suppression of the ρ mode due to the mass diffference $m_D - m_\rho$.

Some time ago theorists would claim that the total nonleptonic rate is also useful to determine the mixing angles. The claim is not so strong now. The point is that the study of the nonleptonic D-decays has brought an unexpected result $\tau(D^+)/\tau(D^0) \gtrsim 3$, in disagreement with the simple-minded quark model estimates. The quark picture should be true asymptotically for very heavy quark. At the moment it is not clear how fast the asymptotics is approached. Various models give corrections of order m_Q^{-1} or m_Q^{-2}, and these corrections are seemingly very important for the charmed quark. For the b-quark non-leptonic decays seem to be safe as far as one is ready to meet a factor of 2 uncertainty.

We have no particular suggestion how to measure experimentally the inclusive charm production. The signals for charm are well-known: leptonic decays, K-mesons in the final state. If the b-quark couples to charm stronger than to the u-quark, charm is produced practically in each event associated with B-meson.

In the case of D-mesons the standard procedure is to identify D-mesons in one direction my means of some particular decay mode, say $D \to \bar{K}\pi\pi$ and study inclusive rates for the D-mesons produced in the opposite direction.

For the B-mesons this should be harder since any particular decay mode for B-mesons has small branching.

Thus, it is up to experimentalists to judge how far can they go with measuring of the mixing angles (or, better to say the ratio of V_{ub}/V_{cb}, to fix the absolute value one needs to measure the life time).

3. B-$\bar{B}$-mixing

The B-$\bar{B}$ mixing is a nice quantum-mechanical phenomenon similar to that observed in the K-$\bar{K}$ system. The calculation of the mixing involves several parameters, however, and at present it is difficult to predict the magnitude of the effect.

In some reasonable approximation the corresponding mass difference can be written in the form[8]:

$$\Delta m_{B_q} \simeq \frac{G_F^2}{6\pi^2} m_t^2 f_B^2 m_B \eta |V_{tq}|^2 |V_{tb}|^2$$

where q stands for d or s-quark (the mixing can occur both for the B_d and B_s mesons), f_B is the matrix element for annihilation of the meson by the axial current and η is the correction factor, arising from the exchange of gluons. The quantity relevant to the observation of the B-$\bar{B}$ oscillations is $\Delta m/\Gamma$ which is roughly given by

$$\frac{\Delta m_{B_q}}{\Gamma} \simeq \frac{32\pi}{9} \frac{m_t^2}{m_B^2} \frac{f_B^2}{m_B^2} \frac{|V_{tq}|^2 |V_{tb}|^2}{|V_{ub}|^2 + |V_{cb}|^2 f\left(\frac{m_c^2}{m_b^2}\right)}$$

(where we set $\eta = 1$ though it could be not a very good approximation; other uncertainties seem to be worse, however).

The value of f_B is bound by the QCD sum rules[9]:

$$f_B \lesssim 230\ MeV$$

and, respectively,

$$\frac{\Delta m_{B_q}}{\Gamma} \lesssim 0.9 \cdot \left(\frac{m_t}{30\ GeV}\right)^2 \frac{|V_{tq}|^2 |V_{tb}|^2}{|V_{ub}|^2 + |V_{cb}|^2 f\left(\frac{m_c^2}{m_b^2}\right)}$$

The actual number depends on the top-quark mass and on the ratio of the KM mixing angles. Both are very difficult to guess at the moment. Note, however, that if the heavy quarks are coupled primarily to the heavy ones then the ratio of the angles should be small. The estimate of the latter ratio as of $\sin^2\theta_c$ which seems real would also lead to insignificant effect of the mixing.

4. CP Violation

So far CP violation has been observed only in the K-$\bar{K}$ system and there is a number of theoretical suggestions to look for the same kind of phenomena in the B-$\bar{B}$ oscillations. First it was inferred[7] that the CP violation in the B mesons should be of order $tg\, 2\delta$ since the phase of the V_{tb} in the standard notations is roughly equal to δ. This is a kind of mirage, however, since any genuine observable CP odd effect arises only from interference of four different matrix element of the KM matrix: e.g.:

$$Im\,(V_{tb} V_{cd} V_{td}^* V_{cb}^*) = \xi$$

(at least partly the result is readily understandable since for any vanishing mixing angle the situation goes actually back to the four-quark case, with no CP violation at all). Second, the CP- odd effects in the B-$\bar{B}$ mixing are not proportional to m_t^2 unlike the case of the CP-even mass difference. The reason is that in limit $m_c = m_u \simeq 0$ the CP violation again disappears (can be rotated away through redefinition of the quark fields). As a result, the quantity $2\,Re\,\varepsilon/(1+|\varepsilon|^2)$ which represents the observable CP-violation in the B-$\bar{B}$ mixing is estimated[10] of order 10^{-4}, the number which is to be compared with 2×10^{-3} in the case of the K-meson. Combined with the expected smallness of the $\Delta m/\Gamma$ it shows that this source of CP violation is very unlikely to become a subject of an experimental study in the near future.

Probably, a more reasonable, though by no means simple way to look for CP violation is to study direct decays of the b-quarks[11]. The effect is manifested as a nonvanishing difference of the inclusive decay rates:

$$\Delta_1 = \Gamma(b \to c\bar{c}s) - \Gamma(\bar{b} \to c\bar{c}\bar{s}),$$

$$\Delta_2 = \Gamma(\bar{b} \to u\bar{u}\bar{s}) - \Gamma(b \to u\bar{u}s),$$

$$\Delta_3 = \Gamma(\bar{b} \to c\bar{c}\bar{d}) - \Gamma(b \to c\bar{c}d),$$

$$\Delta_4 = \Gamma(b \to u\bar{u}d) - \Gamma(\bar{b} \to u\bar{u}\bar{d}).$$

The CPT theorem ensures that up to higher order in weak interactions $\Delta_1 = \Delta_2$, while the unitarity of the 3 x 3 KM matrix requires that

$$\Delta_1 = \Delta_3 \quad and \quad \Delta_2 = \Delta_4$$

so that all the differences Δ_i are equal to each other. A straightforward calculation of the CP-odd interference of the diagrams in Fig.1 gives[12]:

$$\Delta_i = \frac{8}{9}\alpha_s(m_b)\frac{G_F^2 m_b^5}{192\pi^3} I \xi \simeq 3 \times 10^8 \, sec^{-1} \qquad (2)$$

Here $\alpha_s(m_b) \simeq 0.15$ is the QCD coupling constant normalized at the b-quark mass and I is a kinematical integral roughly equal to the phase space damping factor in the decay $b \to c\bar{c}q$ due to the nonvanishing c-quark mass $I \simeq 0.33$.

From the estimate (2) it is seen that the CP-odd difference can be as large as few per cent in the Cabibbo suppressed modes such as $b \to c\bar{c}d$ and $b \to u\bar{u}s$.

REFERENCES

1. M. Kobayashi and T. Maskawa; Progr.Theor.Phys., 49, 652, (1973).
2. R.E.Schrock and L.L.Wang; Phys.Rev.Lett., 41, 1692, (1978).
3. R.E.Schrock, S.B.Treiman and L.L.Wang; Phys.Rev.Lett., 42, 1589, (1979).
4. V.Barger, W.F.Long and S.Pakvasa; Phys.Rev.Lett., 42, 1585, (1979), R.E.Schrock and M.B.Voloshin; Phys.Lett., 87B, 375, (1979).
5. H.Fritzsch; Phys.Lett., 73B, 317, (1978).
6. F.Wilczek and A.Zee; Phys.Rev.Lett., 42, 421, (1979).
7. J.Ellis et al., Nucl.Phys., B131, 285, (1977), A.Ali and Z.Z.Aydin, ibid, B148, 165, (1979).
8. Originally an estimate of this kind was done for the K-$\bar{K}$ system by A. Vainshtein and I. Khriplovisch, JETP Lett., 18, 83, (1973). A more elaborated discussion is contained in V.Novikov et.al., Phys.Rev., D16, 223, (1977).
9. V.Novikov et al.; Phys.Reports 41C, 1, (1978) and Proc. of the Neutrino-78 Conf., West Laffayette, ed. by E. Fowler, Purdue Univ. (1978), page C278
10. E.Ma.W.A.Simmons and S.F.Tuan; Phys.Rev.D20, 2888, (1979).
11. M.Bander, D.Silverman and A.Soni; Phys.Rev.Lett., 43, 242, (1979).
12. In this form the relation was obtained by L.B.Okun and M.B.Voloshin (1979).

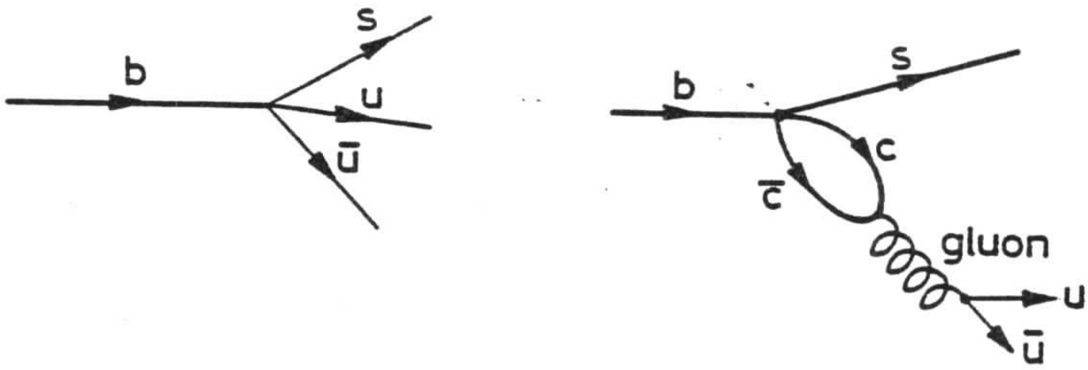

Fig.1 Interference of these two diagramms contains a CP-odd term which gives rise to the difference Δ_2 .