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Covariant Perturbation Expansion in Chiral Theories with Pions and Nucleons

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A covariant perturbation scheme is developed to give a coordinate independent perturbation expansion of the chiral invariant pion model with nucleons. On the mass shell the covariant approach is shown to be equivalent to the standard perturbation theory.

INTRODUCTION

In a series of papers [1, 2] a nonlinear chiral $SU(2) \times SU(2)$ invariant Lagrangian (function of the pion fields only) was studied within the framework of coordinate independent perturbation expansion. This model was then used to calculate in a coordinate independent manner the phase shifts for pion–pion scattering at low energies in the effective range approximation. However, it is clear that the pion–nucleon scattering problem at low energies as well as the calculations on the corrections for the axial current coupling constant lie beyond the framework of the covariant formalism developed in [1, 2]. To deal with such problems one has to developed a covariant perturbation expansion of $SU(2) \times SU(2)$ invariant Lagrangians which are functions of the pion as well as the nucleon fields. In this paper we develop such a covariant formalism. Furthermore we show that on the mass shell, the covariant formalism yields results which are completely equivalent with the results of the standard perturbation expansion. The on mass shell

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equivalence between covariant and noncovariant perturbation theory for the case of chiral invariant Lagrangians which are functions of the pion fields only was demonstrated in [7]. Our proof for the equivalence theorem is very similar to the one given in [7], that is, we show explicitly how one can express covariant graphs by contributions of noncovariant ones and vice versa.

I. The Model

In this paper we study the chiral $SU(2) \times SU(2)$ nonlinear pion model with nucleons within the framework of coordinate independent perturbation expansion (Ecker, Honerkamp [1, 2]).

The pion fields, which form an isovector, transform nonlinearly under chiral $SU(2) \times SU(2)$ transformations, and are taken to be the co-ordinates of a curved manifold, which is a 3-sphere, $\mathscr{S}^3$, of radius F_π, F_π being the pion decay constant. The nucleon fields, on the other hand, transform in a quasilinear manner, and form an isospinor corresponding to isospin $\frac{1}{2}$. The "standard form" [3] of such a realization is given by

$$g \in SU(2) \times SU(2)\colon \pi \to \pi', \qquad \psi \to \psi' = D(e^{u'(\pi)\cdot V})\,\psi \tag{1}$$

where D is a linear two-dimensional representation of $SU(2)$, and

$$g e^{\pi\cdot A} = e^{\pi'\cdot A} e^{u'(\pi)\cdot V}$$

where V_i and A_i $(i = 1, 2, 3)$ are, respectively, the vector and axial vector generators of $SU(2) \times SU(2)$. Any arbitrary nonlinear chiral realization is obtained from the standard form (1) by a redefinition of the fields (π, ψ), e.g.,

$$\pi \to \pi' = \pi f(\pi), \qquad f(0) = 1 \tag{2}$$

where $f(\pi)$ is a $SU(2)$ scalar analytic function of π.

Following the prescription of Callan, Colleman, Wess, and Zumino [4] we write down an $SU(2) \times SU(2)$ invariant Lagrangian with pions and nucleons in the form

$$\mathscr{L} = \tfrac{1}{2}\, g_{ij}(\pi)\, \partial_\mu \pi^i \partial^\mu \pi^j + \bar{\psi}(i\gamma^\mu \Delta_\mu - m)\,\psi$$

where $g_{ij}(\pi)$ is the metric in $\mathscr{S}^3$ (isospace of constant curvature F_π^{-2}). The quantity $\partial_\mu \pi^i$ is a contravariant vector under the pion field redefinition (2), whereas $g_{ij}(\pi)$ transforms like a covariant tensor [5]. We also remark that Δ_μ corresponds to the operation of covariant differentiation on the nucleon fields [4, 5]. We shall show later on that all the interaction terms in $\mathscr{L}$ are due to the curvature of $\mathscr{S}^3$ and vanish in the flat space limit. However, it is possible to add further terms that do not have this property. Such terms must be themselves chiral-invariant, because the minimal form of the Lagrangian given above is already chiral-invariant. We shall add one such term to the minimal form for the Lagrangian and take the Lagrangian density to be

$$\mathscr{L} = (1/2)\, g_{ij}(\pi)\, \partial_\mu \pi^i \partial^\mu \pi^j + \bar{\psi}(i\gamma^\mu \Delta_\mu - m)\, \psi + (1/2F_\pi)\, \bar{\psi}\gamma_5\gamma_\mu \Gamma_i(\pi)\, \psi \partial^\mu \pi^i \quad (3)$$

where $\Gamma^i(\pi)$ are 2×2 matrices depending on the pion field and satisfying the Clifford algebra

$$\{\Gamma^i(\pi), \Gamma^j(\pi)\}_+ = 2g^{ij}(\pi).$$

A representation of this algebra can be obtained in the form

$$\Gamma_i(\pi) = e_{ai}(\pi)\, \tau_a$$

where e_{ai} $(a = 1, 2, 3)$ are dreibein fields satisfying

$$(e_{ai} e_{aj})(\pi) = g_{ij}(\pi)$$
$$e_a{}^i e_{bi} = \delta_{ab}$$

Like $\partial_\mu \pi^i$, $\Gamma^i(\pi)$ transforms as a contravariant vector under pion field redefinitions (2). It is, therefore, clear that $\mathscr{L}$ is a coordinate scalar.

II. Covariant Expansion of the Action

Consider the total action $S = \int d^4x \mathscr{L}(x)$. Our aim is to construct a covariant perturbation expansion of S with the terms of the expansion transforming covariantly under pion field redefinitions of the type (2). To this end we follow [1], and write

$$S(\pi, \psi, \bar{\psi}) = S_1(\pi) + S_2(\pi, \psi, \bar{\psi})$$

where $S_1(\pi)$, and $S_2(\pi, \psi, \bar{\psi})$ represent the contributions from the first and the last two terms in (3), respectively. $S_1(\pi)$, of course, corresponds to pion self-interactions and its appropriate covariant expansion can be found in [1]. Confining ourselves, therefore, to $S_2(\pi, \psi, \bar{\psi})$ we introduce a classical pion field $\phi^i(x)$ which satisfies the equation

$$\frac{\delta S(\pi, 0, 0)}{\delta \pi^i(x)} + J_i(x) = 0 \quad (5)$$

where $J_i(x)$ is a classical source for the field $\phi^i(x)$. In the following we intend to give a covariant expansion of S_2 around the classical field ϕ^i. Let $\zeta^i(\lambda)$ be the geodesics in $\mathscr{S}^3$ from ϕ^i to π^i where the parameter $\lambda(0 \leqslant \lambda \leqslant s)$ measures the length for this curve, and $\zeta^i(0) = \phi^i$, $\zeta^i(s) = \pi^i$. $\zeta^i(\lambda)$ $(i = 1, 2, 3)$ satisfy the equations

$$\frac{d^2\zeta^i}{d\lambda^2} + \Gamma^i_{kl}\frac{d\zeta^k}{d\lambda}\frac{d\zeta^l}{d\lambda} = 0 \tag{6}$$

where Γ^i_{kl} are the Christoffel symbols, of the second kind, for the metric g_{ij}. Before we proceed any further, however, it is necessary to consider in some detail the covariant spinor differentiation.

Let Ω_l be a 9-component field defined by

$$\Omega_l = \tfrac{1}{8}C_{abl}[\tau_a, \tau_b] \qquad (C_{abl} = -C_{bal}) \tag{7}$$

where C_{abl} (the Weyl connection) satisfies the differential equation [6].

$$e_{ak\cdot l} \equiv e_{ak,l} - \Gamma^m_{kl}e_{am} - C_{abl}e_{bk} = 0 \tag{8}$$

(where $e_{ak,l} \equiv \delta e_{ak}(\pi)/\delta\pi^l$). Under the field transformations $\psi \to S\psi$, $\pi \to \pi'$ we deduce from (8) that Ω_l transforms like

$$\Omega_l \to S\Omega_l S^{-1} + S_{,l}S^{-1}. \tag{9}$$

The covariant spinor differential (or Weyl covariant derivative) is, now, given by

$$\psi_{\cdot l} = \psi_{,l} - \Omega_l\psi. \tag{10}$$

It is clear from (9) that $\psi_{\cdot l}$ transforms like ψ, i.e.,

$$\psi_{\cdot l} \to S\psi_{\cdot l}\,.$$

Similarly we write

$$\psi^+_{\cdot l} = \psi^+_{,l} + \psi^+\Omega_l \tag{11}$$

and

$$\Delta_\mu\psi = \partial_\mu\psi - \Omega_l\partial_\mu\pi^l\psi \tag{12}$$

$$\Delta_\mu\psi^+ = \partial_\mu\psi^+ + \psi^+\Omega_l\partial_\mu\pi^l. \tag{13}$$

We now turn to the problem of the covariant expansion of $S_2(\pi, \psi, \bar{\psi})$. We introduce two spinorial quantities $\theta_\alpha(\lambda)$ and $\theta^{+\alpha}(\lambda)$ satisfying the equations

$$\frac{d\theta_\alpha}{d\lambda} - \Omega^\beta_{\alpha k}\frac{d\zeta^k}{d\lambda}\theta_\beta = 0 \tag{14}$$

$$\frac{d\theta^{+\alpha}}{d\lambda} + \theta^{+\beta}\Omega^\alpha_{\beta k}\frac{d\zeta^k}{d\lambda} = 0 \tag{15}$$

where $\Omega^{\beta}_{\alpha k}$ is the matrix element given by

$$\Omega^{\beta}_{\alpha k} = \tfrac{1}{8} C_{abk}([\tau_a, \tau_b])^{\beta}_{\alpha} \tag{16}$$

and

$$\theta_{\alpha}(0) \equiv \xi_{\alpha}(\phi), \qquad \theta_{\alpha}(s) = \psi_{\alpha}(\pi)$$
$$\theta^{+\alpha}(0) \equiv \xi^{+\alpha}(\phi), \qquad \theta^{+\alpha}(s) = \psi^{+\alpha}(\pi).$$

We remark that in the case of vanishing curvature Eqs. (6), (14), and (15) imply the absence of pion–nucleon interactions arising from the covariant derivative of the nucleon fields. The last term in (3) reduces, in this flat space limit, to the gradient coupling term $(1/2F_{\pi})\, \bar{\psi}\gamma_5\gamma_{\mu}\vec{\tau}\psi \cdot \partial_{\mu}\vec{\ }$. This term, however, gives a vanishing contribution due to the choice of the coupling constant. Now, we can write

$$S_2(\pi, \psi, \bar{\psi}) = S_2(\zeta(\lambda), \theta(\lambda), \bar{\theta}(\lambda))|_{\lambda=s} \tag{17}$$

The functional on the right-hand side of (17) is an ordinary function of λ with a Taylor expansion

$$\begin{aligned} S_2(\zeta(\lambda), \theta(\lambda), \bar{\theta}(\lambda)) = {} & S_2(\zeta(0), \theta(0), \bar{\theta}(0)) \\ & + \lambda((d/d\lambda)\, S_2(\zeta(\lambda), \theta(\lambda), \bar{\theta}(\lambda)))_{\lambda=0} \\ & + (\lambda^2/2!)((d^2/d\lambda^2)\, S_2(\zeta(\lambda), \theta(\lambda), \bar{\theta}(\lambda)))_{\lambda=0} + \cdots \end{aligned} \tag{18}$$

where

$$\frac{d}{d\lambda} = \frac{d\zeta^i}{d\lambda}\frac{\delta}{d\zeta^i} + \frac{d\theta_{\alpha}}{d\lambda}\frac{\overleftarrow{\delta}}{\delta\theta_{\alpha}} + \frac{d\bar{\theta}^{\alpha}}{d\lambda}\frac{\overrightarrow{\delta}}{\delta\bar{\theta}^{\alpha}}$$

with the arrows indicating left and right derivatives. Using Eqs. (6), (14), and (15) Eq. (18) yields

$$\begin{aligned} S_2(\zeta(\lambda), \theta(\lambda), \bar{\theta}(\lambda)) = {} & \bar{\theta}^{\alpha}(0)\, A_{\alpha}{}^{\beta}(\zeta(0))\, \theta_{\beta}(0) + \lambda((d\zeta^i/d\lambda)\, \bar{\theta}^{\alpha}(\lambda)\, A^{\beta}_{\alpha\cdot i}(\zeta(\lambda))\, \theta_{\beta}(\lambda))_{\lambda=0} \\ & + (\lambda^2/2!)((d\zeta^i/d\lambda)(d\zeta^j/d\lambda)\, \bar{\theta}^{\alpha}(\lambda)\, A^{\beta}_{\alpha\cdot ij}(\zeta(\lambda))\, \theta_{\beta}(\lambda))_{\lambda=0} + \cdots \end{aligned} \tag{19}$$

where

$$\begin{aligned} A_{\alpha}{}^{\beta}(\zeta(\lambda)) &= (\overrightarrow{\delta}/\delta\bar{\theta}^{\alpha}(\lambda))\, S_2(\zeta(\lambda), \theta(\lambda), \bar{\theta}(\lambda))(\overleftarrow{\delta}/\delta\theta_{\beta}(\lambda)) \\ A^{\beta}_{\alpha\cdot i}(\zeta(\lambda)) &= (\delta A_{\alpha}{}^{\beta}(\zeta(\lambda))/\delta\zeta^i(\lambda)) - \Omega^{\gamma}_{\alpha i} A_{\gamma}{}^{\beta}(\zeta(\lambda)) + A_{\alpha}{}^{\gamma}(\zeta(\lambda))\, \Omega^{\beta}_{\gamma i} \\ A^{\beta}_{\alpha\cdot ij}(\zeta(\lambda)) &= (\delta A^{\beta}_{\alpha\cdot i}(\zeta(\lambda))/\delta\zeta^j(\lambda)) - \Omega^{\gamma}_{\alpha j} A^{\beta}_{\gamma\cdot i}(\zeta(\lambda)) + A^{\gamma}_{\alpha\cdot i}(\zeta(\lambda))\, \Omega^{\beta}_{\gamma j} - \Gamma^{m}_{ij} A^{\beta}_{\alpha\cdot m}(\zeta(\lambda)) \end{aligned}$$

and so on. Hence by virtue of the fact that $d\xi^i/d\lambda\,|_{\lambda=0} = \Gamma^{\,i}_{\cdot}/s$, where $\Gamma^{\,i}_{\cdot}$ is a chiral bivector defined in [1], we get for $\lambda = s$

$$S_2(\pi, \psi, \bar{\psi}) = \bar{\xi}^{\alpha} A_{\alpha}{}^{\beta}(\phi)\, \xi_{\beta} + \sum_{n=1}^{\infty} (1/n!)\, \bar{\xi}^{\alpha} A^{\beta}_{\alpha\cdot k_1\cdots k_n}(\phi)\, \xi_{\beta} \Gamma^{k_1}_{\cdot} \cdots \Gamma^{k_n}_{\cdot}. \tag{20}$$

The corresponding expression for $S_1(\pi)$ given in [1] has the form

$$S_1(\pi) = S_1(\phi) + \sum_{n=1}^{\infty} (1/n!)\, S_{1;k_1\cdots k_n}(\phi)\, \Gamma_{\cdot}^{k_1} \cdots \Gamma_{\cdot}^{k_n}$$

where the covariant derivatives of the coordinate scalar S_1 are defined in the usual manner, i.e.,

$$S_{1;i} = S_{1,i}$$
$$S_{1;ij} = S_{1,ij} - \Gamma_{ij}^{m} S_{1;m} .$$

Before closing this section it will be instructive to look at the expansion (20) from a slightly different point of view. First we observe that any integral curve of (6) is determined by a point, which is taken to be the point corresponding to ϕ^i and a direction at this point, namely $d\zeta^i/d\lambda\,|_{\lambda=0} = \Gamma_{\cdot}^{\,i}/s$ Thus we have

$$\zeta^i(\lambda) = \zeta^i(0) + \frac{\Gamma_{\cdot}^{\,i}}{s}\lambda + \frac{1}{2!}\frac{d^2\zeta^i}{d\lambda^2}\bigg|_{\lambda=0} \lambda^2 + \cdots .$$

The coefficients of λ^2 and higher powers in λ are given by (6) by differentiation with respect to λ and replacing the second and higher derivatives of ζ^i by means of (6) and the resulting equations. Thus by putting $\lambda = s$ we finally obtain

$$\pi^i = \phi^i - \sum_{n=1}^{\infty} (1/n!)\, \Gamma_{k_1\cdots k_n}^{i} \Gamma_{\cdot}^{k_1} \cdots \Gamma_{\cdot}^{k_n} \tag{21}$$

where $\Gamma_{k_1\cdots k_n}^{i}$ are the generalized Christoffel symbols (symmetric in the lower indices) with $\Gamma_k^{\,i} = -\delta_k^{\,i}$. Similarly we observe that the integral curves of the first-order Eqs. (14) and (15) are determined by a point, which is taken conveniently to be ξ_α, and $\bar{\xi}^\alpha$, respectively. Applying the same procedure as above we obtain

$$\psi_\alpha = \xi_\alpha + \Omega_{\alpha k}^{\beta} \xi_\beta \Gamma_{\cdot}^{\,k} + (1/2!)\, \Omega_{\alpha k_1 k_2}^{\beta} \xi_\beta \Gamma_{\cdot}^{k_1} \Gamma_{\cdot}^{k_2} + \cdots$$

$$\bar{\psi}^\alpha = \bar{\xi}^\alpha + \bar{\xi}^\beta x_{\beta k}^{\alpha} \Gamma_{\cdot}^{\,k} + (1/2!)\, \bar{\xi}^\beta x_{\beta k_1 k_2}^{\alpha} \Gamma_{\cdot}^{k_1} \Gamma_{\cdot}^{k_2} + \cdots$$

where

$$x_{\alpha k}^{\beta} = -\Omega_{\alpha k}^{\beta}$$

$$\Omega_{\alpha kl}^{\beta} = \tfrac{1}{2} P[\Omega_{\alpha k,l}^{\beta} + \Omega_{\alpha k}^{\gamma} \Omega_{\gamma l}^{\beta}] - \Omega_{\alpha m}^{\beta} \Gamma_{kl}^{m}$$

$$x_{\beta kl}^{\alpha} = \tfrac{1}{2} P[-\Omega_{\beta k,l}^{\alpha} + \Omega_{\beta l}^{\gamma} \Omega_{\gamma k}^{\alpha}] + \Omega_{\beta m}^{\alpha} \Gamma_{kl}^{m} .$$

where P before an expression indicates symmetrization with respect to the chiral indices. In general we can write

$$\psi_\alpha = \xi_\alpha + \sum_{n=1}^{\infty} (1/n!)\, \Omega^{\beta}_{\alpha k_1 \cdots k_n} \xi_\beta \Gamma^{k_1}_{\cdot} \cdots \Gamma^{k_n}_{\cdot} \tag{22}$$

$$\bar{\psi}^\alpha = \bar{\xi}^\alpha + \sum_{n=1}^{\infty} (1/n!)\, \bar{\xi}^\beta x^{\alpha}_{\beta k_1 \cdots k_n} \Gamma^{k_1}_{\cdot} \cdots \Gamma^{k_n}_{\cdot} \tag{23}$$

with $\Omega^{\beta}_{\alpha k_1 \cdots k_n}$ and $x^{\alpha}_{\beta k_1 \cdots k_n}$ both symmetric in the chiral indices. Let $\pi^i = \phi^i + \chi^i$ and expand the functional $S(\pi, \psi, \bar{\psi})$ around ϕ^i. Then

$$S(\pi, \psi, \bar{\psi}) = S_1(\phi) + \sum_{n=1}^{\infty} (1/n!)\, S_{1, k_1 \cdots k_n}(\phi)\, \chi^{k_1} \cdots \chi^{k_n}$$

$$+ \bar{\psi}^\alpha A_\alpha{}^\beta(\phi)\, \psi_\beta + \sum_{n=1}^{\infty} (1/n!)\, \bar{\psi}^\alpha A^{\beta}_{\alpha, k_1 \cdots k_n}(\phi)\, \psi_\beta \chi^{k_1} \cdots \chi^{k_n}. \tag{24}$$

From (21) we have

$$\chi^i = \pi^i - \phi^i = -\sum_{n=1}^{\infty} (1/n!)\, \Gamma^{i}_{k_1 \cdots k_n} \Gamma^{k_1}_{\cdot} \cdots \Gamma^{k_n}_{\cdot} \qquad (\Gamma_j{}^i = -\delta_j{}^i). \tag{25}$$

Inserting (22), (23), and (25) in (24) we readily obtain the covariant expansion

$$S(\pi, \psi, \bar{\psi}) = S_1(\phi) + \sum_{n=1}^{\infty} (1/n!)\, S_{1; k_1 \cdots k_n}(\phi)\, \Gamma^{k_1}_{\cdot} \cdots \Gamma^{k_n}_{\cdot}$$

$$+ \bar{\xi}^\alpha A_\alpha{}^\beta(\phi)\, \xi_\beta + \sum_{n=1}^{\infty} (1/n!)\, \bar{\xi}^\alpha A^{\beta}_{\alpha \cdot k_1 \cdots k_n}(\phi)\, \xi_\beta \Gamma^{k_1}_{\cdot} \cdots \Gamma^{k_n}_{\cdot}. \tag{26}$$

We remark that in general summation over repeated indices implies integration over their associated space–time coordinates, e.g.,

$$S_{1, k_1 k_2}(\phi)\, \chi^{k_1} \chi^{k_2} = \int d^4x \int d^4x' \sum_{k_1, k_2} \frac{\delta^2 S_1}{\delta \pi^{k_1}(x)\, \delta \pi^{k_2}(x')} \bigg|_{\vec{\pi} = \vec{\phi}} \chi^{k_1}(x)\, \chi^{k_2}(x')$$

and

$$\bar{\psi}^\alpha A_\alpha{}^\beta(\phi)\, \psi_\beta = \int d^4x \int d^4x' \sum_{\alpha, \beta} \bar{\psi}^\alpha(x) (A_\alpha{}^\beta(\phi))(x, x')\, \psi_\beta(x')$$

where

$$A_\alpha{}^\beta(x, x') = (\vec{\delta}/\delta \bar{\psi}^\alpha(x))\, S_2(\phi, \psi, \bar{\psi}) (\overleftarrow{\delta}/\delta \psi_\beta(x')).$$

III. Covariant Perturbation Scheme

In this section we shall develop a covariant perturbation theory, as well as the noncovariant analogue of it, using functional integral techniques. The starting point is the generating functional for connected Green's functions, $\text{ш}(J, \eta, \bar{\eta})$, given by

$$e^{i\,\text{ш}(J,\bar{\eta},\eta)} = (1/N) \int \prod_x \prod_i d\pi^i (g(\pi))^{1/2} \prod_\alpha d\psi_\alpha \prod_\beta d\bar{\psi}^\beta$$
$$\times \exp i \left\{ S(\pi, \psi, \bar{\psi}) + \int [J_i(x)\, \pi^i(x) + \bar{\psi}^\alpha(x)\, \eta_\alpha(x) + \bar{\eta}^\alpha(x)\, \psi_\alpha(x)]\, dx \right\} \tag{27}$$

where η, $\bar{\eta}$ are spinor sources of the anticommuting type, and N is a normalization factor which is fixed by the condition $e^{i\,\text{ш}(0)} = 1$. We remark that the factor $(g(\pi))^{1/2}$, with $g(\pi) = \mathrm{Det}(g_{ij}(\pi))$ is required to maintain a formal invariance of the functional measure with respect to pion field redefinitions. In general we have, using a condensed notation,

$$\prod_x \prod_i (g(\pi))^{1/2}\, d\pi^i \prod_\alpha d\psi_\alpha \prod_\beta d\bar{\psi}^\beta = e^{\{(1/2)\delta^{(4)}(0) \int \ln g(x) dx\}}\, D\pi\, D\psi\, D\bar{\psi}. \tag{28}$$

To obtain a covariant perturbation expansion one proceeds by inserting the covariant expansion for the action (26), and the expansions (22), (23) and (25) into the expression (27) for the generating functional. After changing the integration variables from π^i to $\Gamma_{\cdot}^{\,i}$ and from $(\bar{\psi}^\alpha, \psi_\alpha)$ to $(\bar{\xi}^\alpha, \xi_\alpha)$ one expands all the exponentials except for the term that involves $\frac{1}{2} S_{1;k_1k_2}(\phi)\, \Gamma_{\cdot}^{k_1} \Gamma_{\cdot}^{k_2} + \bar{\xi}^\alpha A_\alpha{}^\beta(\phi)\, \xi_\beta$. In this way one obtains an expression for the generating functional of connected Green's functions in the covariant theory, which we denote by $\Gamma(J, \eta, \bar{\eta})$. This is written as follows:

$$e^{i\Gamma(J,\eta,\bar{\eta})} = (1/N)\, e^{i(S_1(\phi) + J_i\phi^i)} \int D\Gamma_{\cdot}(\partial\pi/\partial\Gamma_{\cdot})\, g^{1/2} e^{(i/2)\, S_{1;k_1k_2}(\phi)\Gamma_{\cdot}^{k_1}\Gamma_{\cdot}^{k_2}}$$
$$\times \int D\bar{\xi}\, D\xi (\partial\bar{\psi}/\partial\bar{\xi})(\partial\psi/\partial\xi)\, e^{i(\bar{\xi}^\alpha A_\alpha{}^\beta(\phi)\xi_\beta + \bar{\eta}^\alpha \xi_\alpha + \bar{\xi}^\alpha \eta_\alpha)}$$
$$\times \exp \left\{ \left(-i \sum_{n=2}^\infty (1/n!)\, J_i \Gamma_{I_n}^i \Gamma_{\cdot}^{I_n} \right) + \left(i \sum_{n=1}^\infty (1/n!)\, \bar{\eta}^\alpha \Omega_{\alpha I_n}^\beta \xi_\beta \Gamma_{\cdot}^{I_n} \right) \right.$$
$$\left. + \left(i \sum_{n=1}^\infty (1/n!)\, \bar{\xi}^\beta x_{\beta I_n}^\alpha \eta_\alpha \Gamma_{\cdot}^{I_n} \right) \right\}$$
$$\times \left\{ 1 + \sum_{n \geqslant 1} \sum_{(\lambda)_n} i^{\Sigma\lambda_\nu} K(\lambda)_n \prod_{\nu=1}^n (\bar{\xi}^\alpha A_{\alpha\cdot I_\nu}^\beta(\phi)\, \xi_\beta \Gamma_{\cdot}^{I_\nu})^{\lambda_\nu} \right.$$

$$+ \sum_{n\geqslant 3} \sum_{\substack{(\lambda)_n \\ \lambda_1=\lambda_2=0}} i^{\Sigma\lambda_\nu} K(\lambda)_n \prod_{\nu=1}^{n} (S_{1;I_\nu}\Gamma^{I_\nu}_{\cdot})^{\lambda_\nu}$$

$$+ \sum_{n\geqslant 4} \sum_{r=3}^{n-1} \sum_{(\lambda)_{n-r}} i^{\Sigma\lambda_\sigma} K(\lambda)_{n-r} \sum_{\substack{(\mu)_r \\ \mu_1=\mu_2=0}} i^{\Sigma\mu_\rho} K(\mu)_r \prod_{\sigma=1}^{n-r} (\bar{\xi}^\alpha A^\beta_{\alpha\cdot I_\sigma} \xi_\beta \Gamma^{I_\sigma}_{\cdot})^{\lambda_\sigma}$$

$$\times \prod_{\rho=1}^{r} (S_{1;I_\rho}\Gamma^{I_\rho}_{\cdot})^{\mu_\rho} \Bigg\} \tag{29}$$

where we have used the concise notation of [7]. $(\lambda)_n$ is a partition of n, that is, a sequence $\lambda_1, \lambda_2, ..., \lambda_n$ of natural numbers ($\lambda_\nu \geqslant 0$) such that $\sum_{\nu=1}^{n} \nu\lambda_\nu = n$. Such partitions are denoted as follows

$$(\lambda)_n = (1^{\lambda_1} 2^{\lambda_2} \cdots n^{\lambda_n}).$$

Each partition carries its own symmetry number, $K(\lambda)_n$, given by

$$K(\lambda)_n = \frac{1}{\prod_{\nu=1}^{n} \lambda_\nu! \, (\nu!)^{\lambda_\nu}}.$$

The index I_n is a shorthand for indices $k_1 \cdots k_n$. It is clear that, apart from off shell contributions, the right-hand side of (29) is independent of the choice of the pion field coordinates. We also remark that in (29) we have disregarded contributions of the $\delta^{(4)}(0)$ type, which arise from the Jacobian functions $(\partial\pi/\partial\Gamma_{\cdot})$, $(\partial\bar{\psi}/\partial\bar{\xi})$, and $(\partial\psi/\partial\xi)$, as well as from $g^{1/2}$ (see (28)). This is certainly in agreement with the BPH point of view. In (29) we neglect all explicit couplings of the sources to non-linear functions of the fields, since these terms contain no single particle pole in their matrix elements. The functional integral now reduces to a series of functional integrals that can be calculated in a standard way. These integrals are either of the Gaussian type [1], or of the type

$$\int D\bar{\xi}\, D\xi\, e^{i[\bar{\xi}^\alpha A_\alpha{}^\beta(\phi)\xi_\beta + \bar{\eta}^\alpha \xi_\alpha + \bar{\xi}^\alpha \eta_\alpha]} \cdots \xi_\alpha \bar{\xi}^\beta \cdots$$
$$= \mathrm{Det}((iA)^\beta_\alpha)(\vec{\delta}/i\delta\bar{\eta}^\alpha) \cdots \{e^{-(i\bar{\eta})\cdot B(\phi)\cdot(i\eta)}\} \cdots (\vec{\delta}/i\delta\eta_\beta) \tag{30}$$

where B is the operator inverse to (iA), and $\mathrm{Det}((iA)^\beta_\alpha)$ is the functional determinant of (iA). Thus in order to calculate the functional $\Gamma(J, \eta, \bar{\eta})$ we need to know the vertices

$$S_{1;I_n} \equiv S_{1;k_1\cdots k_n} \qquad n = 3, 4, ...$$

and

$$A^\beta_{\alpha\cdot I_n} \equiv A^\beta_{\alpha\cdot k_1 k_2 \cdots k_n} \qquad n = 1, 2, ...$$

From [2] we have

$$S_{1;k_1\cdots k_{2n}}(\phi) = 4^{n-1}\partial_\mu\phi^m R_{mk_1k_2r_1} R^{r_1}_{\ \ k_3k_4r_2} \cdots R^{r_{n-1}}_{\ \ k_{2n-1}k_{2n}r_n}\partial^\mu\phi^{r_n}$$

$$+ 4^{n-1}(D_\mu)^m_{k_1} R_{mk_2k_3r_1} R^{r_1}_{\ \ k_4k_5r_2} \cdots R^{r_{n-1}}_{\ \ k_{2n-2}k_{2n-1}r_n}(D^\mu)^{r_n}_{k_{2n}} \qquad (n \geqslant 2) \quad (31)$$

$$S_{1;k_1\cdots k_{2n+1}} = 4^n\partial_\mu\phi^m R_{mk_1k_2r_1} \cdots R^{r_{n-1}}_{\ \ k_{2n-1}k_{2n}r_n}(D^\mu)^{r_n}_{k_{2n+1}} \qquad (n \geqslant 1)$$

where R_{mnkl} is the Riemann curvature tensor. In the present case, where the curved isospace, $\mathscr{S}^3$, has a constant curvature F_π^{-2}, we have

$$R_{mnkl} = F_\pi^{-2}(g_{mk}g_{nl} - g_{ml}g_{nk})\,\delta(x_m - x_n)\,\delta(x_m - x_k)\,\delta(x_m - x_l).$$

Also $(D^\mu)^i_k$ is the differential operator given by

$$(D^\mu)^i_k = (\delta_k^{\ i}\partial^{\mu(i)} + \Gamma^i_{kl}\partial^\mu\phi^l)\,\delta(x_i - x_k)$$

where $\partial^{\mu(i)}$ indicates differentiation with respect to x_i. We shall now obtain expressions for the vertex functions $A^\beta_{\alpha\cdot I_n}$ $(n \geqslant 1)$. From (7), and (11) we obtain

$$(A_\alpha^{\ \beta})(\phi) = i\gamma^\mu(\Delta_\mu)^\beta_\alpha - m\delta_\alpha^{\ \beta} + (1/2F_\pi)\,\gamma_5\gamma_\mu(\tau_a)^\beta_\alpha\, e_{al}\partial^\mu\phi^l$$

with

$$(\Delta_\mu)^\beta_\alpha = \delta_\alpha^{\ \beta}\partial^{(\alpha)}_\mu - \tfrac{1}{8}C_{abl}\partial_\mu\phi^l([\tau_a, \tau_b])^\beta_\alpha .$$

We first calculate

$$(\Delta_\mu)^\beta_{\alpha\cdot k} = (\Delta_\mu)^\beta_{\alpha,k} - \Omega^\gamma_{\alpha k}(\Delta_\mu)^\beta_\gamma + (\Delta_\mu)^\gamma_\alpha\,\Omega^\beta_{\gamma k} .$$

This is given by

$$(\Delta_\mu)^\beta_{\alpha\cdot k} = \tfrac{1}{8}([\Gamma^m, \Gamma^n])^\beta_\alpha\, R_{mnkl}\partial_\mu\phi^l.$$

Similarly, for

$$(\tau_a e_{al}\partial^\mu\phi^l)^\beta_{\alpha\cdot k} = (\tau_a e_{al}\partial^\mu\phi^l)^\beta_{\alpha,k} - \Omega^\gamma_{\alpha k}((\tau_a)^\beta_\gamma\, e_{al}\partial^\mu\phi^l) + ((\tau_a)^\gamma_\alpha\, e_{al}\partial^\mu\phi^i)\,\Omega^\beta_{\gamma k}$$

We obtain

$$(\tau_a e_{al}\partial^\mu\phi^l)^\beta_{\alpha\cdot k} = (\tau_a)^\beta_\alpha\, e_{al}(D^\mu)^l_k .$$

These results can be generalized to the nth derivative case giving rise to the expressions

$$A^\beta_{\alpha\cdot k_1\cdots k_{2n}}(\phi) = -(1/4)\,\epsilon_{abc}(\tau_c)^\beta_\alpha\, e_a^{\ m} e_b^{\ n} R_{mnk_1r_1} R^{r_1}_{\ \ k_2k_3r_2} \cdots R^{r_{n-1}}_{\ \ k_{2n-2}k_{2n-1}r_n}(\not{D})^{r_n}_{k_{2n}}$$

$$+ (1/2F_\pi)\,\gamma_5(\tau_a)^\beta_\alpha\, e_{ai} R^i_{\ k_1k_2r_1} \cdots R^{r_{n-1}}_{\ \ k_{2n-1}k_{2n}r_n}\not{\partial}\phi^{r_n} \qquad (n \geqslant 1) \quad (32)$$

and

$$A^{\beta}_{\alpha\cdot k_1\cdots k_{2n+1}}(\phi) = -(1/4)\,\epsilon_{abc}(\tau_c)^{\beta}_{\alpha}\,e_a{}^m e_b{}^n R_{mnk_1r_1}\cdots R^{r_n}_{k_{2n}k_{2n+1}r_{n+1}}\not{\partial}\phi^{r_{n+1}}$$

$$+ (1/2F_\pi)\,\gamma_5(\tau_a)^{\beta}_{\alpha}\,e_{ai}R^{i}_{k_1k_2r_1}\cdots R^{r_{n-1}}_{k_{2n-1}k_{2n}r_n}(\not{D})^{r_n}_{k_{2n+1}} \qquad (n \geqslant 1). \quad (33)$$

In deriving these formulae we have used the fact that, due to the constant curvature situation, the covariant derivatives of the Riemann tensor are zero, as well as the fact that the Weyl derivative of the dreibein field is zero due to Eq. (8). Furthermore, owing to the form of the generating functional in (29), only the symmetric part of the vertex functions makes a contribution. Therefore, complete symmetrization of the $k_1 \cdots k_n$ indices is understood in (31), (32), and (33).

In the dreibein field formalism one is not dealing directly with chiral tensors $T_{i_1\ldots i_n}$, but only with their components along the dreibein fields themselves, which form the basis functions of a local 3-D Euclidean space. These components are scalars $T_{a_1\ldots a_n}$ given by

$$T_{a_1\cdots a_n} = e^{i_1}_{a_1}\cdots e^{i_n}_{a_n}T_{i_1\cdots i_n}\,.$$

Thus we have

$$(S_1)_{a_1\cdots a_n} = e^{k_1}_{a_1}\cdots e^{k_n}_{a_n}S_{1;k_1\cdots k_n}$$

where the expressions $(S_1)_{a_1\ldots a_n}$ $(n \geqslant 3)$ are given in [2]. Also from (32), and (33) we obtain

$$(A_\alpha{}^\beta)_{a_1\cdots a_n} = e^{k_1}_{a_1}\cdots e^{k_n}_{a_n}A^{\beta}_{\alpha\cdot k_1\cdots k_n}$$

where

$$(A_\alpha{}^\beta)_{a_1\cdots a_{2n}} = ((-1)^n/2F_\pi^{2n})\,\epsilon_{a_1bc}(\tau_c)^{\beta}_{\alpha}\,\delta_{a_2a_3}\cdots\delta_{a_{2n-2}a_{2n-1}}(\not{D})_{ba_{2n}}$$

$$+ ((-1)^{n-1}/2F_\pi^{2n+1})\,\gamma_5(\tau_a)^{\beta}_{\alpha}\,[\delta_{aa_2}\delta_{ba_1} - \delta_{ab}\delta_{a_1a_2}]\,\delta_{a_3a_4}\cdots\delta_{a_{2n-1}a_{2n}}\not{\partial}\phi_b \quad (34)$$

$$(A_\alpha{}^\beta)_{a_1\cdots a_{2n+1}} = ((-1)^n/2F_\pi^{2n+2})\,\gamma_5(\tau_a)^{\beta}_{\alpha}\,[\delta_{ab}\delta_{a_1a_2} - \delta_{aa_1}\delta_{ba_2}]\,\delta_{a_3a_4}\cdots\delta_{a_{2n-1}a_{2n}}(\not{D})_{ba_{2n+1}}$$

$$+ ((-1)^{n+1}/2F_\pi^{2n+1})\,\epsilon_{abc}(\tau_c)^{\beta}_{\alpha}\,\delta_{aa_1}\cdots\delta_{a_{2n}a_{2n+1}}\not{\partial}\phi^b \qquad (n \geqslant 1) \quad (35)$$

and

$$(A_\alpha{}^\beta)_{a_1} = (1/2F_\pi)\,\gamma_5(\tau_a)^\beta_\alpha\,(\not{D})_{aa_1} + ((-1)/2F_\pi)(\tau_c)^\beta_\alpha\,\epsilon_{a_1bc}\not{\partial}\phi^b \tag{36}$$

where $\partial_\mu\phi^a = e_i{}^a\partial_\mu\phi^i$.

We remark that under dreibein field rotations, which, of course, leave the metric invariant, $(S_1)_{a_1\ldots a_n}$ and $(A_\alpha{}^\beta)_{a_1\ldots a_n}$ transform like Euclidean tensors.

We now turn to the noncovariant perturbation theory, which can be developed along similar lines. The starting point is again the generating functional (27). Inserting (24) in (27) we obtain

$$\begin{aligned}
e^{i\omega(J,\eta,\bar\eta)} &= (1/N)\,e^{i(S_1(\phi)+J_i\phi^i)}\int D\chi\,g^{1/2}e^{(i/2)S_{1,k_1k_2}(\phi)\chi^{k_1}\chi^{k_2}}\\
&\times\int D\bar\psi\,D\psi\,e^{i[\bar\psi^\alpha A_\alpha{}^\beta(\phi)\psi_\beta+\bar\eta^\alpha\psi_\alpha+\bar\psi^\alpha\eta_\alpha]}\\
&\times\Bigg\{1+\sum_{n\geqslant1}\sum_{(\lambda)_n} i^{\Sigma\lambda_\nu}K(\lambda)_n\prod_{\nu=1}^{n}(\bar\psi^\alpha A^\beta_{\alpha,I_\nu}(\phi)\,\psi_\beta\chi^{I_\nu})^{\lambda_\nu}\\
&+\sum_{n\geqslant3}\sum_{\substack{(\lambda)_n\\ \lambda_1=\lambda_2=0}} i^{\Sigma\lambda_\nu}K(\lambda)_n\prod_{\nu=1}^{n}(S_{1,I_\nu}(\phi)\,\chi^{I_\nu})^{\lambda_\nu}\\
&+\sum_{n\geqslant4}\sum_{r=3}^{n-1}\sum_{(\lambda)_{n-r}} i^{\Sigma\lambda_\sigma}K(\lambda)_{n-r}\sum_{\substack{(\mu)_r\\ \mu_1=\mu_2=0}} i^{\Sigma\mu_\rho}K(\mu)_r\\
&\times\prod_{\sigma=1}^{n-r}(\bar\psi^\alpha A^\beta_{\alpha,I_\sigma}(\phi)\,\psi_\beta\chi^{I_\sigma})^{\lambda_\sigma}\prod_{\rho=1}^{r}(S_{1,I_\rho}\chi^{I_\rho})^{\mu_\rho}\Bigg\}.
\end{aligned}\tag{37}$$

At this stage it is convenient to introduce certain abbreviations [7]. Let $\int_{\chi,C}\prod_{\nu=1}^{n}(S_{1,I_\nu}\chi^{I_\nu})^{\lambda_\nu}$ be the contribution to the integral $\int D\chi\exp\{\frac{1}{2}S_{1,ij}(\phi)\,\chi^i\chi^j\}\prod_{\nu=1}^{n}(S_{1,I_\nu}(\phi)\,\chi^{I_\nu})^{\lambda_\nu}$ which corresponds to the connection mapping C associated with the particular pairing for the χ's. It corresponds to a graph with no external lines. The vertices of the graphs are fixed by the partition $(\lambda)_n$ (see [7]), and C describes which vertices have to be connected by full pion propagators iG_{ij}. (A full propagator is the one that includes all possible tree insertions). Similarly $\int_{\mu,\psi,\bar\psi,C}\prod_{\nu=1}^{n}(\bar\psi^\alpha A^\beta_{\alpha,I_\nu}(\phi)\,\psi_\beta\chi^{I_\nu})^{\lambda_\nu}$ is the contribution to the integral $\int D\chi\exp\{\frac{1}{2}S_{1,ij}(\phi)\,\chi^i\chi^j\}\int D\psi D\bar\psi\exp\{i(\bar\psi^\alpha A_\alpha{}^\beta(\phi)\,\psi_\beta+\bar\eta^\alpha\psi_\alpha+\bar\psi^\alpha\eta_\alpha)\}\prod_{\nu=1}^{n}(\bar\psi^\alpha A^\beta_{\alpha,I_\nu}(\phi)\,\psi_\beta\chi^{I_\nu})^{\lambda_\nu}$ which corresponds to the connection mapping C associated with the particular pairing of the χ's as well as the $\psi_\alpha\bar\psi^\beta$, $\eta_\alpha\bar\psi^\beta$, and $\psi_\alpha\bar\eta^\beta$ pairings. Again this corresponds to a graph with no external pion lines, but with possible external nucleon lines coupled to the appropriate nucleon sources. Such external lines arise because of (30). With this notation we obtain for $Ш(J, \eta, \bar{\eta})$ the expression

$$\begin{aligned} i\,Ш(J, \eta, \bar{\eta}) = {} & i\,Ш'_{\text{tree}}(J, \eta, \bar{\eta}) + i\,Ш'_{1\text{loop}}(J, \eta, \bar{\eta}) \\ & + \sum_{n=3}^{\infty} \sum_{\substack{(\lambda)_n \\ \lambda_1=\lambda_2=0}} \sum_{c} i^{\Sigma\lambda_\nu} K(\lambda)_n \int_{\chi, c} \prod_{\nu=1}^{n} (S_{1,I_\nu}\chi^{I_\nu})^{\lambda_\nu} \\ & + \sum_{n=1}^{\infty} \sum_{(\lambda)_n} \sum_{c} i^{\Sigma\lambda_\nu} K(\lambda)_n \int_{\chi,\psi,\bar{\psi},c} \prod_{\nu=1}^{n} (\bar{\psi}^\alpha A^\beta_{\alpha,I_\nu}\psi_\beta\chi^{I_\nu})^{\lambda_\nu} \\ & + \sum_{n=4}^{\infty} \sum_{r=3}^{n-1} \sum_{(\lambda)_{n-r}} \sum_{(\mu)_r} \sum_{c} i^{\Sigma\lambda_\sigma} K(\lambda)_{n-r}\, i^{\Sigma\mu_\rho} K(\mu)_r \\ & \times \int_{\chi,\psi,\bar{\psi},c} \prod_{\sigma=1}^{n-r} (\bar{\psi}^\alpha A^\beta_{\alpha,I_\sigma}\psi_\beta\chi^{I_\sigma})^{\lambda_\sigma} \prod_{\rho=1}^{r} (S_{1,I_\rho}\chi^{I_\rho})^{\mu_\rho}. \end{aligned} \tag{38}$$

The tree $Ш'_{\text{tree}}$ as well as the one loop $Ш'_{1\text{loop}}$ contributions arise from

$$(1/N)\, e^{i(S_1(\phi)+J_i\phi^i)} \int D\chi\, e^{(i/2)S_{1,ij}(\phi)\chi^i\chi^j} \int D\bar{\psi}\, D\psi\, e^{i[\bar{\psi}^\alpha A_\alpha{}^\beta(\phi)\psi_\beta + \bar{\eta}^\alpha\psi_\alpha + \bar{\psi}^\alpha\eta_\alpha]}$$

$Ш'_{\text{tree}}$ and $Ш'_{1\text{loop}}$ contain, respectively, all the tree and one loop contributions due to pion selfinteractions; they also contain some tree and one loop contributions arising from pion–nucleon interactions. The remaining contributions of the latter type arise from the last two terms in (38).

In a similar manner Eq. (29) yields the following expression for the generating functional $\Gamma(J, \eta, \bar{\eta})$

$$\begin{aligned} i\Gamma(J, \eta, \bar{\eta}) = {} & i\Gamma'_{\text{tree}}(J, \eta, \bar{\eta}) + i\Gamma'_{1\text{loop}}(J, \eta, \bar{\eta}) \\ & + \sum_{n=3}^{\infty} \sum_{\substack{(\lambda)_n \\ \lambda_1=\lambda_2=0}} \sum_{c} i^{\Sigma\lambda_\nu} K(\lambda)_n \int_{\Gamma_{\cdot},c} \prod_{\nu=1}^{n} (S_{1;I_\nu}\Gamma^{I_\nu}_{\cdot})^{\lambda_\nu} \\ & + \sum_{n=1}^{\infty} \sum_{(\lambda)_n} \sum_{c} i^{\Sigma\lambda_\nu} K(\lambda)_n \int_{\Gamma_{\cdot},\bar{\xi},\xi,c} \prod_{\nu=1}^{n} (\bar{\xi}^\alpha A^\beta_{\alpha\cdot I_\nu}\xi_\beta\Gamma^{I_\nu}_{\cdot})^{\lambda_\nu} \\ & + \sum_{n=4}^{\infty} \sum_{r=1}^{n-1} \sum_{(\lambda)_{n-r}} \sum_{(\mu)_r} \sum_{c} i^{\Sigma\lambda_\sigma} K(\lambda)_{n-r}\, i^{\Sigma\mu_\rho} K(\mu)_r \\ & \times \int_{\Gamma_{\cdot},\bar{\xi},\xi,c} \prod_{\sigma=1}^{n-r} (\bar{\xi}^\alpha A^\beta_{\alpha\cdot I_\sigma}\xi_\beta\Gamma^{I_\sigma}_{\cdot})^{\lambda_\sigma} \prod_{\rho=1}^{r} (S_{1;I_\rho}\Gamma^{I_\rho}_{\cdot})^{\mu_\rho}. \end{aligned} \tag{39}$$

It is clear from (39) that the basic ingredients of the covariant perturbation expansion (for the on shell connected Green's functions) are the covariant vertices $S_{1;I_n}(\phi)$ and $A^{\beta}_{\alpha \cdot I_n}(\phi)$ given by (31), (32), and (33), and the full propagators $G^{ij}(\phi)$ and $B_{\alpha}{}^{\beta}(\phi)$, which are the inverses of $S_{1;ij}(\phi)$ and $(iA)^{\beta}_{\alpha}(\phi)$, respectively, i.e.,

$$\int d^4x''\, S_{1;ij}(x, x''; \phi)\, G^{ik}(x'', x'; \phi) = \delta_i{}^k \delta(x - x') \tag{40}$$

$$\int d^4x''(iA)^{\beta}_{\alpha}(x, x''; \phi)\, B_{\beta}{}^{\gamma}(x'', x'; \phi) = \delta_{\alpha}{}^{\gamma}\delta(x - x'). \tag{41}$$

The full propagators $G^{ij}(y, y'; \phi)$ and $B_{\beta}{}^{\gamma}(y, y'; \phi)$ can be depicted graphically as shown in Fig. 1. Strictly speaking, the graphical representation shown for G^{ij} corresponds to the full propagator in the noncovariant theory.

We define $\delta^{ij}G_0$ and $\delta_{\alpha}{}^{\beta}S_F$ to be the limits of G^{ij} and $iB_{\alpha}{}^{\beta}$, respectively, when the pion source J is taken to zero.

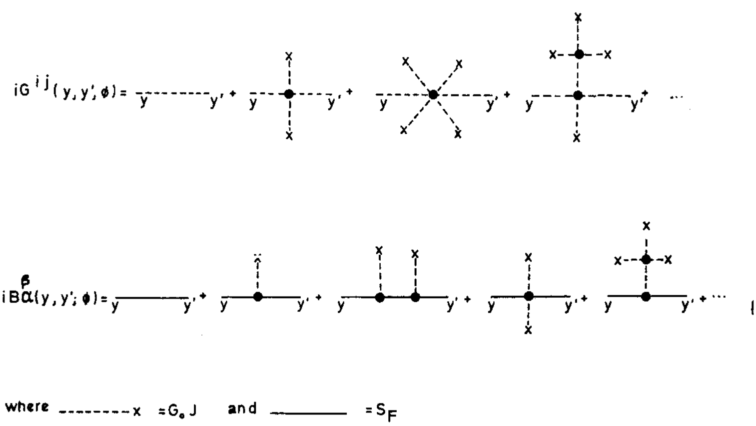


FIGURE 1

Thus

$$G^{ij}(x - x') \to \delta^{ij}G_0(x - x')$$

where

$$G_0(x) = i/4\pi^2(x^2 - i0)$$

and

$$iB_{\alpha}{}^{\beta}(x - x') \to \delta_{\alpha}{}^{\beta}S_F(x - x')$$

where

$$(i\not{\partial}^{(x)} - m)\, S_F(x - x') = \delta(x - x')$$

IV. Relations between Covariant and Noncovariant Derivatives

In this section we shall establish the connection between the covariant and noncovariant derivatives of the total action $S = S_1 + S_2$. For the first part of the action S_1, the required connection is given in [7]. This reads

$$S_{1,I_n}\chi^{I_n} = n! \sum_{(\lambda)_n} K(\lambda)_n \, S_{1;l_i \cdots l_r} \prod_{\nu=1}^{n} (T^{l}_{I_\nu}\chi^{I_\nu})^{\lambda_\nu} \tag{42}$$

with $r = \sum_{\nu=1}^{n} \lambda_\nu$. The coefficients $T^i_{I_\nu} \equiv T^i_{k_1 \cdots k_\nu}$ are symmetric in the lower indices, and can be expressed in terms of the generalized Christoffel symbols $\Gamma^i_{i_1 \cdots i_n}$, which occur in (21). We confine ourselves, therefore, to the second part of the action, S_2. From (24) and (26) we obtain

$$\begin{aligned} S_2(\pi, \psi, \bar{\psi}) &= \bar{\psi}^\alpha A_\alpha{}^\beta(\phi)\, \psi_\beta + \sum_{n=1}^{\infty} (1/n!)\, \bar{\psi}^\alpha A^\beta_{\alpha,I_n}(\phi)\, \psi_\beta \chi^{I_n} \\ &= \bar{\xi}^\alpha A_\alpha{}^\beta(\phi)\, \xi_\beta + \sum_{n=1}^{\infty} (1/n!)\, \bar{\xi}^\alpha A^\beta_{\alpha \cdot I_n}(\phi)\, \xi_\beta \Gamma^{I_n}_{\cdot}. \end{aligned} \tag{43}$$

Now the relations (22), (23), and (25) can be inverted to express $\Gamma^{\,i}_{\cdot}$, ξ_α, and $\bar{\xi}^\alpha$ in terms of χ^i, ψ_α, and $\bar{\psi}^\alpha$, respectively. Thus we obtain

$$\Gamma^{\,i}_{\cdot} = \sum_{n=1}^{\infty} (1/n!)\, T^i_{I_n}\chi^{I_n} \qquad (T_j{}^i = \delta_j{}^i) \tag{44}$$

$$\xi_\alpha = \psi_\alpha + \sum_{n=1}^{\infty} (1/n!)\, \Phi^\beta_{\alpha I_n}\psi_\beta \Gamma^{I_n}_{\cdot} \tag{45}$$

$$\bar{\xi}^\alpha = \bar{\psi}^\alpha + \sum_{n=1}^{\infty} (1/n!)\, \bar{\psi}^\beta X^\alpha_{\beta I_n}\Gamma^{I_n}_{\cdot} \tag{46}$$

where $\Phi^\beta_{\alpha I_n}$ and $X^\alpha_{\beta I_n}$ can be expressed in terms of $\Omega^\beta_{\alpha I_n}$ and $x^\alpha_{\beta I_n}$, respectively. In (45) and (46), of course, $\Gamma^{I_n}_{\cdot}$ has to be expressed in terms of χ^{I_n} via (44). Inserting (44), (45), and (46) in the right-hand side of (43), and equating expressions with the same power of χ on both sides of the equation we obtain a formula analogous to (42), namely,

$$\begin{aligned} \bar{\psi}^\alpha A^\beta_{\alpha,I_n}(\phi)\, \psi_\beta \chi^{I_n} = n!\, \bar{\psi}^\alpha &\sum_{n_1+n_2+n_3=n} X^\gamma_{\alpha l_1 \cdots l_{r_1}} A^\delta_{\gamma \cdot l_1 \cdots l_{r_2}} \Phi^\beta_{\delta l_1 \cdots l_{r_3}} \\ &\times \prod_{i=1}^{3} \left[K(\lambda^{(i)})_{n_i} \prod_{\nu=1}^{n_i} (T^l_{I^{(i)}_\nu}\chi^{I^{(i)}_\nu})^{\lambda^{(i)}_\nu} \right] \psi_\beta \end{aligned} \tag{47}$$

where $v_i = \sum_{\nu=1}^{n_i} \lambda^{(i)}_\nu$ and $n_i = \sum_{\nu=1}^{n_i} \nu\lambda^{(i)}_\nu$.

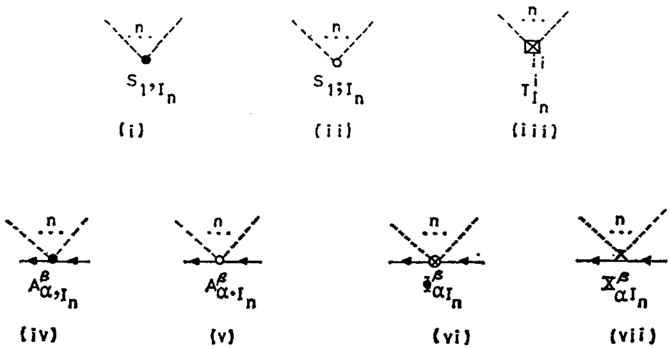


FIGURE 2

Following [7] we introduce, for each symbol, the graphical notation shown in Figs. 2(i)–2(vii). This notation is used to express the formulas (42) and (47) in a graphical manner and examples of this are given in Fig. 3 for small values of n.

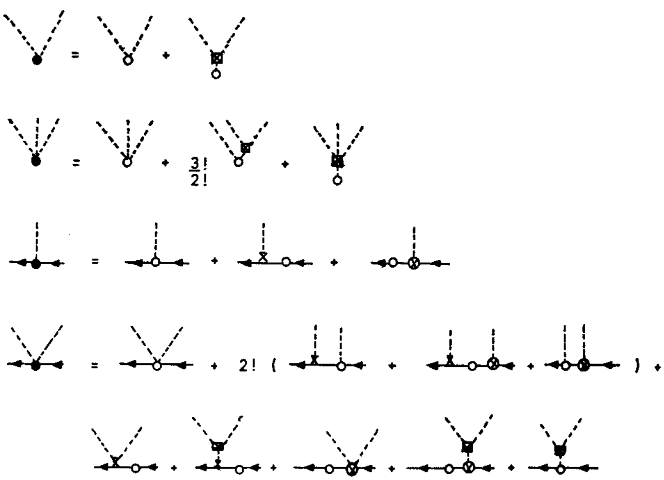


FIGURE 3

For later convenience we introduce the notion of a generalized partition: A sequence $\lambda_1^{(i)},\dots, \lambda_{n_i}^{(i)}$ ($i = 1, 2,\dots, M$) of natural numbers $\lambda_\nu^{(i)} \geqslant 0$ is called a generalized partition of n, if $\sum_{\nu=1}^{n_i} \nu\lambda_\nu^{(i)} = n_i$ and $\sum_{i=1}^{M} n_i = n$. We denote such generalized partitions as follows: $\{\lambda\}_n \equiv \{(1^{\lambda_1^{(1)}} \cdots n_1^{\lambda_{n_1}^{(1)}}) \cdots (1^{\lambda_1^{(M)}} \cdots n_M^{\lambda_{n_M}^{(M)}})\}$. The symmetry number associated with $\{\lambda\}_n$ is denoted by $K\{\lambda\}_n$ where $K\{\lambda\}_n = \prod_{i=1}^{M} K(\lambda^{(i)})_{n_i}$. It is now possible to rewrite (47) in the following way

$$\bar{\psi}^\alpha A^\beta_{\alpha,I_n}(\phi)\, \psi_\beta \chi^{I_n} = n!\, \bar{\psi}^\alpha \sum_{\{\lambda\}_n} K\{\lambda\}_n\, X^\gamma_{\alpha l_1 \cdots l_{r_1}} A^\delta_{\gamma \cdot l_1 \cdots l_{r_2}} \Phi^\beta_{\delta l_1 \cdots l_{r_3}} \times \prod_{i=1}^{3} \left[\prod_{\nu=1}^{n_i} (T^l_{I_\nu^{(i)}} \chi^{I_\nu^{(i)}})^{\lambda_\nu^{(i)}} \right] \psi_\beta \tag{48}$$

where $r_i = \sum_{\nu=1}^{n_i} \lambda_\nu^{(i)}$ $(i = 1, 2, 3)$ with $r_1 = 0$ and $r_3 = 0$ implying $X_\alpha{}^\gamma = \delta_\alpha{}^\gamma$ and $\Phi_\delta{}^\beta = \delta_\delta{}^\beta$, respectively.

V. The Equivalence Theorem

It is desirable to show the equivalence of the noncovariant and the covariant perturbation expansions on the mass shell. That is, we expect that, on the mass shell, the generating functional $\sqcup(J, \eta, \bar{\eta})$ and $\Gamma(J, \eta, \bar{\eta})$ for connected Green's functions in the noncovariant and covariant theory, respectively, are equivalent up to contributions of the type $\delta^{(4)}(0)$. It is the purpose of this section to establish the equivalence in some detail. This equivalence will serve as a possible link between the usual BPH approach and the covariant approach. We shall follow closely the proof of the "Equivalence Theorem" in the case of pure pion selfinteractions given in [7]. To this end we decompose each contribution from (38) into a sum of contributions containing (i) the corresponding covariant contribution from (39), (ii) contributions which vanish on the mass shell, and (iii) contributions which do not vanish on the mass shell and are not covariant. However, it can be shown that contributions of the type (iii) are either of the type $\delta^{(4)}(0)$, or they are cancelled out by analogous contributions, which arise from the decomposition of a finite set of other contributions from (38).

To proceed any further we need to introduce the notion of a double generalized partition. In [7] a double partition, denoted by $[(\lambda)_n^\mu]$ is defined to be the mapping $(\lambda) \to \mu_{(\lambda)}$ which assigns to each partition (λ) a natural number $\mu_{(\lambda)} \geqslant 0$ which is called the multiplicity of (λ). In a similar manner we introduce a double generalized partition, $[\{\lambda\}_n^\mu]$, defined to be the mapping $\{\lambda\} \to \mu_{\{\lambda\}}$ which assigns to each the multiplicity $\mu_{\{\lambda\}} \geqslant 0$. Suppose all the generalized partitions that occur in $[\{\lambda\}_n^\mu]$ are of the form

$$\{\lambda^{(m)}\}_{n_m} = \left\{ \prod_{j=1}^{3} (1^{\lambda_1^{(j)(m)}} \cdots n_j^{(m)\,\lambda_{n_j^{(m)}}^{(j)(m)}}) \right\} \Bigg|_{n_m = \sum_{j=1}^{3} n_j}$$

with $m = 1, 2, \ldots, N$. Then

$$[\{\lambda\}_n^\mu] = \left[\prod_{m=1}^{N} \left\{ \prod_{j=1}^{3} (1^{\lambda_1^{(j)(m)}} \cdots n_j^{(m)\,\lambda_{n_j^{(m)}}^{(j)(m)}}) \right\}^{\mu_m} \right] \tag{49}$$

where the order n of $[\{\lambda\}_n^\mu]$ is given by

$$n = \sum_{m=1}^{N} n_m \mu_m = \sum_{m=1}^{N} \mu_m \left(\sum_{j=1}^{3} \sum_{\nu=1}^{n_j^{(m)}} \nu \lambda_\nu^{(j)(m)} \right).$$

We also define the symmetry number associated with $[\{\lambda\}_n^\mu]$ to be

$$K[\{\lambda\}_n^\mu] = \prod_{m=1}^{N} \frac{1}{\mu_m!} (K\{\lambda^{(m)}\}_{n_m})^{\mu_m} = \frac{1}{\prod_{m=1}^{N} \mu_m! \, (\prod_{j=1}^{3} \prod_{\nu=1}^{n_j^{(m)}} \lambda_\nu^{(j)(m)}! \, (\nu!)^{\lambda_\nu^{(j)(m)}})^{\mu_m}} . \tag{50}$$

The contributions from $Ш'_{\text{tree}}$ and $Ш'_{\text{1loop}}$ coincide (on the mass shell) with the corresponding contributions from Γ'_{tree} and Γ'_{1loop}. Hence it is only necessary to establish the equivalence between $w(J, \eta, \bar{\eta})$ and $\omega(J, \eta, \bar{\eta})$, where

$$w(J, \eta, \bar{\eta}) = Ш(J, \eta, \bar{\eta}) - Ш'_{\text{tree}}(J, \eta, \bar{\eta}) - Ш'_{\text{1loop}}(J, \eta, \bar{\eta})$$

and

$$\omega(J, \eta, \bar{\eta}) = \Gamma(J, \eta, \bar{\eta}) - \Gamma'_{\text{tree}}(J, \eta, \bar{\eta}) - \Gamma'_{\text{1loop}}(J, \eta, \bar{\eta}).$$

From now on, however, we shall neglect the contributions to $w(J, \eta, \bar{\eta})$ which contain only pure pion selfinteractions. Such contributions are dealt with in [7]. Let $\bar{w}(J, \eta, \bar{\eta})$ denote the remaining contributions in $w(J, \eta, \bar{\eta})$. Inserting (42), and (48) in (38) we obtain the following expression for $\bar{w}(J, \eta, \bar{\eta})$

$$\begin{aligned}
i\bar{w}(J, \eta, \bar{\eta}) = & \sum_{n\geqslant 1} \sum_{[\{\lambda\}_n^\mu]} \sum_{c} i^{\Sigma_{m=1}^{N} \mu_m} K[\{\lambda\}_n^\mu] \\
& \times \int_{x,\psi,\bar{\psi},c} \prod_{m=1}^{N} \Bigg[\bar{\psi}^\alpha X^\gamma_{\alpha l_1 \cdots l_{r_1^{(m)}}} A^\delta_{\gamma \cdot l_1 \cdots l_{r_2^{(m)}}} \Phi^\beta_{\delta l_1 \cdots l_{r_3^{(m)}}} \psi_\beta \\
& \times \prod_{j=1}^{3} \left(\prod_{\nu=1}^{n_j^{(m)}} (T^l_{I_\nu^{(m)(j)}} \chi^{I_\nu^{(m)(j)}})^{\lambda_\nu^{(m)(j)}} \right) \Bigg]^{\mu_m} \\
& + \sum_{n\geqslant 4} \sum_{r=3}^{n-1} \sum_{[\{\lambda\}_{n-r}^\mu]} \sum_{[(\sigma)_r^\rho]} \sum_{c} i^{\Sigma_{m=1}^{M} \mu_m} K[\{\lambda\}_{n-r}^\mu] \, i^{\Sigma_{i=1}^{N} \rho_i} K[(\sigma)_r{}^\rho] \\
& \times \int_{x,\psi,\bar{\psi},c} \prod_{m=1}^{M} \Bigg[\bar{\psi}^\alpha X^\gamma_{\alpha l_1 \cdots l_{r_1^{(m)}}} A^\delta_{\gamma \cdot l_1 \cdots l_{r_2^{(m)}}} \Phi^\beta_{\delta l_1 \cdots l_{r_3^{(m)}}} \psi_\beta \\
& \times \prod_{j=1}^{3} \left(\prod_{\nu=1}^{n_j^{(m)}} (T^l_{I_\nu^{(m)(j)}} \chi^{I_\nu^{(m)(j)}})^{\lambda_\nu^{(m)(j)}} \right) \Bigg]^{\nu_m} \\
& \times \prod_{i=1}^{N} \left[S_{1; l_1 \cdots l_{r_1}} \prod_{\nu=1}^{n_i} (T^l_{I_\nu^{(i)}} \chi^{I_\nu^{(i)}})^{\sigma_\nu^{(i)}} \right]^{\rho_i}
\end{aligned} \tag{51}$$

where in the first term $n = \sum_{m=1}^{N} \mu_m (\sum_{j=1}^{3} \sum_{\nu=1}^{n_j(m)} \nu\lambda_\nu^{(j)(m)})$ and $r_j^{(m)} = \sum_{\nu=1}^{n_j(m)} \lambda_\nu^{(m)(j)}$ $(j = 1, 2, 3)$, whereas in the second term $n - r = \sum_{m=1}^{M} \mu_m (\sum_{j=1}^{3} \sum_{\nu=1}^{n_j(m)} \nu\lambda_\nu^{(j)(m)})$, $r_j^{(m)} = \sum_{\nu=1}^{n_j(m)} \lambda_\nu^{(m)(j)}$ $(j = 1, 2, 3)$ and $r = \sum_{i=1}^{N} \rho_i (\sum_{\nu=1}^{n_i} \nu\sigma_\nu^{(i)})$ with $r_i = \sum_{\nu=1}^{n_i} \sigma_\nu^{(i)}$. Each term in the first part of (51) is characterised by a double generalized partition $[\{\lambda\}_n^\mu]$ and a connection mapping C. It corresponds to a graph (with no external pion lines, but with possible external nucleon lines, coupled to the sourced η and $\bar{\eta}$) where each vertex of the type shown in Fig. 2(iv) is replaced by the corresponding vertex shown in Fig. 4(ii) with the same number of pion lines (χ^i). The connection mapping C acts on the χ^i's as well as the ψ's and $\bar{\psi}$'s. The multiplicity of each type of vertex is determined by μ_m $(m = 1, 2, \ldots, N)$. Similarly each term in the second part of (51) is characterized by a pair of double partitions $[\{\lambda\}_{n-r}^\mu]$ and $[(\sigma)_r^\rho]$, and a connection mapping C. It can be represented by a graph (with no external pion lines, but with possible external nucleon lines coupled to η and $\bar{\eta}$) where each vertex of the type shown in Figs. 2(i) and 2(iv) are replaced respectively, by the corresponding vertices shown in Figs. 4(i) and 4(ii), with the same number of legs (χ^i).

FIGURE 4

We consider first the contribution to $\bar{w}(J, \eta, \bar{\eta})$ characterized by the particular double partitions

$$[\{\lambda\}_n^\mu] = \left[\prod_{m=1}^{N} \{(0)(1^{n_m})(0)\}^{\mu_m}\right], \qquad n = \sum_{m=1}^{N} \mu_m n_m$$

and

$$[\{\lambda\}_{n-r}^\mu] = \left[\prod_{m=1}^{M} \{(0)(1^{n_m})(0)\}^{\mu_m}\right]$$

$$[(\sigma)_r^\rho] = \left[\prod_{i=1}^{N} (1^{n_i})^{\rho_i}\right]$$

with $n - r = \sum_{m=1}^{M} \mu_m n_m$, and $r = \sum_{i=1}^{N} \rho_i n_i$. By examining the symmetry factors $K[\{\lambda\}_n^\mu]$, $K[\{\lambda\}_{n-r}^\mu]$, and $K[(\sigma)_r^\rho]$ we can infer that this part of $\bar{w}(J, \eta, \bar{\eta})$ is in 1-1 correspondence with $\omega(J, \eta, \bar{\eta})$.

Let us consider now the contribution to $\bar{w}(J, \eta, \bar{\eta})$ arising from the second part in (51), when the double partition $[(\sigma)_r^\rho]$ contains at least one partition $(\sigma^{(i)})$ with $\sum_{\nu=1}^{n_i} \sigma_\nu^{(i)} = 1$. Such partitions give rise to vertices of the type $S_{1,l}T_{i_1 i_2 \ldots}^l$. However, because of (5) $S_{1;l} = -J_l$ and graphs containing at least one such vertex, therefore, give vanishing contributions on the mass shell.

Let $\Delta\bar{w}(J, \eta, \bar{\eta})$ denote the remaining contributions to (51). Then

$$
\begin{aligned}
i\Delta\bar{w}(J, \eta, \bar{\eta}) = & \sum_{n\geqslant 1} \sum_{[\{\lambda\}_n^\mu]}{}' \sum_{c} i^{\Sigma_{m=1}^N \mu_m} K[\{\lambda\}_n^\mu] \\
& \times \int_{\chi,\psi,\bar{\psi},c} \prod_{m=1}^{N} \Bigg[\bar{\psi}^\alpha X^\gamma_{\alpha l_1 \cdots l_{r_1^{(m)}}} A^\delta_{\gamma\cdot l_1\cdots l_{r_2^{(m)}}} \Phi^\beta_{\delta l_1\cdots l_{r_3^{(m)}}} \psi_\beta \\
& \times \prod_{j=1}^{3} \left(\prod_{\nu=1}^{n_j^{(m)}} (T^l_{I_\nu^{(m)(j)}} \chi^{I_\nu^{(m)(j)}})^{\lambda_\nu^{(m)(j)}} \right) \Bigg]^{\mu_m} \\
& + \sum_{n\geqslant 4} \sum_{r=1}^{n-1} \sum_{[\{\lambda\}_{n-r}^\mu]}{}' \sum_{[(\sigma)_r^\rho]}{}'' \sum_{c} i^{\Sigma_{m=1}^M \mu_m} K[\{\lambda\}_{n-r}^\mu] \, i^{\Sigma_{i=1}^N \rho_i} K[(\sigma)_r^\rho] \\
& \times \int_{\chi,\psi,\bar{\psi},c} \prod_{m=1}^{M} \Bigg[\bar{\psi}^\alpha X^\gamma_{\alpha l_1 \cdots l_{r_1^{(m)}}} A^\delta_{\gamma\cdot l_1\cdots l_{r_2^{(m)}}} \Phi^\beta_{\delta l_1\cdots l_{r_3^{(m)}}} \psi_\beta \\
& \times \prod_{j=1}^{3} \left(\prod_{\nu=1}^{n_j^{(m)}} (T^l_{I_\nu^{(m)(j)}} \chi^{I_\nu^{(m)(j)}})^{\lambda_\nu^{(m)(j)}} \right) \Bigg]^{\mu_m} \\
& \times \prod_{i=1}^{N} \left[S_{1;l_1\cdots l_{r_i}} \prod_{\nu=1}^{n_i} (T^l_{I_\nu^{(i)}} \chi^{I_\nu^{(i)}})^{\sigma_\nu^{(i)}} \right]^{\rho_i}
\end{aligned}
\tag{52}
$$

where the summations Σ' indicate that the double generalized partitions $[\{\lambda\}_n^\mu]$ and $[\{\lambda\}_{n-r}^\mu]$ should not consist of generalized partitions of the type $\{(0)(1^k)(0)\}$ alone. Also the summation Σ'' indicates that the double partition $[(\sigma)_r^\rho]$ should not (i) consist of partitions of the type (1^S) alone, and (ii) contain partitions of the type $(\sigma^{(i)})$ with $\sum_{\nu=1}^{n_i} \sigma_\nu^{(i)} = 1$.

The equivalence between covariant and noncovariant perturbation expansions will be established, if we can show that, up to terms which contain $\delta^{(4)}(0)$ factors, $\Delta\bar{w}(J, \eta, \bar{\eta})$ vanishes identically on the mass shell. In order to prove this we shall make use of the following list of identities (due to (40), and (41)):

(i) $$S_{1;i_1\cdots}(x;\phi)\, iG^{i_1 l}(x, x''; \phi)\, S_{1;l i_2}(x'', x'; \phi)\, T^{i_2}_{\cdots}(x') = iS_{1;i_1\cdots}(x;\phi)\, T^{i_1}_{\cdots}(x), \tag{53}$$

(ii) $$A^\beta_{\alpha\cdot i_1\cdots}(x;\phi)\, iG^{i_1 l}(x, x''; \phi)\, S_{1;l i_2}(x'', x'; \phi)\, T^{i_2}_{\cdots}(x') = iA^\beta_{\alpha\cdot i_1\cdots}(x;\phi)\, T^{i_1}_{\cdots}(x), \tag{54}$$

(iii) $$\begin{aligned} & A^\beta_{\alpha\cdot i_1\cdots}(x;\phi)\, B_\beta{}^\gamma(x, x''; \phi)\, A_\gamma{}^\delta(x'', x'; \phi)\, \Phi^\epsilon_{\delta i_2\cdots}(x') \\ & = (-i)\, A^\beta_{\alpha\cdot i_1\cdots}(x;\phi)\, \Phi^\epsilon_{\beta i_2\cdots}(x), \end{aligned} \tag{55}$$

(iv) $$\begin{aligned} & X^\gamma_{\alpha i_1\cdots} A_\gamma{}^\delta(x, x''; \phi)\, B_\delta{}^\beta(x'', x'; \phi)\, A^\epsilon_{\beta\cdot i_2\cdots}(x';\phi) \\ & = (-i)\, X^\beta_{\alpha i_1\cdots}(x)\, A^\epsilon_{\beta\cdot i_2\cdots}(x;\phi). \end{aligned} \tag{56}$$

The above identities show the possibility of contractions of full pion as well as nucleon propagators. The factors appearing on the right-hand side of (53)–(56) play a very important role in the proof of the equivalence theorem. In general we shall have to examine the change of factors of i in $\Delta\bar{w}$ due to contractions. It is clear that each contraction reduces by one the number of vertices of the graph under consideration leaving, therefore, a factor of i (see factors of i in (51)). In (53) and (54) there is a factor of i due to the fact that full pion propagators always appear in the form iG and not simply G. This comes about because of the functional integral

$$\frac{1}{N}\int D\chi\, e^{(i/2)S_{1,ij}(\phi)\chi^i\chi^j}\chi^{k_1}\cdots\chi^{k_{2n}} = \text{const}\,\frac{1}{[\mathrm{Det}(S_{1,ij})]^{1/2}}\sum iG^{k_{j_1}k_{j_2}}\cdots iG^{k_{j_{2n-1}}k_{j_{2n}}} \tag{57}$$

where the sum in (57) is over all possible pairings of the χ^i,s. Thus every contraction corresponding to (53) and (54) gives rise to an overall factor $i^2 = -1$. Equations (55) and (56), on the other hand, correspond to full nucleon propagator contractions. Now, from (30) it is clear that in order to get an internal nucleon propagator corresponding to $\overbrace{\psi_\alpha\bar{\psi}^\beta}$ both left and right derivatives $\overrightarrow{\delta}/i\delta\bar{\eta}^\alpha$ and $\overleftarrow{\delta}/i\delta\eta_\beta$ have to act on the same term $(i\bar{\eta})^\gamma B_\gamma{}^\delta(\phi)(i\eta)_\delta$ in the expansion of $\exp[-(i\bar{\eta})\cdot B\cdot(i\eta)]$. Thus, we pick up an additional factor (-1). Taking now into account the factor of $(-i)$ appearing on the right-hand side of (55) and (56) we see that, once again, the overall

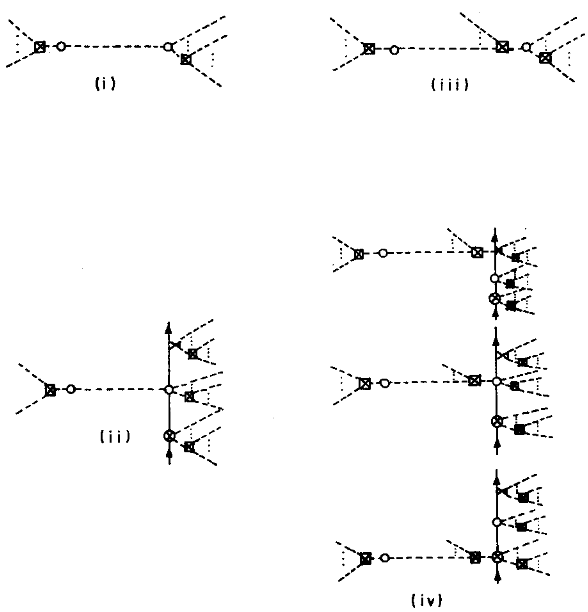


FIGURE 5

factor is $i^2 = -1$. Hence we reach the conclusion that every contraction of a full internal propagator gives rise to a factor (-1), which must be taken into account.

We remark that (53) and (54) correspond to contractions described in [7]. From the work of [7] we know that the vertices $S_{1;ij}$ giving rise to contractions arise necessarily from partitions of the type $(1, k)$ $k \geqslant 2$. Each partition $(1, k)$ contained in $[(\sigma)^\rho]$ gives rise to a vertex factor $S_{1;il}T^l_{I_k}\chi^i\chi^{I_k}$. Let χ^i be connected to χ^j by the action of C. Then, either χ^j is connected to $S_{1;j\ldots}$ or to a nontrivial T, i.e., $T^m_{j\ldots}$ (but not $T_j{}^m$). In general we have the graphical representations shown in Fig. 5. A vertex or a graph containing $S_{1,ij}$ of the kind corresponding to 5(i) and 5(ii) is called contractible. Now let us examine the cases due to internal nucleon propagator contractions. We note that vertices $A_\alpha{}^\beta$ may arise from the following types of generalized partition: (i)$\{(\lambda^{(1)})_{n_1}(0)(0)\}$, (ii) $\{(\lambda^{(1)})_{n_1}(0)(\lambda^{(3)})_{n_3}\}$ and (iii) $\{(0)(0)(\lambda^{(3)})_{n_3}\}$. Each generalized partition of the type (i) implies a vertex factor $\bar{\psi}^\alpha X^\gamma_{\alpha\cdot l_1\ldots} A_\gamma{}^\beta \prod_{\nu=1}^{n_1} (T^l_{I_\nu}\chi^{I_\nu})^{\lambda^{(1)}_\nu}\, \psi_\beta$. Let ψ_β be connected to $\bar{\psi}^\delta$ by the connection mapping C. Then there exist two possibilities. Either $\bar{\psi}^\delta$ is connected to a vertex described by a generalized partition of the form $\{(0)(\lambda^{(2)\prime})_{n_2'}(\lambda^{(3)\prime})_{n_3'}\}$ or to a vertex $\{(\lambda^{(1)\prime})_{n_1'}(\lambda^{(2)\prime})_{n_2'}(\lambda^{(3)\prime})_{n_3'}\}$. Graphically we have the representations shown in Figs. 6(i) and 6(ii). A graph of the kind 6(i) containing $A_\alpha{}^\beta$ is again called contractible. Similarly in the case when $A_\alpha{}^\beta$ arises from $\{(0)(0)(\lambda^{(3)})_{n_3}\}$ we have the two possibilities shown in Figs. 6(iii) and 6(iv). In this case it is only 6(iii) that leads to contractible vertices or graphs. Finally the case where $A_\alpha{}^\beta$ arises from $\{(\lambda^{(1)})_{n_1}(0)(\lambda^{(3)})_{n_3}\}$ leads to no contractible vertices (or graphs).

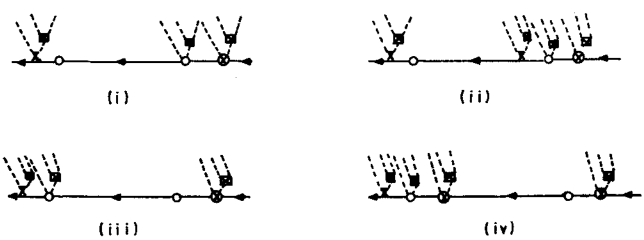


FIGURE 6

In general $\Delta\bar{w}(J, \eta, \bar{\eta})$ contains contractible graphs. If one applies the identities (53) — (56) to the contractible vertices of such a graph, one gets an uncontractible graph. It is understood of course, that the connection mapping of the original contractible graph has to be restricted to the remaining ψ_α, $\bar{\psi}^\alpha$ and χ^i. Now (52) shows that this contribution of such a contractible graph to $\Delta\bar{w}$ is equal to the contribution of the associated uncontractible graph, explicitly present in (52), apart from a sign and a combinatorial factor (due to the presence of the symmetry factors $K[\{\lambda\}^\mu_n]$, or $K[\{\lambda\}^\mu_{n-r}] \cdot K[(\sigma)^\rho_r]$ and the possibility that the different connection mappings of the contractible graph give rise to the same connection mapping on restriction to the uncontractible graph). The sign arises in the way described in the remarks made following Eq. (56). Following [7] we remark that the uncontractible graphs of $\Delta\bar{w}$ give rise to an equivalence relation among the totality of the graphs. The equivalence classes consist of all graphs, which, after complete contraction (by means of repeated applications of (53)–(56)), lead to a fixed uncontractible graph. Thus, what we have to do, is to deal with a general equivalence class, $\epsilon(g_0)$ corresponding to an uncontractible graph g_0, and show that $\Delta\bar{w}\,|_{\epsilon(g_0)} = 0$, where $\Delta\bar{w}\,|_{\epsilon(g_0)}$ denotes the partial sum of $\Delta\bar{w}$ taken over the class $\epsilon(g_0)$. It is shown in [7] that, in the case of pure pion selfinteractions alone the corresponding class $\epsilon(g_0)$, which is in general quite large, can be divided into smaller classes with vanishing partial sums. The same is true in our case, and this is what we would like to show in the remaining part of this section.

Let g_0 be an uncontractible graph, and $\epsilon(g_0)$ the class generated by it. $\epsilon(g_0)$ is precisely obtained by doing all admissible blow ups (a blow up is an operation corresponding to the inverse application of (53)–(56)). Let there be N_0 admissible blow ups on g_0, then $N_0 = \#\epsilon(g_0)$. Thus one way of dividing $\epsilon(g_0)$ into smaller classes is to divide all blow ups on g_0 into a sequence of independent types. The application of a blow up of a given independent type (keeping everything else fixed) then gives rise to one such smaller class. This is in fact the construction of (s, x) equivalence classes in [7]. To this end we choose a vertex from g_0, which allows blow ups. If this choice corresponds to a vertex arising from pion selfinteractions, then the results of [7] are directly applicable with only trivial modifications. Without loss of generality we assume that our choice corresponds to a vertex $\{x\} \neq \{(0)(1^k)(0)\}$. Let $\{x\}$ be given by $\{x\} = \{(x^{(1)})_{\bar{n}_1}\,(x^{(2)})_{\bar{n}_2}\,(x^{(3)})_{\bar{n}_3}\}$. The vertex $\{x\}$ allows in general, three types of blow up with blow up factors given by (i) the generalized partition $\{(x^{(1)})_{\bar{n}_1}\,(0)(0)\}$, (ii) the generalized partition $\{(0)(0)(x^{(3)})_{\bar{n}_3}\}$, and (iii) a nontrivial T of the form $T^{l}_{i_1 i_2 \cdots}$. Now, let $\mu_{\{x\}}$ be the multiplicity of $\{x\}$ in g_0. Next we proceed as follows. We blow up the vertices $\neq \{x\}$ of g_0 in an arbitrary way. Then we blow up the vertices $\{x\}$ of g_0 arbitrarily except for factors $\{(x^{(1)})_{\bar{n}_1}\,(0)(0)\}$ and $\{(0)(0)(x^{(3)})_{\bar{n}_3}\}$. Thus, each of the $\mu_{\{x\}}$ partially blown up vertices is described by a generalized partition

$$\{(x^{(1)})_{\bar{n}_1}\,(1^{x_1^{(2)\prime}} \cdots \bar{n}_2^{x_{\bar{n}_2}^{(2)\prime}})(x^{(3)})_{\bar{n}_3}\}$$

with

$$x_i^{(2)\prime} \leqslant x_i^{(2)} \quad (i \geqslant 2) \qquad \text{and} \qquad \sum_{i=1}^{\bar{n}_2} x_i^{(2)\prime} = \sum_{i=1}^{\bar{n}_2} x_i^{(2)}.$$

The last relation fixes $x_1^{(2)\prime}$ uniquely. Next we define

$$(\bar{x}^{(2)})_{\bar{n}_2} = \max(1^{x_1^{(2)\prime}} \cdots \bar{n}_2^{x_{\bar{n}_2}^{(2)\prime}}),$$

the maximum taken with respect to the lexicographic order of the partitions $(1^{x_1^{(2)\prime}} \cdots \bar{n}^{x_{\bar{n}2}^{(2)\prime}})$. Let n_0 of the $\mu_{\{x\}}$ vertices be of this maximal type. They define a double generalized partition

$$[\{(x^{(1)})_{\bar{n}_1}(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(x^{(3)})_{\bar{n}_3}\}^{n_0}]$$

This set of vertices can be distinguished uniquely from the remaining $\mu_{\{x\}} - n_0$ nonmaximal partially blown up vertices. We then blow up an arbitrary set of these nonmaximal vertices in the way that gives blow up factors of the form $\{(x^{(1)})_{\bar{n}_1}(0)(0)\}$ and $\{(0)(0)(x^{(3)})_{\bar{n}_3}\}$. Our partially blown up graph is now described by the following double generalized partition:

$$\begin{aligned}
&[\{(x^{(1)})_{\bar{n}_1}(0)(0)\}^{M_0}\{(0)(0)(x^{(3)})_{\bar{n}_3}\}^{N_0}\{(x^{(1)})_{\bar{n}_1}(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(x^{(3)})_{\bar{n}_3}\}^{n_0}\\
&\times \{(0)(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(x^{(3)})_{\bar{n}_3}\}^{M_1}\{(x^{(1)})_{\bar{n}_1}(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(0)\}^{N_1}\{(0)(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(0)\}^{P_1}]\\
&\times [\{R_1\}^{r_1}][(R_2)^{r_2}]
\end{aligned} \tag{57}$$

where the multiplicities are determined by the condition that the remainder factors $[\{R_1\}^{r_1}][(R_2)^{r_2}]$ are disjoint from the first factor in (57). The above double generalized partition together with a connection mapping C_0 (specified during the blow up processes) define our $\{x\}$-collapse graph. Define an $\{x\}$-contraction to be a contraction on $g \in \epsilon(g_0)$ given by one of the following cases (i) a contraction of a vertex $\{(x^{(1)})_{\bar{n}_1}(0)(0)\}$ with a vertex $\{(0)(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(x^{(3)})_{\bar{n}_3}\}$, (ii) a contraction of a vertex $\{(0)(0)(x^{(3)})_{\bar{n}_3}\}$ with a vertex $\{(x^{(1)})_{\bar{n}_1}(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(0)\}$, and (iii) a contraction of two vertices $\{(x^{(1)})_{\bar{n}_1}(0)(0)\}$ and $\{(0)(0)(x^{(3)})_{\bar{n}_3}\}$ with a vertex $\{(0)(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(0)\}$. Our $\{x\}$-collapse graph does not admit any $\{x\}$-contractions. Two graphs in $\epsilon(g_0)$ are called $\{x\}$-equivalent, if they become equal (up to a numerical factor) after performing all the appertaining $\{x\}$-contractions. This equivalence relation splits $\epsilon(g_0)$ into smaller classes. Each of these classes contains exactly one $\{x\}$-collapse graph. Now for each $\{x\}$-collapse graph we can do the remaining blow ups, which give rise to blow up factors $\{(x^{(1)})_{\bar{n}_1}(0)(0)\}$ and $\{(0)(0)(x^{(3)})_{\bar{n}_3}\}$. Each such blow up results in a graph, which is described by one of the following double generalized partitions.

$$\begin{aligned}
&[\{(x^{(1)})_{\bar{n}_1}(0)(0)\}^{M_0+s+u}\{(0)(0)(x^{(3)})_{\bar{n}_3}\}^{N_0+t+u}\{(x^{(1)})_{\bar{n}_1}(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(x^{(3)})_{\bar{n}_3}\}^{n_0-s-t-u}\\
&\times \{(0)(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(x^{(3)})_{\bar{n}_3}\}^{M_1+s}\{(x^{(1)})_{\bar{n}_1}(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(0)\}^{N_1+t}\\
&\times \{(0)(\bar{\bar{x}}^{(2)})_{\bar{n}_2}(0)\}^{P_1+u}][\{R_1\}^{r_1}][(R_2)^{r_2}]
\end{aligned} \tag{58}$$

with $0 \leqslant s + t + u \leqslant n_0$. The next step is to evaluate how many connection mappings C exist, for a given double generalized partition (58), which will give C_0 after restriction to the $\{x\}$-collapse graph. This is straightforward. We obtain the following factor

$$\frac{(-1)^{s+t}\,(M_0 + s + u)!\,(N_0 + t + u)!\,(M_1 + s)!\,(N_1 + t)!\,(P_1 + u)!}{M_0!\,N_0!\,M_1!\,N_1!\,P_1!\,s!\,t!\,u!} \tag{59}$$

where the factor $(-1)^{s+t}$ arises from the fact that every contraction gives rise to a factor $i^2 = -1$. Now it is clear from (52) that we have to calculate also the symmetry factor associated with the blown up graphs described by (58). It is important, however, to remember that the disjoint factors $[\{R_1\}^{r_1}][(R_2)^{r_2}]$ give rise to numerical factors, which are independent of s, t, and u, and therefore, can be omitted. Multiplying the required symmetry factor with (59), and summing over all the elements of the $\{x\}$-class we get:

$$\frac{\text{Constant factor independent of } s,\, t,\, u}{n_0!} \sum_{0 \leqslant s+t+u \leqslant n_0} \frac{n_0!\,(-1)^{s+t}}{(n_0 - s - t - u)!\,s!\,t!\,u!}$$
$$= \frac{\text{Const factor}}{n_0!}\,(1 + 1 - 1 - 1)^{n_0} = 0.$$

There remains the exceptional case where the choice of a vertex leads to a generalized partition of the form $\{x\} = \{(0)(x^{(2)})_{\bar{n}_2}(0)\}$, with $(x^{(2)})_{\bar{n}_2} \neq (1^{\bar{n}_2})$. In this case one will proceed in a manner completely analogous to [7]. Here we sketch the main steps. First we blow up the vertices $\neq \{x\}$ of g_0. Then we blow up the vertices $\{x\}$ of g_0 except for the factors $T^l_{i_1 \cdots i_s}$ (s being a fixed given natural number). Now each of the $\mu_{\{x\}}$ partially blown up vertices is described by a generalized partition

$$\{(0)(1^{x_1^{(2)\prime}} \cdots s^{x_s^{(2)}} \cdots \bar{n}_2^{x_{\bar{n}_2}^{(2)\prime}})(0)\}$$

with $x_i^{(2)\prime} \leqslant x_i^{(2)}$ $(i \geqslant 2)$, and $\sum_{i=1}^{\bar{n}_2} x_i^{(2)\prime} = \sum_{i=1}^{\bar{n}_2} x_i^{(2)}$ which fixes $x_1^{(2)\prime}$. Then we define

$$(2^{\bar{x}_2^{(2)}} \cdots \hat{s}^{x_s^{(2)}} \cdots \bar{n}_2^{\bar{x}_{\bar{n}_2}^{(2)}}) = \max(2^{x_2^{(2)\prime}} \cdots \hat{s}^{x_s^{(2)}} \cdots \bar{n}_2^{x_{\bar{n}_2}^{(2)\prime}})$$

and

$$(\bar{x}^{(2)}) = (1^{\bar{x}_1^{(2)}} \cdots s^{\bar{x}_s^{(2)}} \cdots \bar{n}_2^{\bar{x}_{\bar{n}_2}^{(2)}}), \qquad \text{with} \qquad \bar{x}_s^{(2)} = x_s^{(2)}, \qquad \sum_{i=1}^{\bar{n}_2} \bar{x}_i^{(2)} = \sum_{i=1}^{\bar{n}_2} x_i^{(2)}$$

where ^ implies omission and the maximum is again taken with respect to the lexicographic order of the partitions $(2^{x_2^{(2)\prime}} \cdots \bar{n}_2^{x_{\bar{n}_2}^{(2)\prime}})$. Now, let $\bar{n}_0$ be the number of these maximal vertices. They define the following double generalized partition

$$[\{(0)(1^{\bar{x}_1^{(2)}} \cdots s^{x_s^{(2)}} \cdots \bar{n}_2^{\bar{x}_{\bar{n}_2}^{(2)}})(0)\}^{n_0}].$$

We consider now the remaining $\mu_{\{x\}} - n_0$ nonmaximal vertices, and we fix them by blowing an arbitrary set of them in a way that gives blow up factors $T^l_{i_1\cdots i_s}$. Thus, our partially blown up graph is described by the following double generalized partition

$$[(1, s)^{N_0}]\left[\prod_{\nu=0}^{x_s^{(2)}} \{(0)(1^{\bar{x}_1^{(2)}+\nu} \cdots s^{x_s^{(2)}-\nu} \cdots \bar{n}_2^{\bar{x}_{\bar{n}_2}^{(2)}})(0)\}^{n_\nu}\right] [\{R_1\}^{r_1}][(R_2)^{r_2}] \tag{60}$$

where, once again, $[\{R_1\}^{r_1}][(R_2)^{r_2}]$ are disjoint from the first two factors in (60). The above double generalized partition (60) together with the conncetion mapping C_0 (specified during the blow up process) define our $(s, \{x\})$-collapse graph. Such a graph does not admit any $(s, \{x\})$-contractions, where an $(s, \{x\})$-contraction is defined to be the contraction in $g \in \epsilon(g_0)$ which contracts a vertex $(1, s)$ with a vertex $\{(0)(1^{\bar{x}_1^{(2)}+\nu} \cdots s^{x_s^{(2)}-\nu} \cdots n_2^{\bar{x}_{\bar{n}_2}^{(2)}})(0)\}$. Such contractions characterize the $(s, \{x\})$ equivalence class. Each class contains exactly one $(s, \{x\})$-collapse graph. For each $(s, \{x\})$-collapse graph one can do, now, the remaining blow ups of the remaining n_0 maximal vertices, giving rise to blow up factors $T^l_{i_1\cdots i_s}$. Each blow up results in a graph described by a generalized double partition

$$\begin{aligned}[(1, s)^{N_0+\sum_{\nu=1}^{x_s^{(2)}} \nu l_\nu}] & \left[\prod_{\nu=1}^{x_s^{(2)}} \{(0)(1^{x_1^{(2)}+\nu} \cdots s^{x_s^{(2)}-\nu} \cdots \bar{n}_2^{x_{\bar{n}_2}^{(2)}})(0)\}^{n_\nu+l_\nu}\right. \\ & \left.\times \{(0)(1^{x_1^{(2)}} \cdots s^{x_s^{(2)}} \cdots \bar{n}_2^{x_{\bar{n}_2}^{(2)}})(0)\}^{n_0-l}\right] [\{R_1\}^{r_1}][(R_2)^{r_2}] \end{aligned} \tag{61}$$

with $l = \sum_{\nu=1}^{x_s^{(2)}} l_\nu$ and $0 \leqslant l \leqslant n_0$. As in the previous case one has to evaluate how many connection mappings C exist for a given double generalized partition (61), which will give rise to C_0 on restriction to the corresponding $(s, \{x\})$-collapse graph. From [7] we know that this is given by

$$(-1)^{\sum_1^{x_s^{(2)}} l_\nu} \frac{(N_0 + \sum_{\nu=1}^{x_s^{(2)}} \nu l_\nu)!}{N_0!} \prod_{\nu=1}^{x_s^{(2)}} \frac{(n_\nu + l_\nu)!}{n_\nu!\, l_\nu!\, (\nu!)^{l_\nu}} \prod_{\rho=1}^{\nu} (\bar{x}_1 + \rho)^{l_\nu}. \tag{62}$$

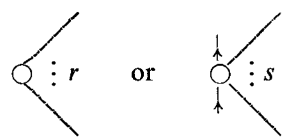
Multiplying (62) by the symmetry factor associated with (61), and summing over all the elements in the $(s, \{x\})$-class one readily obtains

$$\frac{\text{Const factor independent of } l_\nu}{n_0!} \sum_{\substack{0 \leqslant l \leqslant n_0 \\ l = \sum l_\nu}} \frac{n_0!}{(n_0 - l)!} \prod_{\nu=1}^{x_s^{(2)}} \frac{\left[(-1)^\nu \binom{x_s^{(2)}}{\nu}\right]^{l_\nu}}{l_\nu!}$$

$$= \frac{\text{Const factor}}{n_0!} \left\{1 + \sum_{\nu=1}^{x_s^{(2)}} (-1) \binom{x_s^{(2)}}{\nu}\right\}^{n_0} = \{(1 - 1)^{x_s^{(2)}}\}^{n_0} = 0$$

We have, now, to examine certain cases when the general cancellation procedure is not applicable. Supposing for all choices of $\{x\} \in g_0$ one gets $\{x\} = \{(x^{(1)(i)})_{n_i}$ $(0)(0)\}$ This vertex does not allow any blow ups. Hence $\epsilon(g_0)$ consists of only one element. In this case g_0 is characterized by the double generalized partition $[\{(x^{(1)(1)})_{n_1}(0)(0)\}^{M_1} \cdots \{(x^{(1)(i)})_{n_i}(0)(0)\}^{M_i} \cdots]$ and a connection mapping. The latter gives rise to three possibilities:

(i) Vertices of the type $\{(x^{(1)(i)})_{n_i}(0)(0)\}$ form a closed fermion loop. Such a graph is of the $\delta^{(4)}(0)$-type, and is, therefore, disregarded. We remark, however, that such loops may also occur in graphs containing vertices $\{(x^{(1)})_{\bar{n}_1}(x^{(2)})_{\bar{n}_2}$ $(x^{(3)})_{\bar{n}_3}\}$ for which the general cancellation procedure works. They formally cancel out. If, however, the graph under consideration contains only such loops besides pure



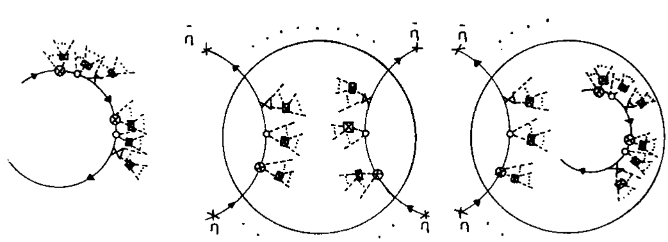
vertices, its class $\epsilon(g)$ contains just one element and its contribution does not vanish, but can be ignored being of the $\delta^{(4)}(0)$-type.

(ii) Vertices of the type $\{(x^{(1)(i)})_{n_i}(0)(0)\}$ are all coupled to the appertaining nucleon sources. Such graphs give contributions which vanish on the mass shell.

(iii) Some vertices are coupled to nucleon sources and some form closed loops. Again such graphs are of the $\delta^{(4)}(0)$-type and are disregarded.

The same three possibilities occur, when, for all choices of $\{x\} \in g$, one gets $\{x\} = \{(0)(0)(x^{(3)(i)})_{n_i}\}$. Once again the contributions arising from these possibilities are either of the $\delta^{(4)}(0)$-type, or vanish on the mass shell.

Finally we have the same three possibilities in the case when $\{x\} = \{(x^{(1)(i)})_{n_i^{(1)}}$ $(0)(x^{(3)(i)})_{n_i^{(3)}}\}$
These three possibilities are shown graphically in Fig. 7(i) to 7(iii).



(i) (ii) (iii)

FIGURE 7

The cases 7(i) and 7(iii) are disregarded being of the $\delta^{(4)}(0)$-type. However, it is clear that in case 7(ii) blow ups giving rise to factors of the form $\{(x^{(1)(i)})_{n_i^{(1)}}(0)(0)\}$ and $\{(0)(0)(x^{(3)i})_{n_i^{(3)}}\}$ can take place, and the general cancellation procedure is now applicable.

We remark that the above exceptional cases cover all the possibilities for generalized vertices, which do not allow any blow ups. We see, therefore, that in the cases, when there exist no possibilities for blow ups (and, therefore, the general cancellation procedure does not work), we always get contributions of the $\delta^{(4)}(0)$-type, or contributions vanishing on the mass shell. We may conclude that, on the mass shell, $\Delta\bar{w} = 0$ up to contributions of the $\delta^{(4)}(0)$-type. This concludes the proof of the Equivalence theorem.

It is easy to classify the graphs which will contribute to the cancellation of the noncovariant parts in $\Delta\bar{w}$ arising from a given graph, g, in the noncovariant expansion. First we assume that g has no external nucleon lines. Now, let $n \geqslant 1$ and consider all partitions of the form

$$(\alpha) \qquad (\lambda)_{2n} = \left(\prod_{\nu=1}^{2n} \nu^{\lambda_\nu}\right)$$

$$(\beta) \qquad (\lambda)_{n_1} = \left(\prod_{\nu=1}^{n_1} \nu^{\lambda_\nu}\right)$$

$$(\rho)_{n_2} = \left(\prod_{\sigma=1}^{n_2} \sigma^{\rho_\sigma}\right) \qquad \text{with} \quad \rho_1 = \rho_2 = 0$$

and $\sum_{\sigma=1}^{n_2} (\sigma - 2)\,\rho_\sigma = n_2$, where $n_1 + n_2 = n$. Take all the graphs in the noncovariant expansion, which are described by partitions of the type (α) or (β).

First we shall examine more closely the graphs of the type (α). They correspond to graphs arising from pion-nucleon interactions only. Let g' be a graph of this type. Let $P(g')$ be the power of F_π^{-1} associated with g', Then

$$P(g') = \sum_{\nu=1}^{2n} \nu\lambda_\nu = 2n = 2L_B(g')$$

where $L_B(g')$ is the number of pion lines in g'. In general, however, $N(g') = L(g') - V(g') + 1$, where $N(g')$ is the number of loops in g'. $L(g') = L_B(g') + L_F(g')$ with $L_F(g')$ being the number of nucleon lines in g', and $V(g')$ is the total number of vertices in g'. Since g' has no external nucleon lines $L_F(g') = V(g')$. Hence $N(g') = L_B(g') + 1$, which implies that

$$P(g') = 2n = 2(N(g') - 1) = 2(N(g) - 1).$$

Thus the graphs of the type (α) are all the connected graphs with $n + 1$ loops arising from pure pion–nucleon interactions.

We proceed now to examine the graphs of the type (β). Let g'' be such a graph. Then

$$P(g'') = \left(\sum_{\nu=1}^{n_1} \nu\lambda_\nu + \sum_{\sigma=1}^{n_2} (\sigma - 2)\, \rho\sigma\right)\bigg|_{n_1+n_2=2n} = 2n. \tag{63}$$

Now $V(g'') = V^{(1)}(g'') + V^{(2)}(g'')$, where $V^{(1)}(g'')$ is the number of vertices in g'' due to pion–nucleon interactions, and $V^{(2)}(g'')$ the number of vertices due to pion self-interactions. Also $L_B(g'') = \sum_{r=1}^{3} L_B^{(r)}(g'')$ where $L_B^{(1)}$ is the number of pion lines connecting vertices belonging to the set $V^{(1)}$, $L_B^{(2)}$ is the number of pion lines connecting vertices belonging to the set $V^{(2)}$, and $L_B^{(3)}$ is the number of lines connecting pairs of vertices one of which belongs to $V^{(1)}$ and the other to $V^{(2)}$. Since g'' has no external nucleon lines

$$N(g'') = L(g'') - V(g'') + 1 = L_B(g'') - V^{(2)}(g'') + 1 \tag{64}$$

From (63) and (64) we obtain

$$P(g'') = 2n = 2L_B(g'') - 2V^{(2)}(g'') = 2(N(g'') - 1).$$

Thus the graphs of the type (β) are all the connected graphs with $n + 1$ loops arising from pion–nucleon as well as pion self-interactions. The totality of the graphs of the type (α) and (β) constitute the class E_{2n}.

EXAMPLE.

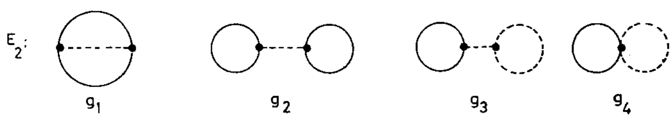


FIGURE 8

In general the sum over E_{2n} of graphs with noncovariant vertices is equal (apart from contributions which vanish on the mass shell, and contributions of the type $\delta^{(4)}(0)$) to the corresponding sum of graphs with covariant vertices.

Consider the example of the class E_2 shown in Fig. 8. Then on the mass shell we have:

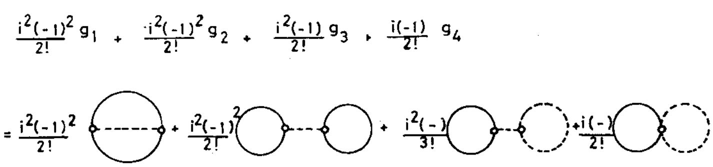


FIGURE 9

We turn now to the case when the given graph g has external nucleon lines. In order to get all the graphs, which contribute to the cancellation of the noncovariant parts arising from g, we proceed as follows. First we construct a graph $\bar{g}$ obtained from g by closing the external nucleon lines in an arbitrary (but admissible) way in order to form loops. Thus $\bar{g}$ has only internal nucleon lines, the number of which is $V^{(1)}(g)$. Then we consider the appropriate class E_{2n}, which contains $\bar{g}$. We now split the nucleon loops of every element in E_{2n} in all possible ways, and select those graphs which have the same number of external nucleon lines, and the same number of loops as in g. These are the graphs that will contribute to the cancellation of the noncovariant parts in g. Let this set be denoted by $\mathscr{C}(g)$. We remark that the same element in $\mathscr{C}(g)$ may arise from different elements in the class $E_{2n}(\bar{g})$ Furthermore the sum over $\mathscr{C}$ of graphs with noncovariant vertices is equal, on the mass shell, to the corresponding sum of graphs with covariant vertices (up to contributions of the $\delta^{(4)}(0)$-type).

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