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DO WE SEE ANOMALOUS DIMENSIONS IN e^+e^- INCLUSIVE ANNIHILATION AT $Q = 4$ GeV?

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It is shown in the framework of scale invariant models that a simple ansatz for the anomalous dimensions allows us to determine the experimentally observed variation with Q of the one-charged particle inclusive cross section.

Our calculation sheds no light on the behaviour of σ_{had} with Q since we have worked with the inclusive cross section normalized to the experimentally measured σ_{had}.

It may sound odd to check experimental predictions of scale invariant theories for the one-particle inclusive spectrum when the safest prediction of these models $\sigma_{\mathrm{had}} \sim Q^{-2}$ seems to be ruled out by the CEA-SPEAR data [1,2]. We feel however that at the present stage of the game any hint that at least some predictions of "conventional" theories are compatible with the data could be useful.

We consider the one-charged particle inclusive distribution which we assume to be isotropic and given mostly by charged pions in agreement with experiment [2]. We thus deal with only one structure function.

It is useful to define the distribution

$$h(y, Q^2) = \frac{1}{\bar{n}_{\mathrm{c}}\sigma_{\mathrm{had}}} \frac{\mathrm{d}\sigma}{\mathrm{d}y} , \tag{1}$$

where $y = \ln (Q/2E)$, E is the pion c.m. energy and $\bar{n}_{\mathrm{c}}$ is the average multiplicity of charged pions. The function $h(y,Q^2)$ satisfies the sum rules:

$$\int_0^\infty h(y,Q^2)\, \mathrm{d}y = 1,$$

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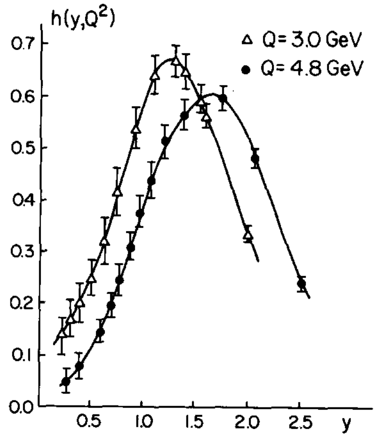

Fig. 1. The distribution $h(y, Q^2)$ defined by eq. (1) at $Q = 3$ and 4.8 GeV. The data are from ref. [2].

$$\int_0^\infty e^{-y}\, h(y, Q^2)\, dy = 2\xi_c/\bar{n}_c, \tag{2}$$

where $\xi_c Q$ represents the fraction of energy carried by charged pions. From the experimental data for $(1/\sigma_{had})\, d\sigma/dx$ ($x = 2p/Q$, p is the magnitude of the pion 3-momentum) we can determine $h(x, Q^2)$. Unfortunately there are no data for $0 < x < 0.1$ and $0.8 < x < 1$. We have thus extrapolated by eyeball the cross sections to $x = 0$ and $x = 1$, knowing that $d\sigma/dx$ should be zero for both of these values. In this way we have obtained $\bar{n}_c = 3$ for $Q = 3$ GeV and $\bar{n}_c = 3.7$ for $Q = 4.8$ GeV. Had we taken the quoted numbers [2] for the average multiplicity $\bar{n}_c = 3.7 \pm 0.4$ and respectively $\bar{n}_c = 4.2 \pm 0.4$, our conclusions would remain unaffected as we have checked. The function $h(y, Q^2)$ for $Q = 3$ GeV and 4.8 GeV is shown in fig. 1.

Knowing the function $h(y, Q^2)$ at $Q = 3$ GeV we will try to determine it theoretically at $Q = 4.8$ GeV. Since we are looking for the variation of the function $h(y, Q^2)$ with Q^2 at fixed y, we have not considered the data at $Q = 3.8$ GeV since in this case the variation of $h(y,Q^2)$ with respect to the $Q = 3$ GeV or $Q = 4.8$ GeV data would be smaller and the effect would drown in the error bars.

Scale invariant theories, among which we include Polyakov's informal bootstrap scheme [3], ϕ^4 field theory with an eigenvalue [4] and branching processes [5], predict the following sum-rules for the function $h(y, Q^2)$:

$$\lim_{Q^2\to\infty} \int_0^\infty e^{-ny}\, h(y, Q^2)\, dy = Z(n)\, e^{-\delta(n)k}, \tag{3}$$

where $k = k(Q^2)$ and asymptotically $k = a \ln (Q^2/M^2)$. (a is an unknown constant and M is an unknown mass scale). The function $\delta(n)$, related to the anomalous dimensions and the short-distance behaviour of the model, is an increasing function of n, vanishes at the origin and $\lim_{n\to\infty} \delta(n)$ = finite [6]. $Z(n)$, which depends on the long distance behaviour of the theory, is unknown and satisfies only the constraint $Z(0) = 1$. In the class of models we consider the average energy per particle increases like a power of Q^2:

$$\frac{\xi_c Q}{\bar{n}_c} \sim (Q^2)^{\frac{1}{2}-a\delta(1)}. \tag{4}$$

We use Parisi's trick [7] to compute the variation with Q^2 of the function $h(y, Q^2)$ for a given value of y. Taking the derivative with respect to k in eq. (3) and using the convolution theorem for Laplace transforms we have:

$$\frac{\partial h}{\partial k} = \int_0^y \Delta(y - y')\, h(y', Q^2)\, \mathrm{d}y', \tag{5}$$

where

$$\int_0^\infty \mathrm{e}^{-ny}\, \Delta(y)\, \mathrm{d}y = -\,\delta(n). \tag{6}$$

In this way we got rid of the unknown function $Z(n)$ and the variation of h with Q^2 is given by h and the anomalous dimensions only. We make the approximation

$$\frac{\partial h}{\partial k} = \frac{1}{k'}\, \frac{\delta h(y, Q^2)}{\delta Q^2} = \frac{1}{k'}\, \frac{h(y, Q^2 = 23) - h(y, Q^2 = 9)}{14}, \tag{7}$$

where $k' = \mathrm{d}k/\mathrm{d}Q^2$ is an unknown constant.

The experimental values of $\delta h/\delta Q^2$ are shown in fig. 2. It is interesting to note that the shape of the function $\delta h(y, Q^2)/\delta Q^2$ looks very much like the predictions of scale invariants theories [6] for electroproduction (the correspondence is $h(y, Q^2) \to \nu W_2/\omega, y \to \ln \omega = \ln(2m\nu/Q^2)$): it starts with negative values and changes sign.

In order to use eq. (5) we have to know $h(y, Q^2 = 9)$ and $\delta(n)$. We approximate $h(y, Q^2)$ at $Q = 3$ GeV by a Gaussian:

$$h(y, Q^2) = \frac{1}{\sqrt{2\pi}\sigma}\, \mathrm{e}^{-(y-\bar{y})^2/2\sigma^2},$$

with $\bar{y} = 1.3$ and $\sigma = 0.6$. The simplest model that we can think of for the anomalous dimensions which satisfies the above mentioned constraints is

$$\delta(n) = 1 - \frac{\lambda}{\lambda + n}, \tag{8}$$

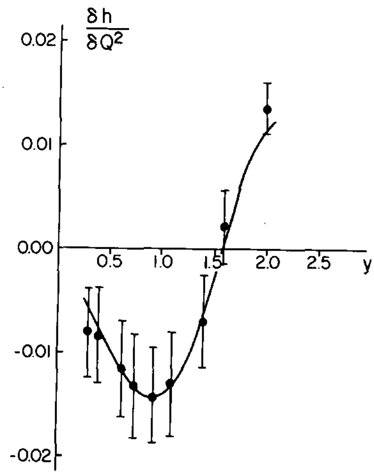

Fig. 2. The variation $\delta b/\delta Q^2$ computed from the data presented in fig. 1. The solid curve is the result of our model.

where λ is a free parameter. The expression (8) for $\delta(n)$ can always be multiplied by a factor which we incorporate in k. With the choice (8) for $\delta(n)$ we have

$$\Delta(y) = -\delta(y) + \lambda\, e^{-\lambda y} \quad , \tag{9}$$

and the integration (5) can be done analytically. Taking $\lambda = 3$ and normalizing at $y = 0.9$ (since we don't know k') we obtain the curve shown in fig. 2, which can be considered an excellent fit to the data.

We would like to warn an enthousiastic reader about several points. First, had we normalized $h(y, Q^2)$ (see eq. (1)) not to σ_{had} but to Q^{-2} we would have found total disagreement with experiment. In case asymptotia has not yet been reached for σ_{had} it seems reasonable to proceed in our way and thus satisfy the sum rules (2) at any Q^2. It is also important to stress that eq. (5) has been obtained from eq. (3) under an assumption of uniformity for the right hand side of eq. (3). This may not be a bad assumption if we consider the overall trend of the data the way we did and don't specialize to some asymptotic domain.

It could be that the excellent agreement between our theoretical curve and the data shown in fig. 2 is an accident, but we have considered it worth mentioning in the hope that it will be confirmed at SPEAR II and thus revive interest in scale invariant models for e^+e^- annihilation.

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