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OFF-DIAGONAL GENERALIZED VECTOR DOMINANCE AND INELASTIC ep SCATTERING

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Abstract By also taking into account off-diagonal contributions to the virtual forward Compton amplitude, scaling in ep scattering is obtained from scaling of the total cross section for e^+e^- annihilation into hadrons The parameter which sets the scale for the q^2 dependence in ep scattering is calculated to be $\bar{m}^2 \simeq 0\,61\, m_\rho^2 \simeq 0\,36\ \mathrm{GeV}^2$

As has been shown almost two years ago [1] * generalized vector dominance (GVD) quite successfully quantitatively accounts for the data on deep inelastic electron nucleon scattering. There is, however, one principal feature within this approach, which became more and more unsatisfactory as progressively higher energy data on $e^+e^- \to$ hadrons became available these showing roughly a $1/s$ law in the Frascati [3] range, and an even slower fall off for c.m. energies $\sqrt{s}$ beyond about 3.5 GeV has been reported by CEA [4] and more recently by the SLAC-LBL collaboration at SPEAR [5]. The problem, as discussed in [1], with a $1/s$ law for e^+e^- annihilation is that in the usual diagonal form of GVD (i e. neglecting off-diagonal diffraction dissociation type terms $Vp \to V'p$, where V and V′ are distinct vector states) it would seem to imply a $1/m_V^2$ behaviour for the vector meson proton total cross section σ_{Vp}. Otherwise, i.e with a mass-independent vector state nucleon cross section, logarithmic divergences and strong violations of scaling, linear in q^2, are encountered.

The requirement of a $1/m^2$ behaviour ** for the vector meson nucleon total cross section may be somewhat disturbing, however, if one believes strong inter-

* See also ref. [2] for related work

** The $1/m^2$ law has in particular been advocated by Greco [2] Contrary to some statements in the literature, especially in [6], Sakurai and one of the present authors did not advocate a $1/s^2$-dependence in e^+e^- annihilation as a necessary consequence of diagonal GVD Rather it was pointed out in [1] that a $1/s^2$ law within the diagonal framework would be the simplest possibility, keeping in mind alternative ones as e g $\sigma_{e^+e^-} \sim 1/s$ together with the Bjorken's jet picture [7] or some kind of "effective" $1/m^2$ law e g in the shadowing paper [8] and in [9]

action cross sections to be roughly of the same order of magnitude and not depending too much upon the mass of the incident particle. More importantly, when a $1/m^2$ law is assumed for the mass dependence of the imaginary part of the vector state nucleon forward scattering amplitude, the justification for the neglect of off-diagonal contributions to the imaginary part of the virtual forward Compton amplitude becomes intolerably weak. In fact, with a $1/m^2$ law, extremely small amplitudes for hadronic diffraction dissociation (e.g for $\rho_0 p \leftrightarrow \rho_N p, N \geqslant 1$, thinking of a discrete series of vector meson states $N = 0, 1, 2, \ldots$) would still be sufficiently large to yield contributions to the Compton amplitude of the same order of magnitude as the ones from diagonal ($\rho_N p \to \rho_N p$) terms Even if the diffraction dissociation forward amplitude for e.g $\rho_0 p \to \rho_N p$ ($N = 1, 2, \ldots$) were to fall off as rapidly as $1/m_N^3$, its contribution to the Compton amplitude (due to the enhancement of the $\rho_0\gamma$ coupling relative to the $\rho_N\gamma$ coupling by a factor m_N/m_ρ following from $\sigma_{e^+e^-} \sim 1/s$), would still be comparable in magnitude to the diagonal $\rho_N p \to \rho_N p$ contribution A $1/m^3$ behaviour for the diffraction dissociation amplitude would imply a diffraction dissociation cross section $d\sigma/dm^2 \sim 1/m^6$ for $\rho_0 p \to \rho_N p$. This cannot be compared directly with the $d\sigma/dm^2 \sim 1/m^2$ behaviour observed [10] in pp and πp collisions, as there the diffractively produced states also contain spins larger than the spin of the incident particle. Nevertheless, from a reasonable increase of the average spin of the produced states with increasing mass we would expect the spin conserving part to be larger than given by the strong $d\sigma/dm^2 \sim 1/m^6$ law If, for example, the number of spin states available for population were to increase linearly with m^2, one might expect $d\sigma/dm^2 \sim 1/m^4$ for the spin conserving part Clearly, a small fraction of the total diffractive cross section found experimentally * would be sufficient to yield contributions to the Compton amplitude larger than the diagonal ones with a conjectured $\sigma_{V_N p} \sim 1/m_N^2$ law. Thus it seems inconsistent to require $\sigma_{V_N p} \sim 1/m_N^2$, while at the same time keeping the diagonal approximation. Furthermore, if a $1/m_N^2$ law is taken literally already for the lowest mass vector mesons, i.e.

$$\sigma_{\rho_N p} = \sigma_{\rho p} m_\rho^2/m_N^2,$$

then for $\rho''(1600)$ photoproduction one expects

$$[d\sigma/dt(\gamma p \to \rho'' p)]/[d\sigma/dt(\gamma p \to \rho^0 p] = (\gamma_\rho^2/\gamma_{\rho''}^2)\,(m_\rho/m_{\rho''})^4 \simeq 0\,01,$$

where the e^+e^- annihilation result [3] $\gamma_{\rho''}^2/4\pi = (4\,1 \pm 1.3)\,\gamma_\rho^2/4\pi$ has been used in addition Experimentally one finds [11]

$$[d\sigma/dt(\gamma p \to \rho'' p)]/[d\sigma/dt(\gamma p \to \rho^0 p)] \simeq 0.14$$

Thus it seems natural and almost compelling to drop the diagonal approximation and the $\sigma_{V_N p} \sim 1/m_N^2$ law which it engenders and to start rather from the more

* Off-diagonal contributions to the Compton amplitude could be ruled out, if diffraction dissociation would show a dip at $t = 0$ Experimentally no indication for such a behaviour has been found [10].

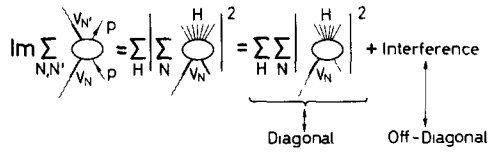

Fig 1 The imaginary part of the virtual forward Compton amplitude or the total virtual photon absorption cross section in GVD

general non-diagonal representation [1] for the imaginary part of the transverse Compton amplitude or σ_T and retain a constant mass-independent cross section $\sigma_{V_N p}$. Off-diagonal transitions correspond to interference terms (compare fig. 1) between different incoming vector mesons. Such contributions are not necessarily positive. They may thus allow the incorporation of the $1/s$ law for e^+e^- annihilation while keeping $\sigma_{V_N p}$ vector meson mass independent, cancelling the logarithmic divergence through destructive interference Indeed, we shall present a simple model that includes negative off-diagonal contributions of reasonable magnitude and allows derivation of scaling in the spacelike region from a $1/s$ behaviour of $e^+e^- \to$ hadrons and a vector meson mass-independent hadronic cross section. The effect on the q^2 dependence in the spacelike region of a possible violation of the $1/s$ law as indicated by CEA [4] and SPEAR [5] data will be discussed elsewhere [12] As these deviations occur at e^+e^- c m energies larger than 3 5 GeV, one expects them to be visible as violations of scaling in the spacelike region at larger values of q^2 than have been explored [13] so far in the diffractive large $\omega' \equiv 1 + W^2/q^2 \gtrsim 8$ region, for which our considerations are expected to be relevant without further corrections.

To formulate off-diagonal GVD *, for definiteness let us assume a Veneziano type spectrum of vector mesons

$$m_N^2 = m_0^2(1 + \lambda N), \; N = 0, 1, \; . \; , \tag{1}$$

m_0 being equal to the $\rho^0(\omega, \phi)$ mass. For the photon vector meson couplings, we assume for the Nth vector meson [2] of isospin $I = 1$ (analogously for $I = 0$)

$$1/\gamma_N^2 = (1/\gamma_\rho^2)\,(m_\rho^2/m_N^2), \tag{2}$$

which in the usual narrow width approximation leads to a $1/s$ behaviour for $e^+e^- \to$ hadrons, the magnitude ** of the cross section being about 2 5 times the μ pair production cross section if one chooses the level spacing $\lambda = 2$ Fig. 2 shows reasonable agreement with the data throughout the Frascati energy range For the vector mes-

* Off-diagonal terms have also been considered in ref [14] in a non-scaling model

** See e g ref [15]

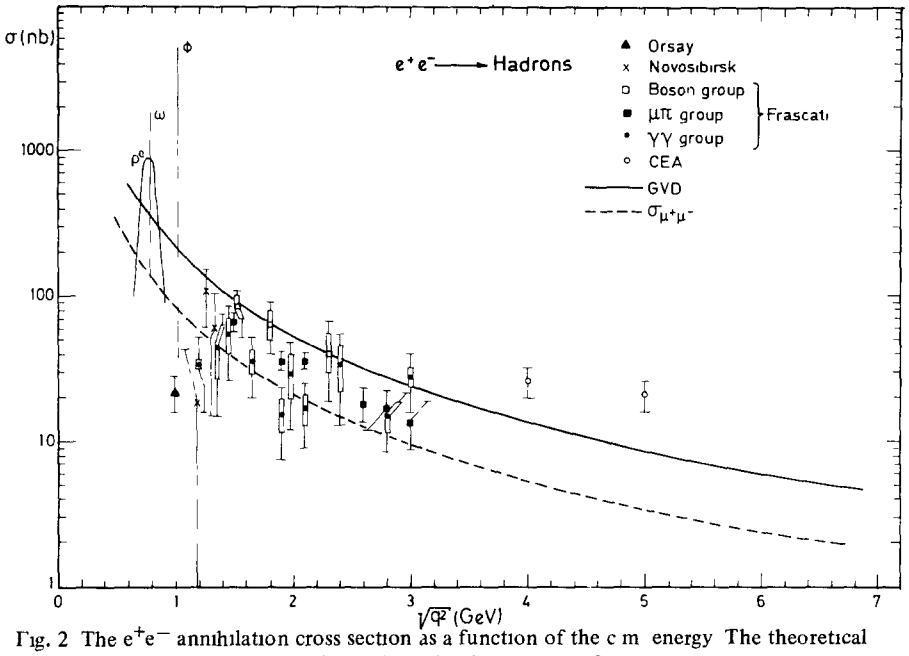

Fig. 2 The e^+e^- annihilation cross section as a function of the c m energy The theoretical curve GVD results from (1) and (2) with the level spacing $\lambda = 2$

on hadron interaction, as mentioned, we will assume $\sigma_{V_N p}$ to be mass independent The Ansatz for the (off-diagonal) diffraction dissociation amplitudes should be motivated from the mentioned power law for $d\sigma/dm^2$, appropriately modified to account for spin conservation. For the purpose of the most simple model calculation to be presented in this work, we will consider (effective) contributions due to transitions $\rho_N p \leftrightarrow \rho_{N+1} p$ to next neighbours only.

Denoting the ratio of the first off-diagonal to the diagonal ($t = 0$) transition amplitude as

$$C_N \equiv T_{\rho_N p \leftrightarrow \rho_{N+1} p} / T_{\rho_N p \to \rho_N p}, \tag{3}$$

we thus obtain the isovector photon part of the transverse virtual photon absorption cross section by writing down the $\rho_N \to \rho_N$, $\rho_N \to \rho_{N+1}$ and $\rho_{N+1} \to \rho_N$ contributions to the imaginary part of the forward Compton amplitude and summing over N

$$\sigma_T^{(I=1)}(W,q^2) = \sigma_{\rho p} \sum_{N=0}^{\infty} \alpha\pi \left[\frac{m_N^2}{\gamma_N} \frac{1}{q^2 + m_N^2}\right]\left[\frac{1}{q^2 + m_N^2} \frac{m_N^2}{\gamma_N} - 2C_N \frac{1}{q^2 + m_{N+1}^2} \frac{m_{N+1}^2}{\gamma_{N+1}}\right] \tag{4}$$

Clearly, for $C_N \equiv 0$ we recover diagonal GVD requiring (with (2)) $\sigma_{\rho_N p} \sim 1/m_N^2$ for convergence and scaling As mentioned, we attempt to cancel the logarithmic divergence by the introduction of negative off-diagonal contributions Therefore we have introduced a minus sign in front of the off-diagonal term, for definiteness assuming C_N to be real and positive (and smaller than 1), and the sign of the $\gamma\rho_N$ coupling to alternate * (i e $\propto (-1)^N, N = 0, 1, 2, \ldots$)

The ratio C_N of first off-diagonal to diagonal transition is not very well known experimentally, except for the fact [10] that C_N for $N = 0$ has to be smaller than 1. Also, although in our ansatz (4) we have explicitly taken into account next neighbour transitions only, C_N should rather be thought of as an effective transition ** standing for the combined effect of all $\rho_N p \leftrightarrow \rho_n p$ ($n \geqslant N + 1$) contributions Anyway, let us suppose a power law for C_N, written with the real parameter δ as

$$C_N = \text{const.}\ (m_N/m_{N+1})^{1+2\delta}, \tag{5}$$

which for large N gives (neglecting order $1/m_N^4$)

$$C_N = \text{const}\ \ (m_N/m_{N+1})\,(1-\delta\lambda\, m_0^2/m_N^2) \tag{6}$$

Then the sum in (4) turns out to be convergent, provided the constant in (5) and (6) is chosen to be $\frac{1}{2}$. Thus inserting (2) and (6) in (4), the result of the summation is easily calculated to be

$$\sigma_T^{(I=1)}(W,q^2) = \frac{\alpha\pi}{\gamma_\rho^2}\ \sigma_{\rho p}\ \frac{1}{\lambda}\left[\frac{q^2 + \lambda(1+\delta)\, m_\rho^2}{(q^2 + m_\rho^2)} - \frac{q^2}{\lambda m_\rho^2}\,\psi^{(1)}\left(\frac{q^2 + m_\rho^2}{\lambda m_\rho^2}\right)\right], \tag{7}$$

where $\psi^{(1)}(z)$ is the derivative of the digamma function $\psi(z)$

$$\psi^{(1)}(z) \equiv (\mathrm{d}/\mathrm{d}z)\,\psi(z) = \sum_{k\,=\,0}^{\infty} 1/(z+k)^2 \sim (1/z) + (1/2z^2) + \mathrm{O}\,(1/z^3). \tag{8}$$

Asymptotically, for $q^2 \to \infty$, σ_T becomes

$$\sigma_T^{(I=1)}(W,q^2) \sim \frac{m_\rho^2(1+2\delta)}{2q^2}\,\frac{\alpha\pi}{\gamma_\rho^2}\,\sigma_{\rho p}\,, \tag{9}$$

the $1/q^2$ behaviour corresponding to scaling of the transverse part of $\nu W_2 \sim q^2\sigma_T$. Thus with the inclusion of off-diagonal transitions (or equivalently by taking into

* Negative signs seem also to be required for the nucleon form factors Alternating signs and relation (2) have been obtained as well in quark model calculations [16]

** This point of view is quite consistent with our investigations of models in which $\rho_n p \leftrightarrow \rho_m p$ transitions with arbitrary $n,m \geqslant 0$ are taken into account [17].

account interference between different vector mesons) we have been able to derive scaling in the spacelike region from a $1/s$ behaviour in the time-like one, while still keeping the reasonable assumption of a constant (mass independent) vector meson nucleon cross section

Generalizing to include the isoscalar parts and evaluating (7) at $q^2 = 0$, where σ_T has to reduce to the total photoproduction cross section $\sigma_{\gamma p}$, we have

$$\sigma_{\gamma p}(W) = \alpha\pi(\gamma_\rho^{-2}\ \sigma_{\rho p} + \gamma_\omega^{-2}\ \sigma_{\omega p} + \gamma_\phi^{-2}\ \sigma_{\phi p})(1+\delta), \tag{10}$$

and may now write our final result in terms of $\sigma_{\gamma p}$

$$\sigma_T(W,q^2) = \frac{1}{\lambda}\left[\frac{\lambda(1+\delta)+q^2/m_\rho^2}{1+q^2/m_\rho^2} - \frac{q^2}{\lambda m_\rho^2}\ \psi^{(1)}\ \left(\frac{1+q^2/m_\rho^2}{\lambda}\right)\right]\frac{\sigma_{\gamma p}}{1+\delta} \tag{11}$$

Asymptotically this becomes

$$\sigma_T(W,q^2) \sim (\bar{m}^2/q^2)\,\sigma_{\gamma p}, \tag{12}$$

with

$$\bar{m}^2 = [(1+2\delta)/(2+2\delta)]\ m_\rho^2 \tag{13}$$

The only free parameter introduced, δ, which fixes the magnitude of the off-diagonal transitions, may now be determined from the normalization (10) at $q^2 = 0$. Empirically ρ^0, ω, ϕ saturate [18] the total photoproduction cross section up to 22% *, i e. $(1+\delta)^{-1} = 0\ 78$ and $\delta = 0\ 28$ The mass scale $\bar{m}^2$, determining the normalization of the asymptotic cross section σ_T, may then be computed from (13) and is obtained to be $\bar{m}^2 = 0.61\ m_\rho^2 = 0\ 36\ \mathrm{GeV}^2$.

Although (11) may easily be evaluated numerically from the tables for $\psi^{(1)}(z)$, it is advantageous to give a much simpler formula for σ_T, which for $\lambda = 2$ approximates (11) extremely well, the error being at most 2% (around $q^2 = 3\ m_\rho^2$ where deviations from (11) are largest) This approximate formula for (11) reads **

$$\sigma_T(W,q^2) = \frac{\bar{m}^2}{(q^2+\bar{m}^2)}\sigma_{\gamma p}, \tag{14}$$

with $\bar{m}^2 = 0.61\ m_\rho^2$ as calculated from (13).

* This number is determined from vector meson photoproduction assuming diagonal vector dominance It is not significantly changed in the off-diagonal model, as photoproduction of ρ^0, ω, ϕ at $t = 0$ is hardly changed by taking into account off-diagonal terms

** This approximative formula is good for $\lambda = 2$ In general (11) depends on λ, although the $q^2 = 0$ and $q^2 \to \infty$ limits are independent of λ

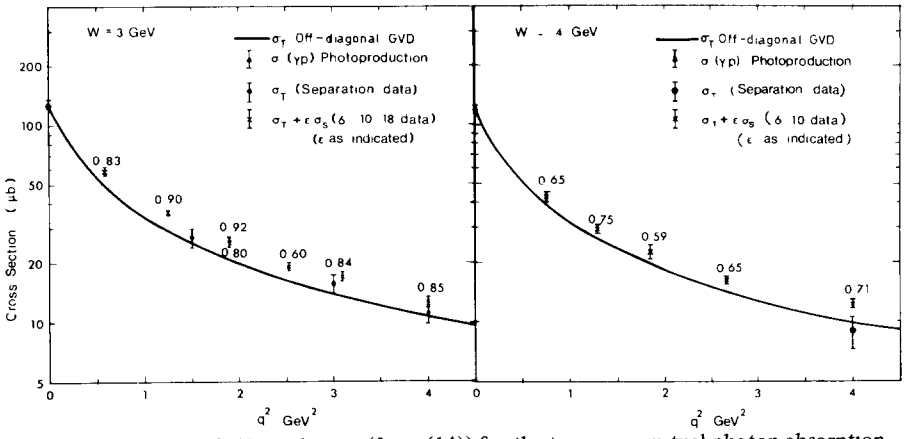

Fig. 3 Off-diagonal GVD prediction (from (14)) for the transverse virtual photon absorption cross section on protons, $\sigma_T(W, q^2)$, as a function of the virtual photon for momentum squared q^2

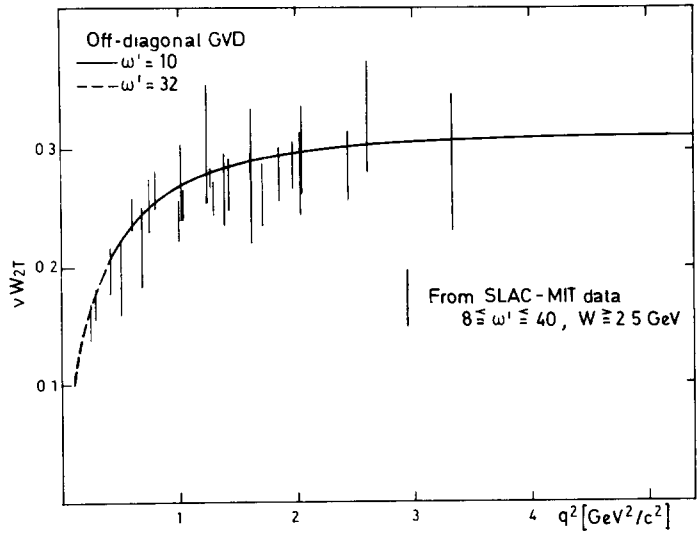

Fig. 4 The transverse part of the proton structure function νW_2 as a function of q^2 in the large ω' diffraction region showing the approach to the scaling limit. The theoretical curve is the off-diagonal GVD prediction calculated from (14) The data points have been computed [9] (see refs. [9] and [1] for details) from the measured 6°, 10° and 18° data [13].

It is amusing to note that the simple pole formula (14) which is equivalent to (11), had previously [9] been shown * to describe extremely well the data for the transverse part of νW_2 in the $\omega' \gtrsim 8$ region From eyeball fits to the data $\bar{m}^2$ had

* The simple pole formula (14) first arose [9] in the diagonal model justified from an effective $\sigma_{VNp} \sim 1/m_N^2$ law as a working hypothesis Putting in off-diagonal terms thus may appear equivalent in practice to an effective $1/m_N^2$ law We cannot adovcate such a point of view, however, as this equivalence is lost for other physical processes, e g ρ^0 electroproduction

been obtained to be $\bar{m}^2 = (0.611)^2 = 0\ 37\ \mathrm{GeV}^2$, compared with $\bar{m}^2 = 0\ 36\ \mathrm{GeV}^2$ as calculated in this paper. Also, as remarked by Sakurai [15], the pole formula (14) implies that the modified scaling variable $\omega_W \equiv (2M\nu + b^2)/(q^2 + a^2)$ of Rittenberg and Rubinstein [19] is a good variable, provided their parameter $a^2 \equiv \bar{m}^2$ In the fits [20] of the data a^2 has been found to be $0.37 \leqslant a^2 \lesssim 0\ 42$ in good agreement with our calculated value of $\bar{m}^2 = 0.36\ \mathrm{GeV}^2$. Agreement of (11) and (14) with the data is explicitly displayed in figs.3 and 4.

Concerning the small longitudinal part of the total cross section σ_S, let us just remark that inclusion of off-diagonal terms is straightforward, no novel features arise, and the conclusions of ref [1] for the ratio $R \equiv \sigma_S/\sigma_T$ are substantially unchanged.

From our ansatz (5), we note that diffraction dissociation, as exemplified by a single, effective off-diagonal term parametrized with C_N, increases with N, becoming a constant fraction of the elastic reaction. This feature seems a necessary one for convergence and scaling We have checked that a constant, N independent C_N gives a logarithmically divergent non-scaling expression except for the singular point $C_N \equiv \frac{1}{2}$, for which case the result is convergent, but also non-scaling with a leading term proportional to $(1/q^4) \ln (q^2/m_\rho^2)$.

To summarize, we are proposing a simple model for inelastic ep scattering in the large ω' diffraction region based upon off-diagonal GVD The model accounts for scaling behaviour for spacelike q^2 for the transverse part of νW_2 from a $1/s$ scaling law for the total cross section $e^+e^- \to$ hadrons. The only free parameter introduced is related to be magnitude of hadronic diffraction dissociation and is adjusted to yield the correct normalization to the total photoproduction cross section at $q^2 = 0$. We obtained excellent quantitative agreement with experiment, and in fact derived the correct numerical value for the mass parameter $\bar{m}^2 \simeq 0\ 61\ m_\rho^2 \simeq 0\ 36$ GeV^2, which sets the scale for the q^2 dependence and had previously been obtained from fits to the data Precocity of scaling follows naturally from the smallness of $\bar{m}^2$

Although we have thus stressed the numerical success of the model, and which indeed is not dissatisfying, we do not necessarily adhere too strictly to the specific way we have introduced off-diagonal transitions. It has been our main point to show by means of a constructive example that inclusion of interference between different vector states within the incoming photon allows us to obtain scaling in the spacelike region for ep scattering from scaling in $e^+e^- \to$ hadrons, while keeping the vector meson nucleon cross section independent of the vector meson mass. Deviations from the $1/s$ behaviour in e^+e^- annihilation, above 3.5 GeV c m energy as recently indicated [5] are expected [12] to also show up as violations of scaling for sufficiently large spacelike q^2. Further tests of off-diagonal GVD can be obtained from photo- and electroproduction of vector mesons.

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References

[1] J.J. Sakurai and D. Schildknecht, Phys Letters 40B (1972) 121, 41B (1972) 489, 42B (1972) 216,
J.J Sakurai, Proc of the 1972 McGill University Summer School, p 435,
D Schildknecht, Proc of the Eighth Rencontre de Moriond, ed J Tran Thanh Van, Vol. 1 p 181 (1973).
[2] A Bramon, E. Etim and M Greco, Phys Letters 41B (1972) 609,
M Greco, Nucl. Phys. B63 (1973) 398.
[3] $\mu\pi$ group
F Ceradini et al , Phys. Letters 42B (1972) 501, 47B (1973) 80,
$\gamma\gamma$ group
C. Bacci et al , Phys. Letters 44B (1973) 533,
Boson group
B Bartoli et al , Phys. Rev. D6 (1972) 2374,
V Silvestrini, Proc. of the 16th Int. Conf on high-energy Physics, Chicago-Batavia 1972 vol. 1, p. 1, ed J.D. Jackson and A Roberts
[4] A Litke et al , Phys. Rev. Letters 30 (1973) 1189, 1349E,
G. Tranopolsky et al , Phys. Rev. Letters 32 (1974) 432
[5] B Richter, Invited talk presented at the Irvine Conf , Dec 1973, and at the 17th Int Conf on high-energy physics, London 1974, SLAC-PUB-1478 (1974)
[6] M Greco, LNF-73/3(P) (1974).
[7] J. Bjorken, Proc of the Summer Institute of Particle Physics, 1973, SLAC report 167,
J D. Bjorken and J. Kogut, Phys. Rev. D8 (1973) 1341
[8] D Schildknecht, Nucl. Phys B66 (1973) 398
[9] B. Gorczyca and D Schildknecht, Phys Letters 47B (1973) 71.
[10] L. Foá, Proc. 2nd Int. Conf. on elementary particles, Aix-en-Provence, 1973, p 317,
D Leith, Diffractive processes, SLAC-PUB-1330 (1973).
[11] K.C Moffeit, Proc. of the Int Symp. on electron and photon interactions at high energies, Bonn 1973, North-Holland, Amsterdam, 1974)
[12] S Iwai and D Schildknecht, DESY 74/42 (1974), Phys Letters, submitted,
D Schildknecht and F Steiner, DESY 74/55 (1974), Phys Letters, submitted
[13] G. Miller et al., Phys Rev D5 (1972) 528,
E.D Bloom et al , Phys. Rev Letters 23 (1969) 930
[14] G Cocho et al., Phys. Letters 48B (1974) 63
[15] J J. Sakurai, Erice lectures 1973, UCLA/73/TEP/90
[16] M. Böhm, H. Joos and M Krammer, Acta Phys. Austriaca 38 (1973) 123
[17] H. Fraas, B. Read and D. Schildknecht, in preparation.
[18] G Wolf, Proc of the Int Symp on electron and photon interactions, Cornell University 1971
[19] V. Rittenberg and H R Rubinstein, Phys. Letters 35B (1971) 50.
[20] F.W. Brasse et al , Nucl Phys. B39 (1972) 521