DETERMINING THE $K_S \to 3\pi$ AMPLITUDE IN $\bar{p}p$ (OR e^+e^-) $\to K^0\bar{K}^0$

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The amplitude ratio $\langle 3\pi|T|K_S\rangle/\langle 3\pi|T|K_L\rangle$ can be well determined in e^+e^- (or low energy $\bar{p}p$) $\to K^0\bar{K}^0$ from the decay time-distribution when each produced kaon $\to 3\pi$, other unknown parameters of the distribution being obtainable from corresponding observations involving known channels like $\pi\pi$.

The ratio ζ_n of the amplitudes for decay of the short (K_S) - and long (K_L) - lived kaons

$$\zeta_n \equiv 1/\eta_n = \langle n|T|K_S\rangle/\langle n|T|K_L\rangle \tag{1}$$

into the decay channel n = 3π is comparable in interest to η_n for n = 2π. We denote the channels n = $\pi^+\pi^-\pi^0$, $3\pi^0$, $\pi^+\pi^-$ and $\pi^0\pi^0$ by subscript symbols c, o, +– and oo respectively. Firstly, in the superweak model (which is consistent with present data), there are no significant *CP*-violating decay amplitudes, giving‡1 $\zeta_o = \eta_{+-}$ as a test of the model, assuming *CPT*-invariance (as we shall do in our analysis). Secondly, if‡1 $\zeta_o \neq \eta_{+-}$, one can consider the possibility of *CP*-violating interactions or amplitudes having mixed parity properties and therefore contributing differently to $\zeta_{c,o}$ and to η_{+-}, the $3\pi(2\pi)$ channel being parity-conserving (violating). If the *CP*-violating interaction is purely [1] parity-conserving‡2, it would not significantly influence the 2π channel, would give the experimentally indicated equality $\eta_{+-} = \eta_{oo}$ and also would account for the smallness of the neutron dipole moment. Thirdly, the present evidence for *T*-violation and for consistency with *CPT*-invariance in K^0 decays depends on numerical evaluations of the overlap $\langle K_L|K_S\rangle$, using the Bell-Steinberger unitarity relation. The present imprecise knowledge of $\zeta_{c,o}$ contributes significantly to uncertainties [3] in these evaluations. As compared to η_{+-}, the numbers for ζ_c and the limit on ζ_o are at present [4] rather poor.

The difficulty which makes the determination of $\zeta_{3\pi}$ imprecise with an initial K^0 or $\bar{K}^0$ or K_L beam can be described as follows. For an initially created state $p_S|K_S\rangle + p_L|K_L\rangle$ where $p_{S,L}$ are production amplitudes for the $K_{S,L}$ components, the time distribution in any decay channel n is

$$I_n = |p_S S_n \theta_S + p_L L_n \theta_L|^2 \tag{2}$$

$$= e^{-\gamma_S t}|p_S S_n|^2 + e^{-\gamma_L t}|p_L L_n|^2 + 2e^{-\gamma t}\,\mathrm{Re}(p_L^* p_S L_n^* S_n\, e^{iMt}) \tag{3}$$

where the time t is measured in the rest system of the decaying kaon; $M = m_L - m_S$; $\gamma = \frac{1}{2}(\gamma_L + \gamma_S)$; $m_{L,S}$ and $\gamma_{L,S}$ are the masses and decay-widths of the $K_{L,S}$ mesons; $\theta_{S,L} = \exp[t(-im_{S,L} - \frac{1}{2}\gamma_{S,L})]$; S_n, L_n denote the decay amplitudes $\langle n|T|K_S, K_L\rangle$ normalised so that $|S_n|^2$ and $|L_n|^2$ are respectively the K_S and K_L partial decay widths in the channel n. Because $\gamma_S \approx 600\gamma_L$, $|\theta_S| < |\theta_L|$ for $t \neq 0$, and the exponential damping of the first term in (3) is the quickest. If $S_n \approx L_n$ (e.g., n=$\pi\ell\nu$) or if $S_n \gg L_n$ (e.g., n=$\pi\pi$), one can make the three terms in (3) observably comparable in suitable ranges of t for K^0 beams ($p_S = p_L$) or for $\bar{K}^0$ beams ($p_S = -p_L$). In K_S regeneration from K_L beams, p_S/p_L can be advantageously varied and is usually small; the case of $S_n \gg L_n$ is again quite suitable for observing the various terms in (3). The difficulty in getting ζ_n for channels with $L_n \gg S_n$ (as expectedly for n = 3π) with K^0 or $\bar{K}^0$ beams arises because the second term in (3) is then overwhelmingly

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‡1 Because of a (presumably small) *CP*-even component in the $\pi^+\pi^-\pi^0$ state, the superweak model gives $\zeta_c = \eta_{+-}$ only in the approximation of neglecting this component.

‡2 Recent $K^\pm \to 3\pi$ data [2] do not exclude such *CP*-violating effects of at least the same order as η_{+-}.

dominant, especially for appreciable times t; the K_S term and the K_S-K_L interference term are too weak to be easily observed. The situation is worsened if p_S/p_L is small, as for K_L beams after regeneration. Making p_S/p_L large, i.e., relatively enhancing the production of the K_L component in the initial state, would help. Regenerating a small K_L component in a K_S beam could do the job, but K_S beams are not available. This relative enhancement (and therefore, effectively producing a K_S beam) can be achieved in e^+e^- or low energy $\bar{p}p$ annihilation into a neutral kaon pair.

Since the experimental distribution I_n determines only the combination $p_S\zeta_n/p_L$, one needs p_S/p_L to deduce ζ_n. Since p_S/p_L depends only on the production of the initial state, and is the same for all decay channels, one can get p_S/p_L by observing the distribution (3) for a channel m for which ζ_m is known. A corresponding thing can be achieved in the above annihilation processes.

Unless angular integrations are done to make them incoherent, the C-$\binom{\text{odd}}{\text{even}}$ states $|K^0\bar{K}^0 \pm \bar{K}^0K^0\rangle$ of the $K^0\bar{K}^0$ pair are, in general, produced coherently so that the initial state is

$$a|K_LK_S - K_SK_L\rangle + b|K_SK_S - K_LK_L\rangle \qquad (4)$$

where the coefficients a and b are associated with the C-odd and C-even states respectively. Experimentally [5], the C-even state is produced only weakly in low energy $\bar{p}p$ annihilation so that roughly, $b/a < 0.1$. In $e^+e^- \to K^0\bar{K}^0$ also, one expects the C-odd state to be produced more strongly than the C-even state: $b/a \approx \alpha = 1/137$ because the C-odd (even) component of (4) arises from intermediate states of an odd (even) number of photons. This suppression[‡3] of the term in (4) can produce a relative enhancement of the K_S component in a time pattern resembling (2), as we shall now see.

To illustrate the enhancement of p_S/p_L, and to indicate the determination of $\zeta_{3\pi}$, consider[‡4] the decay rate R_{nm} of the state (4) into the *mode* (n, m) where the *channels* n and m are detected for decay of the first (time t_1) and the second (time t_2) kaon respectively, $t_{1,2}$ being measured in the rest frame of the relevant kaon.

$$R_{nm} = |a(L_nS_m\theta_L^1\theta_S^2 - S_nL_m\theta_S^1\theta_L^2) + b(S_nS_m\theta_S^1\theta_S^2 - L_nL_m\theta_L^1\theta_L^2)|^2 \qquad (5)$$

$$= |a(\zeta_m\theta_L^1\theta_S^2 - \zeta_n\theta_S^1\theta_L^2) + b(\zeta_n\zeta_m\theta_S^1\theta_S^2 - \theta_L^1\theta_L^2)|^2\,|L_nL_m|^2 \qquad (6)$$

$$= |a(\eta_n\theta_L^1\theta_S^2 - \eta_m\theta_S^1\theta_L^2) + b(\theta_S^1\theta_S^2 - \eta_n\eta_m\theta_L^1\theta_L^2)|^2\,|S_nS_m|^2 \qquad (7)$$

where the superscripts on $\theta_{S,L}$ refer to the times t_1 and t_2. For the mode $(3\pi, (3\pi)')$ where $(3\pi)'$ may also be the same as (3π), the term $b\zeta_n\zeta_m$ in (6) is negligible for b/a small because $\zeta_{c,o}$ are presumably small and also because of the time-dependence of this term. This gives

$$R_{3\pi,(3\pi)'} \Rightarrow |a(\zeta_{(3\pi)'}\theta_L^1\theta_S^2 - \zeta_{3\pi}\theta_S^1\theta_L^2) - b\,\theta_L^1\theta_L^2|^2\,|L_{3\pi}L_{(3\pi)'}|^2 \qquad (8)$$

which shows that for b/a small, the b and the a terms can be suitably comparable. For simplicity of illustration, take t_1 large so that $|\theta_S^1| \ll |\theta_L^1|$, keeping t_2 not very large, so that the $a\zeta_{3\pi}$ term in (8) can be dropped to get[‡5]

$$R_{3\pi,(3\pi)'} \Rightarrow |\zeta_{(3\pi)'}\,\theta_S^2 - \frac{b}{a}\theta_L^2|^2\cdot|\theta_L^1|^2\,|aL_{3\pi}L_{(3\pi)'}|^2 \qquad (9)$$

In the form (9), the t_2-dependence can be seen to be capable of exhibiting a useful interference pattern because the coefficient b/a of the K_L term is small and expectedly comparable to the coefficient $\zeta_{(3\pi)'}$ of the K_S term. Comparing this time dependence with (2), $p_L/p_S = -b/a$, a small number. This proves the point about the relative enhancement of the K_S component. $\zeta_{c,o}$ and b/a are not uniquely predictable,

‡3 Information on neutral kaon decays in the cases when $b/a \sim 1$ or $b/a \gg 1$ or when the $K^0\bar{K}^0$ pair is produced along with other particles is considered in ref. [6] which also includes further details of the present paper.

‡4 See, for example, refs. [7–9].

‡5 Obviously, interchanging $t_1 \leftrightarrow t_2$ does not matter to this illustration.

but the situation is decidedly better than the corresponding distribution (2) with $|p_S| = |p_L|$ which holds for a K^0 or $\bar{K}^0$ beam. Of course, the simplifications in getting (8) and (9) are not obligatory; these were meant only to illustrate the effective enhancement of p_S/p_L. One should determine $\zeta_{c,o}$ by fitting the full expression (6) to the observed time-distribution in the $(3\pi, (3\pi)')$ mode.[‡6] For b/a *negligibly* small, the a term in the $(3\pi, (3\pi)')$ mode is seen in (8) to be capable of determining $|\zeta_{c,o}|$ and *only* the relative phase of ζ_c and ζ_o.

To obtain b/a needed in the above determination of $\zeta_{c,o}$, one can consider the rate (5) for modes where $S_{n,m}$ and $L_{n,m}$ are known so that the only essential unknown in R_{nm} is b/a. For the mode $(2\pi, (2\pi)')$ where $(2\pi)'$ may also be the same as (2π), conditions are suitable [7] to determine b/a by using the known η_{+-} and η_{oo}. For $t_{1,2}$ not very large, the $b\theta_S^1\theta_S^2$ term in (7) can be comparable to the $a\eta_{n,m}$ terms, the $b\eta_n\eta_m$ term being then unimportant. This makes the observation of the b term in the full time-distribution (7) convenient.

To illustrate the possibility of getting b/a using R_{nm} for the modes $(3\pi, \pi\ell\nu)$ and $(\pi\ell\nu, (\pi\ell\nu)')$ where $(\pi\ell\nu)'$ may also be the same as $\pi\ell\nu$, we neglect for simplicity $\Delta S = -\Delta Q$ corrections of relative order x where $|x|$ is at most a few per cent experimentally [10]; these corrections are easy to incorporate. The amplitudes S_n and L_n for the $\pi\ell\nu$ channel with positively (denoted ℓ^+) and negatively (denoted ℓ^-) charged leptons become

$$S_{\ell^+} = L_{\ell^+} = (p/q)S_{\ell^-} = -(p/q)L_{\ell^-} = fp \tag{10}$$

where the real parameter $f = \langle \ell^+|T|K^0\rangle$; $K_{S,L} = pK^0 \pm q\bar{K}^0$; $|p|^2 + |q|^2 = 1$; $\langle K_L|K_S\rangle = |p|^2 - |q|^2 \approx 10^{-3}$. The rate for the mode $(3\pi, \ell^+)$ is

$$R_{3\pi,\ell^+} = |fpL_{3\pi}|^2\, |a(\theta_L^1\,\theta_S^2 - \zeta_{3\pi}\,\theta_S^1\,\theta_L^2) + b(\zeta_{3\pi}\,\theta_S^1\,\theta_S^2 - \theta_L^1\,\theta_L^2)|^2. \tag{11}$$

To get b/a, one combines a fit of the observed distributions to (11) and to $R_{3\pi,(3\pi)'}$. To see the suitability of (11), one can drop the $\zeta_{3\pi}$ terms which are relatively small, especially for t_1 not small, giving

$$R_{3\pi,\ell^+} \Rightarrow |fpL_{3\pi}|^2 \cdot |\theta_L^1|^2 \cdot |a\theta_S^2 - b\theta_L^2|^2 \tag{12}$$

which is convenient because b/a is small. Considering, in addition, the $(3\pi, \ell^-)$ mode offers the advantage that some interference terms have different signs (relative to (11)) because of (10). The modes $(\pi\ell\nu, (\pi\ell\nu)')$ offer similar possibilities[‡7] to get b/a. The rate for the (ℓ^+, ℓ^-) mode is

$$R_{\ell^+,\ell^-} = |f^2pq|^2 \cdot |a(\theta_S^2\,\theta_L^1 + \theta_S^1\,\theta_L^2) + b(\theta_S^1\,\theta_S^2 + \theta_L^1\,\theta_L^2)|^2 \tag{13}$$

which again is suitable for observation for $t_{1,2}$ not very small, b/a being small; (13) involves only b/a as the essential unknown. Considering also the modes $(\ell^\pm, \ell^\pm)$ offers, because of (10), the advantage of signchange (relative to (13)) of some interference terms.

For determining b/a, one can also consider "inclusive" modes of decay of (4), a channel n being observed for one kaon (time t_1), but no specific channel for the other kaon; however, these modes seem [6] more favourable for $b/a \sim 1$ than for $b/a \ll 1$ or $b/a \gg 1$.

Some remarks on the rates R_{nm}. For modes where the channels m and n are different, one obviously can consider $(R_{nm} \pm R_{mn})$ in order to get the appropriate information. For channels with n = m, the C-odd term of (4) drops out [e.g. 9] for $t_1 = t_2$; this offers a way to get (b). Though our arguments used only the time-dependences of R_{nm}, the overall factors (like $|L_n L_m|^2$ in (6)) in R_{nm} are known to be appreciable in the cases considered.

By using different $\bar{p}$ energies, one can vary the relative strength b/a of the C-even term; it is obviously desirable to determine $\zeta_{c,o}$ by using many different values of the phase and magnitude of b/a; these energies should preferably be low so that b/a is small[‡3]. The e^+e^- case seems unfavourable [6] with present luminosities, but the $\bar{p}p$ case is hopeful because of larger cross-sections in general. For e^+e^-, it is useful to work close[‡8] to a resonance (like ϕ, f′ mesons) energy; this also allows variations in b/a. Neglecting

[‡6] Other determinations of $\zeta_{c,o}$ by using (4) are less favourable [6] for b/a small.

[‡7] For details and other modes to get b/a and $\zeta_{3\pi}$, see ref. [6].

[‡8] This is also because, for the one-photon diagram, the coupling is due to only SU_3-breaking effects which are enhanced at a resonance like ϕ-meson.

corrections of relative second (and higher) order in α in amplitude, one retains only the 1- and 2-photon diagrams for $e^+e^- \to K^0\bar{K}^0$; then one knows [e.g. 11] the angular distribution of the $|a|^2$, $|b|^2$ and (a^*b) terms. Varying the angle θ between a kaon and $e^\pm$ then offers another way to vary the effective b/a; the b term vanishes for $\theta = \pi/2$ and the (a^*b) term is a maximum for $\theta \approx 55^\circ$, a convenient value. In the $\bar{p}p$ case, the corresponding angular distributions are not uniquely known, but the fact that they are different for the a and the b terms can again be utilized to vary effective b/a.

In summary, we have considered the $(3\pi, (3\pi)')$ mode of the $K^0\bar{K}^0$ pair for determining the $K_S \to 3\pi$ amplitude; this can avoid the difficulty behind determinations with a K^0 or $\bar{K}^0$ or K_L beam. The relative enhancement of the K_S component in an 'effective beam' is due to the weakness of the C-even component b of the initial $K^0\bar{K}^0$ state. The unknown production amplitude b/a needed in the above determination can be obtained within the experiment by considering the time distribution for a variety of other known modes. Some advantages of the present method are due to two time coordinate $t_{1,2}$ being independently variable and due to the variability of b/a by varying 1) the $\bar{p}$ (or $e^\pm$) beam energy and 2) the angle between one kaon and the beam.

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