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NEW FORM OF VECTOR MESON DOMINANCE AND THE DEPENDENCE ON THE VECTOR MESON MASS SQUARED

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Abstract: We propose a new form of VMD which relates the reactions $\pi N \to VN$ and $\gamma N \to \pi N$. Our form fits the experimental results within errors. As our restriction is strong enough, we can discuss the k_μ^2 dependence. Unexpectedly the k_μ^2 dependence is larger and this suggest the necessity of generalized vector dominance.

1. Introduction

It is well known that vector meson dominance [1] (VMD) works well in the region $|k_\mu^2| \lesssim m_\rho^2$ and explains the experiment within a factor of 2. On the other hand e^+e^- annihilation experiments [2] force one to generalize VMD by introducing more vector mesons than the originally observed ρ^0, ω and ϕ. At first, the application of VMD [3] to the following processes

$$\text{“}\gamma\text{”} + N \to \pi + N\,, \qquad\qquad \pi + N \to V + N\,,$$

succeeded quite well and the different angular distribution was explained by considering the helicity dependence of the cross section. However, more detailed analysis on pion photoproduction by linearly polarized photons [4] shows some contradictions to VMD.

In this note we propose a new form of VMD, and by comparing with experiment we discuss the k_μ^2 dependence of the amplitude (observable quantities). We use the unitarity relation, and describe the observable quantities in terms of the three particle

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forward scattering amplitudes [5] and then we apply Cho-Sakurai type [6] arguments to these discontinuities. That is, we expand the discontinuities up to the first order of k_μ^2 and apply the gauge condition in each order of k_μ^2. As the discontinuities themselves are related to the observable quantities linearly, our new form of VMD is more useful than the Cho-Sakurai type argument. For example, in our form we need not know the experimental results of the complete experiment on pion photoproduction, and moreover, we have enough restriction to discuss the k_μ^2 dependence in our form.

In sect. 2 we explain the formalism of the new form of VMD. In the third section we analyze the experiments using our formula and discuss the k_μ^2 dependence of the amplitude. In sect. 4 a short discussion is given.

2. New relations of vector meson dominance †

In this section we show the new relations between the processes

$$\text{“}\gamma\text{”} + \mathrm{N} \to \pi + \mathrm{N}\,, \tag{1}$$

$$\pi + \mathrm{N} \to \mathrm{V} + \mathrm{N}\,, \tag{2}$$

using VMD. The derivation of the new relations becomes possible by representing the cross sections and the vector-meson density matrix elements of these processes, (1) and (2), in terms of the discontinuities of the three-particle forward scattering amplitudes;

$$M_{\mu_1\nu_1}^{\mu_2\nu_2}(\text{“}\gamma\text{”}\ \pi) \equiv \mathrm{Disc}\,\langle p\nu_2,\, k\mu_2; q|M|p\nu_1,\, k\mu_1; q\rangle\,, \tag{3}$$

$$M_{\mu_2\nu_1}^{\nu_2\mu_1}\ (\pi\ \mathrm{V}) \equiv \mathrm{Disc}\,\langle p\nu_2,\, q; k\mu_1|M|p\nu_1,\, q; k\mu_2\rangle\,, \tag{4}$$

where q, p and k are the energy-momentum four-vector of pion, initial nucleon and vector particle, respectively, ν_i and μ_i represent the helicity of nucleon and vector particle, respectively.

When the nucleon is unpolarized, we represent the discontinuities (3) and (4) as the sum of the following ten (non-gauge invariant) terms;

$$\mathrm{Disc}\, T = \epsilon_{\mu_2}^{*}\, M_{\mu_1}^{\mu_2}\, \epsilon^{\mu_1} \tag{5}$$

$$= A_+a_+ + Bb + Cc + Gg + F_+f_+ + iF_-f_- + J_+j_+ + iJ_-j_- + H_+h_+ + iH_-h_-\,,$$

where ‡

$$a_+ = (\epsilon^*\epsilon) = \tfrac{1}{2}\,[(\gamma\epsilon^*)(\gamma\epsilon) + (\gamma\epsilon)(\gamma\epsilon^*)]\,, \tag{6}$$

$$b = (p\epsilon^*)(p\epsilon)\,, \tag{7}$$

† The result in this section is a special case of that for the inclusive reaction derived in ref. [5].
‡ ϵ is the polarization vector of photon (vector meson), and we use the abbreviation $(ab) \equiv \boldsymbol{a}\cdot\boldsymbol{b} - a_0b_0$.

$$c \ = (q\epsilon^*)(q\epsilon)\,, \tag{8}$$

$$g \ = (k\epsilon^*)(k\epsilon)\,, \tag{9}$$

$$f_+ = (p\epsilon^*)(q\epsilon) + (q\epsilon^*)(p\epsilon)\,, \tag{10}$$

$$f_- = (p\epsilon^*)(q\epsilon) - (q\epsilon^*)(p\epsilon)\,, \tag{11}$$

$$h_+ = (k\epsilon^*)(q\epsilon) + (q\epsilon^*)(k\epsilon)\,, \tag{12}$$

$$h_- = (k\epsilon^*)(q\epsilon) - (q\epsilon^*)(k\epsilon)\,, \tag{13}$$

$$j_+ = (k\epsilon^*)(p\epsilon) + (p\epsilon^*)(k\epsilon)\,, \tag{14}$$

$$j_- = (k\epsilon^*)(p\epsilon) - (p\epsilon^*)(k\epsilon)\,. \tag{15}$$

In the expression (5) all the coefficients A_+, B, ... are taken to be real.

Gauge invariance requires the relation

$$k_{\mu_2} M^{\mu_2}_{\mu_1} \epsilon^{\mu_1} = \epsilon^*_{\mu_2} M^{\mu_2}_{\mu_1} k^{\mu_1} = 0\,, \tag{16}$$

that is, the coefficients satisfy the following equations

$$A_+ + k_\mu^2 G + (kq) H_+ + (kp) J_+ = 0\,, \tag{17}$$

$$(kq) H_- + (kp) J_- = 0\,, \tag{18}$$

$$(kq)\, C + (kp) F_+ + k_\mu^2 H_+ = 0\,, \tag{19}$$

$$(kp) F_- + k_\mu^2 H_- = 0\,, \tag{20}$$

$$(kp) B + (kq) F_+ + k_\mu^2 J_+ = 0\,. \tag{21}$$

Then we can choose the following five discontinuities as independent; A_+, C, B, F_+ and F_-.

The cross sections and the vector meson density matrix elements ρ_{ij} are given in terms of these Lorentz invariant discontinuities as follows.

(i) $\pi + \mathrm{N} \to \mathrm{V} + \mathrm{N}$:

$$\rho_{11}\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi\mathrm{V}} = \frac{1}{4wq}\,[2A_+ + q^2\sin^2\theta B + q^2 \sin^2\theta C - 2q^2\sin^2\theta F_+]\,, \tag{22}$$

$$\rho_{00}\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi\mathrm{V}} = \frac{1}{4wq}\,\Big[2A_+ + \frac{2(qk_0\cos\theta + kp_0)^2}{m_\mathrm{V}^2} B + \frac{2(qk_0\cos\theta - kq_0)^2}{m_\mathrm{V}^2} C - \frac{4(qk_0\cos\theta + kp_0)(qk_0\cos\theta - kq_0)}{m_\mathrm{V}^2} F_+ \,, \tag{23}$$

$$\rho_{1-1}\left(\frac{d^3\sigma}{dk}\right)^{\pi V} = \frac{1}{4wq}\,[-q^2\sin^2\theta B - q^2\sin^2\theta C + 2q^2\sin^2\theta F_+]\ , \tag{24}$$

$$\mathrm{Re}\,\rho_{10}\left(\frac{d^3\sigma}{dk}\right)^{\pi V} = \frac{\sqrt{2}\,q\sin\theta}{4wq}\left[\frac{qk_0\cos\theta + kp_0}{m_V}B + \frac{qk_0\cos\theta - kq_0}{m_V}C - \frac{2qk_0\cos\theta - kq_0 + kp_0}{m_V}F_+\right]\ , \tag{25}$$

$$\mathrm{Im}\,\rho_{10}\left(\frac{d^3\sigma}{dk}\right)^{\pi V} = -\frac{\sqrt{2}\,q\sin\theta}{4wq}\ \frac{k(p_0+k_0)}{m_V}F_-\ , \tag{26}$$

where

$$dk = \frac{d^3k}{2k_0}\ , \tag{27}$$

and q, p and k are magnitudes of momentum of pion, initial nucleon and vector meson in the cm system, respectively, and q_0, p_0 and k_0 are the energies of the corresponding particles. W is the total energy and θ is the scattering angle in the cm system.

(ii) $\gamma + N \to \pi + N$:

$$\left(\frac{d^3\sigma}{dq}\right)^{\gamma\pi}_{\perp} = \frac{1}{2}\left[\left(\frac{d^3\sigma}{dq}\right)^{\gamma\pi}_{11} + \left(\frac{d^3\sigma}{dq}\right)^{\gamma\pi}_{1-1}\right] = \frac{1}{4wk}A_+\ , \tag{28}$$

$$\left(\frac{d^3\sigma}{dq}\right)^{\gamma\pi}_{//} = \frac{1}{2}\left[\left(\frac{d^3\sigma}{dq}\right)^{\gamma\pi}_{11} - \left(\frac{d^3\sigma}{dq}\right)^{\gamma\pi}_{1-1}\right] = \frac{1}{4wk}\,(A_+ + q^2\sin^2\theta C)\ . \tag{29}$$

(iii) $e + N \to e + \pi + N$:

$$\left(\frac{d^6\sigma}{dE'\,d\Omega_e\,dq}\right) = \Gamma_t\,[\sigma_{11} + \epsilon\sigma_{00} - \epsilon\cos 2\varphi\,\sigma_{1-1} - 2\sqrt{\epsilon(1+\epsilon)}\cos\varphi\ \mathrm{Re}\,\sigma_{10}]\ , \tag{30}$$

$$\left(\frac{d^6\sigma}{dE'\,d\Omega_e\,dq}\right)_{P_e} - \left(\frac{d^6\sigma}{dE'\,d\Omega_e\,dq}\right) = \Gamma_t\frac{2P_e}{m_e}\sqrt{\epsilon(1-\epsilon)}\sin\varphi\ \mathrm{Im}\,\sigma_{10}\ , \tag{31}$$

$$\sigma_{11} = \frac{1}{4wk^\gamma}\ (2A_+ + q^2\sin^2\theta\ C)\ , \tag{32}$$

$$\sigma_{1-1} = \frac{1}{4wk^{\gamma}} (-q^2 \sin^2\theta\, C)\,, \tag{33}$$

$$\sigma_{00} = \frac{1}{4wk^{\gamma}} \left[2A_+ + \frac{2(k_0 q\cos\theta - kq_0)^2}{m_V^2} C + \frac{2k^2(p_0+k_0)^2}{m_V^2} B + \frac{4(k_0 q\cos\theta - kq_0)\, k(p_0+k_0)}{m_V^2} F_+ \right], \tag{34}$$

$$\mathrm{Re}\,\sigma_{10} = \frac{1}{4wk^{\gamma}} \left[-\frac{\sqrt{2}\, q\sin\theta (k_0 q\cos\theta - kq_0)}{m_V} C - \frac{\sqrt{2}\, q\sin\theta\, k(p_0+k_0)}{m_V} F_+ \right], \tag{35}$$

$$\mathrm{Im}\,\sigma_{10} = \frac{1}{4wk^{\gamma}} \left[\frac{\sqrt{2}\, kq\sin\theta\,(p_0+k_0)}{m_V} F_- \right], \tag{36}$$

where

$$\Gamma_t = \frac{\alpha}{2\pi^2} \frac{E'}{E} \frac{K}{k_\mu^2} \frac{1}{1-\epsilon}\,, \quad m_V^2 = |k_\mu^2|\,, \tag{37}$$

$$K = \frac{1}{2M} (\bar{W}^2 - M^2) = \frac{\bar{W}k^{\gamma}}{M}\,. \tag{38}$$

P_e is the polarization of the electron in the direction of momentum in the lab system (L system). E and E' are the initial and final electron energy in the L system, respectively. $\mathrm{d}\Omega_e$ is the solid angle of the final electron in the L system, and ϵ is the usual transverse linear polarization parameter. In the above representation it should be noted that photon-vector meson coupling constant and factor $m_V^2/(k_\mu^2 + m_V^2)$ are contained in the amplitude for the reaction "γ" $+ \mathrm{N} \to \pi + \mathrm{N}$ (or $\mathrm{e} + \mathrm{N} \to \mathrm{e} + \pi + \mathrm{N}$).

Using the above representation, we make the following two kinds of approximation:

(i) Assuming that the invariant discontinuities are independent of k_μ^2, we get the usual VMD relations as

$$\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}q}\right)_{\perp}^{\gamma\pi} = \frac{q}{k}\, g_{\gamma V}^2\, (\rho_{11} \pm \rho_{1-1}) \left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi V}. \tag{39}$$

We know that these relations are correct within a factor of 2; however, we may even say that the usual VMD relations are not consistent with experiment.

(ii) In the region $|k_\mu^2| \lesssim m_\rho^2$, we expand the Lorentz invariant amplitudes up to the first order in k_μ^2;

$$A_+ = A_+^{(0)} + k_\mu^2 A_+^{(1)}\,, \qquad B = B^{(0)} + k_\mu^2 B^{(1)}\,, \qquad \text{etc.} \tag{40}$$

where $A_+^{(0)}$, $A_+^{(1)}$, . . . are k_μ^2 independent, and we assume the higher orders of discontinuities are zero (eqs. (41)–(43) are not altered by this assumption). Requiring that the gauge conditions are satisfied for each power of k_μ^2, we get the following three additional subsidiary conditions among the ten k_μ^2 independent amplitudes;

$$A_+^{(0)} = \tfrac{1}{4}(t-\mu^2)^2 C^{(1)} + \tfrac{1}{4}(u-M^2)^2 B^{(1)} + \tfrac{1}{2}(t-\mu^2)(u-M^2) F_+^{(1)}\,, \tag{41}$$

$$B^{(0)} = -\frac{t-\mu^2}{u-M^2}\, F_+^{(0)} \tag{42}$$

$$C^{(0)} = -\frac{u-M^2}{t-\mu^2}\, F_+^{(0)}\,. \tag{43}$$

Now the number of k_μ^2 independent amplitudes is smaller than that of the experimentally observable quantities.

3. Comparison with experimental results

When the initial nucleon and lepton are unpolarized and we do not measure the polarization of the final nucleon, it is enough to take the eight k_μ^2 independent amplitudes $A_+^{(0)}$, $A_+^{(1)}$, $B^{(0)}$, $B^{(1)}$, $C^{(0)}$, $C^{(1)}$, $F_+^{(0)}$ and $F_+^{(1)}$. We have three subsidiary relations (41)–(43) between them, so we have only five k_μ^2 independent amplitudes. For fixed s and t, we have the following experimentally observable quantities:

$$(\gamma \mathrm{N} \to \pi \mathrm{N}) \quad : \left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}q}\right)_{\perp}^{\gamma\pi}, \left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}q}\right)_{\parallel}^{\gamma\pi}, \tag{44}$$

$$(\pi \mathrm{N} \to \mathrm{VN}) \quad : \rho_{11}\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi \mathrm{V}}, \rho_{00}\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi \mathrm{V}}, \rho_{1-1}\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi \mathrm{V}}, \mathrm{Re}\,\rho_{10}\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi \mathrm{V}}, \tag{45}$$

$$(\mathrm{eN} \to \mathrm{e}\pi \mathrm{N}) \quad : \sigma_{11}, \sigma_{00}, \sigma_{1-1}, \mathrm{Re}\,\sigma_{10} \quad \text{for each } k_\mu^2\,. \tag{46}$$

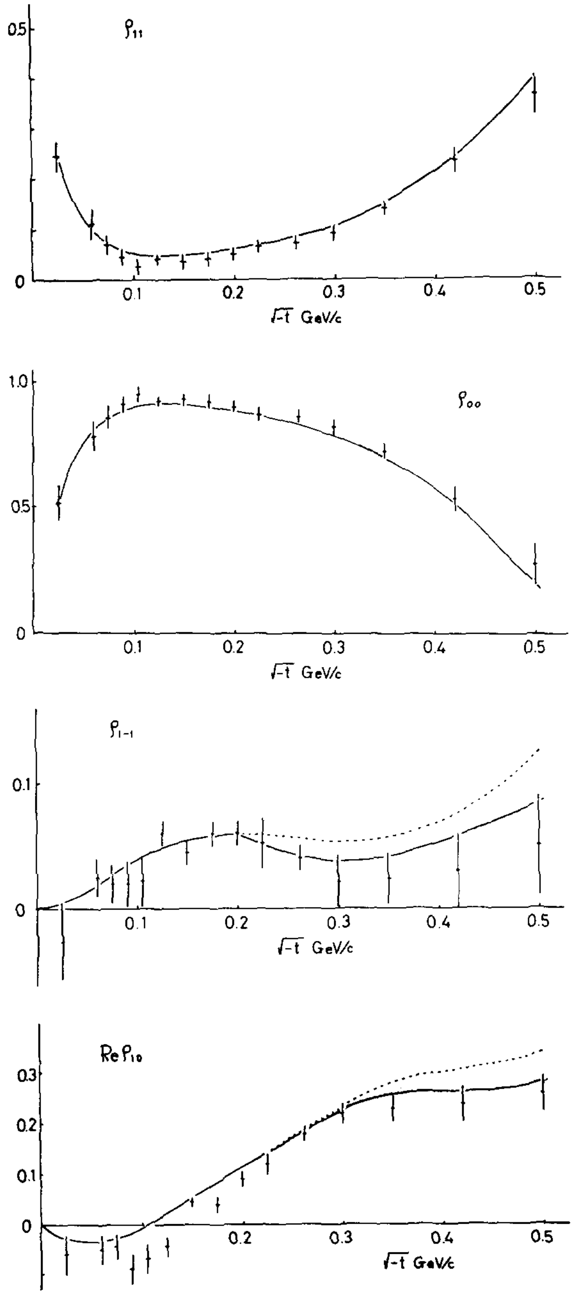
Using the following γV coupling constant [7]

$$\frac{\gamma_\mathrm{V}^2}{4\pi} = \frac{1}{16\pi}\,\frac{e^2}{g_{\gamma\pi}^2} = 0.64 \pm 0.05\,, \tag{47}$$

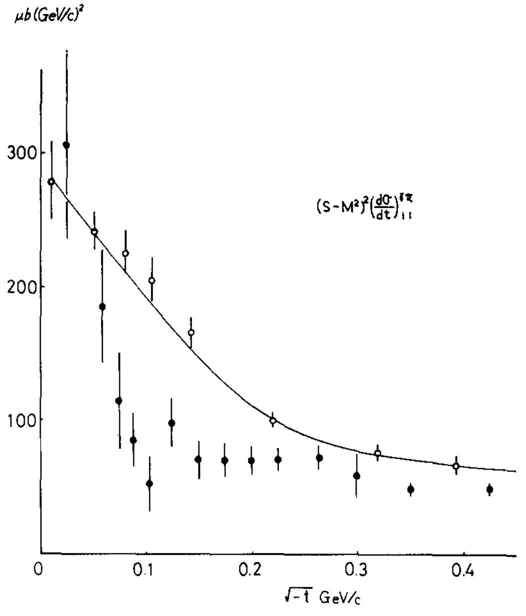
we can fit the six observable quantities,

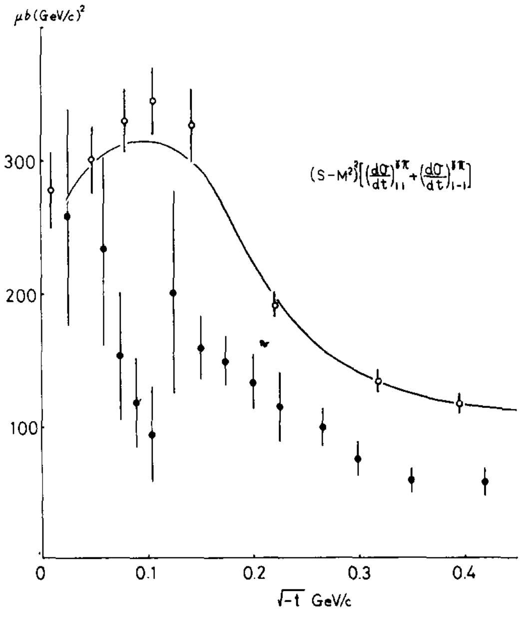
$$\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}q}\right)_{\perp}^{\gamma\pi}, \quad \left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}q}\right)_{\parallel}^{\gamma\pi}, \quad \rho_{11}\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi \mathrm{V}}, \quad \rho_{00}\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi \mathrm{V}},$$

$$\rho_{1-1}\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi \mathrm{V}}, \quad \mathrm{Re}\,\rho_{10}\left(\frac{\mathrm{d}^3\sigma}{\mathrm{d}k}\right)^{\pi \mathrm{V}},$$



Figs. 1, 2, 3 and 4. The ρ^0 density-matrix elements in the helicity frame. The solid (dashed) curve is a fit with our predictions for $\gamma_\rho^2/4\pi = 0.64$ (0.5). Experimental data are taken from Bulos et al.[8].





Figs. 5 and 6. Differential cross section for single-pion photoproduction [8] with our prediction (solid curve). For comparison the usual predictions of VMD from $\pi N \to \rho^0 N$ data are given (closed circle).

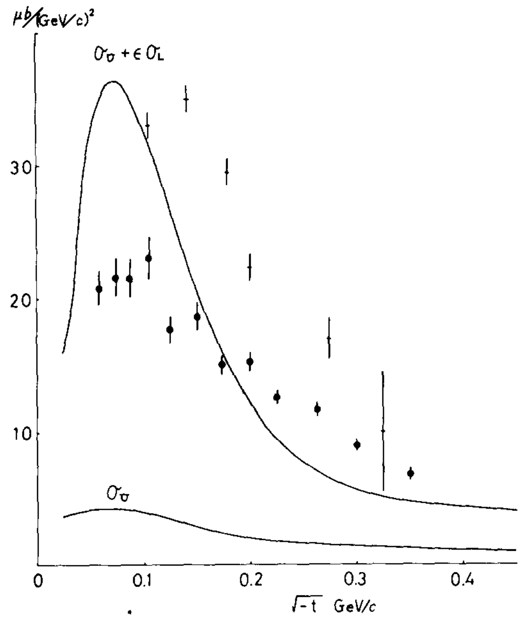


Fig. 7. Prediction for electroproduction. Data are taken from Driver et al. [8] (for k_μ^2 = 0.26 GeV^2). For comparison the prediction by Fraas and Schildknecht is given (closed circle).

by adjusting the five independent amplitudes $A_+^{(1)}$, $B^{(1)}$, $C^{(1)}$, $F_+^{(0)}$ and $F_+^{(1)}$. The fits to the data are shown on figs. 1–6. For comparison we plot the prediction of usual VMD in figs. 5 and 6. This shows our fit is much better than that by the usual VMD, and is consistent with experiment within errors. Also we can make a prediction for the pion electroproduction cross section as in fig. 7.

We may classify the ampltudes into two classes; the amplitudes A and C where the zeroth order component has the same magnitude and shape as the first order component (multiplied by GeV^2), and the amplitudes B and F_+ where the zeroth order component has much smaller magnitude compared with the first order component. Anyway it is very remarkable that these amplitudes have quite a large k_μ^2 dependence.

4. Discussion

In this note we fit the six observable quantities on the $\gamma N \to \pi N$ and $\pi N \to VN$ processes by using five k_μ^2 independent amplitudes. The fit is quite satisfactory and this becomes a very nice modification of the usual VMD. The most striking and remarkable results are that the amplitudes have quite a large k_μ^2 dependence. This is a very unexpected result.

On the other hand, experimental indications recently have been found for at least one more vector meson, $\rho'(1600)$. Many authors are trying to fit the photo-processes considering an infinite series of vector mesons; this is called generalized vector meson dominance (GVMD) [9].

If the GVMD is correct in our processes, the following quantities should have some relation with the k_μ^2 dependence of the amplitudes:

$$\left[\frac{1}{\gamma_{\rho'}}\sqrt{\left(\frac{\mathrm{d}\sigma}{\mathrm{d}k}\right)^{\pi\rho'}}+\frac{1}{\gamma_{\rho''}}\sqrt{\left(\frac{\mathrm{d}\sigma}{\mathrm{d}k}\right)^{\pi\rho''}}\mathrm{e}^{i\varphi}+\ldots\right]\Bigg/\frac{1}{\gamma_\rho}\sqrt{\left(\frac{\mathrm{d}\sigma}{\mathrm{d}k}\right)^{\pi\rho}}\,. \tag{48}$$

At present we do not have enough data in order to compare with GVMD.

We also calculated the pion electroproduction cross section. Our fit looks a little bit better than the usual VMD, in particular in the magnitude of the cross section. The increasing amount of data on pion electroproduction will be very interesting in order to check our new form of VMD.

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