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Time-Reversal Invariance-Like Relations for Spin-Effects in Elastic and Inelastic Reactions; Vector-Meson Photoproduction Versus Compton Scattering from Nucleons

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For elastic scattering, relations between spin-effects (for example, the well-known asymmetry-polarization equality) follow from time-reversal invariance. We show that if certain amplitude combinations vanish, there are strikingly similar relations between spin-effects for elastic and also inelastic reactions. This vanishing of amplitude combinations (denoted M-purity) corresponds asymptotically to purely natural or purely unnatural parity in the crossed channel. The M-purity relations hold for spin-configurations much more general than do the corresponding time-reversal invariance relations.

The experimental evidence for purely natural parity exchanges in high energy vector meson photoproduction from nucleons is shown to be good for all amplitudes involving nonzero meson helicity, but less conclusive for the zero helicity ones. Using time-reversal invariance and a vector meson-dominance argument, this implies no unnatural parity contributions in high energy Compton scattering from nucleons.

Because of this empirical evidence for M-purity in these two processes, a detailed application to spin-effects in Compton scattering and in vector meson photoproduction is made. Some time-reversal invariance relations in Compton scattering resemble the corresponding M-purity relations though the applicability of the two is different, and there are examples where only one of the two exists. Out of our illustrations, the only M-purity relations which change in form due to the extra amplitudes present in the inelastic reaction are the M-purity analogue and extensions of the asymmetry-polarization equality (of Compton scattering) referring to the photon; the change is the appearance of the elements ρ^{00} of the vector meson density-matrix ρ. Our other examples of M-purity relations do not change in form in going over from the elastic reaction (Compton scattering) to the inelastic reaction (vector meson photoproduction).

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1. Introduction

For elastic processes, time-reversal-invariance (denoted hereafter as T-invariance) leads to the equality [1] between the recoil polarization with an unpolarized target and the cross-section asymmetry with a polarized target. In general, there is no such result for an inelastic reaction. We show that if certain combinations of amplitudes vanish, there are such relations for even an inelastic reaction. We denote the vanishing of these amplitude-combinations by M-purity which corresponds, asymptotically, to having pure normality (natural or unnatural parity, but not a mixture) in the crossed channel. Spin effects in Compton scattering ($\gamma N \to \gamma N$) and in vector-meson-photoproduction ($\gamma N \to VN$) from nucleons are considered in detail in order to bring out the comparison between the relations following from T-invariance and from M-purity and in order to illustrate the changes in going from an elastic to an inelastic process. Though our emphasis is on the application to these two processes, M-purity relations can be seen (e.g. Section 2) to be quite general.

It is worth emphasizing that T-invariance is a general principle, while M-purity is a model that becomes empirically interesting in certain cases. We are pointing out the similarity of some M-purity relations to some T-invariance relations, but clearly, there are many nonoverlapping implications of M-purity and of T-invariance. Even when M-purity relations resemble the corresponding T-invariance relations, the regions of applicability of the two are not always the same, as will be discussed in detail.

A. *Definition of M-purity; Comparison with Restrictions on Helicity Amplitudes following from T-Invariance*

For the s-channel helicity amplitudes f_{ba}^{dc} of the process $ab \to cd$ where a, b, c, and d also denote helicities of the corresponding particles, parity conservation gives [2]

$$f_{-b-a}^{-d-c} = \eta(-)^{a-c-(b-d)} f_{ba}^{dc}, \tag{1.1}$$

where η depends on spins and intrinsic parities of a, b, c, and d. For elastic processes and for $\gamma N \to VN$, $\eta = +1$. We define

$$n_{ba}^{dc} = \tfrac{1}{2}(f_{ba}^{dc} + (-)^{d-b} f_{-ba}^{-dc}), \tag{1.2a}$$

$$u_{ba}^{dc} = \tfrac{1}{2}(f_{ba}^{dc} - (-)^{d-b} f_{-ba}^{-dc}), \tag{1.2b}$$

so that

$$n_{-ba}^{-dc}/n_{ba}^{dc} = -u_{-ba}^{-dc}/u_{ba}^{dc} = (-)^{b-d}. \tag{1.2c}$$

For the amplitude f^{dc}_{ba}, M-purity is defined[1] as $M = +1$ or $M = -1$:

$$M = -1 \quad \text{means} \quad n^{dc}_{ba} = 0, \tag{1.3a}$$

$$M = +1 \quad \text{means} \quad u^{dc}_{ba} = 0. \tag{1.3b}$$

The n- and u-type amplitudes will sometimes be called $M = +1$ and $M = -1$ amplitudes, respectively. Using the parity relation (1.1), n^{d-c}_{b-a} is related to n^{dc}_{ba} and u^{d-c}_{b-a} to u^{dc}_{ba}. The amplitudes of Eqs. (1.2) correspond,[2] asymptotically [3], to pure normality in the crossed channel $a\bar{c} \to \bar{b}d$. However, we shall need only the definitions (1.2) and (1.3) which hold for general values of the kinematical variables.

For the elastic process $ab \to cd$, T-invariance gives the relation [2]

$$f^{ba}_{dc} = (-)^{a-c-(b-d)} f^{dc}_{ba}, \tag{1.4}$$

where the particles a and c are the same and so are b and d. The T-invarance relation (1.4) and the corresponding parity relation (1.1) for elastic scattering overlap for only the helicity combinations $b = -d$, $a = -c$. Similarly, the T-invariance relation (1.4) overlaps with the M-purity restrictions (1.3) only for $(a = c, b = -d)$ and similarly, for $(a = -c, b = d)$. In fact, (1.4) implies $M = +1$ for the amplitudes f^{ba}_{-ba} and f^{ba}_{b-a}:

$$u^{ba}_{-ba} = 0, \qquad M = +1, \tag{1.5a}$$

$$u^{ba}_{b-a} = 0, \qquad M = +1. \tag{1.5b}$$

The T-invariance relation (1.4) does not restrict any of the n^{dc}_{ba} amplitudes, and restricts *only some* of the u^{dc}_{ba} amplitudes. We shall call M-purity for the amplitudes f^{ba}_{-ba} and f^{ba}_{b-a} as the M-purity of type 1; T-invariance gives $M = +1$ for this type in elastic processes; M-purity for other helicity amplitudes will be called M-purity of type 2.

For $\gamma N \to \gamma N$, Eqs. (1.5) are the only restrictions that T-invariance puts on helicity amplitudes, if one takes (as we shall do) parity-conservation (1.1) for granted. The fact that M-purity and T-invariance restrictions are related is contained in (1.5). While (1.4) is for elastic processes like $\gamma N \to \gamma N$, the M-purity restrictions can hold for any general reaction like $\gamma N \to VN$. It is now easy to imagine, for any general reaction, M-purity relations which resemble the T-invariance relations between spin-effects in elastic scattering. This is further

[1] This notation should not be confused with the Toller quantum number 'M.' For example, Toller poles with the quantum number 'M' = 1 conspire, and do not have M-purity in our sense.

[2] For elastic scattering and for $\gamma N \to VN$, the n and u amplitudes refer to natural and unnatural parity contributions, respectively.

illustrated in Section 2 where the asymmetry-polarization equality following from T-invariance is compared with its M-purity analogs.

While M-purity implies $M = +1$ or $M = -1$, T-invariance makes only some $M = -1$ amplitudes vanish. Of course, M-purity may also hold for only certain particular amplitudes.

B. *Interest in M-purity Relations; Why the Processes* $\gamma N \to \gamma N$ *and* $\gamma N \to VN$?

The only established evidence against T-invariance comes from neutral kaon decays; there, too, the relative strength of the T-noninvariant amplitude is very small $\sim 10^{-3}$. So, T-invariance is quite general, but M-purity is a model which is interesting in certain situations.[3] A good point about the M-purity relations is their applicability to any general reaction even if the corresponding time-reversed process be very hard to achieve experimentally, e.g., projectile + nucleon $\to$ nucleon + anything. Secondly, very high energy diffractive processes may have a dominant natural parity contribution in the crossed channel; there are many experimentally interesting reactions in this category. In fact, present data on $\gamma N \to \phi N$ support this hypothesis for the normality of the Pomeron.[4] Thirdly, combinations of contributions which are expected to be M-pure can be formed in different processes, and M-purity relations of the type we consider can be used as tests of the M-purity of these combinations. In this sense, M-purity relations are relevant also to the dynamical interpretation of amplitude-analyses.

Our detailed application of the comparison between T-invariance relations and M-purity relations to $\gamma N \to \gamma N$ and $\gamma N \to VN$ is motivated by several reasons. These reactions are easily accessible experimentally and offer enough spin-complications so as to make this comparison feasible; some spin-effects are relatively easy to study experimentally because the vector meson density-matrix, polarized photons and polarized targets are already available. In contrast, in πN elastic scattering, for example, T-invariance and also the vanishing of unnatural parity contributions already follow from the parity relation (1.1) because of the spinlessness of the pion—thus making the desired comparison impossible. One wants to consider an example in which there are some nonoverlapping restrictions on helicity amplitudes due to T-invariance and to M-purity, and in which T-invariance and M-purity do imply restrictions that go beyond those already implied by parityconservation. One such experimentally interesting case is $\bar{\Lambda} N \to \bar{\Lambda} N$ where $\bar{\Lambda}$ is a spin-$\frac{1}{2}$ baryon, but because both $\bar{\Lambda}$ and γ have only two helicities, this example is not essentially different from $\gamma N \to \gamma N$ which we are

[3] There are known instances where M-purity is not a good approximation; for example, high energy two-body processes involving significant pion-exchange contributions.

[4] If the Pomeron factorizes, it must have M-purity, even if it be not a Regge pole [4].

considering in detail. From $\bar{A}N \to \bar{A}N$ data, there is no positive experimental evidence of M-purity, while our example has the advantage that $\gamma N \to VN$ data (and to a large[5] extent, also $\gamma N \to \gamma N$) give evidence of being dominated by natural parity ($M = +1$) contributions. Next, the M-purity relations in $\gamma N \to VN$ (and $\gamma N \to \gamma N$) predict some observables, and are therefore testable by future measurements. Looked at the other way, since the value of these predicted observables follows from only M-purity (which is experimentally supported), their measurement will not reveal[6] further dynamical information for this process; this statement is already of some interest.

Though $\gamma N \to VN$ looks very similar to $\gamma N \to \gamma N$, the helicity zero component of the vector meson does give this process some essential features of an inelastic reaction so that a comparison between the M-purity relations in $\gamma N \to VN$ and in $\gamma N \to \gamma N$ is capable of showing some possible modifications due to inelasticity. Comparison between the M-purity relations and the T-invariance relations in $\gamma N \to \gamma N$ shows how the M-purity ones go beyond the T-invariance ones.

We restrict our illustrations of the various comparisons to the following spin effects for an initial state polarized in generality: cross-section asymmetries, recoil nucleon and final photon polarizations, and the vector meson decay density-matrix. We shall not consider the correlations between the polarizations of the final nucleon and the final photon/vector meson; these correlations are difficult to measure experimentally. Our purpose is to mention some simple illustrations of the resemblance between M-purity relations and T-invariance relations among observable spin-effects, rather than to consider the complete set of these relations.

The plan of the paper: Section 2 gives the standard asymmetry-polarization equality and its M-purity analogs, and compares the two; the generality of the polarization configuration under which this analog holds has been pointed out. Section 3 is devoted to some further notation used for $\gamma N \to \gamma N$ and $\gamma N \to VN$ in the subsequent parts of the paper. In Section 4, we consider, for $\gamma N \to \gamma N$, some T-invariance relations and compare them with their M-purity analogs. The experimental evidence for M-purity in $\gamma N \to VN$, its comparison with and implications (based on a vector dominance argument) for $\gamma N \to \gamma N$ are considered in Section 5. The subsequent section is devoted to the M-purity relations for $\gamma N \to VN$, closely compared with those (of Section 4) for $\gamma N \to \gamma N$; some comments on the modifications due to the inelastic feature of $\gamma N \to VN$ are included. In Section 7, an overall summary of the paper and a

[5] This uses an argument based on the vector-dominance model; the available [5] density-matrix data for $\gamma N \to VN$ indicate that in $\gamma N \to \gamma N$, even the unnatural parity contributions allowed by T-invariance are zero, or at most very small; Section 5.

[6] The discussion of cross-section asymmetries with general initial state polarizations, given in the appendix, is relevant in this context. Many useful predictions about these asymmetries follow from only parity-conservation, and further ones from M-purity; see remarks 1 to 4 in the appendix.

short discussion are given. The Appendix gives a general treatment of cross-section asymmetries in $\gamma N \to \gamma N$ and $\gamma N \to VN$, in particular, the information obtainable from these asymmetry-measurements; only parity-conservation has been assumed.

For a first reading of this paper, Sections 4–6 and the appendix may be omitted. For the reader who is not interested in details, the tables are a convenient summary of Sections 4.A, 4.B, and 6.

2. The Asymmetry-Polarization Equality for Elastic Processes and its M-Purity Analogues

Here, we consider the cross-section asymmetry A with the spin-$\frac{1}{2}$ particle b polarised perpendicular to the plane of the reaction

$$a + b \to X + b' \tag{2.1}$$

and compare it with the polarization $\bar{P}$ (perpendicular to the plane of the reaction) of the spin-$\frac{1}{2}$ particle b' when b is unpolarized. The polarization state of the system X is not observed and the system a may be arbitrarily polarized.

The polarization $\bar{P}$ is

$$\bar{P} = -2\,\mathrm{Im}\Big(\sum_{j,\alpha,\beta,\theta} f_{j\beta}^{+\alpha}\rho_a^{\beta\theta} f_{j\theta}^{-\alpha *}\Big)\Big/D, \tag{2.2}$$

$$D = \sum_{l,j,\alpha,\beta,\theta} f_{j\beta}^{l\alpha}\rho_a^{\beta\theta} f_{j\theta}^{l\alpha *}, \tag{2.3}$$

where ρ_a is the density-matrix for the particle a; the helicities β, j, α and l in the amplitude $f_{j\beta}^{l\alpha}$ for the process (2.1) refer to the particles a, b, X, and b', respectively; the helicities $\pm\frac{1}{2}$ are denoted by $\pm$. The asymmetry A is

$$A = p\Big[\sum_{l,j,k,\alpha,\beta,\theta} f_{j\beta}^{l\alpha}(\sigma_2)^{jk}\,\rho_a^{\beta\theta} f_{k\theta}^{l\alpha *}\Big]\Big/D, \tag{2.4}$$

where σ_2 is the Pauli matrix and p is the polarization of b.

Using the hermiticity of ρ_a, one can show that

$$A = Mp\bar{P} \tag{2.5}$$

if either

$$f_{-\theta}^{-\alpha} = Mf_{+\theta}^{+\alpha}, \tag{2.6}$$

or/and

$$f_{-\theta}^{+\alpha} = -Mf_{+\theta}^{-\alpha}, \tag{2.7}$$

where $M = +1$ or -1. The conditions (2.6) and (2.7) obviously mean M-purity for these amplitudes; α and θ are arbitrary. One may note that for the corresponding elastic process, T-invariance gives $M = +1$ in only (2.7) for only $\alpha = \theta$; T-invariance does not refer to (2.6).

For the case when the particle a is unpolarized, the matrix ρ_a is essentially the unit matrix and using the T-invariance relation (1.4), one gets the standard asymmetry-polarization equality [1] for elastic scattering

$$A = p\bar{P}. \tag{2.8}$$

When the particle a is polarized, T-invariance relates A not to $\bar{P}$, but to some elements of the final-state joint density-matrix which measure correlations between the two final particles.

In the simple case when a and X are spinless, just the parity relation (1.1) gives $M = +1$ (-1) when the product of the intrinsic parities of a and b is the same as (opposite to) the corresponding product for X and b'; the relation (2.5) then gives (2.8) for $\pi N \to \pi N$, $\pi N \to K\Lambda$, $\epsilon N \to \epsilon N$, etc.; and

$$A = -p\bar{P} \tag{2.9}$$

for $\pi N \to \epsilon N$, where ϵ is a $J^P = 0^+$ particle; no reference to T-invariance need be made. For elastic scatterings $\pi N \to \pi N$ and $\epsilon N \to \epsilon N$, T-invariance gives $M = +1$ for only the relation (2.7) with helicities $\alpha = \theta = 0$, leading to (2.8), or (2.5) with $M = +1$. On the other hand, the M-purity relation (2.5) would hold even if (2.7) be not valid, but only (2.6) is true; this is an example of the standard equality (2.8) being satisfied even if T-invariance did not hold [6].

So, the M-purity relation (2.5) holds in a much more general situation than the T-invariance relation (2.8), the differences being in the requirements on (1) the nature of a and X, and of b and b', and (2) the spin-states of a and X. The T-invariance relation (2.8) requires the particles b and b' (and similarly, a and X) to be the same, and a = unpolarized.[7] In getting (2.5), however, there is no such restriction on the identity of the particles b and b' (or of a and X), nor on the polarization state of a; a and X could have different spins which need not even be known; b and b' could have all quantum numbers (except spin) different; the relation (2.5) would hold for the rather general process

$$\text{anything} + b \to b' + \text{anything}, \tag{2.10}$$

where b and b' have spin $= \frac{1}{2}$.

[7] See, however, the remark (3) in the Appendix; for some special polarizations of a, the asymmetry may be the same as for unpolarized a.

We shall not discuss extensions of these considerations to other spin values for b and/or b'; the case of unequal spins for b and b' offers no interesting comparison to the T-invariance relation (2.8). In $\gamma N \to \gamma N$ and $\gamma N \to VN$, we shall encounter M-purity relations similar to (2.5), but involving spin-1 effects, others involving spin-$\frac{1}{2}$ effects, and also those involving both spin-1 effects and spin-$\frac{1}{2}$ effects.

The example of this section illustrates the point that M-purity relations for spin-effects can hold for more general configurations than do the corresponding T-invariance ones. In some cases, there may be no corresponding T-invariance relation; this is the case for relations due to M-purity of type 2. It is interesting that *any* statement resembling the asymmetry-polarization theorem is at all possible for a general inelastic reaction; in fact, the generality of the configuration for which (2.5) holds is remarkable as compared to that for (2.8).

The importance of the theorem (2.8) can hardly be overestimated: a measurement of A avoids the need of a subsidiary experiment to measure $\bar{P}$. Similar remarks would hold for M-purity relations of the type of (2.5) for situations in which there is evidence for M-purity. Looked at the other way, a measurement of both sides of (2.8) would test the underlying symmetry; similar is the case of testing M-purity by using relations like (2.5).

3. Some Further Notation for $\gamma N \to VN$ and $\gamma N \to \gamma N$

We use s-channel helicity amplitudes $f_{i\alpha}^{i'\alpha'}(s, t)$; four-momenta and helicities are defined as follows: (p_μ, i) for the target nucleon N; (p_μ', i') for the recoil nucleon N'; (k_μ, α) for the initial photon γ; and (k_μ', α') for the final photon γ' or vector meson V. The invariants are $s = -(p + k)^2$ and $t = -(p' - p)^2$.

As in Refs. [7] and [8], the polarization of the photon beam and of the target nucleon are described by the conventional density-matrices

$$\rho_\gamma = \tfrac{1}{2}(1 + \mathbf{P} \cdot \boldsymbol{\sigma}) \equiv \tfrac{1}{2} \sum_{\mu=0}^{3} P_\mu \sigma_\mu , \tag{3.1}$$

and

$$\rho_N = \tfrac{1}{2}(1 + \boldsymbol{\zeta} \cdot \boldsymbol{\sigma}) \equiv \tfrac{1}{2} \sum_{\mu=0}^{3} \zeta_\mu \sigma_\mu , \tag{3.2}$$

where the "four-vector" notation implies $P_0 = \zeta_0 = 1$, $\sigma_0 =$ the unit matrix, and $\boldsymbol{\sigma}$ represents the three Pauli matrices. The vector

$$\mathbf{P} = |\mathbf{P}|(-\cos 2\phi, -\sin 2\phi, 0)$$

describes linearly polarized photons with an angle ϕ between the reaction plane (taken as the XZ plane) and the polarization vector

$$\epsilon = (\cos\phi, \sin\phi, 0)$$

of the photons; P_3 corresponds to circular polarization. For the target nucleon, ζ_1 and ζ_2 are the transverse polarizations, respectively, in and normal to the reaction plane, and ζ_3 is similarly the degree of longitudinal polarization. The parameters P_j and ζ_j are restricted

$$|\zeta_j| \leqslant 1 \text{ and } |P_j| \leqslant 1 \ (j = 1, 2, 3).$$

The (unnormalised) joint density-matrix of the vector meson–nucleon final state is

$$\rho_{N',V}^{i'j',\alpha'\beta'} = \sum_{i,j,\alpha,\beta} f_{i\alpha}^{i'\alpha'} \rho_N^{ij} \rho_\gamma^{\alpha\beta} f_{j\beta}^{j'\beta'*}, \tag{3.3a}$$

where the vector-meson V can also be the final photon (γ'). The (unnormalized) density-matrices $\rho^{\alpha'\beta'}$ of the vector-meson and $\rho_{N'}^{i'j'}$ of the final nucleon are, respectively,

$$\begin{aligned} \rho^{\alpha'\beta'} &= \sum_{i'} \rho_{N',V}^{i'i',\alpha'\beta'} \\ &= \sum_{i',i,j,\alpha,\beta} f_{i\alpha}^{i'\alpha'} \rho_N^{ij} \rho_\gamma^{\alpha\beta} f_{j\beta}^{i'\beta'*} \end{aligned} \tag{3.3b}$$

and

$$\begin{aligned} \rho_{N'}^{i'j'} &= \sum_{\alpha} \rho_{N',V}^{i'j',\alpha'\alpha'} \\ &= \sum_{\alpha',i,j,\alpha,\beta} f_{i\alpha}^{i'\alpha'} \rho_N^{ij} \rho_\gamma^{\alpha\beta} f_{j\beta}^{j'\alpha'*}. \end{aligned} \tag{3.3c}$$

The (unnormalized) density-matrix $\rho_{\gamma'}$ of the final photons in $\gamma N \to \gamma N$ is defined in the same way as ρ for the vector meson in $\gamma N \to VN$. The normalization of the helicity amplitudes is provided by the differential cross-section σ as

$$C\sigma = \sum_{i',\alpha'} \rho_{N',V}^{i'i',\alpha'\alpha'} = \sum_{i'} \rho_{N'}^{i'i'} = \sum_{\alpha'} \rho^{\alpha'\alpha'} = \operatorname{tr}\rho, \tag{3.4}$$

$$C = (2\pi/E^*)^{-2},$$

where E^* is the initial photon energy in the C.M. system. The polarization $\mathbf{P}'$ of the final photon and the polarization $\boldsymbol{\zeta}'$ of the final nucleon are described analogously to the corresponding initial polarizations in (3.1, 2):

$$\begin{aligned}
\zeta_1' &= 2 \operatorname{Re} \rho_{N'}^{+-}/\operatorname{tr} \rho_{N'} , \\
\zeta_2' &= -2 \operatorname{Im} \rho_{N'}^{+-}/\operatorname{tr} \rho_{N'} , \\
\zeta_3' &= (\rho_{N'}^{++} - \rho_{N'}^{--})/\operatorname{tr} \rho_{N'} ; \\
P_1' &= 2 \operatorname{Re} \rho_{\gamma'}^{+-}/\operatorname{tr} \rho_{\gamma'} , \\
P_2' &= -2 \operatorname{Im} \rho_{\gamma'}^{+-}/\operatorname{tr} \rho_{\gamma'} , \\
P_3' &= (\rho_{\gamma'}^{++} - \rho_{\gamma'}^{--})/\operatorname{tr} \rho_{\gamma'}
\end{aligned} \qquad (3.5)$$

where + and − stand for the nucleon helicities $+\frac{1}{2}$ and $-\frac{1}{2}$, and for the photon[8] helicities +1 and −1, respectively.

For a given initial polarisation configuration (P_m, ζ_n), (m, n) = fixed, but $\neq 0$, we expand the cross-sections σ and the density-matrices $\rho_{N',V}$, ρ, $\rho_{N'}$, and $\rho_{\gamma'}$ as for example

$$\rho(P_m , \zeta_n) = \rho(0, 0) + P_m\rho(m, 0) + \zeta_n\rho(0, n) + P_m\zeta_n\rho(m, n) \qquad (3.6)$$

where the expansion coefficients $\rho(m, n)$ are very convenient bilinears of amplitudes, and $\rho(P_0 , \zeta_0) = \rho(0, 0)$ for the unpolarized case; in (3.6) the initial density-matrices are

$$\rho_\gamma = \tfrac{1}{2}(1 + P_m\sigma_m), \qquad \rho_N = \tfrac{1}{2}(1 + \zeta_n\sigma_n), \qquad (m, n) = \text{fixed}.$$

For the expansion coefficients, one gets, for example,

$$\rho^{\alpha'\beta'}(m, n) = \tfrac{1}{4} \sum_{i',\alpha,\beta,i,j} f_{i\alpha}^{i'\alpha'}(\sigma_m)^{\alpha\beta} (\sigma_n)^{ij} f_{j\beta}^{i'\beta'*}. \qquad (3.7)$$

The coefficients $\rho(m, n)$, $\rho_{N'}(m, n)$, $\rho_{N',V}(m, n)$,... contain all the information; no loss of generality is incurred by not considering mixtures of these polarizations in the initial state. One can expand only the unnormalized density matrices as in (3.6), but not the normalized ones

$$\mathring{\rho}(P_m , \zeta_n) = \rho(P_m , \zeta_n)/\operatorname{tr} \rho(P_m , \zeta_n), \qquad (3.8)$$

which are not simple polynomials in P_m and ζ_n ; similarly, ζ_j' and P_j' of (3.5)

[8] This notation will be followed also for the vector meson density-matrix ρ.

cannot be so expanded. Our unnormalized "actual" polarizations $\tilde{P}_j'$ and $\tilde{\zeta}_j'$, defined as[9]

$$\tilde{P}_j'(P_m, \zeta_n) = P_j'(P_m, \zeta_n) \operatorname{tr} \rho_{V'}(P_m, \zeta_n) \tag{3.9a}$$

and

$$\tilde{\zeta}_j'(P_m, \zeta_n) = \zeta_j'(P_m, \zeta_n) \operatorname{tr} \rho_{N'}(P_m, \zeta_n), \tag{3.9b}$$

can indeed be expanded like (3.6), as for example

$$\tilde{\zeta}_j'(P_m, \zeta_n) = \tilde{\zeta}_j'(0, 0) + P_m\tilde{\zeta}_j'(m, 0) + \zeta_n\tilde{\zeta}_j'(0, n) + P_m\zeta_n\tilde{\zeta}_j'(m, n). \tag{3.10}$$

Similarly, the "universally normalized actual" polarizations

$$\hat{P}_j'(P_m, \zeta_n) = \tilde{P}_j'(P_m, \zeta_n)/\operatorname{tr} \rho_{V'}(0, 0) \tag{3.11a}$$

and

$$\hat{\zeta}_j'(P_m, \zeta_n) = \tilde{\zeta}_j'(P_m, \zeta_n)/\operatorname{tr} \rho_{N'}(0, 0) \tag{3.11b}$$

can be expanded in terms of their coefficients, as in (3.6). The definitions (3.11) obviously imply

$$\hat{P}_j'(P_0, \zeta_0) = P_j'(P_0, \zeta_0), \tag{3.12a}$$

and

$$\hat{\zeta}_j'(P_0, \zeta_0) = \zeta_j'(P_0, \zeta_0). \tag{3.12b}$$

A universal normalization independent of initial polarisation has been used by Schilling *et al.* [8]:

$$\rho^{m}_{\lambda\lambda'} |_{\text{Theirs}} = \rho^{\lambda\lambda'}(m, 0)/\operatorname{tr} \rho(0, 0)|_{\text{ours}} \tag{3.13a}$$

so that

$$\frac{\rho^{m}_{\lambda\lambda'}}{\rho^{M}_{\Lambda\Lambda'}}\bigg|_{\text{Theirs}} = \frac{\rho^{\lambda\lambda'}(m, 0)}{\rho^{\Lambda\Lambda'}(M, 0)}\bigg|_{\text{Ours}}. \tag{3.13b}$$

This comparison (3.13) is important for our use of $\gamma N \to VN$ data [5] (Section 5) because their [8] notation has been used in Ref. [5].

It is worth noting that for the following combinations of (m, n), parity-conservation makes tr $\rho(= \operatorname{tr} \rho_{N'})$ vanish [7, 8]:

$$\rho_{N'}^{++}(m, n) = -\rho_{N'}^{--}(m, n) \qquad \text{and} \qquad \rho_{V'}^{++}(m, n) = -\rho_{V'}^{--}(m, n) \tag{3.14}$$

[9] One may note that while for some choice of m and n, tr $\rho(m, n)$ can be zero, tr $\rho(P_m, \zeta_n)$ is nonvanishing for all (m, n) because the unpolarized term tr $\rho(0, 0)$ is always nonzero. The same remark applies to the other density-matrices: $\rho_{V'}$, $\rho_{N'}$, $\rho_{N',V}$. In (3.5), the argument of the trace is (P_m, ζ_n).

for

$$m = (0 \text{ or } 1), \qquad n = (1 \text{ or } 3); \qquad \text{and} \quad m = (2 \text{ or } 3), \qquad n = (0 \text{ or } 2). \quad (3.15)$$

The notation for cross-section asymmetries is given in Eqs. (A.10–A.12) of the Appendix.

The notational phrase "nth equality of (5.1 or 5.2)" is defined when Eqs. (5.1 and 5.2) first appear (Section 5).

4. Some T-Invariance and M-Purity Relations in $\gamma N \to \gamma N$

Assuming (as throughout) parity-conservation, there are only eight independent helicity amplitudes for $\gamma N \to \gamma N$:

$$f_{++}^{++}, f_{-+}^{-+}, f_{--}^{++}, f_{+-}^{-+}, f_{-+}^{++}, f_{++}^{-+}, f_{+-}^{++} \text{ and } f_{++}^{+-}. \quad (4.1)$$

Out of these, the first two are not restricted by the T-invariance relation (1.4), while the last four are

$$f_{-+}^{++} = -f_{++}^{-+}, \quad (4.2a)$$

$$f_{+-}^{++} = f_{++}^{+-}, \quad (4.2b)$$

giving

$$u_{-+}^{++} = 0, \qquad u_{++}^{-+} = 0, \quad (4.3a)$$

$$u_{+-}^{++} = 0, \qquad u_{++}^{+-} = 0, \quad (4.3b)$$

as examples of Eqs. (1.5). The relations

$$f_{--}^{++}/f_{++}^{--} = f_{-+}^{+-}/f_{+-}^{-+} = -1 \quad (4.4)$$

involving the remaining two amplitudes of (4.1) are given by both parity-conservation (1.1) and T-invariance (1.4); (4.4) does not restrict any n- or u-type amplitudes. We distinguish the consequences of (4.3) which is characteristic of T-invariance from those of (4.4). Because of (4.3), the only independent nonvanishing u-amplitudes are

$$u_{++}^{++} \text{ and } u_{--}^{++}, \quad (4.5)$$

while the n-amplitudes

$$n_{++}^{++}, n_{--}^{++}, n_{-+}^{++} \text{ and } n_{+-}^{++} \quad (4.6)$$

allowed by parity-conservation are also allowed by T-invariance.

A. *T-Invariance Relations*

Using only parity-conservation which includes the T-invariance constraint (4.4), one gets [7]

$$\tilde{\zeta}_2'(0, 0) = \tilde{P}_1'(1, 2), \tag{4.7}$$

$$\tilde{\zeta}_2'(1, 0) = \tilde{P}_1'(0, 2), \tag{4.8}$$

$$\tilde{\zeta}_2'(0, 2) = \tilde{P}_1'(1, 0), \tag{4.9}$$

and

$$\tilde{\zeta}_2'(1, 2) = \tilde{P}_1'(0, 0). \tag{4.10}$$

These can be expressed in terms of the actual polarizations $\tilde{P}_1'(P_i, \zeta_j)$ and $\tilde{\zeta}_2'(P_l, \zeta_k)$, and also in terms of the cross-section asymmetries defined in the appendix. The four relations (4.7)–(4.10) combine to give

$$\tilde{P}_1'(P_1, \zeta_2) = P_1\zeta_2\tilde{\zeta}_2'(1/P_1, 1/\zeta_2), \tag{4.11}$$

where the argument on the right-hand side expresses the relevant values of P_1 and ζ_2, respectively. Because $| P_1 | \leqslant 1$ and $| 1/P_1 | \leqslant 1$, this relation is meaningful for only $P_1 = \pm 1$ and similarly, for $\zeta_2 = \pm 1$, giving

$$\tilde{P}_1'(P_1, \zeta_2) = P_1\zeta_2\tilde{\zeta}_2'(P_1, \zeta_2) \quad \text{for } P_1 = \pm 1, \quad \zeta_2 = P_1 \text{ or } -P_1. \tag{4.12}$$

Further use of the T-invariance restriction (4.3) in (4.7) and (4.10) gives

$$\tilde{\zeta}_2'(0, 0) = C\sigma(0, 2) \tag{4.13}$$

and

$$\tilde{P}_1'(0, 0) = C\sigma(1, 0) \tag{4.14}$$

which provide examples of the standard asymmetry-polarization theorem for elastic scattering. In fact, using the asymmetries $A(P_0, \pm\zeta_2)$ and $A(\pm P_1, \zeta_0)$ of Eqs. (A.15) and (A.16) of the Appendix, the above two relations read, in terms of the normalized polarizations ζ_2' and P_1' as

$$A(P_0, \pm\zeta_2) = \zeta_2 \cdot \zeta_2'(P_0, \zeta_0) \tag{4.15}$$

and

$$A(\pm P_1, \zeta_0) = P_1 \cdot P_1'(P_0, \zeta_0) \tag{4.16}$$

which are the equality (2.8) corresponding to the nucleon and the photon, respectively.

Another consequence of T-invariance (4.3) is to relate [7] different final photon polarizations

$$\tilde{P}_2'(2, 2) = \tilde{P}_3'(3, 2) \tag{4.17}$$

and

$$\tilde{P}_2'(3, 0) = -\tilde{P}_3'(2, 0), \tag{4.18}$$

which can be expressed in terms of the actual polarizations as, for example

$$P_2[\tilde{P}_3'(P_3, \zeta_2) - \tilde{P}_3'(P_3, \zeta_0)] = P_3[\tilde{P}_2'(P_2, \zeta_2) - \tilde{P}_2'(P_2, \zeta_0)] \tag{4.19}$$

and

$$P_2\tilde{P}_2'(P_3, \zeta_0) = -P_3\tilde{P}_3'(P_2, \zeta_0). \tag{4.20}$$

T-invariance (4.3) also gives relations [7] between the final nucleon polarizations in the reaction plane and final photon polarizations:

$$\tilde{P}_2'(0, 1) = \tilde{\zeta}_1'(2, 0), \tag{4.21}$$

$$\tilde{P}_2'(0, 3) = -\tilde{\zeta}_3'(2, 0), \tag{4.22}$$

$$\tilde{P}_3'(0, 3) = \tilde{\zeta}_3'(3, 0), \tag{4.23}$$

and

$$\tilde{P}_3'(0, 1) = -\tilde{\zeta}_1'(3, 0) \tag{4.24}$$

which may be rewritten in terms of the actual polarizations as, for example

$$P_2\tilde{P}_2'(P_0, \zeta_1) = \zeta_1\tilde{\zeta}_1'(P_2, \zeta_0), \tag{4.25}$$

$$P_2\tilde{P}_2'(P_0, \zeta_3) = -\zeta_3\tilde{\zeta}_3'(P_2, \zeta_0), \tag{4.26}$$

$$P_3\tilde{P}_3'(P_0, \zeta_3) = \zeta_3\tilde{\zeta}_3'(P_3, \zeta_0), \tag{4.27}$$

and

$$P_3\tilde{P}_3'(P_0, \zeta_1) = -\zeta_1\tilde{\zeta}_1'(P_3, \zeta_0). \tag{4.28}$$

B. *M-Purity Relations*

In Section 4.A were considered some relations as consequences of the M-purity ($M = +1$) of type 1, as embodied in (4.3). If one has full M-purity (of type 2 as well), simplifications in these relations occur, and some new relations hold; we consider the two separately.

1. *M-Purity Modifications in the T-Invariance Relations of Section* 4.A

The relations (4.7)–(4.10) involving recoil nucleon polarizations normal to the reaction plane become, under M-purity,

$$\tilde{\zeta}_2'(0, 0) = \tilde{P}_1'(1, 2) = MC\,\sigma(0, 2), \tag{4.29}$$

$$\tilde{\zeta}_2'(1, 0) = \tilde{P}_1'(0, 2) = MC\,\sigma(1, 2), \tag{4.30}$$

$$\tilde{\zeta}_2'(0, 2) = \tilde{P}_1'(1, 0) = MC\,\sigma(0, 0), \tag{4.31}$$

$$\tilde{\zeta}_2'(1, 2) = \tilde{P}_1'(0, 0) = MC\,\sigma(1, 0). \tag{4.32}$$

Out of these only (4.29) and (4.32) have T-invariance analogs in (4.13) and (4.14). In fact, the four cross-section coefficients in (4.29)–(4.32) are the only nonvanishing ones for $\gamma N \to \gamma N$ and $\gamma N \to VN$ under M-purity; all the nonvanishing cross-section asymmetries are then related to recoil nucleon polarizations perpendicular to the reaction plane (or, equivalently, to the corresponding final photon polarizations $\tilde{P}_1'$). Combining the four relations (4.29)–(4.32), full M-purity gives

$$\tilde{\zeta}_2'(P_1, \zeta_2) = MC\, \zeta_2\, \sigma(P_1, 1/\zeta_2) \tag{4.33}$$

and

$$\tilde{P}_1'(P_1, \zeta_2) = MC\, P_1\, \sigma(1/P_1, \zeta_2), \tag{4.34}$$

where the arguments on the right-hand side express the relevant values of P_1 and ζ_2, respectively. Because of the restrictions $| P_1 | \leqslant 1$, $| 1/P_1 | \leqslant 1$, (4.34) is meaningful for only $P_1 = \pm 1$, and similarly, (4.33) for $\zeta_2 = \pm 1$ giving

$$\tilde{\zeta}_2'(P_1, \zeta_2 = \pm 1) = \pm MC\, \sigma(P_1, \zeta_2 = \pm 1) \tag{4.35}$$

and

$$\tilde{P}_1'(P_1 = \pm 1, \zeta_2) = \pm MC\, \sigma(P_1 = \pm 1, \zeta_2), \tag{4.36}$$

which read as

$$\zeta_2'(P_1, \zeta_2 = \pm 1) = \pm M \tag{4.37}$$

and

$$P_1'(P_1 = \pm 1, \zeta_2) = \pm M \tag{4.38}$$

in terms of the normalized polarizations. The M-purity relations (4.37 and (4.38) have the interesting feature that there is no dependence[10] on the polarization of the "other" particle: If the initial nucleon (photon) is fully polarized, so is the final nucleon (photon), independent of the value of the initial photon (nucleon) polarization $P_1(\zeta_2)$, the relative sign between the initial and final nucleon (photon) polarizations being given by M.

The relation of (4.29) and (4.32) to cross-section asymmetries is, apart from the factor M, the same as in (4.15) and (4.16). The relation of (4.30) to various cross-section asymmetries may be written in terms of the double asymmetry $A(\pm P_1, \pm\zeta_2)$ and the single asymmetries $A(P_0, \pm\zeta_2)$ and $A(\pm P_1, \zeta_0)$ as

$$\tilde{\zeta}_2'(1, 0) = \frac{MC\sigma(0, 0)}{P_1\zeta_2} \left\{ -1 + \frac{A(\pm P_1, \zeta_0) + A(P_0, \pm\zeta_2)}{A(\pm P_1, \pm\zeta_2)} \right\} \tag{4.39}$$

[10] The forms (4.37) and (4.38) are suitable for generalization to $\gamma N \to VN$. For $\gamma N \to \gamma N$, taking into account the T-invariance relations (4.11) and (4.12), one gets

$$\zeta_2'(P_1, \zeta_2 = \pm 1) = \pm M\sigma(MP_1, \zeta_2 = \pm M)/\sigma(P_1, \zeta_2 = \pm 1)$$

and $P_1'(P_1 = \pm 1, \zeta_2) = \pm M\sigma(P_1 = \pm M, M\zeta_2)/\sigma(P_1 = \pm 1, \zeta_2)$, where $M = +1$ has been taken for purity of the type 1; in these relations, the right-hand side does depend on P_1 and ζ_2 for $M = -1$ (but not for $M = +1$), M now referring to (4.30) and (4.31).

or equivalently in terms of these single asymmetries and $A(P_1, \pm\zeta_2)$ as

$$\tilde{\zeta}_2'(1, 0) = \frac{MC\sigma(0, 0)}{P_1\zeta_2}\{A(P_1, \pm\zeta_2)(1 + A(\pm P_1, \zeta_0)) - A(P_0, \pm\zeta_2)\}, \quad (4.40)$$

where, of course, the left-hand side may be expressed in terms of the actual polarizations as for example $[\tilde{\zeta}_2'(P_1, \zeta_0) - \tilde{\zeta}_2'(P_0, \zeta_0)]/P_1$ or replaced by the corresponding expressions for $\tilde{P}_1'(0, 2)$. The relations (4.39 and (4.40) expressing the recoil nucleon polarizations in terms of cross-section asymmetries have no T-invariance analog because the "other" particle (the photon in $\tilde{\zeta}_2'(1, 0)$ or the nucleon in $\tilde{P}_1'(0, 2)$) is initially polarized.

The relation (4.31) shows that under M-purity, the final state polarization coefficients $\tilde{\zeta}_2'(0, 2)$ and $\tilde{P}_1'(1, 0)$ when the corresponding initial particle is polarized, but the "other" particle initially unpolarized, are determined by the unpolarized cross-section $\sigma(0, 0)$.

The relations (4.29)–(4.32) also provide interesting M-purity analogues of the depolarization relation of πN scattering[11]

$$\text{Final Polarization} = D \cdot \text{Initial Polarization} + P \quad (4.41)$$

where all the polarizations are perpendicular to the reaction plane; P is the recoil polarization obtained with an unpolarized target; D is the depolarization coefficient; for $\pi N \to \pi N$, $D = +1$ and for $\pi N \to \epsilon N$, $D = -1$. The relation closest to (4.41), arising from (4.31), reads in terms of the "universally normalized actual" polarizations as

$$\hat{\zeta}_2'(P_0, \zeta_2) = M\zeta_2 + \hat{\zeta}_2'(P_0, \zeta_0) \quad (4.42)$$

or equivalently

$$\hat{P}_1'(P_1, \zeta_0) = MP_1 + \hat{P}_1'(P_0, \zeta_0), \quad (4.43)$$

the role of the depolarization coefficient being taken by M now. The corresponding relations arising from (4.29), (4.30), (4.32) look less simple, but are straightforward to write.

Coming to the M-purity analogs of (4.17) and (4.18), one gets

$$\tilde{P}_2'(2, 2) = M\tilde{P}_3'(3, 2), \quad (4.44)$$

$$\tilde{P}_2'(3, 0) = -M\tilde{P}_3'(2, 0). \quad (4.45)$$

Equations (4.17) and (4.18) are the $M = +1$ examples of these relations.

The T-invariance relations (4.21)–(4.24) between final nucleon polarizations in the reaction plane and final photon polarizations involve interference between

[11] For a summary, see Ref. [9].

TABLE I

Summary of the T-Invariance and the M-Purity Relations for $\gamma N \rightarrow \gamma N$ Given in Section 4

No :	Equation; Its source	Quantities related	Remarks
1	(4.7)–(4.12); T-invariance constraint (4.4) which overlaps with parity-conservation	Final nucleon polarizations normal to the reaction plane TO Final photon linear polarizations normal or parallel to the reaction plane when the initial polarizations are also along these directions, or are zero.	Not necessary to invoke T-invariance.
2	(4.13) and (4.15); T-invariance constraint (4.3)	Cross-section asymmetry with initial nucleon polarized normal to the reaction plane and photon unpolarized TO recoil nucleon polarization normal to the reaction plane, the initial nucleon and photon being unpolarized.	Standard asymmetry-polarization equality for elastic scattering.
3	(4.14) and (4.16) T-invariance constraint (4.3)	Same as in No: (2) with nucleon $\leftrightarrow$ photon and with polarization normal to reaction plane $\rightarrow$ linear polarization normal or parallel to reaction plane.	Same as above.
4	(4.17)–(4.20); T-invariance constraint (4.3)	Different final photon polarizations either linear at 45° (135°) to the reaction plane or circular, when the initial photon is also polarized along one of these directions, but the initial nucleon is either unpolarized or polarized normal to the reaction plane.	M-purity analogs are in (4.44) and (4.45).
5	(4.21)–(4.28); T-invariance constraint (4.3)	Final nucleon polarizations in the reaction plane, initial nucleon being unpolarized and photon being polarized either linearly at 45° (135°) to the reaction plane or circularly TO Final photon polarization, either linear at 45° (135°) to the reaction plane or circular, the initial photon being unpolarized and the nucleon polarized in the reaction plane.	M-purity analogues are null identities because both sides are an interference between amplitudes of the n- and u-types.

Table continued

TABLE I (*continued*)

No :	Equation; Its source	Quantities related	Remarks
6a	(4.29) and (4.32); *M*-purity	Cross section coefficients for initial nucleon polarization zero or normal to the reaction plane and photon either unpolarized or polarized linearly normal or parallel to the reaction plane TO Recoil nucleon polarizations normal (or photon linear polarizations normal or parallel) to the reaction plane when the initial polarizations are the same as those for a particular cross section coefficient.	*M*-purity analogs of (4.13) and (4.14).
6b	(4.30) and (4.31); *M*-purity		No *T*-invariance analog.
7	(4.39) and (4.40); The *M*-purity relation (4.30)	Recoil nucleon polarization normal to the reaction plane for initial nucleon unpolarized and photons polarized linearly normal or parallel to the reaction plane TO Cross section asymmetries with photon and nucleon polarization either zero or along the above directions.	Cross-section asymmetry version of (4.30); Can be expressed in terms of recoil photon (instead of nucleon) polarization; No *T*-invariance analog.
8a	(4.42): The *M*-purity relation (4.31)	Final nucleon polarization when initial nucleon is polarized TO that when initial nucleon is unpolarized and TO the initial nucleon polarization, all polarizations being normal to the reaction plane, and photons unpolarized.	Resembles the Depolarization Relation in $\pi N \to \pi N$; No *T*-invariance analog; Similar relations follow from (4.29), (4.30), and (4.32).
8b	(4.43); The *M*-purity relation (4.31)	Same as in no: 8a), with nucleon $\leftrightarrow$ photon, and nucleon polarizations normal to the reaction plane $\rightarrow$ linear photon polarizations normal or parallel to the reaction plane.	Same as above.
9	(4.44) and (4.45); *M*-purity	Same as in no: (4)	*T*-invariance version is in (4.17) and (4.18).
10	(4.46) and (4.47); *M*-purity	Same as above.	No *T*-invariance analog.

amplitudes of the n- and the u-type so that for full M-purity, they become identities of the $0 = 0$ type.

2. *New M-Purity Relations*

Here we mention only two relations resembling (4.44) and (4.45) between final photon polarizations:

$$\tilde{P}_2'(2, 0) = M\tilde{P}_3'(3, 0), \tag{4.46}$$

$$\tilde{P}_2'(3, 2) = -M\tilde{P}_3'(2, 2). \tag{4.47}$$

Taken together, the M-purity relations (4.44)–(4.47) cover the case of both unpolarized targets and targets polarized normal to the reaction plane. Written in terms of the actual polarizations, (4.46) and (4.47) read respectively as

$$P_3\tilde{P}_2'(P_2, \zeta_0) = MP_2\tilde{P}_3'(P_3, \zeta_0), \tag{4.48}$$

and

$$P_3\tilde{P}_3'(P_2, \zeta_2) = -P_2\tilde{P}_2'(P_3, M\zeta_2), \tag{4.49}$$

where the T-invariance result (4.18) has been used in (4.49), and $M\zeta_2$ in the argument on the right-hand side of (4.49) gives the relevant value of ζ_2.

C. *Remarks on the Comparison between the T-Invariance and the M-Purity Relations in $\gamma N \to \gamma N$*

A convenient summary of Sections (4.A) and (4.B) is given in Table I. In some cases, there are only T-invariance relations and no corresponding useful M-purity ones (no: 5); in other cases, there are only M-purity relations but no corresponding T-invariance relations (nos: 6b, 7, 8, 10); in still other cases one has both the T-invariance relations and their corresponding M-purity analogues (nos: 2, 3,6a; 4,9). The fact that M-purity relations like the asymmetry polarization equality can hold even when the "other" particle is polarized is illustrated by item no: 7 which is based on (4.30). Another example where an M-purity relation goes beyond T-invariance is the depolarization-like relations (4.42) and (4.43) based on (4.31).

There is some vector meson-dominance argument (Section (5.B)) for full M-purity (with $M = +1$) in $\gamma N \to \gamma N$; the argument is not model-independent. To decide whether data favour only T-invariance (M-purity of type 1), or full M-purity, one can consider situations where either only T-invariance relations or only M-purity relations exist. For example, are the T-invariance relations [(4.21)–(4.28)] nontrivial, or only null identities as full M-purity would give? Examples of the other type are to test the consequences (4.39), (4.40); (4.42), (4.43); (4.46), (4.47) of full M-purity; these consequences do not follow if one had only T-invariance. The above tests require measurements of final state polarizations.

5. EXPERIMENTAL EVIDENCE FOR M-PURITY IN $\gamma N \to VN$; IMPLICATIONS FOR $\gamma N \to \gamma N$

A. $\gamma N \to VN$

Data on the vector meson density matrix are available [5] for linearly polarized photons, and unpolarized target nucleons. In order to examine the M-purity implications of these data, we recall [4] the M-purity restrictions on elements of the vector meson density matrix for various initial polarizations. Stated in terms of the density-matrix coefficients $\rho^{ij}(m, k)$, these relations read as

$$-\frac{\rho^{00}(1,k)}{\rho^{00}(0,k)} = \frac{\operatorname{Re}\rho^{+-}(0,k)}{\rho^{++}(1,k)} = \frac{\operatorname{Re}\rho^{+-}(1,k)}{\rho^{++}(0,k)} = -\frac{\operatorname{Re}\rho^{+0}(1,k)}{\operatorname{Re}\rho^{+0}(0,k)} = M = 1/M \tag{5.1}$$

and

$$-\frac{\operatorname{Im}\rho^{+-}(2,k)}{\rho^{++}(3,k)} = \frac{\operatorname{Im}\rho^{+-}(3,k)}{\rho^{++}(2,k)} = M = 1/M, \tag{5.2}$$

where k is 0 or 2. The equality of the first, second, third, and fourth expressions in (5.1) to M would be called the first, second, third, and fourth equalities of (5.1); and similarly for (5.2). The denominators in (5.2) do not appear in the decay distribution of the vector meson. If one had M-purity of only type 1, one would obtain the second equality of (5.1) for $k = 0$, the third equality of (5.1) for $k = 2$, the first equality of (5.2) for $k = 2$ and the second equality of (5.2) for $k = 0$. The first and the last equalities of (5.1) obviously involve amplitudes with zero V-helicity.

For our purposes, the available data [5] refer to only (5.1) for $k = 0$ (unpolarised targets). The highest energy (9.3GeV photon energy) data on $\gamma N \to \rho N$ show that all the density-matrix elements in (5.1) except those in its third equality are rather small, and consistent with being zero, especially for small momentum transfers. The elements in the third equality are both equal, giving

$$\operatorname{Re}\rho^{+-}(1,0)/\rho^{++}(0,0) = +1 \tag{5.3}$$

within experimental errors which are not large. The highest energy (9.3GeV photon energy) data on $\gamma N \to \omega N$, ϕN also show [5] the same features, though the errors are larger than for $\gamma N \to \rho N$. Written in terms of the n- and u-type amplitudes, (5.3) reads as

$$(a - b)/(a + b) = +1, \tag{5.4a}$$

where

$$\begin{aligned} a &= a_1 + a_2, \\ a_1 &= |n_{++}^{++}|^2 + |n_{-+}^{++}|^2, \\ a_2 &= |n_{+-}^{++}|^2 + |n_{--}^{++}|^2; \end{aligned} \tag{5.4b}$$

$$\begin{aligned} b &= b_1 + b_2, \\ b_1 &= |u_{++}^{++}|^2 + |u_{-+}^{++}|^2, \\ b_2 &= |u_{+-}^{++}|^2 + |u_{--}^{++}|^2. \end{aligned} \tag{5.4c}$$

Equations (5.4) imply that within the experimental errors,

$$u_{++}^{++} = u_{--}^{++} = u_{-+}^{++} = u_{+-}^{++} = 0 \tag{5.5}$$

in $\gamma N \to VN$. This result is remarkable in that the single ratio (5.3) implies the vanishing of *all* the u-type amplitudes with a nonzero helicity of the vector meson. The n- and u-type amplitudes with a zero helicity of the vector meson are experimentally small, and do not provide such a clear-cut conclusion, though data are consistent with the vanishing of these u-amplitudes also.

The direct experimental evidence for

$$\text{Re}\, \rho^{+-}(0, 0)/\rho^{++}(1, 0) = +1 \tag{5.6}$$

is weak since these density-matrix elements are small and consistent with zero. However, (5.5) implies (5.6). Similarly, (5.2) fully and the second and third equalities of (5.1) for also polarised targets ($k = 2$) are implied by (5.5). One may note that M-purity of only type 1 would not lead to the result (5.3).

Further experimental evidence for M-Purity in $\gamma N \to VN$ comes from data for the asymmetries P_σ and Σ defined as [5, 8]

$$P_\sigma = (\sigma^N - \sigma^U)/(\sigma^N + \sigma^U), \tag{5.7}$$

$$\begin{aligned} \Sigma &= (\sigma_\parallel - \sigma_\perp)/(\sigma_\parallel + \sigma_\perp) \\ &= (\rho^{++}(1, 0) + \text{Re}\, \rho^{+-}(1, 0))/(\rho^{++}(0, 0) + \text{Re}\, \rho^{+-}(0, 0)), \end{aligned} \tag{5.8}$$

where $\sigma^{N,U}$ are contributions to the cross-section from natural and unnatural parity exchanges, respectively, in the t-channel; $\sigma_{\parallel,\perp}$ are cross-sections for producing pseudoscalar meson-pairs (from V-decay) parallel and normal respectively to the photon polarization. To leading order in energy,

$$P_\sigma = (2\, \text{Re}\, \rho^{+-}(1, 0) - \rho^{00}(1, 0))/\text{tr}\, \rho(0, 0). \tag{5.9}$$

The second and third equalities of (5.1), if used in (5.8), lead to

$$\Sigma = M. \tag{5.10}$$

From these definitions, and the fact that $\rho^{++}(1, 0)$, Re $\rho^{+-}(0, 0)$, $\rho^{00}(0, 0)$ and $\rho^{00}(1, 0)$ are experimentally small, it is clear that to a good approximation, P_σ and Σ depend primarily on only the large matrix-elements Re $\rho^{+-}(1, 0)$ and $\rho^{++}(0, 0)$ which occur also in (5.3). *The main experimental evidence for M-purity in* $\gamma N \to VN$ *is, therefore, based on* (5.3) *which implies* $M = +1$ *for all amplitudes with nonzero V-helicity.*

We estimate the experimental errors associated with M-purity in $\gamma N \to VN$ using $\gamma N \to \rho^0 N$ data [5] for unpolarized targets at 9.3 GeV/c at a typical small $|t|$ value, $t = -(0.12 \to 0.18)(\text{GeV/c})^2$. Equation (5.1) then reads as

$$-\frac{-0.05 \pm 0.04}{0.03 \pm 0.02} = \frac{0.01 \pm 0.03}{-0.02 \pm 0.04} = \frac{0.48 \pm 0.05}{0.485 \pm 0.01} = -\frac{-0.01 \pm 0.03}{0.03 \pm 0.02} = M \tag{5.11a}$$

or

$$1.6 \pm 1.7 = -0.5 \pm 1.8 = 0.99 \pm 0.11 = 0.33 \pm 1.02 = M, \tag{5.11b}$$

where the correlations between the various errors have been dropped (also hereafter). Except for the ratio (5.3), the errors in (5.11) are too large to justify a firm conclusion about $M = +1$, though $M = +1$ is allowed. For the ratio (5.3), the quantity b/a is seen to be small. Neglecting second and higher powers of b/a, one gets from (5.11b),

$$b/a = 0.005 \pm 0.055, \tag{5.12}$$

which shows that the $M = -1$ combination b is at most about 5% of the $M = +1$ combination a.

In fact, the numbers for Re $\rho^{+-}(1, 0)$, $\rho^{++}(0, 0)$, and Im $\rho^{+-}(2, 0)$ imply [7] that any of the three combinations a_2, b_1, and b_2 are at most a few percent of the dominant combination a_1:

$$\begin{aligned} a_1 - b_1 &= (0.96 \pm 0.064)\, d, & b_1 + b_2 &= (0.005 \pm 0.051)\, d, \\ a_1 + a_2 &= (0.965 \pm 0.051)\, d, & a_2 + b_1 &= (0.005 \pm 0.041)\, d, \\ a_1 + b_2 &= (0.965 \pm 0.041)\, d, & a_2 - b_2 &= (0.0 \pm 0.064)\, d, \end{aligned} \tag{5.13}$$

where d is the overall normalization. Taking 0.05d as the typical upper limit on b_1 and b_2, the $M = -1$ amplitudes occuring in b are consistent with zero, and bounded in magnitude by about a fifth of the $M = +1$ combination

$$(|n_{++}^{++}|^2 + |n_{-+}^{++}|^2)^{1/2}.$$

It would be nice to have much smaller errors in (5.12). We use $b/a = 0$ in Section 5.B.

Using data on $\rho^{00}(0, 0)$ and $\rho^{00}(1, 0)$, one similarly gets

$$| n_{++}^{+0} |^2 + | n_{-+}^{+0} |^2 = (0.04 \pm 0.022)\, d, \tag{5.14a}$$

$$| u_{++}^{+0} |^2 + | u_{-+}^{+0} |^2 = (-0.01 \pm 0.022)\, d. \tag{5.14b}$$

It is, therefore, difficult to make a firm statement about M-purity of these small (helicity-flip) amplitudes. Because the remaining available [5] density-matrix-elements depend on unknown relative phases between different helicity amplitudes, it is hard to get further numerical estimates for M-purity.

B. *Implications for $\gamma N \to \gamma N$*

The result (5.5) is interesting also for $\gamma N \to \gamma N$. The vanishing of u_{-+}^{++} and u_{+-}^{++} is M-purity ($M = +1$) of type 1, as given by T-invariance in $\gamma N \to \gamma N$, but vanishing of the amplitudes u_{++}^{++} and u_{--}^{++} means M-purity ($M = +1$) of type 2 which is not required by T-invariance in $\gamma N \to \gamma N$. One may regard M-purity ($M = +1$) of type 1 in $\gamma N \to VN$ as support[12] for the vector-meson dominance of the electromagnetic current, since the corresponding purity in $\gamma N \to \gamma N$ is already guaranteed by T-invariance. The smallness of the $\gamma N \to VN$ amplitudes with zero V-helicity may be regarded as another support for vector meson dominance, since these amplitudes are absent in $\gamma N \to \gamma N$. With this support from data, one may turn the argument around: One can use the vector meson dominance model to deduce that the vanishing of u_{++}^{++} and u_{--}^{++} in $\gamma N \to VN$ implies the vanishing of these amplitudes also in $\gamma N \to \gamma N$. Since u_{++}^{++} and u_{--}^{++} are the only independent u-amplitudes in T-invariant $\gamma N \to \gamma N$, *the result* (5.5) *coupled with vector dominance implies that there are no u-type amplitudes in* $\gamma N \to \gamma N$.

This full M-purity in even $\gamma N \to \gamma N$ has obvious implications for the comparison between M-purity relations and T-invariance relations, considered in Section 4.C. An experimental test of these consequences of full M-purity would throw light also on the vector dominance model, as the above argument shows. One should, however, note that at present there are some experimental errors associated with (5.5) on which our argument for full M-purity in $\gamma N \to \gamma N$ is based. It would be nice to reduce these experimental errors.

[12] We note that our considerations involve only ratios of density-matrix elements; these ratios will remain unaffected even if the overall normalization [5] provided by the vector dominance model is not correct. In the vector dominance model, the effective ρ-contribution to the Compton amplitude is an order of magnitude larger than the corresponding ω- and ϕ-contributions so that the errors in $\gamma N \to \omega N$, ϕN data in connection with (5.3) are not very important.

6. M-PURITY RELATIONS IN $\gamma N \to VN$; COMPARISON WITH $\gamma N \to \gamma N$

Section 5.A shows that there is good evidence for M-purity ($M = +1$) for amplitudes with nonzero V-helicity in $\gamma N \to VN$, while that for amplitudes with zero V-helicity is not so good. These latter amplitudes are relatively small [5], making M-purity a good approximation in $\gamma N \to VN$. The extra (= zero) V-helicity in $\gamma N \to VN$ as compared to $\gamma N \to \gamma N$ allows one to study the effect of inelasticity on M-purity relations as a question of principle. As we shall see, this feature of inelasticity does not modify some classes of M-purity relations, exemplifying their generality.

For $\gamma N \to VN$, one needs to consider the vector meson density-matrix instead of the final photon polarization of $\gamma N \to \gamma N$. Using the relation of this polarization to the photon density-matrix, one translates the relations involving these polarizations into the corresponding $\gamma N \to VN$ relations. The relations (4.7)–(4.10), for example, now become [7], respectively,

$$\tilde{\zeta}_2'(0, 0) = 2\mathrm{Re}\, \rho^{+-}(1, 2) - \rho^{00}(1, 2), \tag{6.1}$$

$$\tilde{\zeta}_2'(1, 0) = 2\mathrm{Re}\, \rho^{+-}(0, 2) - \rho^{00}(0, 2), \tag{6.2}$$

$$\tilde{\zeta}_2'(0, 2) = 2\mathrm{Re}\, \rho^{+-}(1,0) - \rho^{00}(1, 0), \tag{6.3}$$

and

$$\tilde{\zeta}_2'(1, 2) = 2\mathrm{Re}\, \rho^{+-}(0, 0) - \rho^{00}(0, 0), \tag{6.4}$$

where the elements ρ^{00} represent the modification due to amplitudes with zero V-helicity; these relations now depend on only parity-conservation (1.1).

We now consider the $\gamma N \to VN$ analogues of the M-purity relations of Section 4. The M-purity analogues of (4.29)–(4.32) can be obtained by combining (6.1)–(6.4) with the first three equalities of (5.1):

$$\tilde{\zeta}_2'(0, 0) = 2\,\mathrm{Re}\, \rho^{+-}(1, 2) - \rho^{00}(1, 2) = MC\, \sigma(0, 2), \tag{6.5}$$

$$\tilde{\zeta}_2'(1, 0) = 2\,\mathrm{Re}\, \rho^{+-}(0, 2) - \rho^{00}(0, 2) = MC\, \sigma(1, 2), \tag{6.6}$$

$$\tilde{\zeta}_2'(0, 2) = 2\,\mathrm{Re}\, \rho^{+-}(1, 0) - \rho^{00}(1, 0) = MC\, \sigma(0, 0), \tag{6.7}$$

$$\tilde{\zeta}_2'(1, 2) = 2\,\mathrm{Re}\, \rho^{+-}(0, 0) - \rho^{00}(0, 0) = MC\, \sigma(1, 0). \tag{6.8}$$

The way in which the cross-section coefficients appear in these relations is the same as in $\gamma N \to \gamma N$. For full M-purity, the relation between recoil nucleon polarizations and these coefficients is also unchanged.

Unlike $\gamma N \to \gamma N$, there is no T-invariance analogue of (6.5) and (6.8) now, but we consider M-purity of only type 1 to illustrate how the inelastic component (i.e., amplitudes with zero V-helicity) behaves so as to leave the relations between

$\tilde{\zeta}'$ and σ's for full M-purity unmodified. For M-purity of only type 1, the interesting elements are Re $\rho^{+-}(1, 2)$ and Re $\rho^{+-}(0, 0)$. The relevant relations (6.5) and (6.8) then become

$$\tilde{\zeta}_2'(0, 0) = 2 \text{ Re } \rho^{+-}(1, 2) - \rho^{00}(1, 2) = MC\, \sigma(0, 2) - [M\rho^{00}(0, 2) + \rho^{00}(1, 2)], \tag{6.9}$$

$$\tilde{\zeta}_2'(1, 2) = 2 \text{ Re } \rho^{+-}(0, 0) - \rho^{00}(0, 0) = MC\, \sigma(1, 0) - [M\rho^{00}(1, 0) + \rho^{00}(0, 0)]. \tag{6.10}$$

If M-purity holds also for amplitudes with zero V-helicity with the same value of M as for the other amplitudes, the first equality of (5.1) shows that the square brackets in (6.9) and (6.10) vanish and one recovers, for full M-purity, the analogues (6.5) and (6.8) of the T-invariance relations (4.13) and (4.14) for even the inelastic reaction $\gamma N \to VN$. A similar remark applies to analogues (6.6) and (6.7) of the full M-purity relations (4.30) and (4.31).

Because the above relations between recoil nucleon polarizations and cross-section coefficients are the same as in $\gamma N \to \gamma N$, the relations (4.33), (4.35), and (4.37) which determine recoil nucleon polarizations hold also for $\gamma N \to VN$; so does the M-purity version of (4.15). Similarly, the relations (4.39) and (4.40) between recoil nucleon polarizations and cross-section asymmetries remain unmodified. Also, the relation (4.42) resembling the depolarization relation for πN scattering is not modified.

Using the definition of $\tilde{P}_j'$ and (3.14), one sees that the $\gamma N \to VN$ analogues of the M-purity relations[13] (4.44)–(4.47) are all contained in (5.2). Similarly, the M-purity relations [like (4.34), (4.36), (4.38), and (4.43)] concerning $\tilde{P}_1'$ all get translated into the $\gamma N \to VN$ case by using the second and third equalities of (5.1).

The M-purity analogs of (4.21)–(4.28) continue to be null identities, as in $\gamma N \to \gamma N$.

Some comments on these ρ^{00} modifications. They occur already in the relevant relations (6.1)–(6.4) following from only parity-invariance, but appear systematically in such a way that for full M-purity, the form of the relations (6.5)–(6.8) between recoil nucleon polarizations and cross-section coefficients is not changed as compared to $\gamma N \to \gamma N$. The M-purity equalities relating Re $\rho^{+-}(i, k)$ to $\rho^{++}(j, k)$, $(i, j) = (0, 1)$, $i \neq j$, $k = (0, 2)$, embodied in (5.1) do not get modified[13] in going over to $\gamma N \to VN$. It is the relation of ρ^{++} to tr ρ (and therefore, to σ) that gets modified by the ρ^{00} contributions. The interesting point is that the ρ^{00} modifications occur in such a way that under full M-purity, the trace of the final photon density

[13] These relations do not involve amplitudes with zero V-helicity, and go over from $\gamma N \to \gamma N$ to $\gamma N \to VN$ unchanged.

matrix $\rho_{\gamma'}$ in $\gamma N \to \gamma N$ gets replaced by exactly the trace of the vector meson density matrix ρ in $\gamma N \to VN$.

There is thus a "*vertex-dependence*" in the effects of inelasticity on M-purity relations: The relations (4.29)–(4.32) between recoil nucleon polarizations and cross-section coefficients are not modified in going over to $\gamma N \to VN$. The corresponding relations (4.29)–(4.32)

$$2 \operatorname{Re} \rho_{\gamma'}^{+-}(i, k) = MC\sigma(j, k), \qquad i \neq j, \quad (i, j) = (0, 1), \quad k = (0, 2) \tag{6.11}$$

of $\gamma N \to \gamma N$ become (6.5)–(6.8)

$$2 \operatorname{Re} \rho^{+-}(i, k) = MC\, \sigma(j, k) + \rho^{00}(i, k) \tag{6.12}$$

for $\gamma N \to VN$. In contrast to the "unexcited vertex" (nucleon $\to$ nucleon), there is a modification at the "inelastic vertex" (photon $\to$ vector meson).

7. Summary and Discussion

We have considered M-purity relations for spin-effects in elastic and in inelastic scattering; the emphasis was on relations which resemble those following from T-invariance in elastic scattering—in particular, the asymmetry-polarization equality of Section 2. The clue to why some M-purity relations resemble T-invariance ones is the fact that T-invariance forbids some $M = -1$ amplitudes in elastic scattering, Eq. (1.5). The M-purity analogs of the asymmetry-polarization theorem hold, however, for spin configurations much more general than that for the theorem, as discussed in Section 2. It is interesting that *any relations* resembling those following from T-invariance do exist between spin effects also for *inelastic* reactions.

Several reasons were given in Section 1.B why M-purity is experimentally interesting, but perhaps the cleanest case where there is already experimental evidence (Section 5) of M-purity (with $M = +1$, especially for amplitudes with nonzero V-helicity) is $\gamma N \to VN$ for which M-purity relations were given in Section 6 and compared with the corresponding ones for $\gamma N \to \gamma N$. The M-purity relations for $\gamma N \to \gamma N$ were compared with the corresponding T-invariance relations in Section 4, the results being summarised in Table I.

Our examples of the T-invariance relations could be divided into three classes: (a) relations between different final state photon polarisations, (4.17) and (4.18), (b) relations between recoil nucleon polarizations in the reaction plane and final photon polarizations (4.21)–(4.24), and (c) relations between recoil nucleon polarizations normal to the reaction plane and final photon polarizations, (4.7)–(4.10). It is to class (c) that the two examples (4.15) and (4.16) of the standard asymmetry-polarization theorem in $\gamma N \to \gamma N$ belong; in fact, the M-purity analogs and extensions (4.29)–(4.32) and (6.5)–(6.8) of the class (c) all resemble the theorem. The M-purity analogs of the class (b) are only null-identities; those of the class (a) are the relations (4.44)–(4.47) for $\gamma N \to \gamma N$ and (5.2) for $\gamma N \to VN$ (and $\gamma N \to \gamma N$). But the M-purity analogs for the classes (a) and (b) do not reveal interesting information about the transition from an elastic to an inelastic reaction; they do not involve the inelastic component (i.e., amplitudes with zero V-helicity), and go over unchanged to $\gamma N \to VN$. The M-purity analogues of the class (c) do offer this information; we discuss this class now.

Consider first the recoil nucleon polarizations and their relation to the corresponding cross-section asymmetries. From T-invariance, one gets the standard theorem (4.13) for $\gamma N \to \gamma N$. The corresponding M-purity relations for $\gamma N \to \gamma N$ and $\gamma N \to VN$ are, respectively,[14] (4.29) and (6.5). The interesting point is that not only do the u-amplitudes forbidden by T-invariance in $\gamma N \to \gamma N$ appear in the proper way so as to give the relations (4.29) and (6.5), but also do the extra amplitudes (of both n- and u-types) with a zero V-helicity appear properly so as to give (6.5). The relations (4.30) and (6.6) provide examples of M-purity relations having no T-invariance analogs; these relations between cross-section asymmetries and recoil polarizations have the "other" particle polarized. The comment about the extra n- and u-amplitudes with a zero V-helicity appearing properly so as to leave the form (4.30) unchanged applies again. Similarly for the transition from (4.31) and (4.32) to (6.7) and (6.8).

While the form of the relation of cross-section asymmetries to recoil nucleon polarizations is not changed in going from $\gamma N \to \gamma N$ to $\gamma N \to VN$, there *is* a change in the form of the corresponding relation to recoil photon polarizations $\tilde{P}_1'$ written as 2 Re $\rho_{\gamma'}^{+-}$. These changes due to the inelastic component are conveniently summarized in Eqs. (6.11) and (6.12) where an extra positive semidefinite quantity ρ^{00} adds on to the appropriate cross-section coefficient.

This implies a "*vertex-dependence*" in the effects of inelasticity on the M-purity analogues and extensions of the asymmetry-polarization theorem: When one considers the recoil polarization and the cross-section asymmetry corresponding to the unexcited vertex (nucleon $\to$ nucleon), there is no effect of inelasticity. For the excited vertex (photon $\to$ vector meson), there is a modification due to inelasticity. While the example of $\gamma N \to \gamma N$, VN is interesting, it provides only a rather simple form of introducing inelasticity. On the basis of only this example, it is difficult to make statements about the changes in M-purity analogs of the asymmetry-polarization theorem in going over to a more generally excited vertex (for example, a spin-change: $J^P = 1^-$ to $J^P = 2^+$). The corresponding relations for the

[14] Of course, (4.29) is interesting mainly for its comparison with $\gamma N \to VN$ because for $\gamma N \to \gamma N$, $M = +1$ is given by T-invariance, leading to (4.13).

TABLE II

Summary of Illustrations of the Three Classes of T-Invariance and M-purity Relations for $\gamma N \to \gamma N$. The examples A-1 and A-2 are the standard asymmetry polarization theorem.

Class	Relations due to: T-invariance	Relations due to: M-purity	Examples	Reference to text (equation number)	Reference to Table I (item number)
A	Exist (use $M = +1$)	Exist	1. $\tilde{P}_1'(0,0) = MC\ \sigma(1,0)$	(4.14), (4.32)	3, 6a
			2. $\tilde{\zeta}_2'(0,0) = MC\ \sigma(0,2)$	(4.13), (4.29)	2, 6a
			3. $\tilde{P}_2'(2,2) = M\tilde{P}_3'(3,2)$	(4.17), (4.44)	4, 9
			$\tilde{P}_2'(3,0) = -M\tilde{P}_3'(2,0)$	(4.18), (4.45)	4, 9
B	Exist	None (only null-identities)	1. $\tilde{P}_2'(0,1) = \tilde{\zeta}_1'(2,0)$	(4.21)	5
			$\tilde{P}_2'(0,3) = -\tilde{\zeta}_3'(2,0)$	(4.22)	5
			$\tilde{P}_3'(0,3) = \tilde{\zeta}_3'(3,0)$	(4.23)	5
			$\tilde{P}_3'(0,1) = -\tilde{\zeta}_1'(3,0)$	(4.24)	5
C	None	Exist	1. $\tilde{P}_1'(0,2) = MC\ \sigma(1,2)$	(4.30)	6b, 7
			$\tilde{P}_1'(1,0) = MC\ \sigma(0,0)$	(4.31)	6b, 8b
			2. $\tilde{\zeta}_2'(1,0) = MC\ \sigma(1,2)$	(4.30)	6b, 7
			$\tilde{\zeta}_2'(0,2) = MC\ \sigma(0,0)$	(4.31)	6b, 8a
			3. $\tilde{P}_2'(2,0) = M\tilde{P}_3'(3,0)$	(4.46)	10
			$\tilde{P}_2'(3,2) = -M\tilde{P}_3'(2,2)$	(4.47)	10

TABLE III

Effect of Inelasticity $[(\gamma N \rightarrow \gamma N) \rightarrow (\gamma N \rightarrow VN)]$ on M-purity relations of the types A and C of Table II. For the type B, the corresponding relations are only null-identities for both these processes.

Class and example		Form for $\gamma N \rightarrow \gamma N$	Form for $\gamma N \rightarrow VN$	Reference to text (equation number)
A	1	$\tilde{P}_1'(0,0) \equiv 2\,\mathrm{Re}\,\rho_{\gamma'}^{+-}(0,0) = MC\,\sigma(1,0)$	$2\,\mathrm{Re}\,\rho^{+-}(0,0) = MC\,\sigma(1,0) + \rho^{00}(0,0)$	(4.32), (6.8, 10)
	2	$\tilde{\zeta}_2'(0,0) = MC\,\sigma(0,2)$	No change	(4.29), (6.5, 9)
	3	$\tilde{P}_2'(2,2) = M\tilde{P}_3'(3,2)$ or $-\mathrm{Im}\,\rho_{\gamma'}^{+-}(2,2) = M\rho_{\gamma'}^{++}(3,2)$	No change: $-\mathrm{Im}\,\rho^{+-}(2,2) = M\rho^{++}(3,2)$	(4.44), (5.2)
		$\tilde{P}_2'(3,0) = -M\tilde{P}_3'(2,0)$ or $\mathrm{Im}\,\rho_{\gamma'}^{+-}(3,0) = M\rho_{\gamma'}^{++}(2,0)$	No change: $\mathrm{Im}\,\rho^{+-}(3,0) = M\rho^{++}(2,0)$	(4.45), (5.2)
C	1	$\tilde{P}_1'(0,2) \equiv 2\,\mathrm{Re}\,\rho_{\gamma'}^{+-}(0,2) = MC\,\sigma(1,2)$	$2\,\mathrm{Re}\,\rho^{+-}(0,2) = MC\,\sigma(1,2) + \rho^{00}(0,2)$	(4.30), (6.6)
		$\tilde{P}_1'(1,0) \equiv 2\,\mathrm{Re}\,\rho_{\gamma'}^{+-}(1,0) = MC\,\sigma(0,0)$	$2\,\mathrm{Re}\,\rho^{+-}(1,0) = MC\,\sigma(0,0) + \rho^{00}(1,0)$	(4.31), (6.7)
	2.	$\tilde{\zeta}_2'(1,0) = MC\,\sigma(1,2)$	No change	(4.30), (6.6)
		$\tilde{\zeta}_2'(0,2) = MC\,\sigma(0,0)$	No change	(4.31), (6.7)
	3.	$\tilde{P}_2'(2,0) = M\tilde{P}_3'(3,0)$ or $-\mathrm{Im}\,\rho_{\gamma'}^{+-}(2,0) = M\rho_{\gamma'}^{++}(3,0)$	No change: $-\mathrm{Im}\,\rho^{+-}(2,0) = M\rho^{++}(3,0)$	(4.46), (5.2)
		$\tilde{P}_2'(3,2) = -M\tilde{P}_3'(2,2)$ or $\mathrm{Im}\,\rho_{\gamma'}^{+-}(3,2) = M\rho_{\gamma'}^{++}(2,2)$	No change: $\mathrm{Im}\,\rho^{+-}(3,2) = M\rho^{++}(2,2)$	(4.47), (5.2)

unexcited vertex (nucleon $\rightarrow$ nucleon) are just an example of Section 2 where these relations were illustrated for the case when the only restriction on the unexcited vertex was the equality of the initial and final spins. The $\gamma \rightarrow V$ vertex goes only a little farther than this. Because the case of a more general excited vertex has no direct T-invariance analog, we do not consider the M-purity analogs of the theorem for such a vertex.

In Table II (which is only another version of Table I) is given a summary of our illustrations of the T-invariance and the M-purity relations for $\gamma N \rightarrow \gamma N$; and in Table III, a summary of the modifications in these M-purity relations in going over to $\gamma N \rightarrow VN$.

The information to be obtained from cross-section asymmetry measurements in $\gamma N \rightarrow \gamma N$, VN has been considered in the appendix. The question: "Which asymmetries are necessary (and which determined therefrom by parity conservation)?" has been answered, see remarks (especially no: 1) there. A simple observation concerning the asymmetry-polarization theorem is that though T-invariance relates the recoil polarization for an unpolarized initial state to the cross-section asymmetry with the "other" particle unpolarized, parity invariance equates this asymmetry also to the corresponding asymmetry with some special nonzero polarizations of the "other" particle; the various asymmetries in Eqs. (A.15) and (A.16) of the appendix provide the relevant examples.

APPENDIX: Cross Section Asymmetries in $\gamma N \rightarrow VN$ and $\gamma N \rightarrow \gamma N$

Some consequences of parity-invariance for cross-section asymmetries in vector meson photoproduction and Compton scattering are now considered. Using an expansion like (3.6) for the complete set of the actual cross-sections $\sigma(P_i, \zeta_k)$, one gets

$$\sigma(P_0, \zeta_0) = \sigma(P_0, \zeta_1) = \sigma(P_0, \zeta_3) = \sigma(P_2, \zeta_0) = \sigma(P_3, \zeta_0) = \sigma(0, 0), \quad \text{(A.1)}$$

$$\sigma(P_0, \zeta_2) = \sigma(P_2, \zeta_2) = \sigma(P_3, \zeta_2) = \sigma(0, 0) + \zeta_2\sigma(0, 2), \quad \text{(A.2)}$$

$$\sigma(P_1, \zeta_0) = \sigma(P_1, \zeta_1) = \sigma(P_1, \zeta_3) = \sigma(0, 0) + P_1\sigma(1, 0), \quad \text{(A.3)}$$

$$\sigma(P_1, \zeta_2) = \sigma(0, 0) + P_1\sigma(1, 0) + \zeta_2\sigma(0, 2) + P_1\zeta_2\sigma(1, 2), \quad \text{(A.4)}$$

$$\sigma(P_2, \zeta_1) = \sigma(0, 0) + P_2\zeta_1\sigma(2, 1), \quad \text{(A.5)}$$

$$\sigma(P_2, \zeta_3) = \sigma(0, 0) + P_2\zeta_3\sigma(2, 3), \quad \text{(A.6)}$$

$$\sigma(P_3, \zeta_1) = \sigma(0, 0) + P_3\zeta_1\sigma(3, 1), \quad \text{(A.7)}$$

$$\sigma(P_3, \zeta_3) = \sigma(0, 0) + P_3\zeta_3\sigma(3, 3), \quad \text{(A.8)}$$

where use has been made of the vanishing [7] (because of parity-invariance) of eight coefficients out of the sixteen possible ones:

$$\sigma(0, 1) = \sigma(1, 1) = \sigma(2, 0) = \sigma(3, 0) = \sigma(2, 2) = \sigma(3, 2) = \sigma(0, 3) = \sigma(1, 3) = 0. \quad (A.9)$$

The cross-section asymmetries are

$$A(P_i, \pm\zeta_k) = \frac{\sigma(P_i, \zeta_k) - \sigma(P_i, -\zeta_k)}{\sigma(P_i, \zeta_k) + \sigma(P_i, -\zeta_k)}, \quad (A.10)$$

$$A(\pm P_i, \zeta_k) = \frac{\sigma(P_i, \zeta_k) - \sigma(-P_i, \zeta_k)}{\sigma(P_i, \zeta_k) + \sigma(-P_i, \zeta_k)}, \quad (A.11)$$

and

$$A(\pm P_i, \pm\zeta_k) = \frac{\sigma(P_i, \zeta_k) - \sigma(-P_i, -\zeta_k)}{\sigma(P_i, \zeta_k) + \sigma(-P_i, -\zeta_k)}, \quad (A.12)$$

where (i and k) can be (0, 1, 2, and 3), but the subscript of the polarization appearing with both signs in the argument of an asymmetry cannot be zero; the asymmetries (A.10) and (A.11) can be called "single" asymmetries in contrast to the "double" asymmetry (A.12). As shown below, the double asymmetry $A(\pm P_1, \pm\zeta_2)$ is interesting, but the other nonzero double asymmetries are very simply given by some single asymmetry because of pairty-invariance.

The parity-conservation results (A.1)–(A.8) give, for the single asymmetries,

$$A(P_0, \pm\zeta_1) = A(P_0, \pm\zeta_3) = A(P_1, \pm\zeta_1) = A(P_1, \pm\zeta_3) = 0, \quad (A.13)$$

$$A(\pm P_2, \zeta_0) = A(\pm P_2, \zeta_2) = A(\pm P_3, \zeta_0) = A(\pm P_3, \zeta_2) = 0, \quad (A.14)$$

$$A(P_0, \pm\zeta_2) = A(P_2, \pm\zeta_2) = A(P_3, \pm\zeta_2) = \zeta_2\sigma(0, 2)/\sigma(0, 0), \quad (A.15)$$

$$A(\pm P_1, \zeta_0) = A(\pm P_1, \zeta_1) = A(\pm P_1, \zeta_3) = P_1\sigma(1, 0)/\sigma(0, 0), \quad (A.16)$$

$$A(\pm P_2, \zeta_1) = A(P_2, \pm\zeta_1) = P_2\zeta_1\sigma(2, 1)/\sigma(0, 0), \quad (A.17)$$

$$A(\pm P_2, \zeta_3) = A(P_2, \pm\zeta_3) = P_2\zeta_3\sigma(2, 3)/\sigma(0, 0), \quad (A.18)$$

$$A(\pm P_3, \zeta_1) = A(P_3, \pm\zeta_1) = P_3\zeta_1\sigma(3, 1)/\sigma(0, 0), \quad (A.19)$$

$$A(\pm P_3, \zeta_3) = A(P_3, \pm\zeta_3) = P_3\zeta_3\sigma(3, 3)/\sigma(0, 0), \quad (A.20)$$

$$A(P_1, \pm\zeta_2) = \zeta_2[\sigma(0, 2) + P_1\sigma(1, 2)]/[\sigma(0, 0) + P_1\sigma(1, 0)], \quad (A.21)$$

$$A(\pm P_1, \zeta_2) = P_1[\sigma(1, 0) + \zeta_2\sigma(1, 2)]/[\sigma(0, 0) + \zeta_2\sigma(0, 2)], \quad (A.22)$$

$$1 + A(P_1, \pm\zeta_2) = [1 + A(\pm P_1, \zeta_2)][1 + A(P_0, \pm\zeta_2)]/[1 + A(\pm P_1, \zeta_0)], \quad (A.23)$$

and, for the double asymmetries,

$$A(\pm P_2, \pm\zeta_1) = A(\pm P_2, \pm\zeta_3) = A(\pm P_3, \pm\zeta_1) = A(\pm P_3, \pm\zeta_3) = 0, \quad (A.24)$$

$$A(\pm P_1, \pm\zeta_1) = A(\pm P_1, \pm\zeta_3) = P_1\sigma(1, 0)/\sigma(0, 0), \quad (A.25)$$

$$A(\pm P_2, \pm\zeta_2) = A(\pm P_3, \pm\zeta_2) = \zeta_2\sigma(0, 2)/\sigma(0, 0), \quad (A.26)$$

$$A(\pm P_1, \pm\zeta_2) = [P_1\sigma(1, 0) + \zeta_2\sigma(0, 2)]/[\sigma(0, 0) + P_1\zeta_2\sigma(1, 2)]. \quad (A.27)$$

The double asymmetries (A.25) and (A.26) are related to the single asymmetries (A.16) and (A.15) which are interesting for T-invariance in $\gamma N \to \gamma N$. One gets

$$A(\pm P_1, \pm\zeta_1) = A(\pm P_1, \zeta_1) = A(\pm P_1, \zeta_0) \quad (A.28a)$$
$$= A(\pm P_1, \pm\zeta_3) = A(\pm P_1, \zeta_3) = A(\pm P_1, \zeta_0), \quad (A.28b)$$

$$A(\pm P_2, \pm\zeta_2) = A(P_2, \pm\zeta_2) = A(P_0, \pm\zeta_2) \quad (A.29a)$$
$$= A(\pm P_3, \pm\zeta_2) = A(P_3, \pm\zeta_2) = A(P_0, \pm\zeta_2). \quad (A.29b)$$

The remaining double asymmetry (A.27) can also be related to single asymmetries, but the relation is not equally simple. One gets

$$A(\pm P_1, \zeta_0) + A(P_0, \pm\zeta_2) = A(\pm P_1, \pm\zeta_2)[1 + P_1\zeta_2\sigma(1, 2)/\sigma(0, 0)]; \quad (A.30)$$

since the factor $P_1\zeta_2\sigma(1, 2)/\sigma(0, 0)$ also occurs in the single asymmetries (A.21) and (A.22), one gets

$$A(P_1, \pm\zeta_2)[1 + A(\pm P_1, \zeta_0)] - A(P_0 \pm \zeta_2)$$
$$= -1 + [A(\pm P_1, \zeta_0) + A(P_0, \pm\zeta_2)]/A(\pm P_1, \pm\zeta_2), \quad (A31)$$

used in (4.39) and (4.40). Because of (A.23), a corresponding relation holds between $A(\pm P_1, \zeta_2)$ and $A(\pm P_1, \pm\zeta_2)$.

Some Remarks

(1) Out of the eight independent nonvanishing cross-section coefficients, the seven independent ratios

$$\sigma(1, 0)/\sigma(0, 0),\ \sigma(2, 1)/\sigma(0, 0),\ \sigma(2, 3)/\sigma(0, 0),\ \sigma(3, 1)/\sigma(0, 0),\ \sigma(3, 3)/\sigma(0, 0),$$
$$\sigma(1, 2)/\sigma(0, 0) \text{ and } \sigma(0, 2)/\sigma(0, 0) \quad (A.32)$$

represent the information obtainable from asymmetry measurements. This information is contained in seven independent asymmetries—for example, the set

$$[A(P_{0,1}, \pm\zeta_2),\quad A(P_{2,3}, \pm\zeta_1),\quad A(P_{2,3}, \pm\zeta_3)] \quad \text{and} \quad A(\pm P_1, \zeta_0), \quad (A.33)$$

or equivalently, the set

$$[A(\pm P_1, \zeta_{0,2}), \quad A(\pm P_2, \zeta_{1,3}), \quad A(\pm P_3, \zeta_{1,3})], \quad \text{and} \quad A(P_0, \pm\zeta_2). \tag{A.34}$$

(2a) Asymmetries requiring a reversal of target polarization in the reaction plane may be regarded unnecessary because (a) some of these vanish, as in (A.13), and (b) the others are related to asymmetries requiring a reversal of the photon (but not nucleon) polarizations, as in (A.17)–(A.20).

(2b) One can make a corresponding statement for asymmetries requiring a reversal of photon polarizations which are either circular, or at 45°(135°) to the reaction plane because (a) some of these vanish, as in (A.14), and (b) the others are related to asymmmetries requiring a reversal of the nucleon (but not photon) polarizations in the reaction plane, as in (A.17)–(A.20).

Of course, either photon or nucleon polarization reversal is necessary to measure the asymmetries (A.17)–(A.20). The same remark applies to (A.21) and (A.22) though the polarization directions now relevant are different.

(3a) The asymmetry (A.15) requiring a reversal of nucleon polarization normal to the reaction plane remains the same whether the "other" particle (photon) is polarized circularly or linearly at 45°(135°) to the reaction plane or not polarized at all. While, T-invariance relates—(4.15)—only $A(P_0, \pm\zeta_2)$ to the recoil nucleon polarization normal to the reaction plane with an unpolarized initial state, the relation (A.15) shows that that recoil polarization is related also to the corresponding cross-section asymmetries for some special[15] nonzero polarizations of the "other" particle; these special polarizations behave as "inactive spectators."

(3b) A corresponding statement holds in the case of the asymmetries (A.16) requiring a reversal of photon polarizations normal (or parallel) to the reaction plane; the initial nucleon may be unpolarized or polarized in the reaction plane; T-invariance relates—(4.16)—only $A(\pm P_1, \zeta_0)$ to the final state photon polarization normal (or parallel) to the reaction plane with an unpolarized initial state.

To obtain $\sigma(0, 2)/\sigma(0, 0)$, it is not necessary to use photon polarizations which are circular or linear at 45°(135°) to the reaction plane; an unpolarized photon beam would do; this supplements the remark (2b) above. Similarly, initial nucleon polarizations in the reaction plane do not go beyond an unpolarized target in determining $\sigma(1, 0)/\sigma(0, 0)$; this supplements the remark (2a) above.

(4) Under M-purity, the asymmetries (A.17)–(A.20) vanish [7], leaving (A.15), (A.16), and (A.21) as the only nonzero independent ones; these occur in Eqs. (4.29)–(4.32) for $\gamma N \to \gamma N$ and (6.5)–(6.8) for $\gamma N \to VN$.

[15] Here, for example, this is not true for photon polarizations normal to the reaction plane.

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