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STICHEL RELATIONS IN THE MUELLER-REGGE THEORY OF INCLUSIVE PHOTO- AND ELECTROPRODUCTION

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Abstract: We develop the Mueller-Regge formalism for inclusive photo- and electroproduction of pions in the photon fragmentation sector. We discuss under what assumptions about the analyticity of the six-point function one can derive a Sommerfeld-Watson representation, which involves integrals over the crossed-channel helicities.

In particular we examine the properties of the so-called helicity-pole limit (H.P.L.), which is relevant in realizing the Mueller-Regge expansion in the fragmentation region. In the case of the four-point function there exist relations among the s-channel helicity amplitudes at high energies, if only Regge singularities of definite normality are exchanged in the t-channel (Stichel relations). One of the main points we make here is that in the case of inclusive distributions, these properties carry over to H.P.L. if we take into account only the leading helicity pole. An important consequence of these relations is that the inclusive photoproduction distributions vanish as $k_T^2 \to 0$ (k_T being the transverse momentum of the pions).

1. Introduction

In recent years much work has been done on the phenomenology of inclusive distributions. However although there have been a number of attempts to understand the detailed way, in which a Regge expansion arises in the case of an inclusive distribution [1–3], some aspects of this problem have not been adequately treated. In particular, when we include external helicity, it is important to see if and how those properties which one normally associates with a Regge pole theory of a two-body process, carry over to the case of an inclusive distribution. Photo- and electroproduction have always been a good testing ground for the helicity dependent properties of Regge theory [4] and for example, it was pointed out by Stichel [5], that when definite normality is exchanged in the t-channel, certain linear combinations of the helicity amplitudes vanish. This has the important consequence that the unpolarized differential cross section for photoproduction vanishes as $t \to 0$ [6]. In a previous work [7] we argued on heuristic grounds (by using elementary exchanges) that completely analogous relations exist for inclusive photo- and electroproduction of pions in the photon fragmentation sector. Further we showed in ref. [7] that as a consequence of these relations in a purely Regge pole model the inclusive photoproduction distributions of pions vanish as $k_T \to 0$ (k_T being the transverse momentum of the pions). Experimentally [8] this and related properties are not in evidence and it was concluded that absorption corrections (i.e. Regge cut contributions) are needed in order to reproduce the data [9]. In order that this interpretation is really binding it is necessary to establish that the symmetry relations mentioned above are indeed a property of the inclusive distributions. We shall attempt here on a more formal level to establish the Stichel relations for inclusive photo- and electroproduction of pions. This we do by deriving a generalized Sommerfeld-Watson representation of the inclusive distribution, starting from a model of the analyticity of the corresponding six-point function. Our analysis differs in certain important points from refs. [1–3] and is most directly related to the approach of White developed for the five-point function [10]. The latter involves a direct generalization of the usual Gribov-Froissart continuation of the partial wave amplitudes and Sommerfeld-Watson transformation of the partial-wave summation *. This approach is the most appropriate if one wants to examine the helicity dependence and the symmetry relations that arise if the process is determined by Regge poles carrying definite quantum numbers, in particular normality.

In sect. 2 we review the definitions and kinematics involved in a Mueller-Regge analysis of inclusive photo- and electroproduction of pions in the photon-fragmentation region. In particular we shall recall why the relevant asymptotic limit for the missing mass discontinuity is a helicity pole limit [1–3]. In sect. 3 we derive the generalized Sommerfeld-Watson representation and discuss how poles in the complex angular momentum plane determine the asymptotic behaviour in the helicity pole limit. We discuss the crossing and normality properties of the helicity amplitudes in sect. 4 and derive the Stichel relations for the inclusive distributions in the helicity pole limit in sect. 5. In the appendix we discuss the model of the analyticity on which our analysis is based.

2. Definitions and kinematics

The essential idea behind a Regge expansion of the one-particle distribution $\gamma p \to \pi X$ is to assume that the t-channel six-line function (fig. 1a):

$$T^{(t)}_{\lambda', \lambda} = T(\gamma(q, \lambda) + \pi(-k) + \gamma(-q', \lambda') + \pi(k') \to \overline{p}(-p) + p(p')) \tag{2.1}$$

can be related by crossing to the physical region of the six-line function in the s-channel (fig. 1b):

$$T^{(s)}_{\lambda',\lambda} = T(\lambda(q, \lambda) + p(p) + \pi(k') \to \gamma(q', \lambda') + p(p') + \pi(k)) \ . \tag{2.2}$$

The latter, through the Mueller optical theorem, determines the one-particle distribution, i.e.

* For a review of Regge theory, see Collins [11].

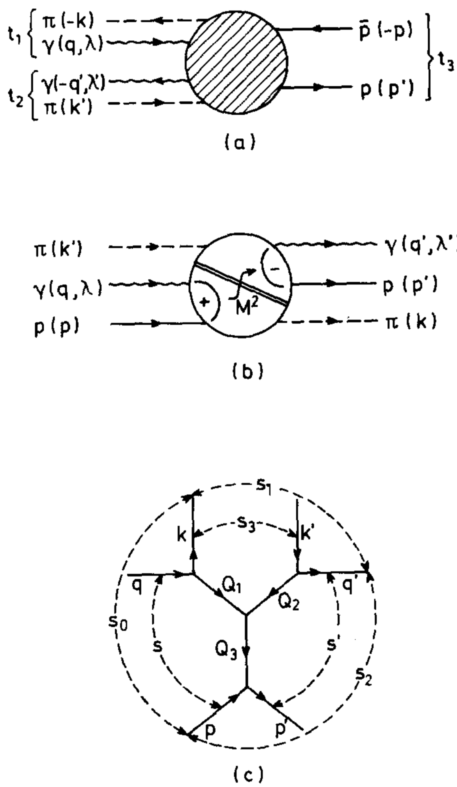


Fig. 1. (a) and (b) show configurations of the six-point function $\gamma p \pi \to \gamma' p' \pi'$, (a) depicting the t- or crossed-channel and (b) the s- or direct-channel missing mass discontinuity.

$$2k_0 \frac{d^3\sigma}{d^3k} = F_{\text{eff}}^{-1} \sum_{\lambda', \lambda} (\text{Disc}_{M_X^2}\, T^{(s)}_{\lambda', \lambda})_{k=k',\, p=p',\, q=q'} \,, \tag{2.3}$$

(where F_{eff} is the effective flux factor for the virtual photon [7]).

In both (2.1) and (2.2) it is understood that we averaged over the proton helicities. The configuration in fig. 1a will be referred to as the crossed channel (or t-channel), while fig. 1b shall be referred to as the direct channel (or s-channel) of the six-line function. A symmetric set of four vectors and invariants for the various channels are defined in fig. 1c, so that

$$s = (p+q)^2 \,, \qquad s' = (p'+q')^2 , \qquad s_0 = M_X^2 = (p+q-k)^2 = (p+Q_1)^2 \,,$$

$$s_1 = (k-k'+q')^2 = (q-Q_3)^2 \,, \tag{2.4}$$

$$s_2 = (p-p'-q')^2 = (q'+Q_3)^2 \,,$$

$$s_3 = (k - k')^2 , \qquad t_1 = (q - k)^2 ,$$
$$t_2 = (q' - k')^2 , \qquad t_3 = (p - p')^2 . \tag{2.4}$$

For defining the kinematics of the crossed channel partial-wave expansions we need the c.m.s. systems "1", "2" and "3" which are defined as follows:

"1" c.m.s. of $\gamma(q, \lambda)\, \pi(-k)$ $[\boldsymbol{q} + (-\boldsymbol{k}) = 0]$: (2.5)

$$Q_1 = q - k = (\sqrt{t_1}, 0, 0, 0) ,$$
$$q = (E_q, \boldsymbol{q}) , \quad \hat{\boldsymbol{q}} = \frac{\boldsymbol{q}}{|\boldsymbol{q}|} = (\theta_1, \varphi_1) , \quad |\boldsymbol{q}| = \frac{1}{2\sqrt{t_1}} \lambda^{\frac{1}{2}}(t_1, q^2, m_\pi^2) ,$$
$$k = (-E_k, \boldsymbol{q}) , \quad E_q = \frac{1}{2\sqrt{t_1}} (t_1 + q^2 - m_\pi^2) , \qquad E_k = \frac{1}{2\sqrt{t_1}} (t_1 + m_\pi^2 - q^2) .$$

"2" c.m.s. of $\pi(k')\, \gamma(-q', \lambda')$ $[\boldsymbol{k}' + (-\boldsymbol{q}') = 0]$: (2.6)

$$Q_2 = k' - q' = (\sqrt{t_2}, 0, 0, 0) ,$$
$$q' = (-E_{q'}, \boldsymbol{q}') , \qquad \hat{\boldsymbol{q}}' = \frac{\boldsymbol{q}'}{|\boldsymbol{q}'|} = (\theta_2, \varphi_2) , \qquad |\boldsymbol{q}'| = \frac{1}{2\sqrt{t_2}} \lambda^{\frac{1}{2}}(t_2, q^2, m_\pi^2) ,$$
$$k' = (E_{k'}, \boldsymbol{q}') , \quad E_{q'} = \frac{1}{2\sqrt{t_2}} (t_2 + q^2 - m_\pi^2) , \quad E_{k'} = \frac{1}{2\sqrt{t_2}} (t_2 + m_\pi^2 - q^2) .$$

"3" c.m.s. of $p(p')\, \bar{p}\,(-p)$ $[\boldsymbol{p}' + (-\boldsymbol{p}) = 0]$: (2.7)

$$Q_3 = p' - p = (\sqrt{t_3}, 0, 0, 0) ,$$
$$p' = (E_p, \boldsymbol{p}) ,$$
$$p = (-E_p, \boldsymbol{p}) , \qquad \hat{\boldsymbol{p}} = \frac{\boldsymbol{p}}{|\boldsymbol{p}|} = (\theta_3, \varphi_3) ,$$
$$E_p = \tfrac{1}{2} \sqrt{t_3} , \qquad |\boldsymbol{p}| = \tfrac{1}{2} (t_3 - 4m^2)^{\frac{1}{2}} .$$

The center-of-mass systems "1" and "2" can be reached by boots along the z-axis, i.e.

$$Q_1^s = B_z(\chi_1)\, (\sqrt{t_1}, 0, 0, 0) = (\sqrt{t_1} \cosh \chi_1, 0, 0, \sqrt{t_1} \sinh \chi_1) ,$$
$$Q_2^s = B_z(\chi_2)\, (\sqrt{t_2}, 0, 0, 0) = (\sqrt{t_2} \cosh \chi_2, 0, 0, \sqrt{t_2} \sinh \chi_2) . \tag{2.8}$$

The boost parameters χ_1 and χ_2 are related to t_1, t_2 and t_3 respectively by

$$\sinh\chi_1 = (\lambda(t_1,t_2,t_3)/4t_1t_3)^{\frac{1}{2}}\ , \qquad \sinh\chi_2 = -(\lambda(t_1,t_2,t_3)/4t_2t_3)^{\frac{1}{2}}\ , \tag{2.9}$$

so that from (2.8)

$$Q_1^s + Q_2^s = (\sqrt{t_1}\cosh\chi_1 + \sqrt{t_2}\cosh\chi_2, 0, 0, 0)\ . \tag{2.10}$$

We define

$$C_i = \cosh\chi_i\ , \qquad S_i = \sinh\chi_i\ . \tag{2.11}$$

Then the four vectors q, k, k', q', p and p' are represented by:

$$q = (C_1E_q + S_1|q|\cos\theta_1, |q|\sin\theta_1\cos\varphi_1, |q|\sin\theta_1\sin\varphi_1, S_1E_q + C_1|q|\cos\theta_1)\ ,$$

$$k = (-C_1E_k + S_1|q|\cos\theta_1, |q|\sin\theta_1\cos\varphi_1, |q|\sin\theta_1\sin\varphi_1, -S_1E_q + C_1|q|\cos\theta_1)\ ,$$

$$q' = (-C_2E_{q'} + S_2|q|\cos\theta_2, |q'|\sin\theta_2\cos\varphi_2, |q'|\sin\theta_2\sin\varphi_2, -S_2E_{q'} + C_2|q'|\cos\theta_2)$$

$$k' = (C_2E_{k'} + S_2|q'|\cos\theta_2, |q'|\sin\theta_2\cos\varphi_2, |q'|\sin\theta_2\sin\varphi_2, S_2E_{q'} + C_2|q'|\cos\theta_2)\ ,$$

$$p = (-E_p, |p|\sin\theta_3\cos\varphi_3, |p|\sin\theta_3\sin\varphi_3, |p|\cos\theta_3)\ ,$$

$$p' = (E_p, |p|\sin\theta_3\cos\varphi_3, |p|\sin\theta_3\sin\varphi_3, |p|\cos\theta_3)\ . \tag{2.12}$$

For defining the t-channel partial-wave expansion we use the eight variables

$$t_1, t_2, t_3, \theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3\ , \tag{2.13}$$

and for the crossing to the direct channel the set of nine symmetric invariants

$$s, s', s_0, s_1, s_2, s_3, t_1, t_2, t_3\ . \tag{2.14}$$

defined in (2.4). Only eight of the above invariants are independent there being one non-linear constraint (arising from the dimensionality of space time). We can avoid this constraint by restricting ourselves to a submanifold, in which the four 4-vectors, whose associated channel invariants are all t variables, are linearly dependent (see sect. 3 and the appendix). The nine invariants (2.4) are related to the eight variables t_1, t_2, t_3, θ_1, φ_1, θ_2, φ_2 and θ_3(by setting $\varphi_3 = 0$):

$$s - m^2 - q^2 = 2pq = -2E_p(C_1E_q + S_1|q|\cos\theta_1) - 2|p|\,|q|\sin\theta_3\sin\theta_1\cos\varphi_1$$
$$- 2|p|\cos\theta_3(S_1E_q + C_1|q|\cos\theta_1)\ ,$$

$$s' - m^2 - q^2 = 2p'q' = -2E_p(C_2E_{q'} - S_2|q'|\cos\theta_2) - 2|p|\,|q'|\sin\theta_3\sin\theta_2\cos\varphi_2$$
$$+ 2|p|\cos\theta_3(S_2E_{q'} - C_2|q'|\cos\theta_2)\ ,$$

$$M_X^2 = m^2 + t_1 - 2E_p\sqrt{t_1}\,C_1 + 2|p|\sqrt{t_1}\,S_1\cos\theta_3\ , \tag{2.15}$$

$$
\begin{aligned}
s_1 &= t_3 + q^2 - 2\sqrt{t_3}\,(C_1 E_q + S_1 |q| \cos\theta_1)\,, \\
s_2 &= t_3 + q^2 - 2\sqrt{t_3}\,(C_2 E_{q'} - S_2 |q'| \cos\theta_2)\,, \\
s_3 &= 2m_\pi^2 + 2\,(C_1 E_k - S_1 |q| \cos\theta_1)(C_2 E_{k'} + S_2 |q'| \cos\theta_2) \\
&\quad + 2\,|q|\,|q'| \sin\theta_1 \sin\theta_2 \cos(\varphi_1 - \varphi_2) - 2\,(S_1 E_k - C_1 |q| \cos\theta_1) \\
&\qquad \times (S_2 E_{q'} + C_2 |q'| \cos\theta_2)\ .
\end{aligned} \tag{2.15}
$$

2.1. Double and triple Regge limits

We are interested in the double (D.R.L.) and the triple (T.R.L.) Regge limits of the forward direct-channel amplitudes defined above. These are given by:

$$
\begin{aligned}
&\text{D.R.L.: } s \to s' \to \infty, \text{ all other variables fixed and} \\
&\qquad s_0 = M_X^2,\ \ t_1 = t_2 = t\,,\quad t_3 = 0\,,\ \ s_1 = s_2 = q^2,\ \ s_3 = 0\,.
\end{aligned} \tag{2.16}
$$

T.R.L.: $s/M_X^2 \to s'/M_X^2 \to \infty$, all other variables fixed as in D.R.L.

Clearly the T.R.L. is a further asymptotic limit to the D.R.L., so that one should consider the latter first. These limits are however problematic. In particular there exists more than one route, by which the D.R.L. can be reached kinematically. For example, if we divide the limit into two steps: L_1 is going to the forward direction ($t_1 \to t_2 = t$, $t_3 \to 0$) and L_2 is the direct asymptotic limit s, $s' \to \infty$, other variables fixed. When realizing L_1 and L_2 in terms of the Regge variables (2.13) one sees that $L_1 L_2$ and $L_2 L_1$ are not necessarily equivalent. In fact by requiring these limits to commute one obtains additional constraints. Let us beginn by analysing the limit L_1. Here the relevant quantities in the relationship between the Regge variables (2.13) and the invariants (2.14) are the boost parameters χ_i ($i = 1, 2$) defined in (2.9), i.e.

$$
\sinh \chi_i = \left\{ \frac{1}{4 t_i t_3} \left(t_3 - (\sqrt{t_1} + \sqrt{t_2})^2\right)\left(t_3 - (\sqrt{t_1} - \sqrt{t_2})^2\right) \right\}^{\frac{1}{2}} . \tag{2.17}
$$

From (2.17) one readily sees that the limit $t_1 \to t_2 \to t, t_3 \to 0$ depends crucially on the path taken. For example:

(a) $\lim\limits_{t_3 \to 0}\ \lim\limits_{t_1 \to t_2}\ \sinh \chi_i = i$,

(b) $\lim\limits_{t_3 \to 0}\ \lim\limits_{t_3 = (\sqrt{t_1} \pm \sqrt{t_2})^2}\ \sinh \chi_i = 0$,

(c) $\lim\limits_{t_1 \to t_2}\ \lim\limits_{t_3 \to 0}\ \sinh \chi_i = \infty$.

The correct limit presumably depends on the dynamics. We shall later use possibility (a), which is consistent with $k = \lambda k'$. In this case

$$C_1 = C_2 = 0\,, \qquad S_1 = S_2 = i\,, \qquad p = im\,,$$

so that one reads from (2.15)

$$\begin{aligned}
s &- m^2 - q^2 = -2\,im\,|\boldsymbol{q}|\sin\theta_3 \sin\theta_1 \cos\varphi_1 + \frac{m}{\sqrt{t}}(t + q^2 - m_\pi^2)\cos\theta_3\,,\\
s' &- m^2 - q^2 = -2\,im\,|\boldsymbol{q}|\sin\theta_3 \sin\theta_2 \cos\varphi_2 - \frac{m}{\sqrt{t}}(t + q^2 - m_\pi^2)\cos\theta_3\,,\\
s_1 &= s_2 = q^2\,,\\
M_X^2 &= m^2 + t - 2m\cos\theta_3\,,\\
s_3 &= 2(m_\pi^2 + q^2)(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\varphi_1 - \varphi_2) + E_k E_q)\,.
\end{aligned} \tag{2.18}$$

Now L_2 corresponds to taking

$$\sin\theta_1\cos\varphi_1\,, \qquad \sin\theta_2\cos\varphi_2 \to \infty\,, \tag{2.19}$$

with the constraint

$$\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\varphi_1 - \varphi_2) = 1\,. \tag{2.20}$$

The degree of ambiguity in (2.20) is clear. In particular, if we take $\sin\theta_1$, $\sin\theta_2 \to \infty$, $\varphi_1 = \varphi_2$ fixed, we have a Regge pole limit, while on the other hand, if we take $\cos\varphi_1$, $\cos\varphi_2 \to \infty$, $\cos\theta_1$, $\cos\theta_2$ fixed, we have the helicity pole limit. To see that the triple Regge limit, from the kinematical point of view, corresponds to the latter, one need only consider the second ordering $L_1 L_2$ (i.e. L_2 first), where L_2 is the limit $s, s' \to \infty$, t_1, t_2, t_3 fixed and $t_3 \neq 0$. Assuming either $\cos\theta_i$ and/or $\cos\varphi_i \to \infty$ ($i = 1, 2$) we have

$$\begin{aligned}
s &= -2E_p S_1 |\boldsymbol{q}|\cos\theta_1 - 2|\boldsymbol{p}|\,|\boldsymbol{q}|\sin\theta_3\sin\theta_1\cos\varphi_1 - 2|\boldsymbol{p}|\,|\boldsymbol{q}|C_1\cos\theta_3\cos\theta_1\,,\\
s' &= 2E_p S_2 |\boldsymbol{q}'|\cos\theta_2 - 2|\boldsymbol{p}|\,|\boldsymbol{q}'|\sin\theta_3\sin\theta_2\cos\varphi_2 - 2|\boldsymbol{p}|\,|\boldsymbol{q}'|C_2\cos\theta_3\cos\theta_2\,,\\
s_1 &= -2\sqrt{t_3}\,S_1|\boldsymbol{q}|\cos\theta_1\,,\\
s_2 &= 2\sqrt{t_3}\,S_2|\boldsymbol{q}'|\cos\theta_2\,,\\
s_3 &= -2|\boldsymbol{q}|\,|\boldsymbol{q}'|(S_1S_2 - C_1C_2)\cos\theta_1\cos\theta_2 + 2|\boldsymbol{q}|\,|\boldsymbol{q}'|\sin\theta_1\sin\theta_2\cos(\varphi_1 - \varphi_2)\,.
\end{aligned} \tag{2.21}$$

Hence we see that if the D.R.L. is reached through $\cos\theta_1$, $\cos\theta_2 \to \infty$, then s_1, s_2 and $s_3 \to \infty$. However in the forward direction we know $s_1 = s_2 = q^2$ and $s_3 = 0$. This means on the grounds of smoothness we must opt for the helicity pole limit, since $\cos\varphi_1$, $\cos\varphi_2 \to \infty$ is not problematic with respect to the interchange of limits.

Since we shall be imposing the constraint $k = \lambda k'$, it is convenient to use the ordering of the limits $L_2 L_1$, by going first to the forward direction $t_1 \to t_2$ then $t_3 \to 0$. This leads to constraints on θ_1 and θ_2 which follow from (2.12). The derivation goes as follows. From $t_1 \to t_2$ we have $|q| = |q'|$, $E_q = E_{q'}$ and $E_k = E_{k'}$. Then we take $k = k'$, which according to (2.12) is equivalent to

$$\begin{aligned}
&(1) \quad -C_1 E_k + S_1 |q| \cos\theta_1 = C_2 E_2 + S_2 |q| \cos\theta_2 \,, \\
&(2) \quad \sin\theta_1 \cos\varphi_1 = \sin\theta_2 \cos\varphi_2 \,, \\
&(3) \quad \sin\theta_1 \sin\varphi_1 = \sin\theta_2 \sin\varphi_2 \,, \\
&(4) \quad -S_1 E_k + C_1 |q| \cos\theta_1 = S_2 E_k + C_2 |q| \cos\theta_2 \,.
\end{aligned} \tag{2.22}$$

From (2) and (3) we see that $\theta_1 = \theta_2 = \theta$, $\varphi_1 = \varphi_2 = \varphi$. When $t_1 = t_2$, $S_1 = -S_2 = S$ and $C_1 = C_2 = C$, so that (4) is automatically fulfilled. From (1) we have

$$(5) \quad 2S\,|q| \cos\theta = 2CE_k \,,$$

and since for $t_3 = 0$, $S = \sinh\chi_1 = i$ and $C = \cosh\chi_1 = 0$, we see from (5) that $\cos\theta = 0$ or $\theta = \frac{1}{2}\pi$. Hence the limit, which determines the inclusive distribution, is

$$\varphi_1 = \varphi_2 = \varphi \,, \qquad \cos\varphi \to \infty \,, \qquad \text{and} \qquad \theta_1 = \theta_2 = \tfrac{1}{2}\pi \quad . \tag{2.23}$$

2.2. t-channel partial-wave expansion

Referring to (2.1) and fig. 1a, the t-channel amplitude is denoted by

$$T^{(t)}_{\mu_2,\mu_1,\lambda',\lambda} = \langle \hat{p}; \mu_2, \mu_1 | T | (\hat{q}', \lambda', 0)_\mathrm{T}, \hat{q}, \lambda, 0 \rangle \,. \tag{2.24}$$

where $(\ldots)_\mathrm{T}$ denotes the time reversed system. We introduce O(3) states in the $\gamma\pi$ channels t_1 and t_2 to obtain the partial-wave expansion

$$T^{(t)}_{\mu_2,\mu_1,\lambda',\lambda} = \sum_{J',J,M',M} \langle \hat{p}, \mu_2, \mu_1 | T | J', M', \lambda', J, M, \lambda \rangle \langle J', M', \lambda' | (\hat{q}', \lambda', 0)_\mathrm{T} \rangle$$
$$\times \langle J, M, \lambda | \hat{q}, \lambda, 0 \rangle \,. \tag{2.25}$$

The expansion coefficients are given by (using the Jacob and Wick convention [12]):

$$\begin{aligned}
&\langle J_1 M_1 \lambda | q, \lambda, 0 \rangle = N_J D^J_{M,\lambda}(\varphi_1\, \theta_1, -\varphi_1) \,, \qquad N_J^2 = (2J+1)/4\pi \,, \\
&\langle J', M', \lambda' | (q', \lambda', 0)_\mathrm{T} \rangle = N_{J'} (-1)^{J'-M'} D^{J'*}_{-M,\lambda'}(\varphi_{2'}\, \theta_{2'}, -\varphi_2) \,,
\end{aligned} \tag{2.26}$$

where we use

$$\langle T\varphi, T\psi \rangle = \langle \varphi, \psi \rangle^* \quad \text{and} \quad T | J', M', \lambda' \rangle = (-1)^{J'-M'} | J', -M', \lambda' \rangle \,.$$

Hence

$$T^{(t)}_{\mu_2,\mu_1,\lambda',\lambda} = \sum_{J',J,M',M} N_{J'} N_J D^{J'*}_{-M',\lambda'}(\varphi_2,\theta_2,-\varphi_2)\, D^J_{M,\lambda}(\varphi_1,\theta_1,-\varphi_1)\, A^{(t)}_{\mu_2,\mu_1,\lambda',\lambda} , \tag{2.27}$$

where

$$A^{(t)}_{\mu_2,\mu_1,\lambda',\lambda} = (-1)^{J'-M'} \langle \hat{p}, \mu_2, \mu_1 | T | J', M', \lambda', J, M, \lambda \rangle . \tag{2.28}$$

We shall use later the truncated amplitudes

$$\widetilde{T}^{(t)}_{\mu_2,\mu_1,\lambda',\lambda} = e^{i(\lambda\varphi_1 - \lambda'\varphi_2)}\, T^{(t)}_{\mu_2,\mu_1,\lambda',\lambda} . \tag{2.29}$$

3. Sommerfeld-Watson representation and helicity poles

In this section we discuss the Sommerfeld-Watson (SW) transformation of the t-channel partial-wave expansions defined in sect. 2. The problem of defining signatured amplitudes or partial-wave amplitudes that satisfy the conditions of Carlson's theorem, is closely related to the underlying analytic structure of the six-point function. This is in general expected to be rather complicated. However some simplified models do exist, which may be good approximations in certain circumstances [13, 14]. The most appropriate for our purposes is a generalized fixed t dispersion representation proposed by Dahmen, Steiner and Konetschny [14], which includes all the discontinuities appropriate to the double Regge expansion and explicitly satisfies the Steinmann constraints [15] for the basic cuts *. This representation is written for the retarded function and is described in the appendix. It fulfills the causal requirements of the retarded six-point function and has the crossing properties one would expect from, for example, the dual six-point function [2]. The corresponding representation for the missing mass discontinuity in which we are interested is given by (see the appendix):

$$T = \frac{1}{\pi^2} \sum_{i,j=1}^{2} \int_{\sigma_0^i}^{\infty} d\sigma_i \int_{\sigma_0^j}^{\infty} d\sigma_j' \frac{\rho_{ij}(\sigma_i, \sigma_j', M^2, \{t\})}{(\sigma_i - s_i + i\epsilon)(\sigma_j' - s_j' - i\epsilon)} , \tag{3.1}$$

where

$$s_1 = s = (p+q)^2 , \qquad s_2 = u = (p-k)^2 ,$$
$$s_1' = s' = (p'+q')^2 , \qquad s_2' = u' = (p'-k')^2 ,$$

and $\{t\}$ denotes in general five t variables; however we shall restrict ourselves to the submanifold $k = \lambda k'$, which is sufficient for our purposes. In general when we include spin, the full amplitude infact is expressible in terms of a number of invariant functions, each having such a representation. However, it is well known [11] from the four point function that, when one removes the kinematic singularities, the helicity amplitudes also have the basic representation. For $k = \lambda k', \{t\} = \{t_1, t_2, t_3\}$ we

* By basic cuts we mean those involving multiparticle normal discontinuities.

write the helicity structure functions $T^{t}_{\lambda',\lambda}$ defined in sect. 2 in the form (neglecting the spin of the proton)

$$T^{(t)}_{\lambda',\lambda} = \frac{1}{16\pi^2} \sum_{M=-\infty}^{\infty} \sum_{J=M}^{\infty} \sum_{M'=-\infty}^{\infty} \sum_{J'=M'}^{\infty} (2J+1)(2J'+1)$$
$$\times z'^{M'} d^{J'}_{-M',\lambda'}(x')\, z^{M} d^{J}_{M,\lambda}(x)\, A^{t}_{\lambda',\lambda}(J',M',J,M,M_X^2,\{t\}) , \tag{3.2}$$

and from (3.1) and (3.2) we see that we can discuss the (SW) transformation of the respective partial-wave summations separately. To this end we write (3.1) in the form

$$T(s,t_1,M_X^2,\upsilon) = \frac{1}{\pi} \int^{\infty} ds' \frac{\rho_s(s',t_1,M_X^2,\upsilon)}{s'-s+i\epsilon} + \frac{1}{\pi}\int^{\infty} du' \frac{\rho_u(u',t_1,M_X^2,\upsilon)}{u'-u+i\epsilon} ,$$

where

$$\rho_s(s,t_1,M_X^2,\upsilon) = \frac{1}{\pi} \sum_{j=1}^{2} \int_{\sigma_0^j}^{\infty} d\sigma_j' \frac{\rho_{1j}(s,\sigma_j',M_X^2,t_1,t_2,t_3)}{\sigma_j' - s_j' - i\epsilon} , \tag{3.3}$$

and similarly for ρ_u replacing ρ_{1j} by ρ_{2j} and (3.2) in the form

$$T_\lambda(z,x,t_1,M_X^2,\upsilon) = \frac{1}{4\pi} \sum_{M=-\infty}^{\infty} \sum_{J=\mathrm{Max}\,|M|,|\lambda|} (2J+1)\, z^M d^J_{M,\lambda}(x)\, a_\lambda(J,M,t_1,M_X^2,\upsilon) \tag{3.4}$$

$$a_\lambda(J,M,t_1,M_X^2,\upsilon) = \frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{z^{M+1}} \int_{-1}^{1} dx\, d^J_{M,\lambda}(x)\, T_\lambda(z,x,t_1,M_X^2,\upsilon) . \tag{3.5}$$

In (3.3) – (3.5) we have used υ to denote the set of unexhibited variables. The connection between (3.3) and (3.4) comes about when we remove the half angle kinematic singularity factor from (3.4) * i.e.

$$T_\lambda = \left(\frac{1-x}{1+x}\right)^{\frac{1}{2}\lambda} T . \tag{3.6}$$

The angular variables in (3.4) are related to the variables s and u defined in (2.15), from which we obtain

$$\frac{s}{M_X^2} = az\,(1-x^2)^{\frac{1}{2}} + bx + c , \qquad z = e^{i\varphi} , \qquad x = \cos\theta ,$$

where

$$a = \frac{1}{2t_1 \sinh \chi_1} \lambda^{\frac{1}{2}}(t_1, q^2, m_\pi^2) + a_0 ,$$
$$b = a\cosh\chi_1 + b_0 , \tag{3.7}$$

* Here we concentrate on only the singularities relevant to the present discussion.

$$c = \tfrac{1}{2}(t_1 + q^2 - m_\pi^2) + c_0 \ , \tag{3.7}$$

(a_0, b_0 and c_0 all vanish like t_3 as $t_3 \to 0$ and are of order M_X^{-2}.) Using (3.7) we can replace (3.3) by

$$T = \frac{1}{\pi} \int_{-u_0(x)}^{\infty} dz' \, \frac{\rho_s}{z' - z} + \frac{1}{\pi} \int_{1-u_0(x)}^{-\infty} dz' \, \frac{\rho_u}{z' - z} + T_r \ , \tag{3.8}$$

where in the limit s, $s/M_X^2 \gg 1$, $T_r = O(1/s)$ and $u_0(x) = (bx + c)/a(1 - x^2)^{\frac{1}{2}}$. (3.8) is sufficient to allow us to define the Gribov-Froissart projections of the partial-wave amplitudes (3.5) into both the complex M and J planes and consequently to make the SW transformation of (3.4). We begin by considering the summation over M, where we essentially repeat the analysis of White [10] and write (3.8) in the form

$$T = \frac{1}{\pi} \left\{ \int_{R_>^+} + \int_{R_>^-} + \int_{R_<^+} + \int_{R_<^-} \right\} dz' \, \frac{\rho}{z' - z} \ , \tag{3.9}$$

where

$$R_>^\pm = \{ |z| > 1, \mathrm{Re}\, z \gtrless 0 \} \ , \qquad R_<^\pm = \{ |z| < 1, \mathrm{Re}\, z \gtrless 0 \} \ ,$$

and ρ can be read off from (3.8). Writing (3.4) and (3.5) in the form (suppressing for the moment the x dependence)

$$T = \sum_{M=-\infty}^{\infty} b_M z^M \ , \tag{3.10}$$

$$b(M) = \frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{z^{M+1}} T(z) \ , \tag{3.11}$$

and inserting (3.9) in (3.11) we see that

$$b(M) = \begin{cases} b_>(M) \ , & M \geqslant 0 \ , \\ b_<(M) \ , & M < 0 \ , \end{cases}$$

where

$$b_\gtrless(M) = b_\gtrless^+(M) + b_\gtrless^-(M) \ ,$$

with

$$b_\gtrless^\tau(M) = \frac{1}{\pi} \int_{R_\gtrless^+} dz' \, z'^{-M-1} \rho - \frac{\tau}{\pi} \int_{R_\gtrless^-} dz' \, (-z')^{-M-1} \rho \ . \tag{3.12}$$

Hence $b(M)$ is separated into four sets $\{b_\gtrless^\pm(M)\}$ according to whether M is even or odd and $M \geqslant 0$ or $M < 0$. From (3.12) it is easy to see that

$$|b^\tau_>(M)| < (e^{\alpha|M|})\,, \qquad \mathrm{Re}\,M > 0\ ,$$

$$|b^\tau_<(M)| < (e^{\alpha|M|})\,, \qquad \mathrm{Re}\,M < 0\ .$$

as $|M| \to \infty$ with $\alpha < \pi$. Hence the $\{b^\tau_\gtrless(M)\}$ satisfy the conditions of Carlson's theorem and therefore can be projected to complex M; correspondingly (3.10) has the SW representation

$$T = 2i \sum_{\tau=\pm} \left\{ \int_{C_>} \frac{dM(-1)^M}{\sin M\pi}\, b^\tau_>(M)\, \tfrac{1}{2}\,(z^M + \tau(-z)^M) \right. + \int_{C_<} \frac{dM(-1)^M}{\sin M\pi}\, b^\tau_<(M)\, \tfrac{1}{2}\,(z^M + \tau(-z)^M)\ , \tag{3.13}$$

where the contours $C_\gtrless$ are shown in fig. 2. In eq. (3.13)

$$b^\tau_\gtrless(M,x) = \sum_{J=\mathrm{Max}\ |M|,|\lambda|} (2J+1)\, a^\tau_{\lambda,\lessgtr}(J,M)\, d^J_{M,\lambda}(x)\ , \tag{3.14}$$

where

$$a^\tau_{\lambda,\gtrless}(J,M) = \int_{-1}^{1} dx\ d^J_{M,\lambda}(x)\, b^\tau_\gtrless(M,x)\ . \tag{3.15}$$

We now consider the problem of continuing $a^\tau_{\lambda,\gtrless}(J,M)$ to complex J, restricting ourselves to the $\lambda = 0$ case (generalization to $\lambda = \pm 1$ is essentially only technical). A representation of the $\{b^\tau_\gtrless(M)\}$ can be read off from eq. (3.8) to (3.12) and it shows that they have a complicated overlapping cut structure in x. Part of this however is induced by the representation itself through the half angle kinematic factor $(1-x^2)^{-\frac{1}{2}|M|}$. We therefore consider $(1-x^2)^{\frac{1}{2}|M|}\, b^\tau_\gtrless(M,x)$, which from the original representation (3.3) we know must be analytic in the upper half x-plane. This means we can represent the function by

$$(1-x^2)^{\frac{1}{2}|M|}\, b^\tau_\gtrless(M,x) = \int_0^\infty f^\tau_\gtrless(M,\alpha)\, e^{i\alpha x}\, d\alpha\ , \tag{3.16}$$

where the specific form of $f^\tau_\gtrless$ is of no interest to us. Inserting (3.16) in (3.15) we obtain

$$a^\tau_\gtrless(J,M) = \int_0^\infty d\alpha\, f^\tau_\gtrless(M,\alpha) \int_{-1}^{1} dx\, e^{i\alpha x}\, N(J,M,0)\,(1-x^2)^M\, P_J^{(M,M)}(x), \tag{3.17}$$

where

$$N(J,M,\lambda) = \left(\frac{\Gamma(J-\lambda+1)\,\Gamma(J+\lambda+1)}{\Gamma(J+M+1)\,\Gamma(J-M+1)}\right)^{\frac{1}{2}}\ , \tag{3.18}$$

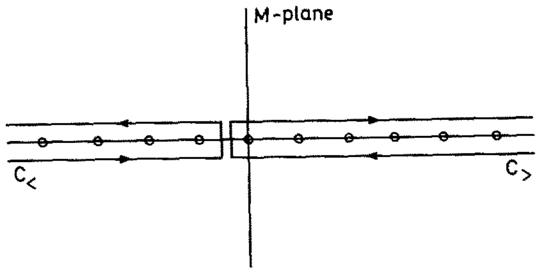


Fig. 2. Configuration of the contours in the M plane.

and $P_n^{(\alpha,\beta)}(x)$ are Jacobi polynomials.

Using the symmetry property

$$P_J^{(M,M)}(x) = (-1)^J P_J^{(M,M)}(-x) \ , \tag{3.19}$$

we can write

$$a^{\tau}_{\gtrless}(J,M) = \begin{cases} 2N(J,M,0) \int_0^\infty \mathrm{d}\alpha f^{\tau}_{\gtrless}(M,\alpha) \int_0^1 \mathrm{d}x \cos \alpha x (1-x^2)^M P_J^{(M,M)}(x) \\ \quad \text{for } J \text{ even} \ , \\ 2i N(J,M,0) \int_0^\infty \mathrm{d}\alpha f^{\tau}_{\gtrless}(M,\alpha) \int_0^1 \mathrm{d}x \sin \alpha x (1-x^2)^M P_J^{(M,M)}(x) \\ \quad \text{for J odd} \ . \end{cases} \tag{3.20}$$

Hence if we define the signatured amplitudes

$$a^{\tau s}_{\gtrless}(J,M) = \int_{-1}^{1} \mathrm{d}x \, d^J_{M,0}(x) \, (b^{\tau}_{\gtrless}(M,x) + s(-1)^J b^{\tau}_{\gtrless}(M,-x)) \ . \tag{3.21}$$

we obtain for them the representation

$$a^{\tau}_{\gtrless}(J,M) = \mathrm{e}^{\frac{1}{2}i\pi J} \left(\frac{\Gamma(J+M+1)}{\Gamma(J-M+1)}\right)^{\frac{1}{2}} \sqrt{2\pi} \int_0^\infty \mathrm{d}\alpha f^{\tau}_{\gtrless}(M,\alpha)\, \alpha^{-M-\frac{1}{2}} J_{J+M+\frac{1}{2}}(\alpha) \ , \tag{3.22}$$

(similarly for $a^{\tau}_{\gtrless}(J,M)$). Using

$$\lim_{\nu\to\infty} J_\nu(z) = \frac{1}{\sqrt{2\pi\nu}} \left(\frac{ez}{2\nu}\right)^\nu \ , \tag{3.23}$$

we see that

$$a^{\tau s}_{\gtrless}(J,M) \underset{|J|\to\infty}{\sim} \frac{\mathrm{e}^{\frac{1}{2}i\pi J}}{\sqrt{J}} \int_0^\infty \mathrm{d}\alpha \ f^{\tau}_{\gtrless}(M,\alpha) \left(\frac{\mathrm{e}\alpha}{2J}\right)^J \ . \tag{3.24}$$

Hence the signatured partial-wave coefficients separately satisfy the conditions of Carlson's theorem and can be projected to complex J.

The SW transformation of the J summation is now straightforward; we write

$$a^{\tau}_{\gtrless}(J, M) = \tfrac{1}{2}(1 + (-1)^J)\, a^{J+}_{\gtrless}(J, M) + \tfrac{1}{2}(1 - (-1)^J)\, a^{\tau-}_{\gtrless}(J, M) \ , \tag{3.25}$$

and

$$b^{\tau}_{\gtrless}(M, x) = -2i \int_{C_0} \frac{\mathrm{d}J\,(2J+1)}{\sin \pi(J-M)}\, a^{\tau}_{\gtrless}(J, M)\, d^{J}_{M,0}(x) \ , \tag{3.26}$$

where the contour C_0 is shown in fig. 3a (dotted curve). Since we expect no singularities to the left of C_0 arising from $(\sin \pi (J-M))^{-1}$, we replace this factor by $\Gamma(J-M)$ and redefine $a^{\tau}_{\gtrless}$ accordingly (see eq. (3.28)). We note if in fig. 3a the contour encloses the point $J = j$, $j = 0, 1, 2, \ldots$, then the M contour $C_>$ and $C_<$ in fig. 2 will be pinched at respectively $M = 0, 1, \ldots, J$ and $M = -1, -2, \ldots, -J$. If one avoids Regge singularities we can without ambiguity simultaneously distort the contours C_0 and $C_{\gtrless}$ to respectively to C (fig. 3a) and the line $\mathrm{Re}\, M = -\frac{1}{2}$ (fig. 3b). We thus obtain the required SW representation

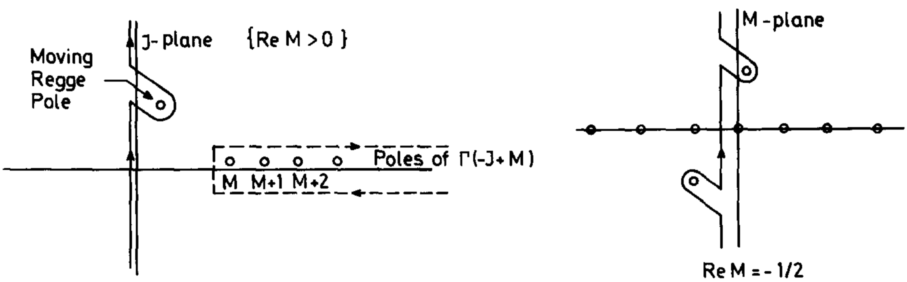
$$T = \tfrac{1}{2} \sum_{\tau = \pm} \int \frac{\mathrm{d}M}{2i \sin \pi M} ((-z)^M + \tau z^M)\, (b^{\tau}_{>}(M) + b^{\tau}_{<}(M)) \ , \tag{3.27}$$

with

$$b^{\tau}_{\gtrless}(M) = \int_{C} \frac{\mathrm{d}J}{2\pi i}\, \Gamma(-J + |M|)\, \widetilde{a}^{\tau}_{\gtrless}(J, M, \nu)\, d^{J}_{M,0}(-x) \ . \tag{3.28}$$

A similar analysis can be carried through for $\lambda \neq 0$. However since the J summation runs from $J = \max(|M|, |\lambda|)$, one should remove the discrete contributions for $J \leqslant |\lambda|$ and treat these as additional background terms.

For the primed J-M summation we return to (3.1) and (3.2) and repeat the above analysis with the important difference that the opposite $i\epsilon$ enters in the dispersion representation, which means



(a) (b)

Fig. 3. (a) Configuration of the distorted contour in the J plane. The dotted curve reproduces the partial-wave summation. (b) The distorted curve in the M plane. The deformation from the $\mathrm{Re}\, M = -\frac{1}{2}$ corresponding to the distortion of the J contour in (a) around the Regge pole.

$$\frac{s'}{M_X^2} = a'z'^{-1}(1-x'^2)^{\frac{1}{2}} + b'x' + c' . \tag{3.29}$$

We are now in a position to write the full double SW representation for the t-channel helicity amplitudes $T^{(t)}_{\lambda',\lambda}$ *viz.*,

$$T^{(t)}_{\lambda',\lambda} = \frac{1}{4}\sum_{\tau',\tau}\int_{\mathrm{Re}M=-\frac{1}{2}} \frac{\mathrm{d}M}{2i\sin\pi M}((-z)^M + \tau z^M)\int_{\mathrm{Re}M'=-\frac{1}{2}}\frac{\mathrm{d}M'}{2i\sin\pi M'}((-z')^{-M'} + \tau z'^{-M'})$$

$$\times \{b^{\tau',\tau}_{\lambda',\lambda,>,>} + b^{\tau',\tau}_{\lambda',\lambda,>,<} + b^{\tau',\tau}_{\lambda',\lambda,<,>} + b^{\tau',\tau}_{\lambda',\lambda,<,<}\} , \tag{3.30}$$

$$b^{\tau',\tau}_{\lambda',\lambda,\gtrless,\gtrless} = \sum_{s,s'}\int_C \frac{\mathrm{d}J'}{2\pi i}\,\Gamma(-J'+|M'|)\int_C\frac{\mathrm{d}J}{2\pi i}\,\Gamma(-J+|M|)(1+s'(-1)^{J'})(1+s(-1)^J)$$
$$\times\, a^{\tau',\tau,s',s}_{\lambda',\lambda,\gtrless,\gtrless}(J',M',J,M,t_1,t_2,t_3)\,d^{J'}_{-M',\lambda'}(-x')\,d^J_{M,\lambda}(-x) \ . \tag{3.31}$$

We end this section by considering the contribution of a Regge-pole dominated mechanism in the H.P.L. (see sect. 2), for which, if we neglect for the moment normality

$$a_{\lambda',\lambda,\gtrless,\gtrless} = \beta^{\alpha_2 *}_{\lambda'}(q^2,t_2)\,\beta^{\alpha_1}_{\lambda}(q^2,t_1)\frac{1}{(J'-\alpha_2(t_2))(J-\alpha_1(t_1))}$$
$$\times\, F^{\alpha_2\alpha_1}(M'_{\gtrless}, M_{\gtrless}, M_X^2, t_1, t_2, t_3) . \tag{3.32}$$

where $F^{\alpha_2\alpha_1}(M, M', M_X^2, t_1, t_2, t_3)$ is the Regge-particle discontinuity function, which depends on the helicity variables M and M'. We perform the J plane integrals by extracting the Regge pole residues. Through the gamma functions, this introduces helicity poles at sense values $M = \alpha_1, \alpha_1 - 1, \ldots$ (similarly for M'). Remembering in the forward H.P.L. $z = z'^{-1} \sim s/M_X^2 \to \infty$ we need only retain the contribution from the leading helicity pole, which yields

$$T^{(t)}_{\lambda',\lambda} = \beta^{\alpha_2 *}_{\lambda'}(q^2,t)\,\beta^{\alpha_1}_{\lambda}(q^2,t)\left(\frac{1+\tau'\,\mathrm{e}^{-i\pi\alpha_2(t)}}{\sin\pi\alpha_2(t)}\right)^*\left(\frac{1+\tau\,\mathrm{e}^{-i\pi\alpha_1(t)}}{\sin\pi\alpha_1(t)}\right)$$
$$\times\left(\frac{s}{M^2}\right)^{\alpha_2(t)+\alpha_1(t)} F^{\alpha_2\alpha_1}(-\alpha_2(t), \alpha_1(t), M_X^2, t, t, 0) \ . \tag{3.33}$$

We notice that the leading behaviour is determined by the maximal helicity flip in the Regge particle forward discontinuity. It is apparent from the above analysis that this is a direct consequence of the $i\epsilon$ prescription involved in a Mueller discontinuity and has little to do with the dynamics of Regge particle amplitudes.

4. Crossing and the normality properties

In this section we discuss the crossing relations and the construction of helicity structure functions of definite normality. We then consider the Stichel theorem for the Regge limit $\cos\theta_1$, $\cos\theta_2 \to \infty$, leaving the derivation of the corresponding relations in the H.P.L. to sect. 5.

The crossing matrix can be constructed by using the well known properties of the helicity states under boosts and rotations [12]. The general crossing relations between the s-channel amplitudes $T^{(s)}_{\{\lambda\}}$ defined in (2.2) and the t-channel amplitudes $T^{(t)}_{\{\lambda\}}$ (2.1) is given by

$$T^{(s)}_{\lambda',\mu_2,\lambda,\mu_1} = \sum_{\nu',\nu,\mu'_1,\mu'_2} T^{(t)}_{\mu'_2,\mu'_1;\nu',\nu} D^{\frac{1}{2}*}_{\mu_2\mu'_2}(R_{p'}) D^{\frac{1}{2}}_{\mu_1\mu'_1}(R_p) D^{1*}_{\lambda'\nu'}(R_{\gamma'}) D^{1}_{\lambda\nu}(R_\gamma) . \tag{4.1}$$

We obtain for the helicity averaged amplitudes in the forward direction $R_{p'} = R_p^{-1}$ namely

$$T^{(s)}_{\lambda',\lambda} = \frac{1}{4}\sum_{\mu_1} T^{(s)}_{\lambda',\mu_1;\lambda,\mu_1} , \tag{4.2}$$

$$T^{(t)}_{\lambda',\lambda} = \frac{1}{4}\sum_{\mu_1} T^{(t)}_{\mu_1,\mu_1;\lambda',\lambda} ,$$

the crossing relation

$$T^{(s)}_{\lambda',\lambda} = \sum_{\nu',\nu} T^{(t)}_{\nu',\nu} D^{1*}_{\lambda'\nu'}(R_{\gamma'}) D^{1}_{\lambda\nu}(R_\gamma) . \tag{4.3}$$

In the Regge limit, where $\cos\theta_1$ and $\cos\theta_2$ are the relevant dynamical variables it is simple to see that the s-channel amplitudes obtained by setting the azimuthal angles φ and φ' equal to zero, are related by crossing to the truncated amplitudes

$$\widetilde{T}^{(t)}_{\lambda',\lambda} = \mathrm{e}^{-i(\lambda\varphi_1 - \lambda'\varphi_2)}\, T^{(t)}_{\lambda',\lambda} . \tag{4.4}$$

In the forward direction $\theta_1 = \theta_2$ the crossing relation reducing correspondingly to

$$\widetilde{T}^{(s)}_{\lambda',\lambda} = \sum_{\nu',\nu} \widetilde{T}^{(t)}_{\nu',\nu} d^{1}_{\lambda'\nu'}(\chi)\, d^{1}_{\lambda\nu}(\chi) , \tag{4.5}$$

where $\cos\chi = E_q/|q|$.

Because of (4.4) the relevant dynamical variables in (4.5) are $\cos\theta_1$ and $\cos\theta_2$. In the H.P.L. we saw in sect. 2 that $\theta_1 = \theta_2 = \frac{1}{2}\pi$ and $\varphi_1 = \varphi_2 = \varphi$ with $\cos\varphi \to \infty$. In this case the t-channel vectors q and q' have a different orientation than that they have in the Regge situation so that the crossing relations have to be correspondingly modified. We shall discuss this in sect. 5 and restrict ourselves here exclusively to the Regge configuration, in which the azimuthal dependence is completely factored out.

From (4.5) it is a simple matter to show that the combinations

$$\widetilde{T}^{(t)}_{\pm} = \widetilde{T}^{(t)}_{1,1} \pm \widetilde{T}^{(t)}_{1,-1} , \tag{4.6}$$

together with $\widetilde{T}^{(t)}_{1,0}$ and $\widetilde{T}^{(t)}_{0,0}$ transform under crossing according to

$$\begin{bmatrix} \widetilde{T}^{(s)}_{+} \\ \widetilde{T}^{(s)}_{-} \\ \widetilde{T}^{(s)}_{1,0} \\ T^{(s)}_{0,0} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2\chi & \sqrt{2}\sin\chi\cos\chi & \sin^2\chi \\ 0 & -\frac{1}{\sqrt{2}}\cos\chi\sin\chi & \cos^2\chi & \frac{1}{\sqrt{2}}\sin\chi\cos\chi \\ 0 & \sin^2\chi & -2\sqrt{2}\sin\chi\cos\chi & \cos^2\chi \end{bmatrix} \begin{bmatrix} \widetilde{T}^{(t)}_{+} \\ \widetilde{T}^{(t)}_{-} \\ \widetilde{T}^{(t)}_{1,0} \\ \widetilde{T}^{(t)}_{0,0} \end{bmatrix} . \tag{4.7}$$

4.1. States of definite normality

Angular momentum states in the $\gamma\pi$ system with definite normality $n = (-1)^J P$ where P is the parity, are defined by

$$|J, M, \lambda, 0, n\rangle = \frac{1}{\sqrt{2}} (|J, M, \lambda, 0\rangle - n|J, M, -\lambda, 0\rangle) . \tag{4.8}$$

so that

$$(-1)^J P|J, M, \lambda, 0, n\rangle = n|J, M, \lambda, 0, n\rangle . \tag{4.9}$$

The partial-wave amplitudes with definite normality are correspondingly given by

$$A^{n',n}_{\mu_2,\mu_1;\lambda',\lambda}(J', M'; J, M; \hat{p}) = \langle \hat{p}, \mu_2, \mu_1 | T | J', M', \lambda', 0, n'; J, M, \lambda, 0, n\rangle , \tag{4.10}$$

and have the following symmetry relations

$$\begin{aligned} A^{n',n}_{\mu_2,\mu_1;-\lambda',\lambda}(J', M'; J, M, \hat{p}) &= -n' A^{n',n}_{\mu_2,\mu_1;\lambda',\lambda}(J', M'; J, M, \hat{p}) , \\ A^{n',n}_{\mu_2,\mu_1;\lambda',-\lambda}(J', M'; J, M, \hat{p}) &= -n A^{n',n}_{\mu_2,\mu_1;\lambda',\lambda}(J', M', J, M, \hat{p}) . \end{aligned} \tag{4.11}$$

With (4.8) we can express the states $|J, M, \lambda, 0\rangle$ in terms of states with definite normality and by making use of (4.10) we can rewrite the partial-wave summation defined in (2.27) in the form

$$\begin{aligned} T^{(t)}_{\mu_2,\mu_1;\lambda',\lambda} = \frac{1}{2} \sum_{J',J,M',M} N_{J'} N_J D^{J'*}_{-M',\lambda'}(\varphi_2, \theta_2-\varphi_2) D^J_{M,\lambda}(\varphi_1, \theta_1-\varphi_1) \\ \times \sum_{n',n} A^{n',n}_{\mu_2,\mu_1;\lambda',\lambda}(J', M'; J, M; \hat{p}) . \end{aligned} \tag{4.12}$$

By virtue of the symmetry relations (4.11) of the partial-wave coefficients with definite normality, we have the following equations for the t-channel helicity amplitudes when either $\lambda = 0$ or $\lambda' = 0$

$$T^{(t)}_{\mu_2,\mu_1;\lambda',0} = -n\, T^{(t)}_{\mu_2,\mu_1;\lambda',0} \quad \text{for} \quad \lambda' = \pm 1, 0\,,$$

and (4.13)

$$T^{(t)}_{\mu_2,\mu_1;0,\lambda} = -n'\, T^{(t)}_{\mu_2,\mu_1;0,\lambda} \quad \text{for} \quad \lambda = \pm 1, 0\,.$$

This means that only $n = -1$ states can contribute to $T^{(t)}_{\mu_2,\mu_1;\lambda',0}$ and $n' = -1$ state to $T^{(t)}_{\mu_2,\mu_1;0,\lambda}$. Furthermore only the term $n = n' = -1$ can contribute to $T^{(t)}_{\mu_2,\mu_1;0,0}$.

4.2. Stichel relations in the limit $\cos\theta_1$, $\cos\theta_2 \to \infty$

If we set $\varphi_1 = \varphi_2 = 0$ and let $\cos\theta_1$, $\cos\theta_2 \to \infty$ we can obtain additional constraints, because the d^J functions satisfy in the limit $\cos\theta \to \infty$ the relation

$$d^J_{M,-\lambda}(\cos\theta) = (-1)^\lambda\, d^J_{M,\lambda}(\cos\theta)\,. \tag{4.14}$$

Using (4.11) and (4.14) we obtain to leading order $(\cos\theta_1)^J (\cos\theta_2)^J$ the relations

$$T^{(t)}_{\mu_2,\mu_1;-\lambda',\lambda} = (-1)^{\lambda'+1}\, n'\, T^{(t)}_{\mu_2,\mu_1;\lambda',\lambda}\,, \quad T^{(t)}_{\mu_2,\mu_1;\lambda',-\lambda} = (-1)^{\lambda+1}\, n\, T^{(t)}_{\mu_2,\mu_1;\lambda',\lambda}\,. \tag{4.15}$$

For $\lambda' = \lambda = 1$ (4.15) tells us that

$$T^{(t)}_{\mu_2,\mu_1;1,-1} = n\, T^{(t)}_{\mu_2,\mu_1;1,1}\,, \qquad T^{(t)}_{\mu_2,\mu_1;-1,1} = n'\, T^{(t)}_{\mu_2,\mu_1;1,1}\,, \tag{4.16}$$

which means the appropriate linear combinations are dominated respectively by positive and negative normality states in the $t_1 - (\gamma(q)\,\pi(-k))$ channel, a corresponding relation holding for the $t_2 - (\gamma(-q')\,\pi(k'))$ channel. The relations (4.13), (4.15) and (4.16) are valid for arbitrary proton helicities μ_2 and μ_1. However if we restrict ourselves to the inclusive distributions averaged over the proton helicities $T^{(t)}_{\lambda',\lambda}$ (see eq. (4.2)), then we obtain additional constraints. By virtue of parity invariance $T^{(t)}_{\lambda',\lambda}$ satisfy

$$T^{(t)}_{-\lambda',-\lambda} = (-1)^{\lambda'-\lambda}\, T^{(t)}_{\lambda',\lambda}\,, \tag{4.17}$$

and together with (4.15) it follows from (4.17) that

(a) $T^{(t)}_{+} = T^{(t)}_{1,1} + T^{(t)}_{1,-1} \neq 0 \qquad \text{for} \quad n = n' = 1\,,$

while

$$T^{(t)}_{-} = T^{(t)}_{1,1} - T^{(t)}_{1,-1} \neq 0 \qquad \text{for} \quad n = n' = -1\,,$$

while for $\lambda' = 1$, $\lambda = 0$ (4.15) and (4.17) lead to the relation

(b) $T^{(t)}_{1,0} = -T^{(t)}_{-1,0} \neq 0 \qquad \text{for} \quad n = n' = -1.$

Together with result for $T^{(t)}_{0,0}$ derived above we have in summary the asymptotic relations:

(1) $T^{(t)}_{+} = T^{(t)}_{1,1} + T^{(t)}_{1,-1}$

is dominated by positive normality states in the $\gamma\pi$ channels;

(2) $T_-^{(t)} = T_{1,1}^{(t)} - T_{1,-1}^{(t)}$, $T_{1,0}^{(t)}$, $T_{0,0}^{(t)}$,

are dominated by negative normality states in the $\gamma\pi$ channels.

(1) and (2), derived here in the Regge limit $\cos\theta_1$, $\cos\theta_2 \to \infty$ is our main result. It is important to note that the helicity averaged amplitudes (4.2) cannot have asymptotically interference terms involving opposite normalities in the respective $\gamma\pi$-t-channels. Such terms would only appear if polarized targets were used. It follows from the crossing relation (4.7) that in the limit $\theta_1 = \theta_2$ the relations (1) and (2) carry over to the s-channel helicity structure functions, which are directly related to observable quantities [7].

Now we analyze the normality content of the t_3 channel in the limit $\cos\theta_3 \to \infty$. For this purpose we expand the state $|\mu_2, \mu_1, \hat{p}\rangle$ into partial waves in the $\bar{\rm p}$p center-of-mass system. The expansion is:

$$\begin{aligned}&\langle\hat{p}; \mu_2, \mu_1 |T| J', M', \lambda', n'; J, M, \lambda, n\rangle \\ &= \sum_j N_j\, d^j_{M'-M, \mu_2-\mu_1}(\theta_3) \langle j, \mu_2, \mu_1 |T| J', M', \lambda', n'; J, M, \lambda, n\rangle \ .\end{aligned} \tag{4.18}$$

As for the other two channels we define states of definite normality τ:

$$|j, m, \mu_2, \mu_1, \tau\rangle = \frac{1}{\sqrt{2}} \{|j, m, \mu_2, \mu_1\rangle + \tau |j, m, -\mu_2, -\mu_1\rangle\} \ , \tag{4.19}$$

so that

$$(-1)^j P\, |j, m, \mu_2, \mu_1, \tau\rangle = \tau\, |j, m, \mu_2, \mu_1, \tau\rangle \ , \tag{4.20}$$

and

$$|j, m, -\mu_2, -\mu_1, \tau\rangle = \tau\, |j, m, \mu_2, \mu_1, \tau\rangle \ . \tag{4.21}$$

With the states (4.18) we express (4.17) in the form

$$\begin{aligned}&\langle p, \mu_2, \mu_1 |T| J', M', \lambda', n'; J, M, \lambda, n\rangle \\ &= \frac{1}{\sqrt{2}} \sum_{j,\tau} N_j\, d^j_{M'-M, \mu_2-\mu_1}(\theta_3) \langle j, \mu_2, \mu_1, \tau |T| J', M', \lambda', n'; J, M, \lambda, n\rangle \, .\end{aligned} \tag{4.22}$$

According to (4.19) the partial-wave amplitudes with definite normality in the t_3 channel obey the relation

$$\begin{aligned}&\langle j, m, -\mu_2, -\mu_1, \tau |T| J', M', \lambda', n'; J, M, \lambda, n\rangle \\ &= \tau \langle j, m, \mu_2, \mu_1, \tau |T| J', M', \lambda', n'; J, M, \lambda, n\rangle \ .\end{aligned} \tag{4.23}$$

With this and the asymptotic relation (for $\cos\theta_3 \to \infty$) for the d^j functions:

$$d^j_{M'-M,-(\mu_2-\mu_1)}(\theta_3) = (-1)^{\mu_2-\mu_1} d^j_{M'-M,\mu_2-\mu_1}(\theta_3) \;. \tag{4.24}$$

we obtain

$$\begin{aligned}&\langle \hat{p}, -\mu_2, -\mu_1 | T | J', M', \lambda', n'; J, M, \lambda, n\rangle \\ &\quad = \tau(-1)^{\mu_2-\mu_1}\langle \hat{p}, \mu_2, \mu_1 | T | J', M', \lambda', n'; J, M, \lambda, n\rangle \,.\end{aligned} \tag{4.25}$$

The result (4.22) tells us that the combinations

$$\begin{aligned}&\langle p, \mu_2, \mu_1 | T | J', M', \lambda', n'; J, M, \lambda, n\rangle \\ &\pm (-1)^{\mu_2-\mu_1}\langle p, \mu_2, \mu_1 | T | J', M', \lambda', n'; \times J, M, \lambda, n\rangle \,,\end{aligned} \tag{4.26}$$

with the ± sign have only positive (negative) normality exchanges in the t_3-channel. In particular, for the helicity averaged amplitudes, defined in (4.15) only states with positive normality can contribute asymptotically. States with negative normality contribute only if the helicity of the target proton is fixed (inclusive polarization experiments). It is clear that the relation (4.26) is useful to select the observables in which different normalities in the t_3 channel can be detected *.

5. The Stichel relations in the helicity pole limit

In the H.P.L. the crossing relation has to be modified, because, compared to the Regge configuration discussed in the last section, the vectors q and q' are orientated differently in the t-channel. However it is sufficient to compute the crossing relation between the two respective t-channel situations, characterized by $q = q_\theta$ (Regge) and $q = q_\varphi$ (H.P.L.), where, referring to sect. 2:

$$q_\theta = (i|q|\cos\theta, |q|\sin\theta, 0, iE_q) \;, \quad q_\varphi = (0, |q|\cos\varphi, |q|\sin\varphi, iE_q), \tag{5.1}$$

q_θ and q_φ are related by a complex boost along the y axis i.e.

$$q_\varphi = B_y(\tfrac{1}{2}i\pi)\, q_\theta \;, \tag{5.2}$$

where in addition we have $\cos\theta = \sin\varphi$. Because of the difficulty of defining the rest frame of the photon, we shall use as the standard frame in the t-channel that defined by

$$q = (E_q, 0, 0, |q|) \,. \tag{5.3}$$

This frame is reached by the complex boost $B_z(\frac{1}{2}i\pi)$ along the z axis. The Wigner rotation corresponding to (5.2) is given by

$$R_W = B_z(-\tfrac{1}{2}i\pi)\, B_y(\tfrac{1}{2}i\pi)\, B_z(\tfrac{1}{2}i\pi) = R_x(\tfrac{1}{2}\pi) \,. \tag{5.4}$$

Thus according to the general transformation law of the helicity states [12] we have

* For a related discussion of polarization effects in inclusive reactions see [16].

$$U(B_y(\tfrac{1}{2}i\pi))|q_\theta,\lambda\rangle = \sum_{\lambda'} |q_\varphi,\lambda'\rangle D^1_{\lambda',\lambda}(R) \ , \tag{5.5}$$

where, if we restrict ourselves to the forward direction in the H.P.L., for which $\theta = \frac{1}{2}\pi$ then R is given by

$$R = R_y(-\tfrac{1}{2}\pi)\,R_x(\tfrac{1}{2}\pi)\,R_y(\tfrac{1}{2}\pi)\,R_z(-\varphi) = R_z(\tfrac{1}{2}\pi-\varphi) \ . \tag{5.6}$$

Hence the crossing relations involve simply a phase

$$U(B_y(\tfrac{1}{2}i\pi))|q_\theta,\lambda\rangle = e^{i(\frac{1}{2}\pi-\varphi)\lambda}\,|q_\varphi,\lambda\rangle \ . \tag{5.7}$$

From (5.7) we see that the φ phase factors out in such a way that the s-channel amplitudes are related through crossing to the reduced t-channel helicity amplitudes defined in (2.29). However we are left with the additional λ dependent phase factor $e^{\frac{1}{2}i\pi\lambda}$ in the H.P.L. Therefore instead of (4.12) we have now the following partial-wave expansion in terms of amplitudes with definite normality:

$$T^{(t)}_{\mu_2,\mu_1;\lambda',\lambda} = \frac{1}{2}\sum_{J',J,M',M} N_{J'}N_J\, z'^{M'} z^M i^{\lambda'+\lambda}\, d^{J'}_{-M',\lambda'}(\tfrac{1}{2}\pi)\, d^J_{M,\lambda}(\tfrac{1}{2}\pi)$$
$$\times \sum_{n',n} A^{n',n}_{\mu_2,\mu_1;\lambda',\lambda}(J',M';J,M) \ , \tag{5.8}$$

where $z = e^{i\varphi}$ and $z' = e^{-i\varphi'}$.

Using the symmetry relation for $M = J$

$$d^J_{J,\lambda}(\tfrac{1}{2}\pi) = d^J_{J,-\lambda}(\tfrac{1}{2}\pi) \ , \tag{5.9}$$

and (4.11) for the partial-wave amplitudes with definite normality we recover for the leading helicity pole in the H.P.L. from (5.8) the relations

$$T^{(t)}_{\mu_2,\mu_1;-\lambda',\lambda} = (-1)^{\lambda'+1}\, n'\, T^{(t)}_{\mu_2,\mu_1;\lambda',\lambda} \ , \qquad T^{(t)}_{\mu_2,\mu_1;\lambda',-\lambda} = (-1)^{\lambda+1}\, n\, T^{(t)}_{\mu_2,\mu_1;\lambda',\lambda} \ , \tag{5.10}$$

which are identical to the relations (4.15), derived in the Regge limit in the last section. It is clear then that all the further consequences derived from (4.15), at the end of sect. 2 are also true in the helicity pole limit. This completes our analysis, in which we have seen that, inspite of the kinematic and analytic peculiarities involved in the reggeization of an inclusive distribution, one recovers the usual properties one has come to associate with a Regge theory, through the study of the four point function.

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Appendix. The generalized fixed t dispersion relation

We record here the generalized fixed t dispersion representation of the six-point function mentioned in sect. 3 and on which our analysis was based. The idea [14] is to write a causal multiple Cauchy representation of the retarded function in the physical region, in which we are interested. This is done by singling out the positive energy vector p, with which one defines the s and u variables. All channel invariants not involving p or its related vector p' define generalized t variables, which can be held below their respective thresholds. We define $p_i, \ldots, 4$, where referring to sect. 2, $p_i = \{-k, q, k', -q'\}$.

The s variables are defined by:

$$s_i = (p - p_i)^2 , \qquad s_{ij} = (p - p_i - p_j)^2 , \qquad s_{ijk} = (p - p_i - p_j - p_k)^2 ,$$

while the t variables are given by

$$t_{ij} = (p_i + p_j)^2 , \tag{A.1}$$

$\{i, j, k\}$ are permutations of $\{1, 2, 3, 4\}$.

(A.1) defines nine variables, which are related through one Gram determinant constraint i.e.

$$\text{Gram det.} \{p, p_1, p_2, p_3, p_4\} = 0 , \tag{A.2}$$

which relates in general the s and t variables. However, if we choose

$$p_4 = \sum_{i=1}^{3} \lambda_i p_i , \qquad \lambda_i = \varphi_i(t_{ij} , t_{4j}) , \tag{A.3}$$

then we satisfy (A.2) by forcing a relation only among the t variables. In this case we can write a multiple Cauchy representation, involving at most triple discontinuities. Such a representation has only the allowed normal threshold (see the dual diagram in fig. 4), i.e. explicitly satisfies the Steinmann constraints [15]. Concentrating on only the triple discontinuity, the representation is given by

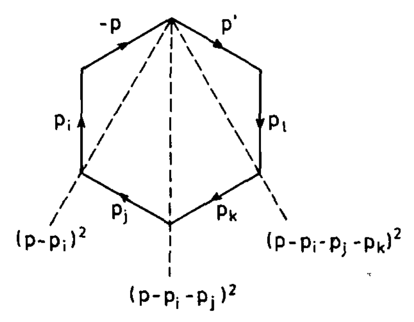

Fig. 4. Dual diagram for a given term in the dispersion relation.

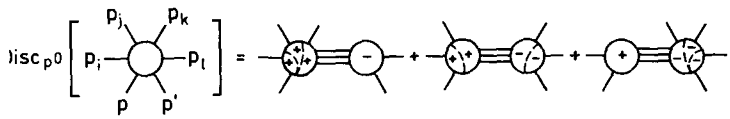

Fig. 5. Discontinuity with respect to p^0 of the retarded function.

$$T_6^{\mathrm{ret}}(p, p_1, p_2, p_3, p_4) = \frac{1}{\pi^3} \sum_{\substack{\mathrm{perm} \\ i,j,k}} \int_{\sigma_0}^{\infty} \mathrm{d}\sigma_1 \frac{1}{\sigma_1 - (p^+ - p_i)^2}$$

$$\times \int_{\sigma_0}^{\infty} \mathrm{d}\sigma_2 \frac{1}{\sigma_2 - (p^+ - p_i - p_j)^2} \int \mathrm{d}\sigma_3 \frac{1}{\sigma_3 - (p^+ - p_i - p_j - p_k)^2}$$

$$\times \rho_6(\sigma_1, \sigma_2, \sigma_3; \{t_{ij}\}) \ , \tag{A.4}$$

where $p^+ = (p^0 + i\epsilon, \boldsymbol{p})$ and for scalar particles $\rho_6(\ldots)$ is given by:

$$\rho_6(s_i, s_{ij}, s_{ijk}, \{t_{ij}\})\, \theta(p_0)\, \theta(p_0 - p_{i0})\, \theta(p_0 - p_{i0} - p_{j0})\, \theta(p_0 - p_{i0} - p_{j0} - p_{k0})$$

$$= \frac{1}{2^3} \int \prod_1^3 \mathrm{d}^4 x_r\, \mathrm{e}^{-i(p_i \cdot x_1 + p_j \cdot x_2 + p_k \cdot x_3)} \tag{A.5}$$

$$\times \langle p | j(x_1) j(x_2) j(x_3) j(0) | p' \rangle \ .$$

If we consider the total discontinuity of T_6^{ret} with respect to p^0 i.e.

$$\mathrm{Disc}_{p_0} T_6^{\mathrm{ret}}(p, p_1, p_2, p_3, p_4) = \frac{1}{\pi^3} \sum_{\mathrm{perm}} \int_{\sigma_0}^{\infty} \mathrm{d}\sigma_1 \int_{\sigma_0}^{\infty} \mathrm{d}\sigma_2 \int_{\sigma_0}^{\infty} \mathrm{d}\sigma_3\, \rho_6(\sigma_1, \sigma_2, \sigma_3, \{t_{ij}\}) \tag{A.6}$$

$$\times \mathrm{Disc}_{p_0} \left\{ \frac{1}{(\sigma_1 - (p - p_i)^2)(\sigma_2 - (p - p_i - p_j)^2)(\sigma_3 - (p - p_i - p_j - p_k)^2)} \right\} .$$

Then denoting $D_i^{\pm} = (\sigma_i - (p^{\pm} - p_i)^2)^{-1}$ we have

$$\mathrm{Disc}_{p_0} D_1 D_2 D_3 = D_1^+ D_2^+ D_3^+ - D_1^- D_2^- D_3^- = D_1^+ D_2^+ (D_3^+ - D_3^-) + D_1^+ (D_2^+ - D_2^-) D_3^-$$

$$+ (D_1^+ - D_1^-) D_2^- D_3^- \ , \tag{A.7}$$

i.e. we can decompose this discontinuity into the three basic discontinuities shown in fig. 5. The middle term fig. 4c is of the Mueller type and we must extract from the representation all such terms, i.e. by setting $s_{ij} = (p - p_i - p_j)^2 = M_X^2$. It is simple to see that there are four such terms and collecting these together we obtain the representation for the Mueller discontinuity given in sect. 3.

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