DEUTSCHES ELEKTRONEN - SYNCHROTRON DESY

A POSITRON FOCUSING SYSTEM
FOR
LINEAR ACCELERATORS

by

H. Wiedemann

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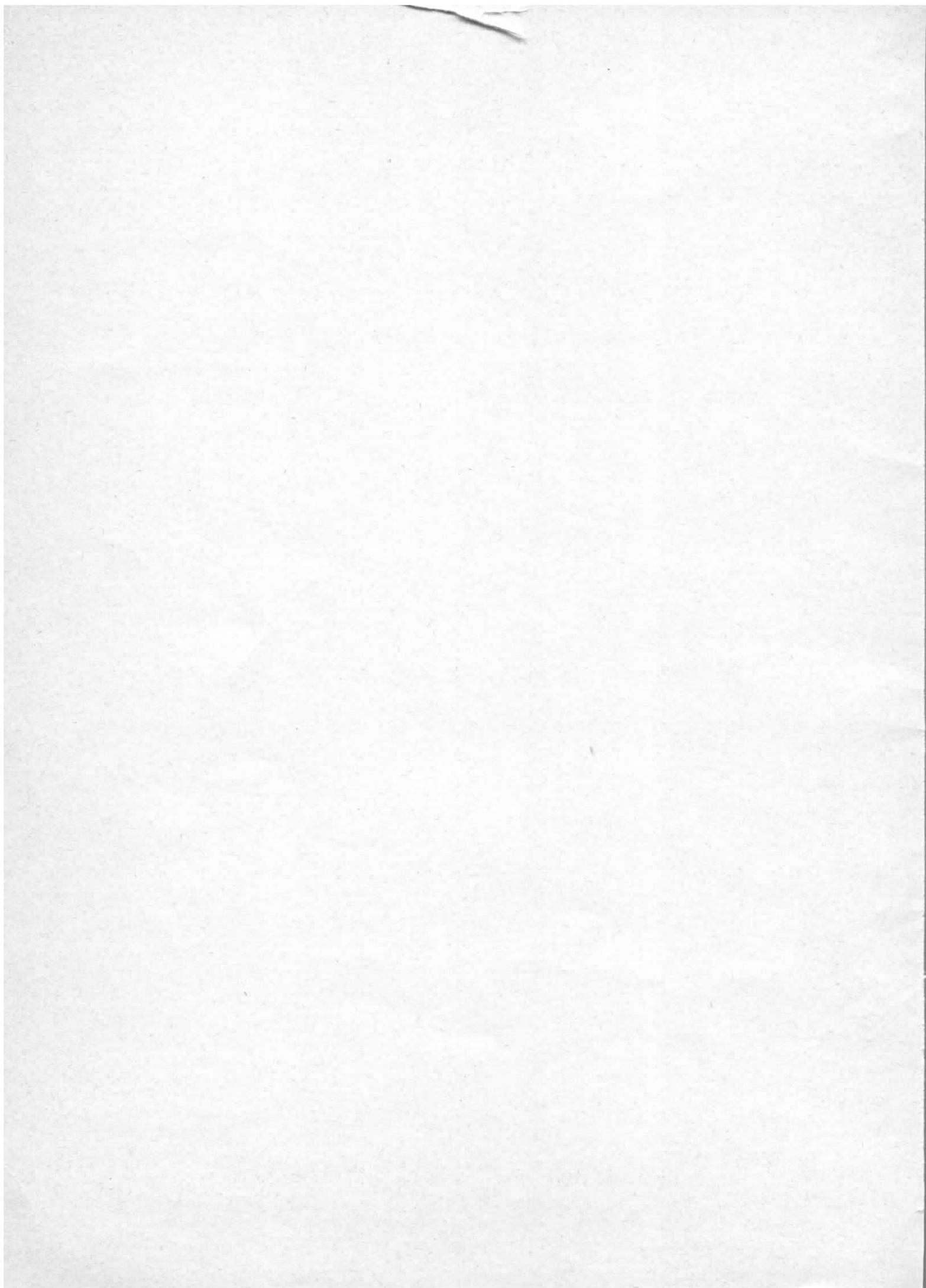
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I n d e x

Introduction

The new 300 MeV linear accelerator as an injector for DESY
is planned to accelerate both electrons and positrons.

In this paper, a target and a focusing system is described
which yields a high intensity analysed positron current of
a few 2 mA at the end of the linac. The accelerator gives an
electron pulse power of more than 50 MW on the converter tar-
get, and accelerates the positrons in seven following sections
to 360 MeV.

This paper essentially is a translation of the DESY-note H9,
August 1965. However, new calculations and informations on
the positron horn have been included in addition.

I. Converter Target

For positron production a converter target is needed, which
withstands an electron pulse power of 50 MW and 3 to 7 kW
average for a long time.

The converter targets built and proposed so far consist of a
moving tungsten annulus. This metal combines a high Z with
a high melting point and a rather good heat conduction. But
like all other solid target materials, tungsten has the
disadvantage that the target has to be rapidly moved with
its cooling system in the high vacuum in order to avoid too
high temperatures. Besides this, tungsten is very brittle and
will be mechanically destroyed by the point heating.

Both disadvantages can be avoided by a fluid metal target in
which the heat will be transported by forced convection and the
mechanical destruction of the housing may be strongly reduced

by taking a low Z metal (e.g. steel). The following calculations assume Wood's metal as converter target.

For simplicity we assume that the thermal energy deposited on the target by the electron pulse is equal to the lost ionisation energy of the electrons. The calculation results in:

$$(1) \qquad\qquad - \frac{\Delta E}{\Delta x} = 17 \frac{MeV}{cm}$$

As we show in a later section, the optimum target-thickness is about 1.5 radiation length, or for Wood's metal l = 0.75 cm.

The deposited thermal energy is

$$(2) \qquad\qquad \Delta Q = 0.9 \; cal/pulse$$

for a pulse duration of 1 μsec. This corresponds to a temperature rise of

$$(3) \qquad\qquad \Delta T = \frac{\Delta Q}{\rho \cdot V \cdot c} = 137^{\circ}C$$

(target volume $V = \pi \frac{d^2}{4} \cdot l$; d = 2 mm) if the effective target cross section is taken equal to the electron beam cross section.

This temperature rise per pulse does not appear too large for a steel target housing.

The Wood's metal will be made fluid by electric heating and will be pumped by an electromagnetic pump without rotating elements.

II. Initial Positron Focusing

1. Positron intensities

There are results of two experiments which allow to calculate
the yield of the positron conversion. In Orsay, T.L. Aggson
and L. Burnot (AG62) measured positron intensities in the
foreward direction ($\delta = \pm 2^{\circ}$) for various positron energies
and various electron energies between 55 MeV and 220 MeV.
Since we like to focus the positrons over a wide solid angle,
we cannot take quantitative results from these measurements.
Qualitatively, the measurement yields a broad intensity
maximum for positron energies between 5 and 10 MeV for all
electron energies. A second important result for us is the
optimal target thickness which is about 1.5 radiantion lengths
independent of the electron energy.

The most important source for our intensity calculations are
measurements of De Staebler (STAE 63) in Stanford. He measured
the angular distribution of positrons at various positron
energies. The electron energy was 1 GeV. De Staebler also
found an intensity maximum for positron energies between
5 and 10 MeV (Fig. 1).

As for the absolute values, De Staebler interprets them as
low value limits. He assumes that by scattering mainly of
the low energy positrons in the spectrometer windows, about
30 % of the positrons are lost. In spite of this, we take the
figures of De Staeblers report for our further calculations.

Integration over the angle of De Staeblers figures gives
positron intensities for various solid angles (Fig. 2) for
an electron energy of 1 GeV and positron energies between
5 and 10 MeV.

Our new linear accelerator gives at the converter an electron pulse current of 322 mA at 177 MeV, i.e. a pulse power of P = 57 MW. The positrons are accelerated in the second part of the linear accelerator to 360 MeV. For this power, for an initial energy width of ΔE_+ = 3,6 MeV for the positrons, corresponding to a relative energy range of 1 % at the end of the linac and a solid angle given by δ = 40° one gets a positron current of

$$(4) \qquad J_+ = (\frac{1}{E_O} \frac{\Delta n_+}{\Delta E_+}) \; P_- \; \cdot \; \Delta E_+ \; = \; 15 \text{ mA}$$

This is the lower limit; the real positron current may be up to 50 % higher. Supposing that the focusing efficiency of the positron horn is about 50 % and assuming that one third of the positrons (Am 63) captured in the linac will remain in an energy range of 1 % during acceleration this yields in a positron pulse current of

$$(5) \qquad J_+ = 2.5 \text{ mA}$$

which can be injected in the synchrotron. This current corresponds to an effective converting efficiency of

$$(6) \qquad n_+ = 0.05 \; \frac{e^+}{e^- \cdot \text{GeV}}$$

into an energy range of $\Delta E_+/E_+$ = ± 0,5 % and into an emittance of 1,6 πmradcm (see following section)

2. Optics of the positron horn

The design aim of the positron focusing system is to focus positrons of 6 MeV and production angles up to 40 - 45 degrees into an emittance area of 100 πmradcm, which gives at 360 MeV an emittance of 1.67 πmradcm corresponding to the acceptance of the synchrotron.

In the positron linear accelerators built and proposed so far
the positron target is immersed in a strong solenoid field
(Am 63, Mi 65) which yields a parallel positron beam from a
point source. To focus positrons up to a production angle
of 14° a magnetic field strength of 17,7 kGauss is needed.
To go to even higher magnetic fields seems to be technically
difficult.

In order to focus positrons over a larger angle one has to
use a magnetic field normal to the positron trajectories,
which can be produced by an axial electric current used in
the Neutrino-experiment. This is done at CERN by the so called
"Magnetic Horn" (ME 61, ME 62, AS 65). In Novosibirsk (BU 63)
they use a parabolic lens which is essentially a "horn" to
focus a 700 MeV electron beam on a target and to make the
divergent positron beam parallel. In our case, we cannot use
such a parabolic lens because the efficiency is too small.
Therefore, we propose a horn which is similar to the CERN-type
(Fig. 3).

For the particle trajectories in a coaxial magnetic field
one gets (e.g. from the equation (19)) the differential
equations in polar coordinates $\{r, \phi, z\}$:

$$r'' - r\phi'^2 + \varepsilon\frac{z'}{r} = 0$$

$$\frac{d}{ds}(r^2\phi') = 0$$

$$z'' - \varepsilon\frac{r'}{r} = 0$$

$$\varepsilon = \frac{2eI}{cE}$$

For focusing the positrons on a point, one has to shape the
horn such that each particle escape the magnetic field when
the tangent of its trajectory aims at this focus. However,
this is only possible for monoenergetic particles from a

ring source. To "focus" particles of an energy range of
some MeV originating from an extended source, one has to
give the horn such a form that as many positrons as possible
aim at a spot of about 2 cm diameter corresponding to the
aperture of the linear accelerator.

For this purpose the differential equations (8) have been
written as follows:

$$(8) \qquad r^2 \phi' = \frac{\rho^2 \phi_O}{c} = \alpha$$

ρ : radius of the target

$m\rho^2 \phi_O$: angular momentum

With (8) eq. (7) reduces to

$$r^3 r'' - \alpha^2 + \varepsilon z' r^2 = 0$$
$$(9)$$
$$z'' r - \varepsilon r' = 0$$

These equations have been solved on the analog computer for
various initial conditions $\{r_o, \phi_o, z_o\}$.

The magnetic field has been chosen such that particles of
6 MeV and an initial angle of 40 degrees are bent to the
forward direction within about 1 cm distance from the axis.
This high field is necessary to avoid too large differences
in path length and can be realized with a pulse current of
15000 Amperes.

Typical particle trajectories for different initial conditions
are shown in Figures 4 and 5. From a large number of cal-
culated trajectories a favourable horn-profile as shown in
Fig. 6 has been derived. This horn-profile should be a good
compromise between the ideal profiles for different energies
and source points.

With this horn profile (Fig. 6) many particle trajectories
have been calculated to get the emittance area of the
horn (Fig. 7) at z = 8 cm. This plane has been found to
have the highest particle density. The ellipses shown in
Fig. 7 give the acceptance of the downstream solenoid for
different energies (see later sections).

The Figure 7 shows that with the positron horn positrons
can be captured over a large solid angle and a broad
energy range. The energy spectrum for these captured
positrons is shown in Fig. 8. For comparison the energy
spectrum accepted by a solenoid is given, for which
Amman (Am 63) obtains the equation (see Fig. 8a):

$$(10) \qquad \left(\frac{\delta}{\delta_o}\right)^2 = \frac{(V_o/V)^2}{(H_1/H_2)^2 \cos\left(\frac{\pi}{2}\frac{V_o}{V}\right) + \sin\left(\frac{\pi}{2}\frac{V_o}{V}\right)}$$

$$(V_o = 10 \text{ MeV}, \; H_1 = 17{,}7 \text{ k}\Gamma, \; H_2 = 4 \text{ k}\Gamma, \; \delta_o = 14^o)$$

The calculation shows that the positron horn does not
only yield an essentially higher positron intensity, but
also a broader energy spectrum, which is advantageous for
the acceleration in a linear accelerator. There, the single
positron bunches are broadened (e.g. by phase errors and
field inhomogenities) so that only about 30 % (Am 63) of
the initial positron intensity remains in an energy band
of ± 0.5 %. This percentage increases if the energy band
of the positrons is large.

Another advantage connected with the positron horn are the
small phase differences between particles on different
trajectories. Fig. 9 shows these phase differences for
different energies and initial angles. All positrons be-
tween 4 and 8 MeV are within a phase angle of ± 4 degree.

The primary electrons have the same phase spread, so that the positrons are within ± 8° that means within an energy gap of ± 0.5 % for an ideal 3000 mc linac.

3. Technical design of the positron horn

Because of the very low positron energy, we cannot apply the lens types used at CERN and Novosibirsk; we have to approximate the coaxial magnetic field by a few thin current carrying wires as already shown in Fig. 3.

The actual field as shown in figures 10 and 11 was obtained by digital computer calculation. The field is very distorted in the vicinity of the wires. However, the azimuthal deflection of particles due to radial field components is not serious (less than 35 mrad). The calculation of the positron current in eq. (4) is based on an effective horn transparency of only 70 %; that means the vicinity of the wires has been neglected for intensity calculations.

The positron horn is connected over a spark gap to a delay line. The pulse charge is 0.06 Asec, 50 times per sec. The ohmic losses rapidly rise the temperature of the wires so that water cooling at the ends of the horn is needed. The material of the horn wires is the silver alloy Argodur 32. This alloy has the high and constant tensile strength of 53 kp/mm up to a temperature of 650°C. This material seems to be the best for our purpose and will withstand the magnetic forces also at high temperatures. The electrical resistance is about 13 % higher compared with pure silver.

The energy deposited on 1 cm of wire length is:

(11) $\Delta Q = 1,0 \cdot 10^{-3}$ cal/cm · pulse

For a wire length of 4.5 cm this gives a total energy of
0.009 cal which has to be transported by heat conduction
to the ends of the horn within 20 msec. For silver this
results in a temperature gradient of

(12) $\Delta T = 160^{\circ}C/cm$

The temperature in the middle of the wire therefore is
$360^{\circ}C$. This is an upper limit because radiation has been
neglected.

4. Mechanical forces by the pulsed magnetic field

We regard a wire which is supported only at its ends. For
such a wire the eigenfrequencies are:

(13) $\omega_n = \varkappa_n^2 \, c_w \, \dfrac{h}{\sqrt{12}}$

      h: height of the wire (here 1 mm)

      c_w: E/ρ

$\varkappa_n$: eigenvalues of the frequency equation $1 - \cos\ l \cdot \cosh\ l = 0$
The first three eigenvalues are:

      $\varkappa_1 l$ = 4,7300

(14) $\varkappa_2 l$ = 7,8532

      $\varkappa_3 l$ = 10,9956

$h = 1$ mm, $l = 4{,}5$ cm, $E = 8000$ kp/cm^2 (Argodur 32)
$\rho = 10{,}5$ g/cm^2 yields for the three first eigenoscillations:

      ω_1 = $7{,}6 \cdot 10^3$ sec^{-1}

(15) ω_2 = $21{,}2 \cdot 10^3$ sec^{-1}

      ω_3 = $35{,}5 \cdot 10^3$ sec^{-1}

These frequencies are very small compared with the pulse
frequency of $\omega_p = 1,6 \cdot 10^6 \text{ sec}^{-1}$ (2 μsec pulse length)
that means both the pulse frequency and the pulse sequence
frequency of $\omega_o = 314 \text{ sec}^{-1}$ are far away from the resonance
frequencies. The wires are not excited to considerable
oszillation. A calculation yields a maximum oscillation
amplitude of 10^{-5} cm. That means that, in spite of the
high current in a high magnetic field no additional
supports besides the end supports are needed.

III. Positron focusing in the linear accelerator

1. Solenoid and quadrupole focusing

The positrons will be accelerated up to $\hat{E} = 360$ MeV in
seven sections following the converter and will then be
injected into the DESY synchrotron. The acceptance of the
synchrotron is $\varepsilon = 2$ πmradcm. According to Liouville's
theorem we get a useful emittance of the positron horn of

$$(16) \qquad \pi rr' = 2 \pi \frac{\hat{E}}{E_o} = \pi \frac{720}{E_o} r \quad \text{mradcm}$$

i.e. with $E_o = 6$ MeV and $r = 1$ cm, a positron divergence
up to 120 mrad can be accepted. Safely, we have used
$r' = 100$ mrad in our calculation (see Fig. 7).

This corresponding emittance of 100 πmradcm is to be trans-
formed to the end of the linac with the lowest possible
focusing power.

There are two essential methods for positron focusing:

a) Focusing with the aid of an axial solenoidal magnetic
   field.

b) Focusing with magnetic quadrupole lenses.

The efficiency of the quadrupole focusing is higher than that of a solenoid. While the pure solenoid magnetic field has to be maintained over the whole accelerator length, the spacing between the single lenses in quadrupole focusing increases adiabatically. This difference is more and more observable with higher particle momentum. However we will see that there are cases in which only solenoids can be used.

The particles are assumed to be highly relativistic $(\gamma \gg m_o c^2)$.

The general equations of motion in a Cartesian coordinate system are

$$(17) \qquad \frac{d}{dt} (m\vec{v}) = e\vec{E} + \frac{e}{c} \circ [\vec{v} \times \vec{B}]$$

Transforming to the particle trajectories by inserting

$$(18) \qquad \frac{d}{dt} = \frac{ds}{dt} \frac{d}{ds} = c \circ \frac{d}{dt}$$

one gets with (17)

$$(19) \qquad \vec{r}'' + \frac{(mc^2)'}{mc^2} \vec{r}' - \frac{e\vec{E}}{mc^2} - \frac{e}{mc^2} [\vec{r}' \times \vec{B}] = 0 \qquad \vec{r}(x,z,s)$$

Primes are derivatives with respect to the trajectory lengths s.

With

$$(20) \qquad mc^2 = m_o c^2 (\alpha s + \gamma_o) = m_o c^2 \circ \alpha \circ (s + \lambda)$$

γ_o: initial energy at the entrance in the focusing system

α: energy gradient $(MeV/m/m_o c^2)$

and

$$(21) \qquad e\,|\vec{E}| \;=\; m_o c^2 \cdot \alpha$$

we get

$$(22) \qquad \vec{r}'' + \frac{1}{s+\gamma}\,\vec{r}' - \frac{e\,\vec{E}}{\alpha(s+\gamma)m_o c^2} - \frac{e}{\alpha(s+\gamma)m_o c^2}(\vec{r}' \times \vec{B}) = 0$$

To simplify the differential equation system (22) we change to the system $\{x, z, \tau\}$ of the accelerated particle:

$$(23) \qquad \gamma_o\,ds \;=\; \gamma\,d\tau$$

$$(24) \qquad \frac{d^2}{d\tau^2}\,\vec{r} - \frac{\gamma \cdot e\vec{E}}{\gamma_o \cdot m_o c^2} - \frac{e}{\gamma_o \cdot m_o c^2}\;\frac{d}{d\tau}(\vec{r} \times \vec{B}) \;=\; 0$$

We will use this system of differential equations for the two types of magnetic fields:

a) Solenoid focusing , characterized by

$\vec{B}\,(0,\ 0,\ B)$
$\vec{E}\,(0,\ 0,\ E),$

(24) yields

$$\ddot{z} - \frac{e}{m_o c^2 \cdot \gamma_o}\,\dot{x}\,B = 0$$

$$(25)$$

$$\ddot{x} + \frac{e}{m_o c^2 \cdot \gamma_o}\,\dot{z}\,B = 0$$

(Points are derivatives with respect to τ)

Separation of the variables yields with (20) and (21)

$$\dddot{z} + \left(\frac{B}{\lambda E}\right)^2 \dot{z} = 0$$

(26)

$$\dddot{x} + \left(\frac{B}{\lambda E}\right)^2 \dot{x} = 0$$

A main solution for (26) is

(27)
$$y = A \cdot \cos \frac{B}{\lambda E} \tau$$

Going back again to the laboratory system by $\tau = \lambda \log \frac{s+\lambda}{\lambda}$ one gets

(28)
$$y = A \cdot \cos \left(\frac{B}{E} \log \frac{s + \lambda}{\lambda}\right)$$

and

(29)
$$y' = - A \frac{B}{E} \cdot \frac{1}{s+\lambda} \sin \left(\frac{B}{\lambda E} \log \frac{s+\lambda}{\lambda}\right)$$

The particles oscillate undamped in both coordinates with increasing wave length. That means in a solenoid field the acceleration reduces the beam divergence only. As the beam cross section and the radial particle momentum stay constant, the magnetic field has to be carried in its full strength up to the end of the accelerator.

b) Focusing with quadrupoles

With
$$\vec{B} \, (B_z, \, B_x, \, 0)$$

(24) yields

$$\ddot{z} + \frac{e}{m_o c^2 \gamma_o} \dot{s} \, B_x = 0$$

(30)

$$\ddot{x} - \frac{e}{m_o c^2 \gamma_o} \dot{s} \, B_z = 0$$

These equations are simplified by (23)

$$(31) \qquad B_x = gz; \quad B_z = gx; \quad g = g(s)$$

giving

$$(32) \qquad \ddot{y} \pm \frac{e \cdot \gamma}{m_o c^2 \cdot \gamma_o} \, g \cdot y = 0$$

(y is taken for x and z)

For thin lenses (32) further reduces by introducing the quantity Q

$$(33) \qquad Q = \frac{\delta B_x}{\delta z} \, ds = g \cdot L_{Q_o}$$

where L_{Q_o} is the quadrupole length in the laboratory system. For a relativistic particle the quadrupole length is reduced to

$$(34) \qquad L_Q = \frac{\gamma_o}{\gamma} \, L_{Q_o}$$

where L_Q is the quadrupole length in the particle system.

(33) and (34) gives

$$g = \frac{Q}{L_{Q_o}} = \frac{\gamma_o}{\gamma} \frac{Q}{L_Q}$$

To get a periodic solution for (32) the product $\gamma \cdot g$ has to be constant. That means $Q/L_Q = g_o$ and

$$(36) \qquad g = \frac{\gamma_o}{\gamma} \, g_o$$

With the lens strength k_o:

$$(34) \qquad k_o = \frac{e}{p_o} g_o = \frac{ec}{m_o c^2 \cdot \gamma_o} g_o$$

we get

$$(38) \qquad \ddot{y} \pm k_o y = 0$$

Thus, one has a motion similar to that of unaccelerated particle in a periodic quadrupole channel if one keeps the quadrupole strength k_o and length L_{Q_o} constant and monotonically increases the spacing between quadrupole centers according to eq. (23). This shows the advantage of the quadrupole focusing for long accelerators.

2. FODO-Channel

A suitable quadrupole focusing system for the positrons in a linear accelerator is the periodic FODO-channel.

For the determination of the optimum quadrupole arrangement Steffen (STE 65) gave the following formulae:

$$(39) \qquad \phi = 1 \sqrt{k}$$

$$(40) \qquad m^2 = \frac{1 + \tanh\phi(\tan\phi + \frac{L}{l}\phi)}{1 - \tan\phi(\tanh\phi + \frac{L}{l}\phi)}$$

$$(41) \qquad \epsilon^2 = \frac{R^4}{l^2} \frac{m^2}{(1+m^2)^2} \frac{\left[\tan\phi(ctanh\phi + \frac{L}{l}\phi) - 1\right]\phi^2}{1 + ctanh\phi(\tan\phi + \frac{L}{l}\phi)}$$

k: lens strength
m: beat factor
ϵ: acceptance of the FODO-channel
R: minimum radius of the irishole
l: lens length
L: distance between lenses

In order to find the optimum parameter for a given acceptance
and lens length, we calculated on the digital computer the
functions $\varepsilon_{max}(L,l)$, $m_{opt}(L,l)$ and $k_{opt}(L,l)$ (see Fig. 12).
The calculation yielded a constant beat factor $m_{opt} = 2,3$.
The maximum acceptance increases with decreasing L and l.
This shows the limitation of quadrupole focusing. In order
to get a proper quadrupole field, the quadrupole length
should be about twice as big as the aperture diameter.
Since in the linac the quadrupole aperture has to include
the accelerating structure, the diameter must be of the
order of 15 cm and, accordingly, the lens length must be
at least 25 to 30 cm. Because of the quadrupole coil ends
the distance L has to be larger than 15 cm. This require-
ment restricts the application of quadrupole focusing, and
figure 12 shows that it is impossible to start with the
FODO-channel directly behind the positron horn. Therefore,
we are obliged to focus the positrons in the first two
accelerator sections following the converter by means of
a strong solenoid field.

3. Solenoid field in the first two positron linac sections

The positrons spiral in the solenoid field with a radius
given by

$$(42) \qquad\qquad \rho = \frac{E_o r_o{}'}{eB}$$

E_r/c: radial momentum
E_o: initial energy
B: solenoid field strength

For $\rho \leqslant 0,5$ cm and $E_o = 6,0$ MeV and $r_o{}' = 100$ mrad we need
a magnetic field of 4Kr.

The phase area $F = \pi r r'$ decreases with increasing energy
and we can change to quadrupole focusing the earlier the
faster the particles are accelerated.

With a klystron power of 20 MW each the positrons reach
an energy of 114 MeV after the first two accelerator sections.
At this energy the phase area has reduced from 100 πmradcm
to 5,26 πmradcm.

Thus the emittance is small enough to change to quadrupole
focusing.

Acceptance and Emittance of the Solenoid

For a positron entering the solenoid fringe field at A
(see Figure) with the angle ϕ to the axis the maximum
radius of the spiral in the solenoid field is given by
the radius ρ resulting from

(43) $\qquad (r_o - \rho\sin\phi)^2 + \rho^2\cos^2\phi = (R-\rho)^2$

or

(44) $\qquad 2\rho_{max} = \dfrac{R^2 - r_o^2}{R - r_o \sin\phi}$

solenoid-
aperture
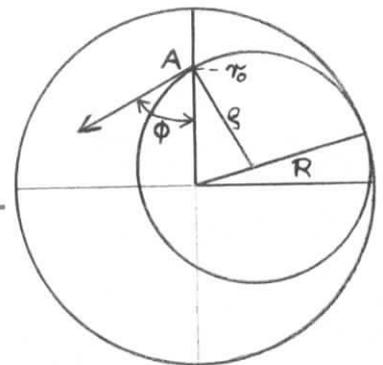
In the fringe field the positrons get an azimuthal momentum
$\Delta p = p_o \cdot \sin\Delta\phi$ while the radial momentum $p_r = p_o \cdot \sin\phi_r$
remains constant.

The particle momentum normal to the magnetic field lines
is given by

(45) $\qquad p_\perp = p_o\sqrt{\sin^2\phi_r + \sin^2\Delta\phi}$
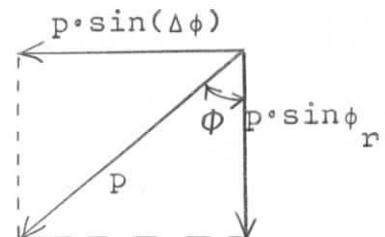
With (46)
$$\frac{1}{\rho_{max}} = \frac{eB}{cp_{\perp}}$$

and (44) this yields

(47)
$$p_{\perp} = \frac{r_o}{R} p_o \sin\Delta\phi + \frac{R^2 - r_o^2}{R} \frac{eB_o}{c}$$

The approximation $\sin\Delta\phi \approx \Delta\phi$ and $\sin\phi_r \approx \phi_r$ leads with (45) to

(48)
$$\phi_r = \pm \frac{1}{2} \frac{R}{R_o} \sqrt{1 - (\frac{r_o}{R})^2}$$

where

$$R_o = \frac{cp}{eB} \quad \text{and} \quad \Delta\phi = \frac{1}{2} \frac{eB}{cp} r \quad (Am\ 63)$$

For $B = 4$ kG and $R = 1$ cm we get

(48a)
$$\phi_r \leqslant \pm \frac{0,6}{p_o(\text{MeV/c})} \sqrt{1 - r_o^2}$$

The solenoid acceptance therefore is

(49)
$$\alpha_{sol} = \frac{0,6}{p_o} \cdot R\pi = \frac{600 \cdot \pi}{p_o(\text{MeV/c})} \text{ mradcm.}$$

The emittance at the end of the solenoid is within a few per cent the same for all energies because of the adiabatic damping during acceleration. The beam cross section remains constant while the divergence decreases with increasing energy. With E_1 being the positron energy at the end of the solenoids, the emittance for all initial momenta is given by

(50)
$$\varepsilon_{sol} = \frac{600\ \pi}{\frac{E_1}{c}(\text{MeV/c})} \text{ mradcm}$$

In our case the emittance of the solenoid for E_1 = 114 MeV
is

(50a) ε_{sol} = 5,26 πmradcm

This emittance is small enough to be matched into the
FODO-channel which is done by a quadrupole doublet.

4. FODO-channel in the remaining 5 positron linac sections

For further focusing following a FODO-channel is used
which has the following parameters (see Fig. 12)

$$\varepsilon = 5,28 \quad mradcm$$
$$L = 30 \ cm$$
$$2l = 30 \ cm$$
$$K = 9,86 \ m^{-2}$$
$$g = 374 \ G/cm$$

With a useful iris diameter 2R = 2,2 cm of the accelerating
structure and a beat factor of 2,3 we get a rectangular
beam cross section with half sides a = 1,01 cm and
b = 0.43 cm.

The free distances L have to be transformed to get the
quadrupole separations in the laboratory system.

The distance L_n of the center of the nth quadrupole from
the plane s = o is given by

(51) $L_n = \lambda(e^{nL'/\lambda} - 1)$

With an acceleration α = 0,18 $m_o c^2$/1/cm, γ_o " 224 and
L' = L + 2l = 60 cm one needs 25 quadrupoles to focus the
positrons up to an energy of 360 MeV.

Here, however, detailed calculations showed that the trans-
formation (23) is not exactly correct. In the derivation of
eq. (38) we used in the coefficient a "thin lens" approxi-
mation by the substitution (35). One gets the precise
FODO-structure by reducing the beam phase area according
to the increase of energy, taking from Fig. 13 the correspon-
ding quadrupole distances and with these from Fig. 14 the
lens strengths. In Fig. 15 the corrected quadrupole distances
and lens strengths are compared with those calculated with
eq. (51). One can see that the lens strengths and lens
separation have to be slightly smaller.

Acknowledgement

Literature

AG 62 :   T.L. Aggson - L. Burnot, LAL 27, Orsay 1962

AM 63 :   F. Amman - R. Andreani LNF - 63/46

AS 65 :   A. Asner, CERN 65 - 17

BU 63 :   G.I. Budker, Conference Report, DUBNA 1963

ME 61 :   S. van der Meer, CERN 61-7

ME 62 :   S. van der Meer, CERN 62 - 16

Mi 65 :   M.C. Crowley-Milling EL/TM/27

STAE 63: De Staebler SLAC-TN-65-23

STE 65:   K.G. Steffen, High Energy Beam Optics, I. Wiley 65

fig.1 Angular Distribution of Positrons (SLAC-de-Staebler)

$\frac{2\pi}{E_0^-} \cdot \frac{dn}{dE \cdot d\Omega}$

$\left[\frac{e^+/e^-}{GeV \cdot MeV \cdot sterad}\right]$

Target: Pb 2.9 r.l.

ϑ

0°

7°

15°

20°

25°

30°

35°

E_+ [MeV]

fig.2 Positron Intensity

$$J_+ \, [mA] = \left(\frac{1}{E_0^-} \cdot \frac{\Delta n_+}{\Delta E_+} \right) \cdot P_- \, [MW] \cdot \Delta E_+ \, [MeV]$$

$E_+ = 5{-}10 \; MeV$
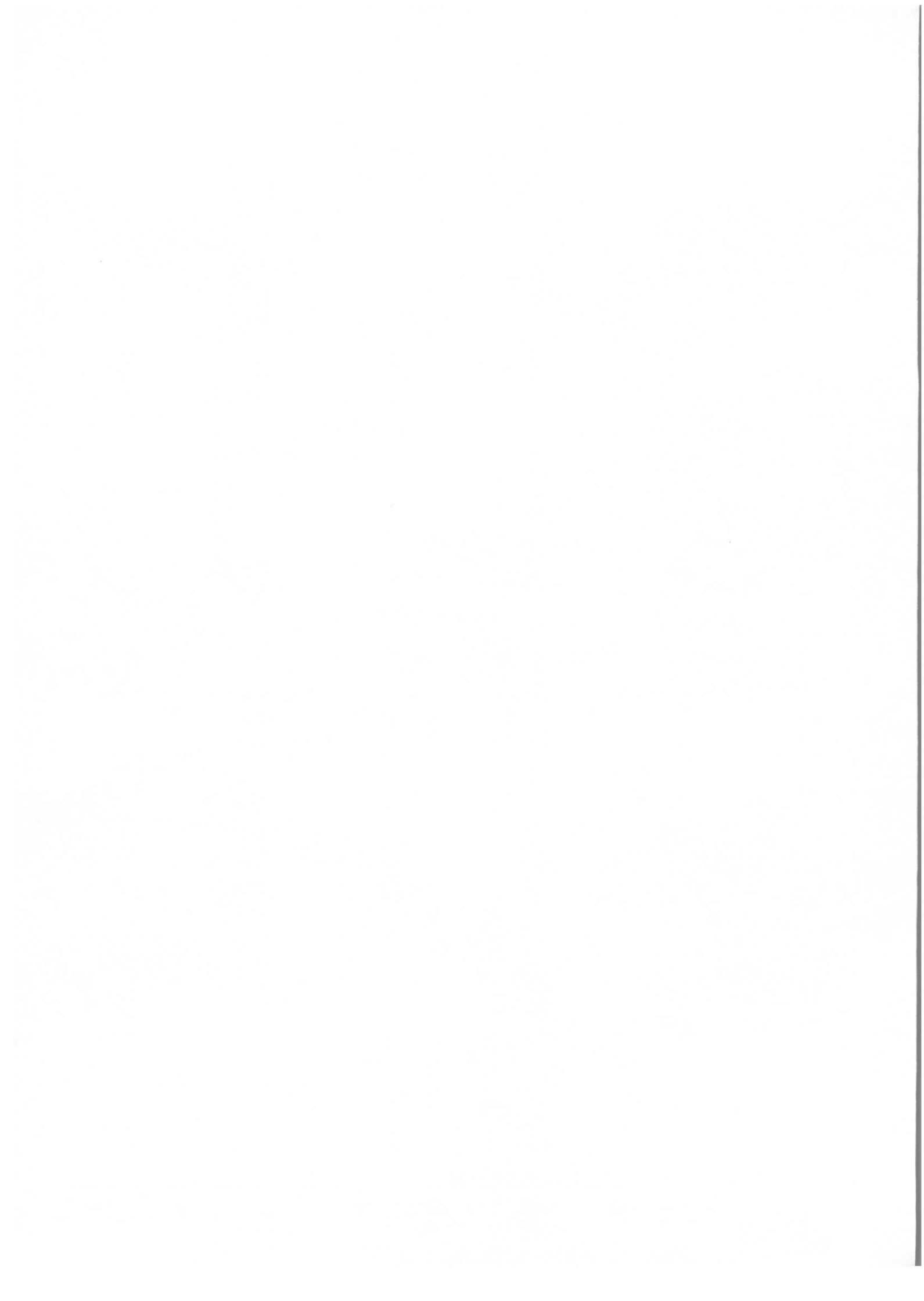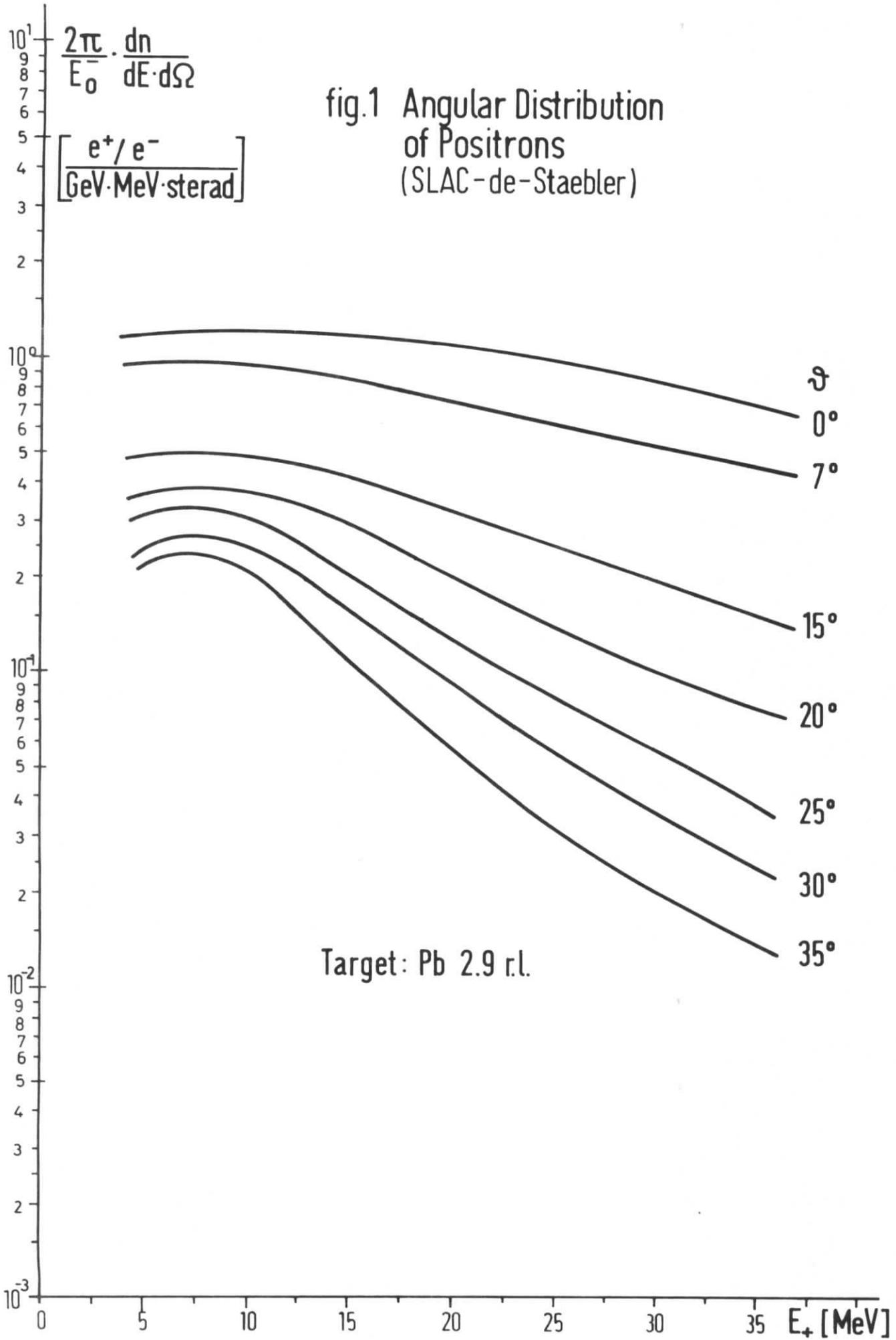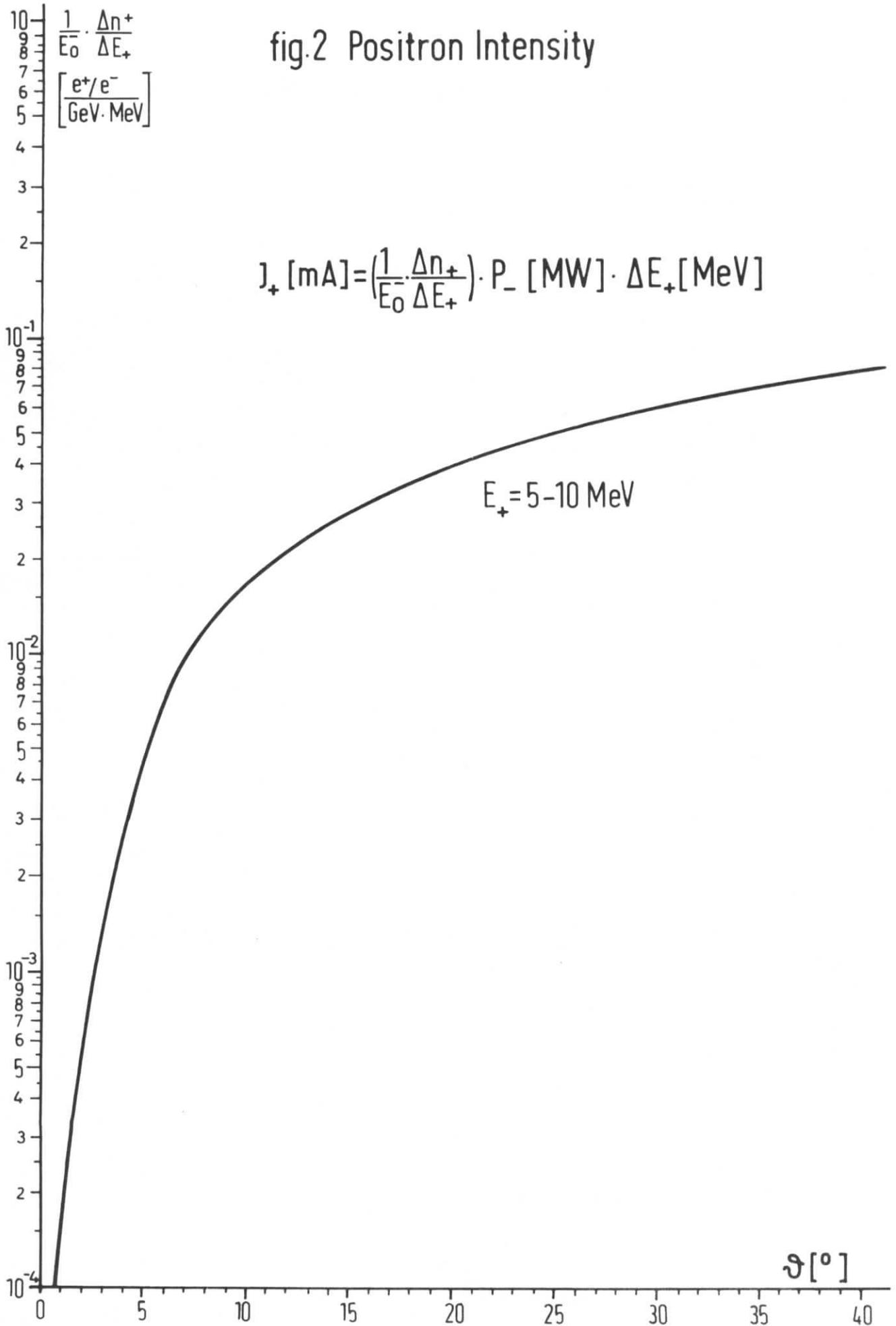
Positron Horn (schematic)

A Horn-Cross-Section

fig.3

fig.4 Typical Axial Trajektories
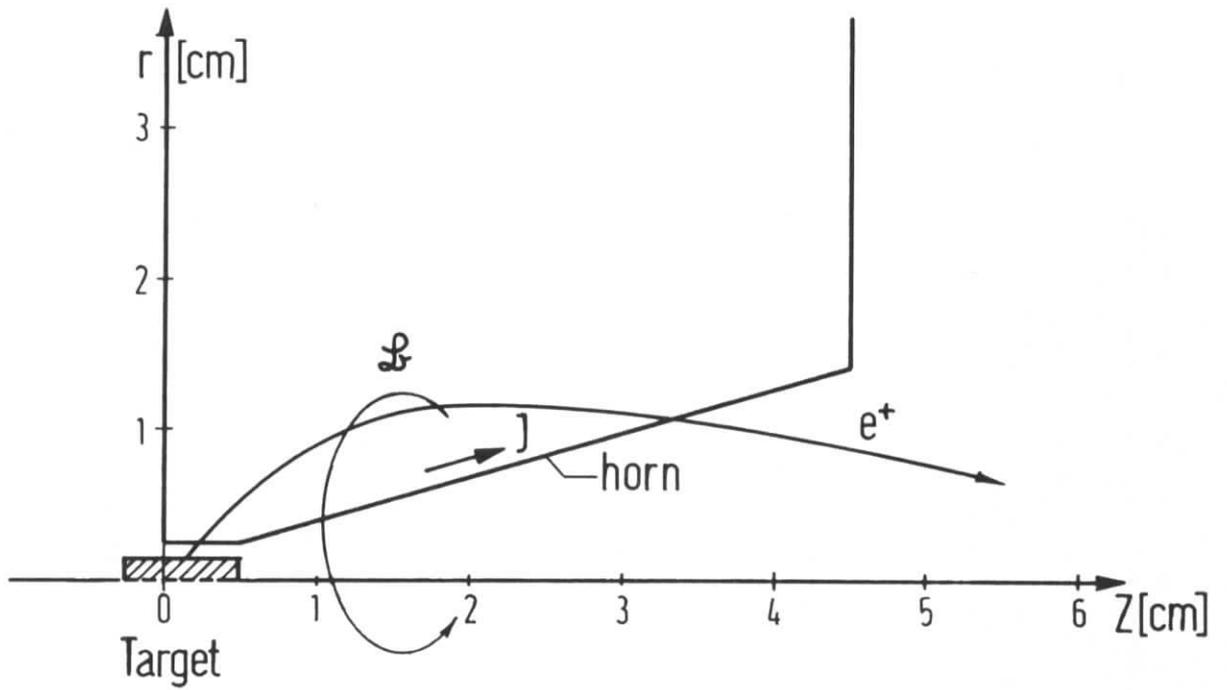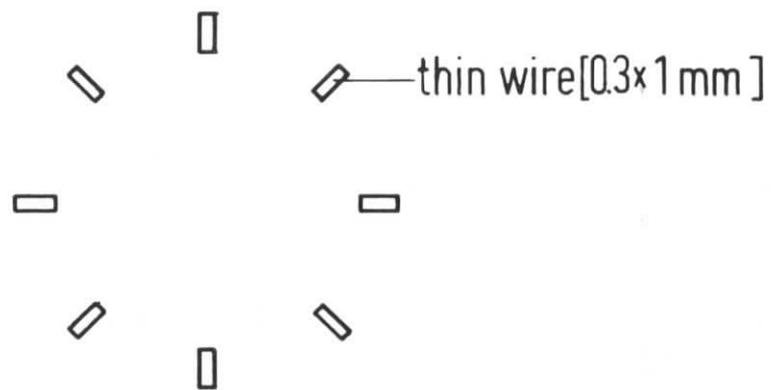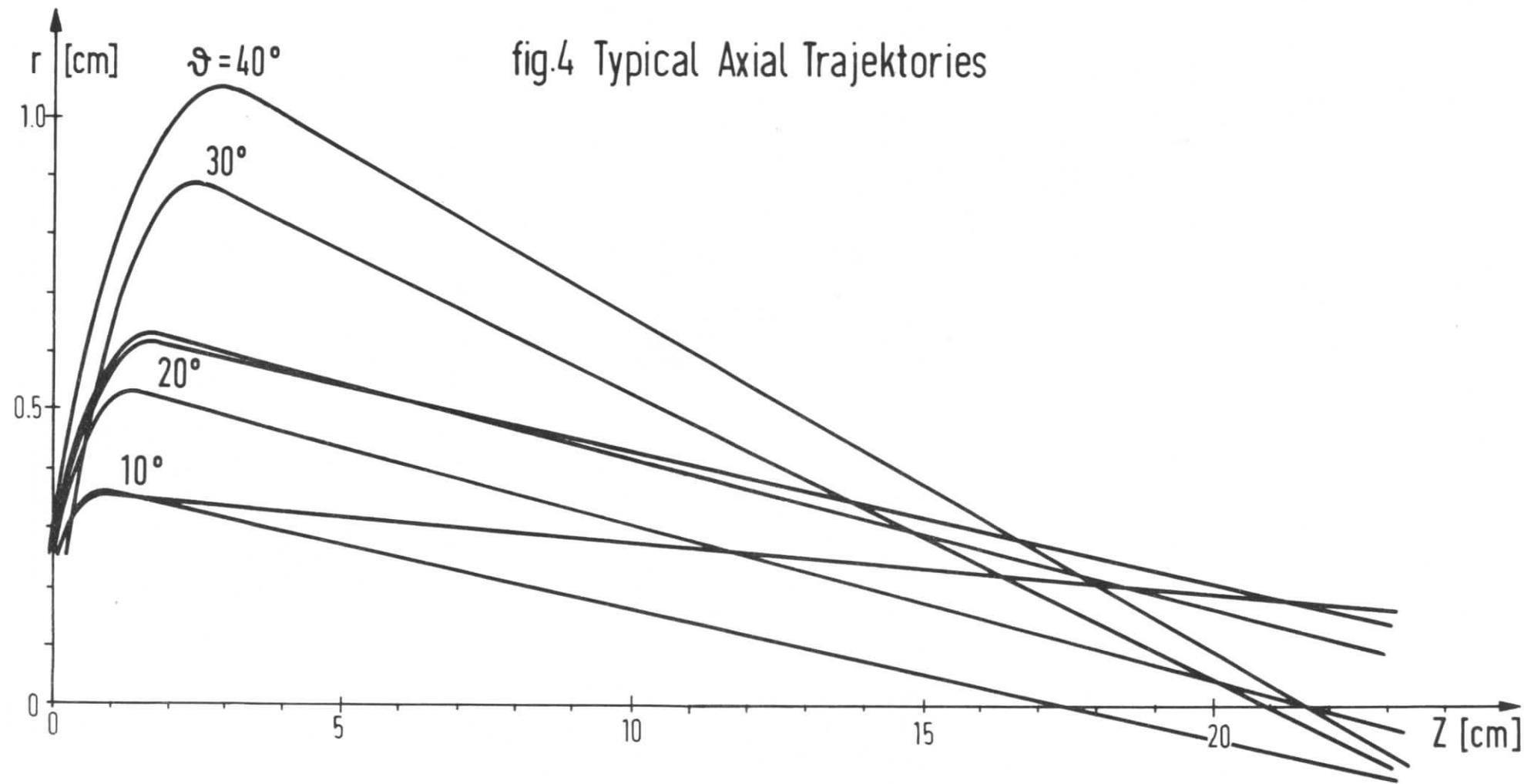
fig.5 Typical off Axis Trajektories
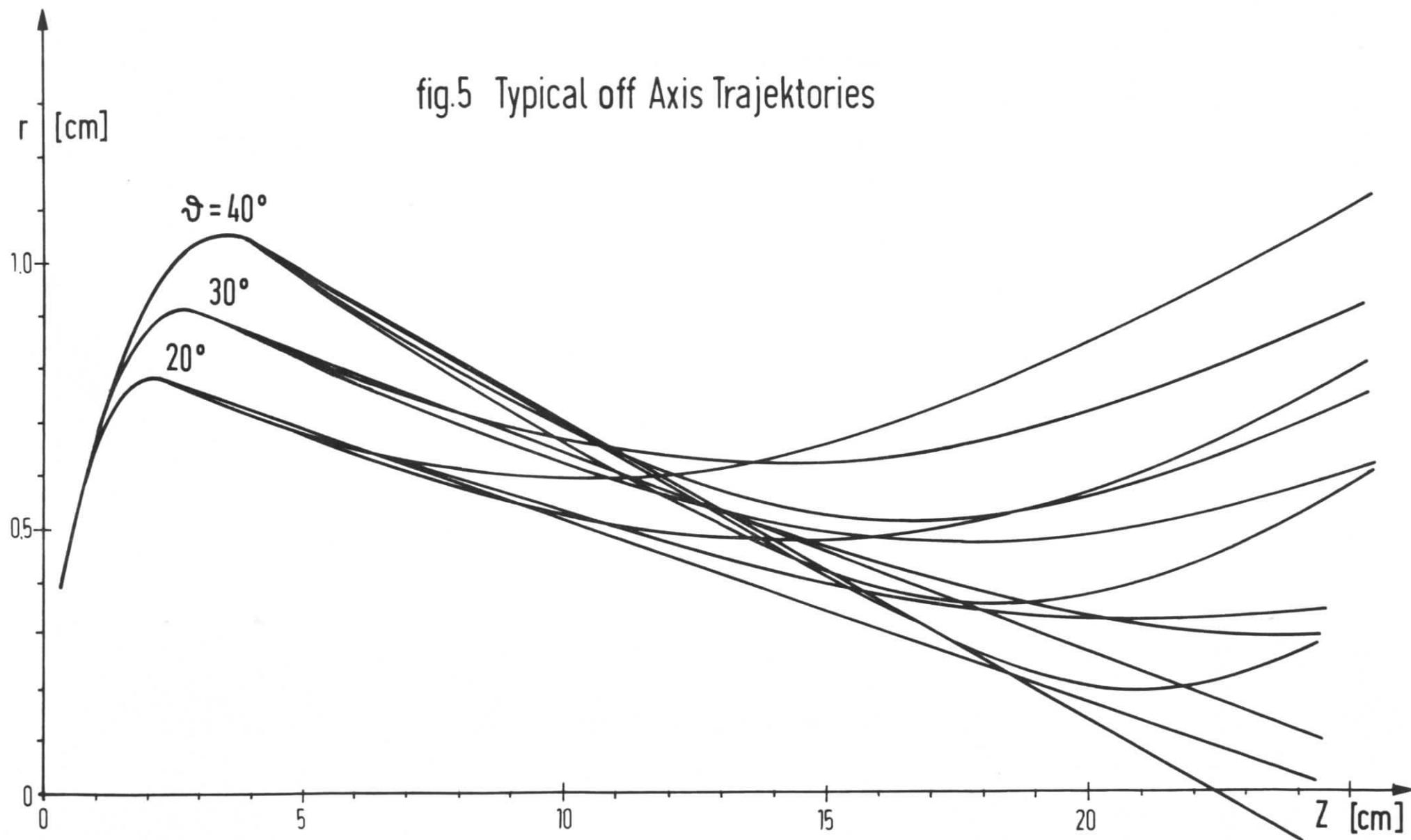
fig.6 Profile of the Positronhorn

The points on the curves are initial slopes in steps of two degrees!
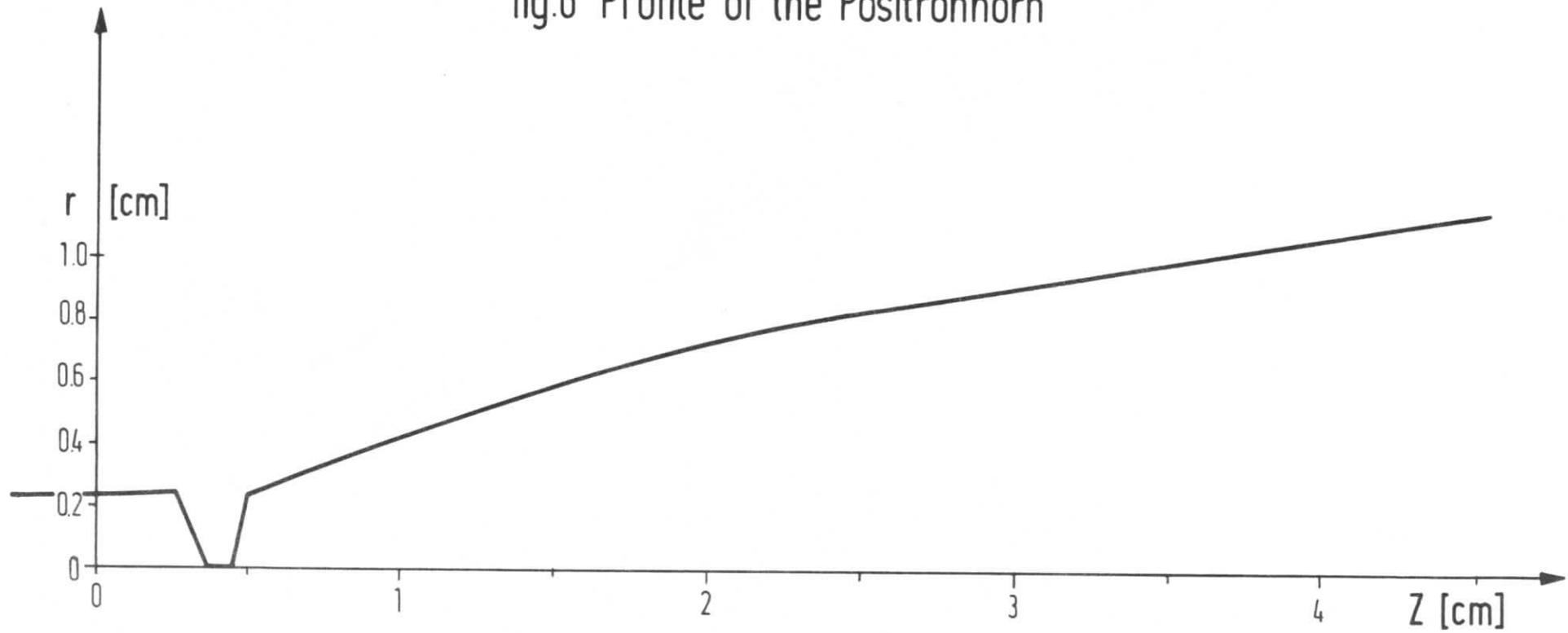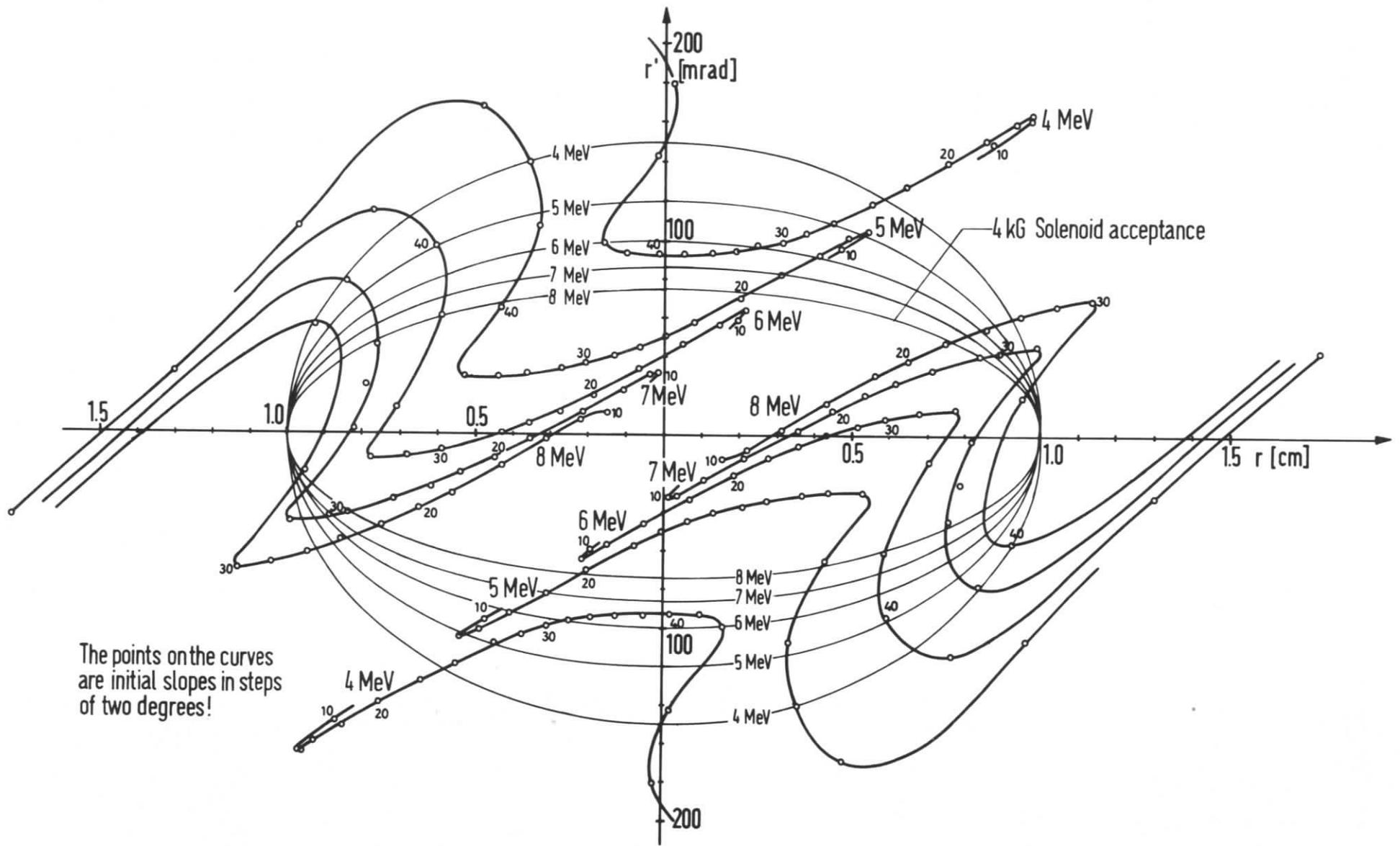
fig.7 Emittance of the Positron Horn for Various Positronenergies at Z=8 cm

fig.8 Energy Distribution

$$J_+ [mA] = \left(\frac{1}{E_0^-} \cdot \frac{\Delta n^+}{\Delta E_+}\right) \cdot P_- [MW] \cdot \Delta E_+ [MeV]$$

$\frac{1}{E_0^-} \frac{\Delta n^+}{\Delta E_+}$

$\left[\frac{e^+/e^-}{GeV \cdot MeV}\right]$

Positronhorn

3.6 MeV
$[E_{0+} = 6 \text{ MeV}]$

$\frac{E_+}{E_{0+}}$
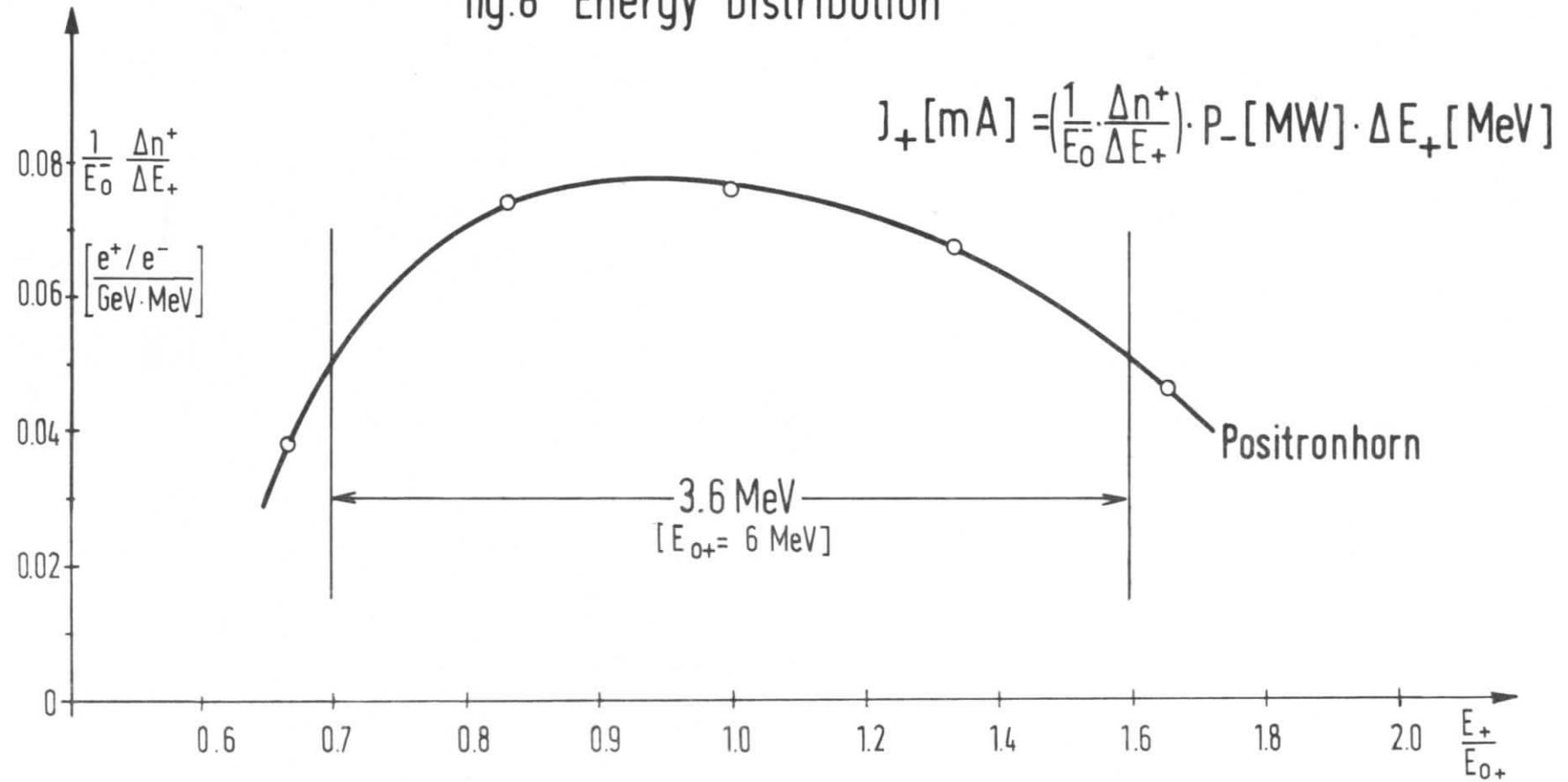
fig. 8a Energy Distribution
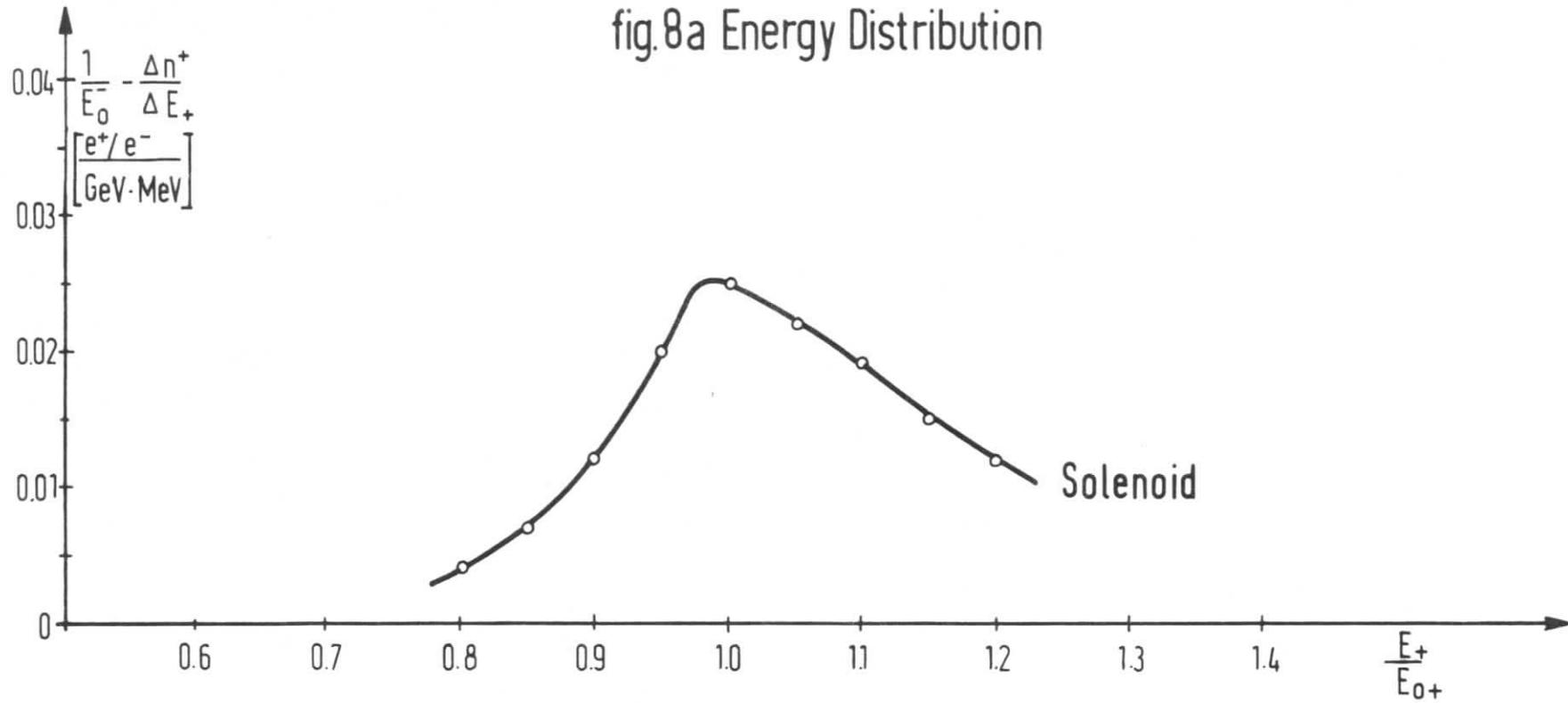
fig.9 Pathlengthdifference at Z = 8 cm
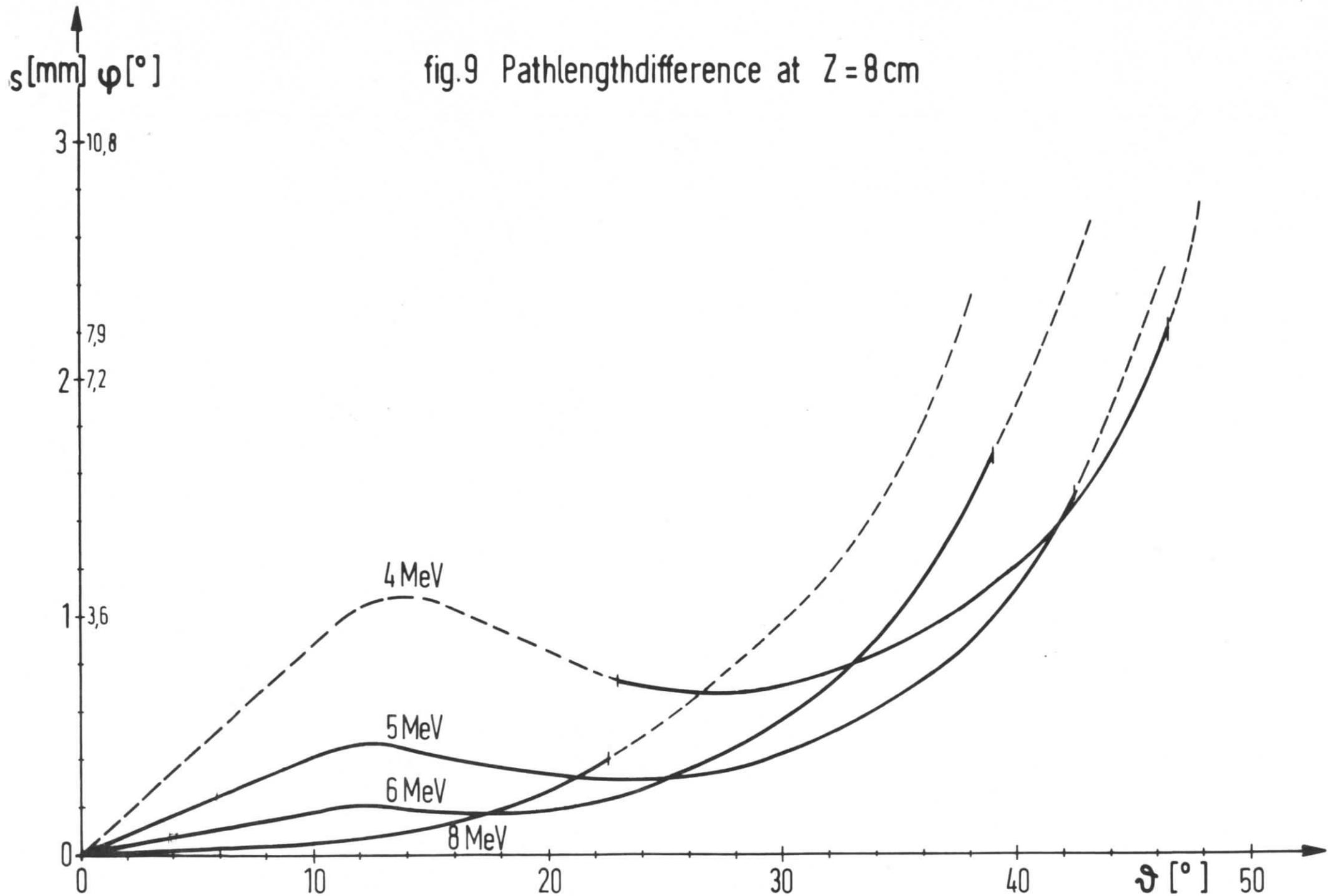
fig.10 Radial Fielddistribution in the Positron Horn
8 wires
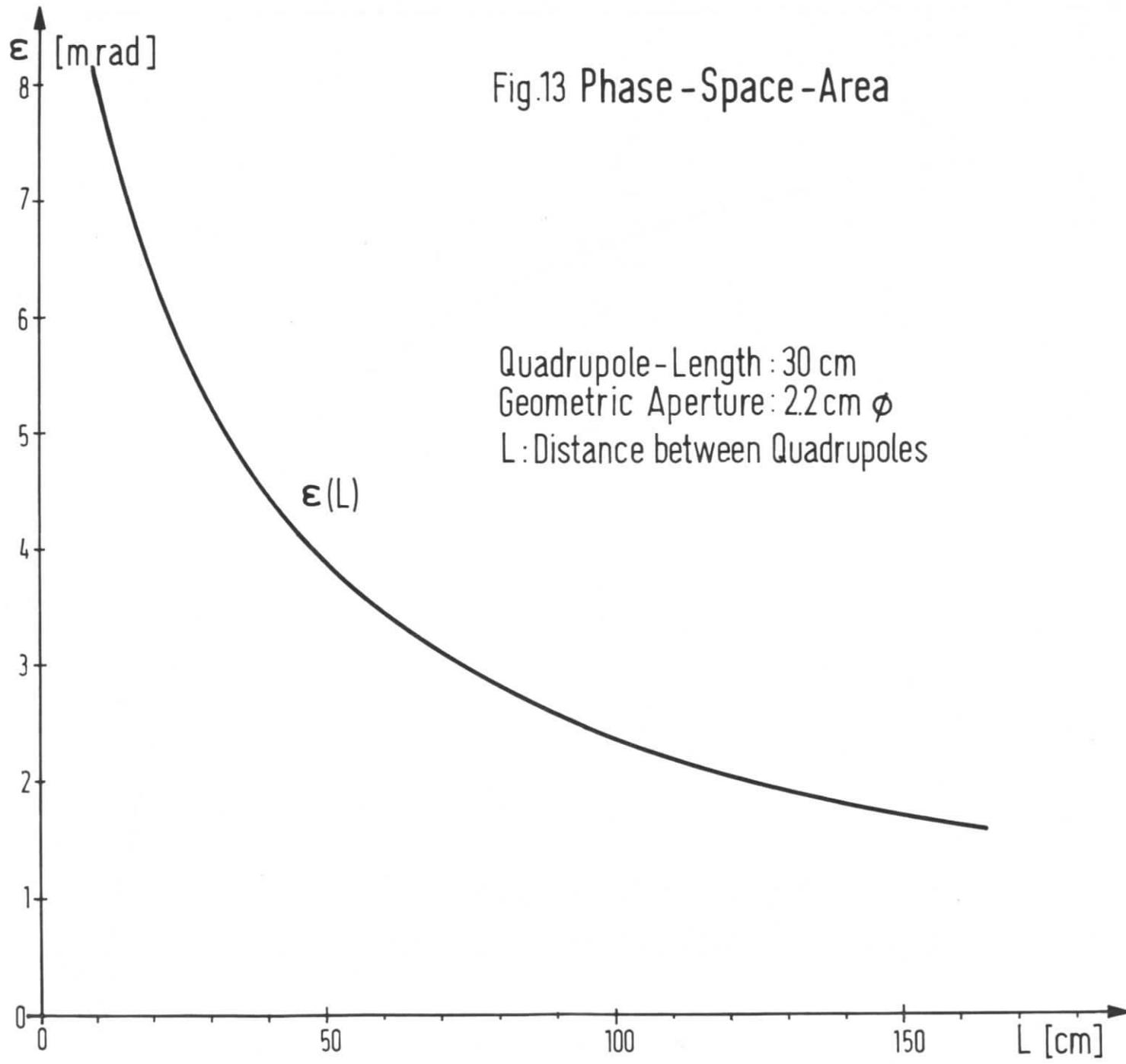
Horn Cross-Section
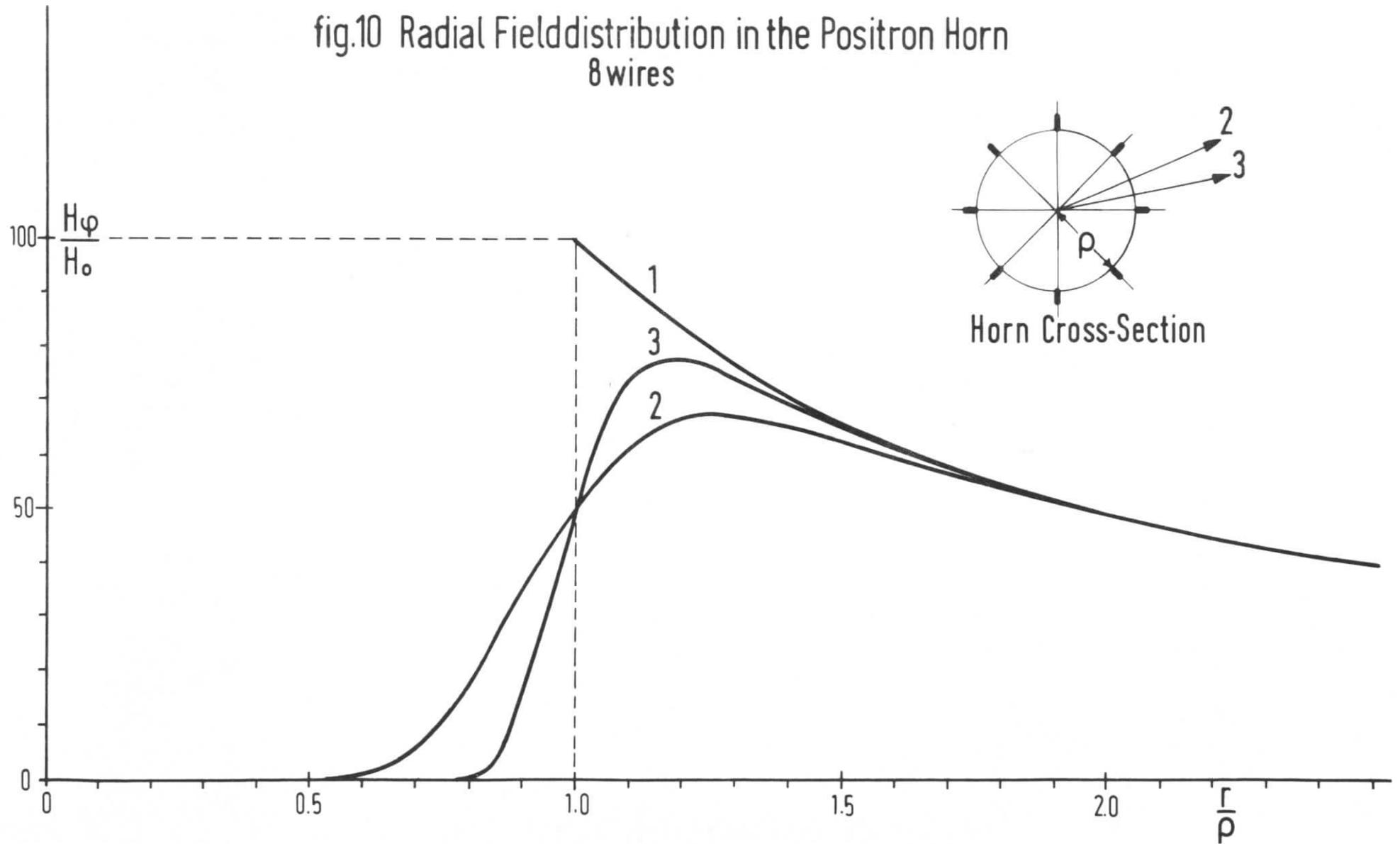
fig.11 Azimuthal Fielddistribution
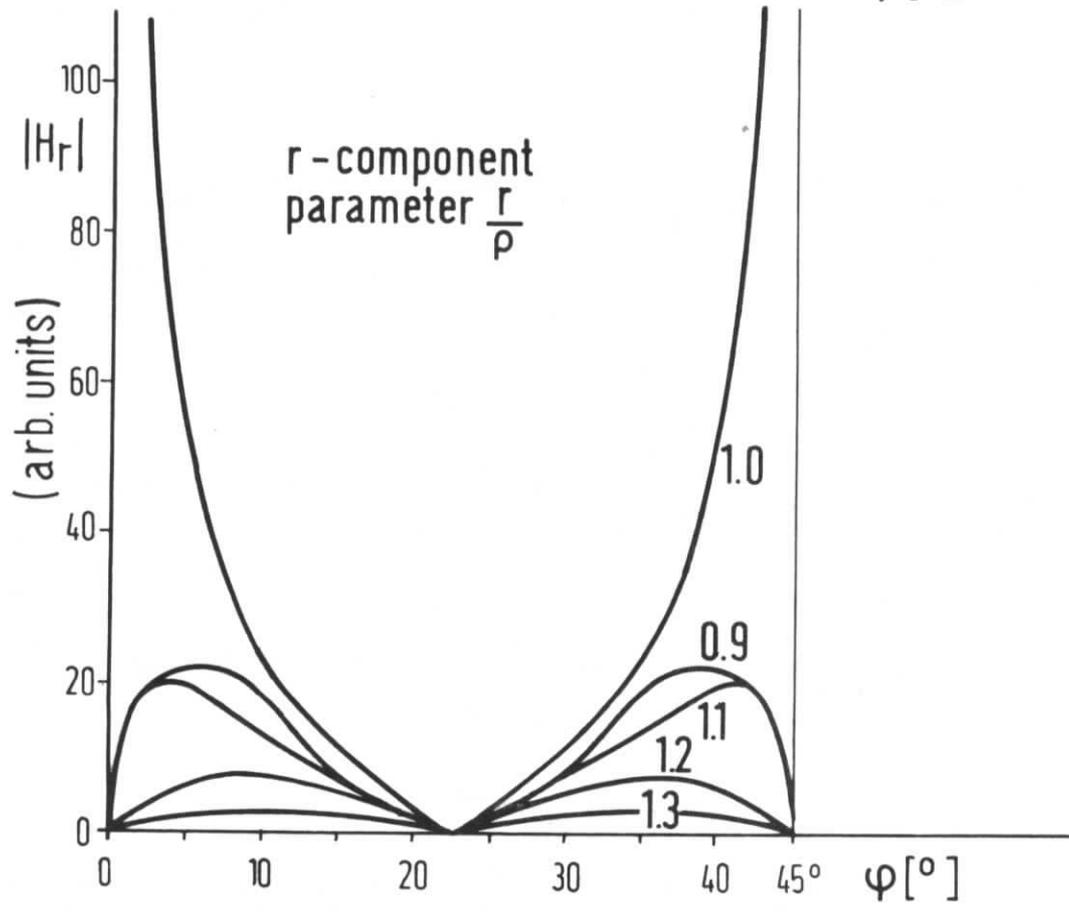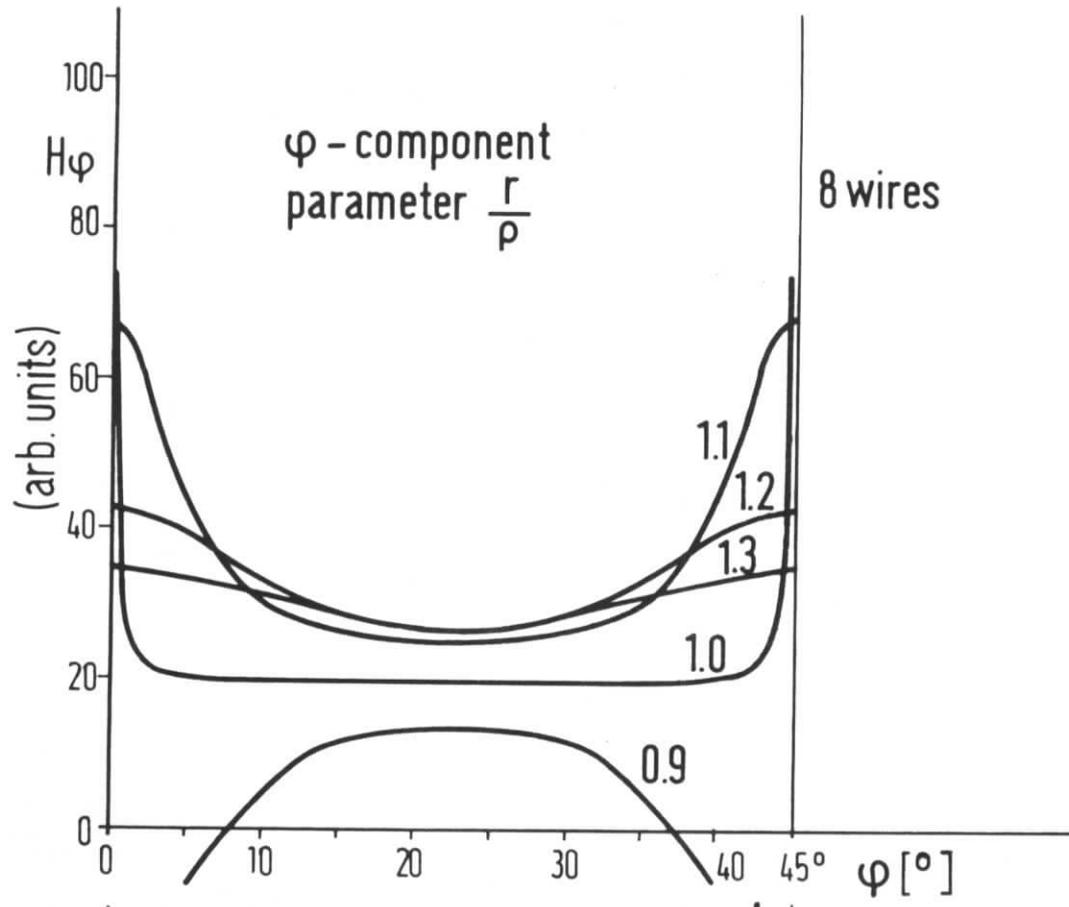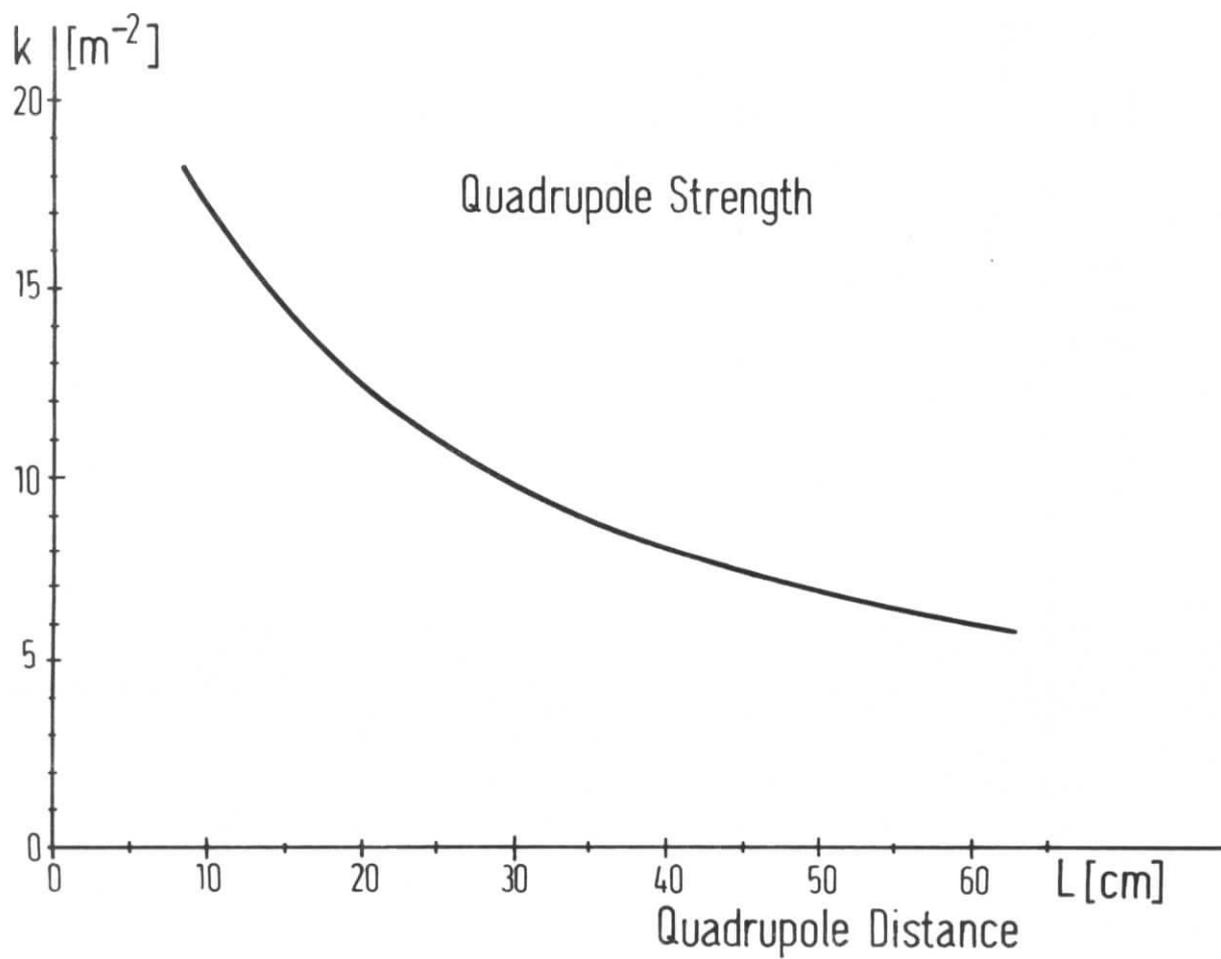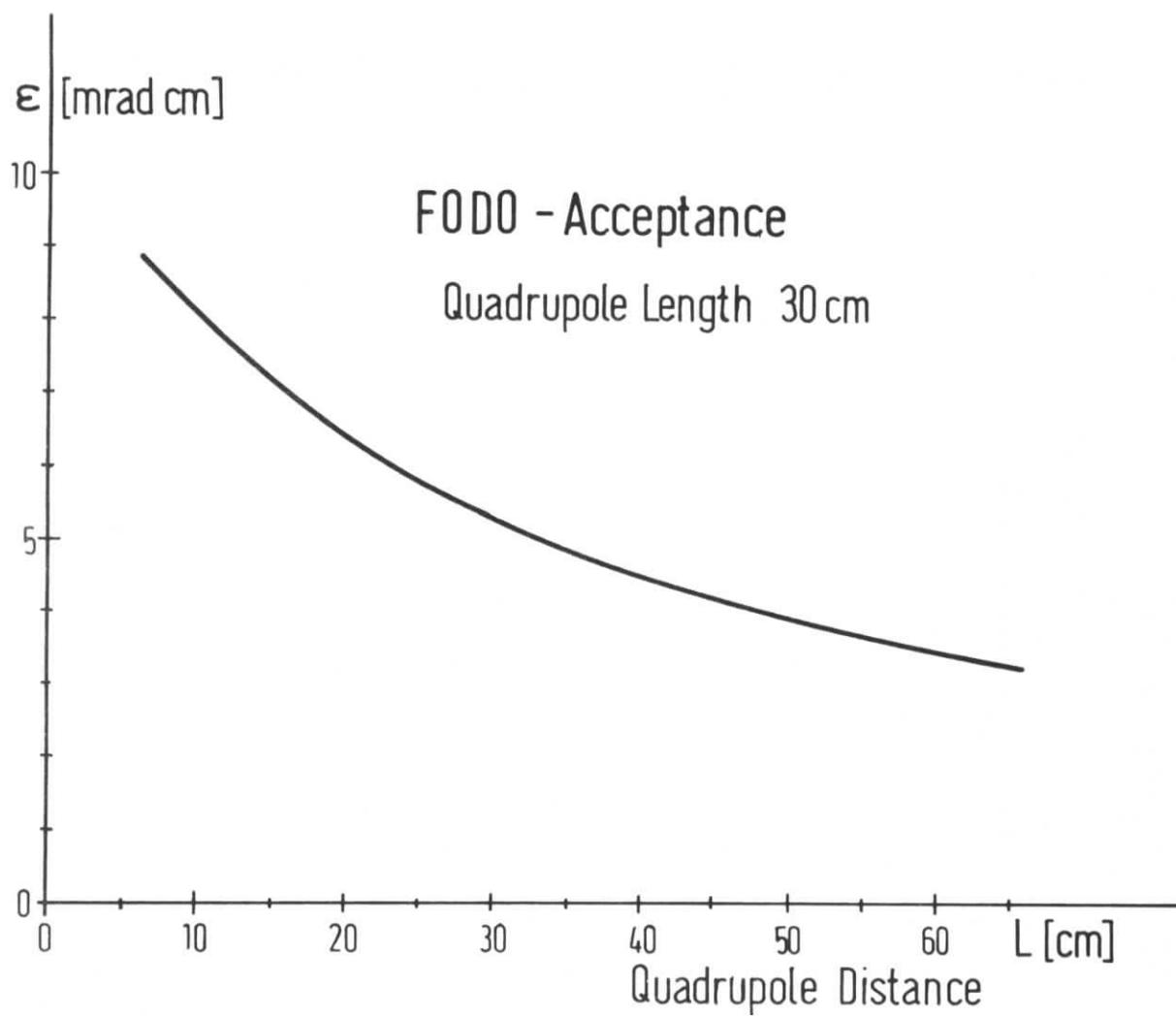
fig. 12

Fig.13 Phase-Space-Area

Quadrupole-Length : 30 cm
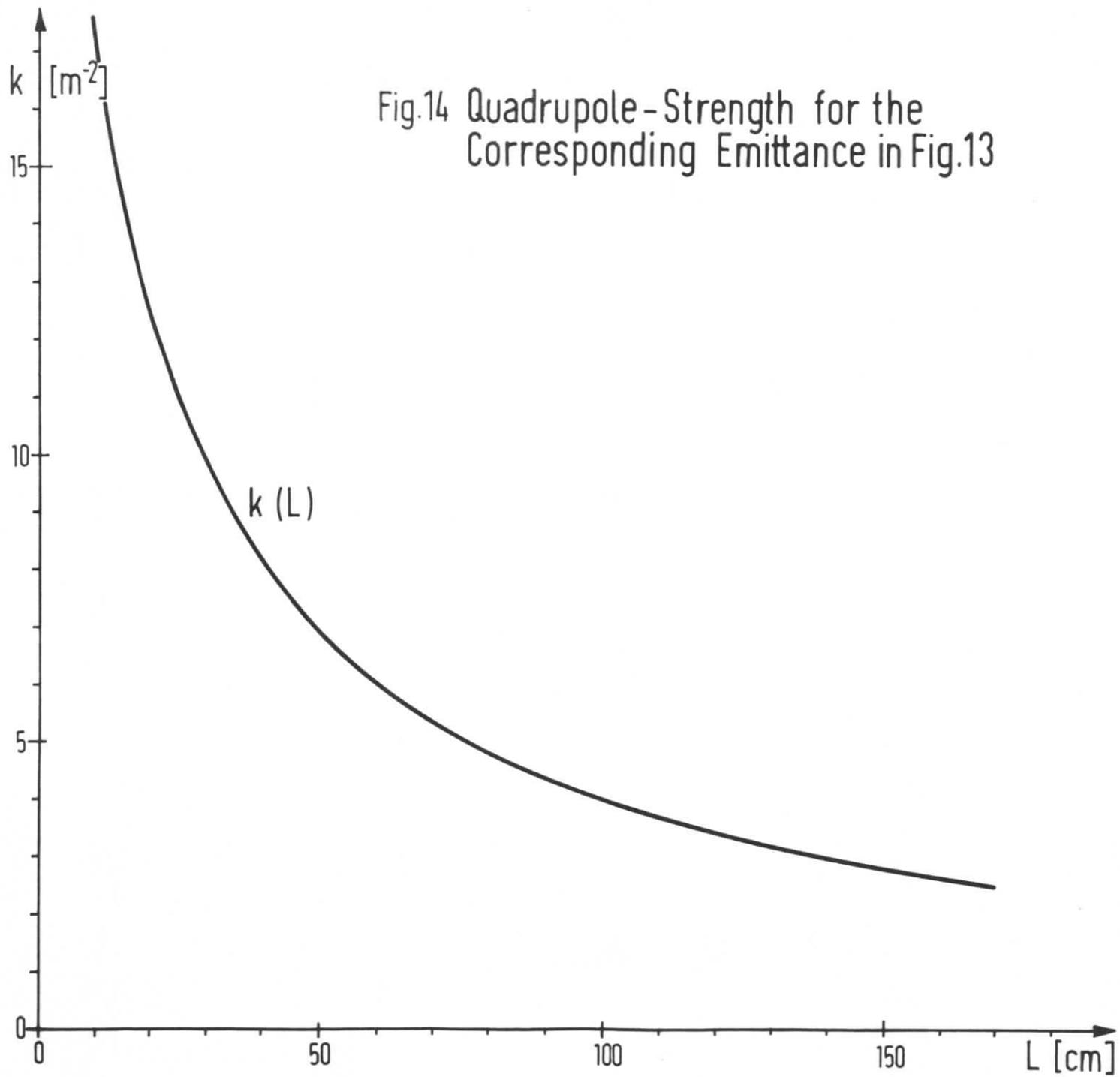Geometric Aperture : 2.2 cm ∅
L : Distance between Quadrupoles
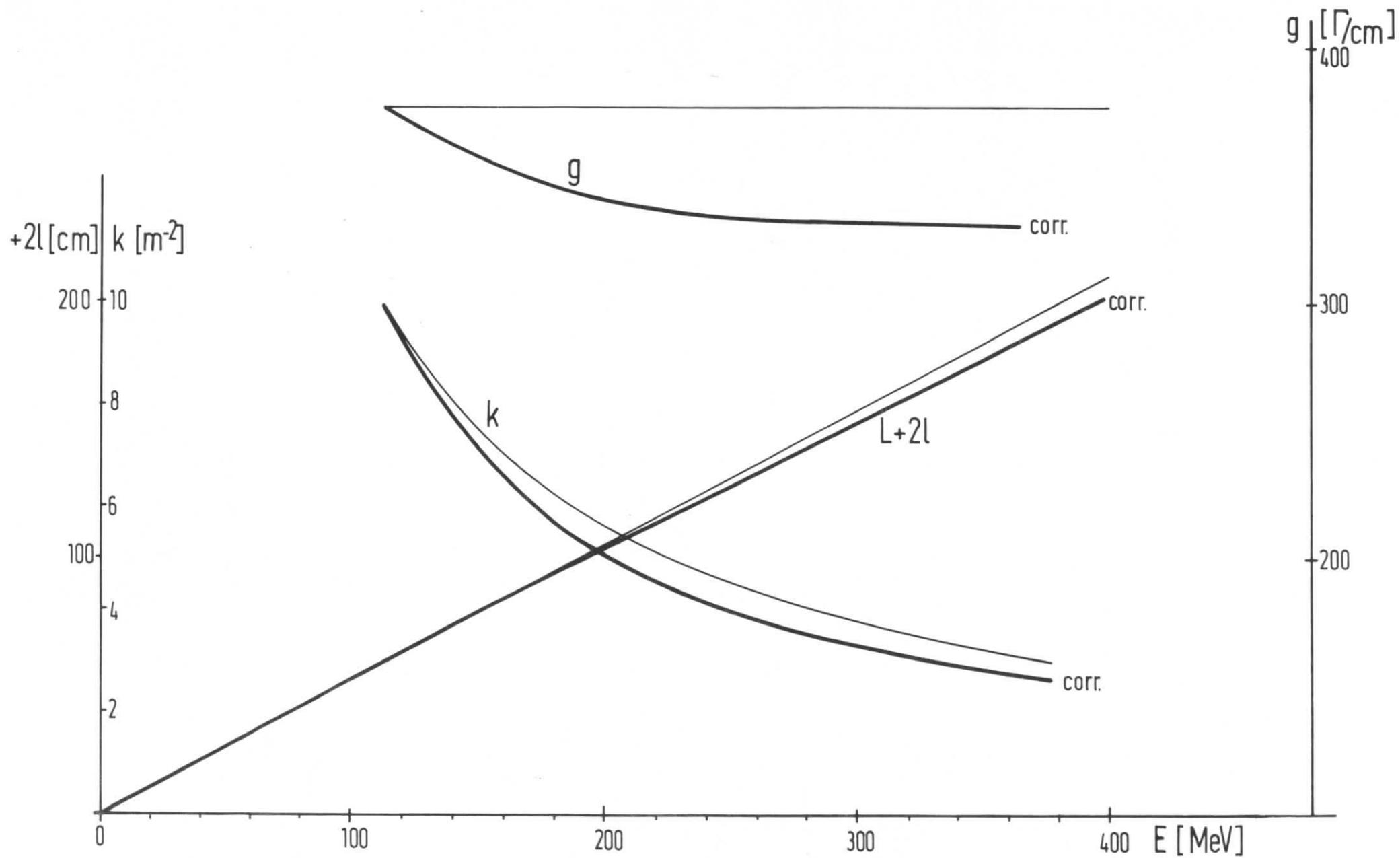
Fig.14 Quadrupole-Strength for the Corresponding Emittance in Fig.13

Fig.15 FODO-Channel over the Accelerator