ISOSPIN BOUNDS AND BOSE CONDENSATION IN e^+e^- ANNIHILATION

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We derive isospin bounds for two particle correlations in $e^+e^- \to \gamma \to N\pi$, discuss the physical significance of saturation of the upper bound and present various multiplicity distributions for this case.

Recent e^+e^- annihilation experiments do not seem to agree well with popular preconceptions. Noteworthy in this respect is the behaviour of the total CM energy into charged particles, E_c. Most naive considerations lead one to expect that the final state would consist principally of pions and that the number of neutral pions is about half the number of charged pions. If the mean π^0 and $\pi^\pm$ momenta are the same this leads to $E_c/E_{tot} = 2/3$. Experimentally one observes that $E_c/E_{tot} \sim 1/2$ or $E_{neutral}/E_c \sim 1$ at the highest energies [1]. If one assumes that the final state consists mostly of directly produced pions, for which there is some experimental evidence, then one can entertain two extreme alternatives. Either neutral pions carry off more energy on the average than the charged pions [2] ("energy crisis" [3]) or there are simply more of them (population explosion). At present one is restricted to considering possibilities which account for E_c/E_{tot}, and trying to find experimental checks which might help to clarify the situation. We will fix here on the second possibility that there are many neutral pions present, saturating or nearly saturating the isospin upper bound of [4]. We shall argue that this is not so unlikely as has been generally believed.

Assuming that one photon annihilation is responsible for the observed effect and that the photon has odd charge conjugation and isospin $I = 0, 1$ only, we will first consider rigorous results following from isospin conservation and Bose statistics for pions, and extend the isospin bounds to the two particle correlations. Then we shall consider some interesting consequences of saturation or near saturation of the upper bounds and, finally, we shall comment on the physical significance of the bounds ‡.

Using the methods of Chacon and Moshinsky [6], we can reduce the average number of neutral and charged pions for fixed $N = \langle n_0 \rangle + \langle n_c \rangle$ as well as the averages $h_{ij} = \langle n_i n_j - \delta_{ij} n_i \rangle$ $(i;j = +, -, 0)$ to the form of matrix elements of an isotensor operator Q_0 and its square. Namely [6],

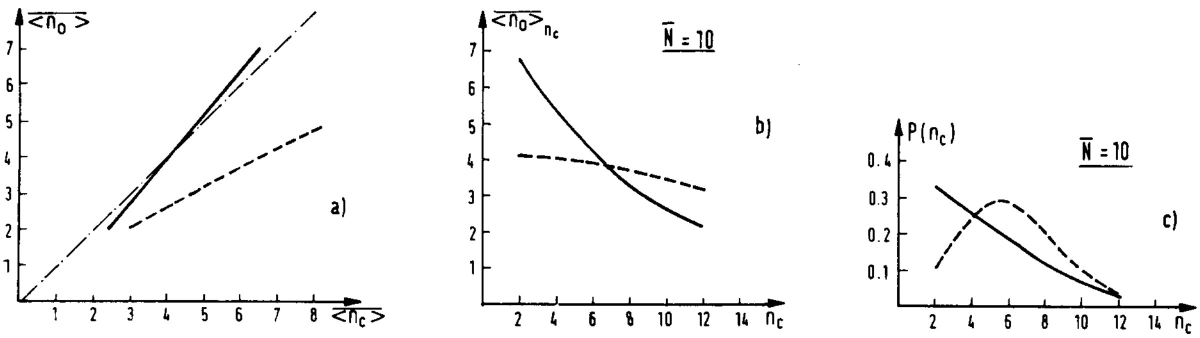
$$\langle n_0 \rangle = \tfrac{1}{3}(\langle Q_0 \rangle + N), \qquad \langle n_c \rangle = \tfrac{2}{3}(N - \tfrac{1}{2}\langle Q_0 \rangle), \tag{1}$$

and

$$\begin{aligned} h_{00} &= \tfrac{1}{9}\{\langle Q_0^2 \rangle + (2N-3)\langle Q_0 \rangle + N(N-3)\}, \qquad & h_{0+} &= \tfrac{1}{9}\{-\tfrac{1}{2}\langle Q_0^2 \rangle + \tfrac{1}{2}N\langle Q_0 \rangle + N^2\}, \\ h_{++} &= \tfrac{1}{9}\{\tfrac{1}{4}\langle Q_0^2 \rangle - \tfrac{1}{2}(2N-3)\langle Q_0 \rangle + N(N-3)\}, \qquad & h_{+-} &= \tfrac{1}{9}\{\tfrac{1}{4}\langle Q_0^2 \rangle - N\langle Q_0 \rangle + N^2\}. \end{aligned} \tag{2}$$

The calculation of $\langle Q_0 \rangle$ and $\langle Q_0^2 \rangle$ requires information on the $N\pi$ states. These are labelled by representations $[N] = [N_1, N_2, N_3]$ of the permutation group S_N corresponding to a Young tableau with three rows of lengths

‡ Much of this stems from work of Pais [5]. A different method for deriving bounds is due to Chacon and Moshinsky [6], who use results of Elliott [7]. Bounds using isospin sum rules have also been derived [8].



Figs. 1a–c. Average number of neutral versus average number of charged pions (1a); average number of π^0 as a function of n_c for $\bar{N} = 10$ (1b) and the charged multiplicity $P(n_c) = \sigma(n_c)/\sigma_{tot}$ as a function of n_c for $\bar{N} = 10$ (1c) for the symmetrized wave function (solid lines) and the unsymmetrized wave function (dashed lines).

N_1, N_2, N_3 where $N = N_1 + N_2 + N_3$ and $N_1 \geqslant N_2 \geqslant N_3$. The Young tableau $[N]$ contains the isospin content and Bose symmetry of the Nπ states [5]. The effect of Q_0 on these states has been given by Elliott [7] resulting in the matrix elements

$$\langle Q_o \rangle^{I=0} = 0; \qquad \langle Q_o^2 \rangle^{I=0} = \tfrac{4}{5}(\hat{N}_1^2 + \hat{N}_2^2 - \hat{N}_1\hat{N}_2 + 3\hat{N}_1)\,, \tag{3}$$

and

$$\langle Q_o \rangle^{I=1} = \tfrac{2}{5}\begin{cases} -\hat{N}_1 - \hat{N}_2 - 3 & \hat{N}_1 \text{ odd}, \quad \hat{N}_2 \text{ odd} \\ 2\hat{N}_1 - \hat{N}_2 + 3 & \hat{N}_1 \text{ odd}, \quad \hat{N}_2 \text{ even} \\ -\hat{N}_1 + 2\hat{N}_2 & \hat{N}_1 \text{ even}, \quad \hat{N}_2 \text{ odd} \end{cases}$$

$$\langle Q_o^2 \rangle^{I=1} = \tfrac{4}{35}\begin{cases} 5\hat{N}_1^2 + 5\hat{N}_2^2 + \hat{N}_1\hat{N}_2 + 21\hat{N}_1 + 12\hat{N}_2 - 9 & \hat{N}_1 \text{ odd}, \hat{N}_2 \text{ odd} \\ 11\hat{N}_1^2 + 5\hat{N}_2^2 - 11\hat{N}_1\hat{N}_2 + 33\hat{N}_1 - 12\hat{N}_2 - 9 & \hat{N}_1 \text{ odd}, \hat{N}_2 \text{ even} \\ 5\hat{N}_1^2 + 11\hat{N}_2^2 - 11\hat{N}_1\hat{N}_2 + 9\hat{N}_1 - 27 & \hat{N}_1 \text{ even}, \hat{N}_2 \text{ odd} \end{cases}\,, \tag{4}$$

where $\hat{N}_1 = N_1 - N_3$, $\hat{N}_2 = N_2 - N_3$. By an exercise in fortitude like that required to find the bounds for $\langle n_o \rangle / \langle n_c \rangle$ [4], we get for $h_{oc} = h_{o+} + h_{o-}$, $h_{cc} = 2h_{++} + 2h_{+-}$ the following bounds: For $I = 0$, hence N odd,

$$\frac{N^2 - 3N + k}{4N^2 - 6N + k} \leqslant \frac{h_{oo}}{h_{cc}} \leqslant \frac{3(N-3)}{8N - 14} \leqslant \tfrac{3}{8} \qquad N \geqslant 5\,, \tag{5}$$

where $k = 0$ or 8 according as $N/3$ is an integer or not. For $I = 0$ one has $\langle Q_o \rangle = 0$ which implies that $h_{oc} = 2h_{cc} - h_{oo}$, hence h_{oo}/h_{oc} is not independent of (5). For $I = 1$, hence N even, one gets

$$\frac{2N^2 - 9N + 10 + k}{23N^2 - 2N - 18 + k} \leqslant \frac{h_{oc}}{h_{cc}} \leqslant \tfrac{5}{2}\,\frac{3N^2 - 9N + 2}{4N^2 - 5N - 9} \leqslant \tfrac{15}{8}\,,$$

$$\frac{2N^2 - 5N + 2}{12N^2 - 2N - 9} \leqslant \frac{h_{oo}}{h_{cc}} \leqslant \frac{3N^2 + 5N + 2}{4N^2 - 5N - 9} \leqslant \tfrac{3}{4}\,, \tag{6}$$

where $N \geqslant 4$ and $k = 0$ or 8 according as $N/2$ is odd or even. The important point for us is that the upper bounds in (5) and (6) are saturated by the same partition $[N-2, 1, 1]$ which gives the upper bound for $\langle n_o \rangle / \langle n_c \rangle =$

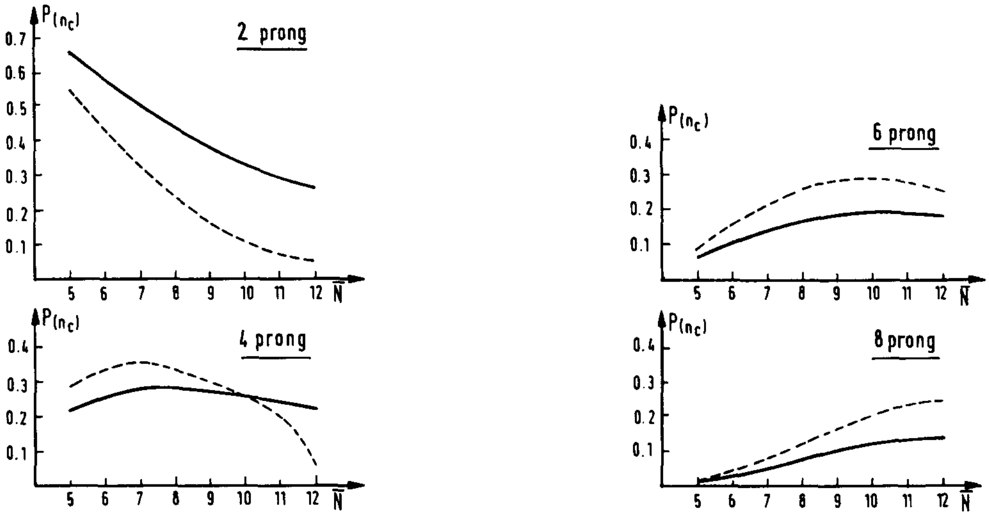


Fig. 2. Prong distributions as a function of $\bar{N}$ for the symmetrized (solid lines) and unsymmetrized (dashed lines) wave functions.

$(3N-2)/(2N+2)$ and the lower bound of the prong inequalities of Pais [9]. A large $\langle n_0\rangle/\langle n_c\rangle$ then implies large correlations among neutrals.

Let us investigate the consequences of assuming that the upper bound is saturated and that the dynamical mechanism is such that the partition $[N-2,1,1]$ gives the dominant contribution. We can now obtain more detailed results than are possible in the general case. For example if $[N-2,1,1]$ alone is present

$$h_{+-} - h_{++} = \begin{cases} \frac{1}{5}(N+1) & N \text{ even} \\ \frac{1}{3}N & N \text{ odd} \end{cases}, \tag{7}$$

which averaged over N could provide useful information on the average number of particles from measurements involving charged particles only. Further, the probability to observe n_0 neutral and n_c charged pions is ($N = n_0 + n_c$)

$$P(n_0, n_c) = P_N \Gamma_{[N-2,1,1]}(n_0, n_c), \tag{8}$$

where P_N is the probability distribution for producing N pions and $\Gamma_{[N]}$ their branching ratio with $\Sigma_{n_0,n_c}\Gamma_{[N]}(n_0, n_c) = 1$ at fixed N. Now the $\Gamma_{[N-2,1,1]}$ can be calculated with the state vector realisation ϕ_S to be discussed below or with the methods of [5].

$$\Gamma_{[N-2,1,1]}(n_0, n_c) = \frac{[(N-4)(N-6)\ldots n_0]\, 1\cdot 3\cdot 5\ldots(n_0-1)}{5\cdot 7\cdot 9\ldots(N-1)} \qquad (N\text{ even})$$

$$\Gamma_{[N-2,1,1]}(n_0, n_c) = \frac{[(N-3)(N-5)\ldots(n_0+1)]\, 1\cdot 3\cdot 5\ldots(n_0-2)}{1\cdot 3\cdot 5\ldots(N-2)} \qquad (N\text{ odd}) \tag{9}$$

where the square bracket is to be replaced by 1 for $n_0 > N-4$.

Defining the average of a function $f(N)$ by

$$\bar{f} = g_0 \sum_{N=\text{odd}} f(N) + g_1 \sum_{N=\text{even}} f(N), \tag{10}$$

where $g_0(g_1)$ are the isospin weights for $I=0$ ($I=1$), satisfying $g_0 + g_1 = 1$, $g_{0,1} \geqslant 0$, we can calculate with the knowledge of the branching ratios and a choice for P_N the following quantities: The prong distribution $\sigma(n_c)/\sigma_{tot} \equiv P(n_c) = P(N-n_c, n_c)$, the mean number of π^0 $\langle n_o\rangle_{n_c}$ as a function of n_c defined by $\langle n_o\rangle_{n_c} P(n_c) = n_o P(n_o, n_c) = (N-n_c)P(N-n_c, n_c)$ and the total mean number of neutrals $\langle n_o\rangle = \Sigma_{n_c}\langle n_o\rangle_{n_c} P(n_c)$ [10]. In fig. 1 and 2 (solid lines) we present numerical results when $g_1/g_0 = 3$ is the usual SU(3)-value and when P_N is a Poisson distribution with mean value $\bar{N}$, corresponding to an independent emission of the pions [11]. We want to call attention to the large two prong component $P(2)$ and the steeply falling $\langle n_o\rangle_{n_c}$. The trend of the prong cross section versus $\bar{N}$ given in fig. 2 resembles that in the data [11]. We have checked using a Gaussian P_N that these results do not depend sensitively on either the width of the distribution or on g_0/g_1.

So far we have just seen what would happen if the isospin bounds were saturated. Is there any physics in this? We can demonstrate by construction that there is and, further, that the bounds are saturated not by weird isospin combinations but simply by long-familiar Bose condensation effects.

Consider the hadronic decay of the virtual photon which cascades down in mass by emitting $I = J = 0$ ϵ-like $\pi\pi$-states of low momentum plus an ω or an $\omega\pi^0$ state (necessary to get the correct total I). This is in the spirit of a linear thermodynamic bootstrap [12] or of cascade (chain-emission) models [2]. The isospin wave functions of such a chain are $\phi^1 = \omega_{123}\,\pi_4^0\,\epsilon_{56} \ldots \epsilon_{N-1,N}$ for N even and $\phi^0 = \omega_{123}\,\epsilon_{45} \ldots \epsilon_{N-1,N}$ for N odd where $\omega_{123} = (\pi_1 \times \pi_2)\cdot\pi_3$ and $\epsilon_{ij} = \pi_i \cdot \pi_j$. For these states the branching ratios can be calculated to be

$$\Gamma_{\phi^1}(n_o, n_c) = (\tfrac{1}{3})^{(N-4)/2}\begin{bmatrix}(N-4)/2\\ k\end{bmatrix} 2^k$$

$$\Gamma_{\phi^0}(n_o, n_c) = (\tfrac{1}{3})^{(N-3)/2}\begin{bmatrix}(N-3)/2\\ k\end{bmatrix} 2^k \qquad n_c = 2k+2 = N - n_o \tag{11}$$

giving $\langle n_o\rangle/\langle n_c\rangle = (N+2)/(2N-2) \to 1/2$ for $N \to \infty$, N even. If the final pions have low relative momenta it is necessary to symmetrize the isospin states. In this symmetrization the ω can be ignored as it is completely antisymmetric and thus we can construct the properly symmetrized states $\phi_S^1 = \omega_{123} S\pi_4^0\epsilon_{56} \ldots \epsilon_{N-1,N}$ for N even, and $\phi_S^0 = \omega_{123} S\epsilon_{45} \ldots \epsilon_{N-1,N}$ for N odd, where S means symmetrization of momentum labels among pions of like charge. The branching ratios (9) can be directly calculated from the ϕ_S by writing them in the charge basis and counting charge status using the binomial expansions for the product of the ϵ-mesons. As a result, the isospin upper bound is saturated. Mathematically this is because the ϕ_S are vectors belonging to the representation $[N-2, 1, 1]$. Physically this is because the presence of the extra π^0 in ϕ_S^1 stimulates, through the symmetrization, the ϵ-states to "decay" preferentially into neutral pion modes. Thus, saturation of the upper bound is a kind of Bose condensation effect. We demonstrate the consequences of this by showing in the figures the analogous results for the unsymmetrized chain (dashed lines). Notice, in particular in fig. 1a, the striking effect of symmetrization. In contrast to the symmetrized case the results for the unsymmetrized chain depends on the width of the distribution P_N. For instance, a narrow distribution gives the falling $\langle n_o\rangle_{n_c}$ but at the same time gives a smaller two prong component [13]. Much of what we have said should be independent of the detailed dynamics for multiparticle production so long as the dominant low energy $\pi\pi$ correlations are isoscalar and a π^0 carries the photon's isospin. We should remark that the effects discussed here many set in only for large $\bar{N}$; the symmetrized and unsymmetrized chains differ not at all for $N = 4$ and minimally for $N = 6$. It appears to us that the unexpectedly large ratio E_{neutral}/E_c can be explained by an excess of neutral pions with the mechanism we have discussed; the observed prong distributions are consistent with this; future experiments will decide the issue.

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