$J(3.1)$ and $\psi(3.7)$: How about color?

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The new particles $J(3.1)$ and $\psi(3.7)$ are interpreted as colored ω and ϕ mesons, respectively. By relating the radiative decay $J(3.1)\to\eta\gamma$ to 2γ decays of the color singlet mesons π^0, η, X^0, we find that the radiative decay $J(3.1)\to\eta\gamma$ is suppressed (in contrast to simple expectations from $\omega\to\pi^0\gamma$). Via "strong new duality" we predict the recurrences $J'(4.18\pm0.08)$, $J''(5.03\pm0.13)$,... and $\psi'(4.63\pm0.08)$, $\psi''(5.41\pm0.13)$,... .

I. INTRODUCTION

There are various theoretical schemes, which enlarge the traditional I-B-Y degrees of freedom of hadronic matter by new ones, such as charm or color. The particles $J(3.1)$ (see Ref. 1) and $\psi(3.7)$ (see Ref. 2) can be the milestones (stumbling blocks?) for them. The difference between the charm[3] and the color[4] interpretation of the new particles is that in the first case the new degree of freedom is hidden, while in the second it is manifest. In group-theoretical language one has to discriminate between the two main possibilities of extending SU(3): Charm corresponds to $SU(3)\to SU(n)$, specifically SU(4),[3] whereas color extends SU(3) to $SU(3)\times\mathcal{G}$, where $\mathcal{G}$ may again be identified[4] with SU(3).

The mentioned theoretical alternatives have certain similarities, but also very characteristic differences. In both cases, when identifying the new particles with either hidden charm or color octet vector mesons, decays into normal hadrons are forbidden. In the charm identification this prohibition of the decay is obtained via the Zweig or duality diagram selection rule[5] in close analogy to the case of $\phi(1019)$. In the color case, transitions to normal hadrons are forbidden, as they would correspond to SU(3) color $[\equiv SU(3)^c]$ octet$\to$ singlet transitions. Differences become important as soon as transitions of the new states into photons plus hadrons are considered: The hidden charm particle cannot lose its hidden charm by photon emission; the colored (color-octet) vector meson, however, can turn into a normal (color-singlet) hadron just by radiating away its color via a color-octet photon (Fig. 1). While available phase space for photon emission may thus be small in the charm case $[m(\eta_{c\bar{c}})\cong 3$ GeV has been suggested[3]], phase space is enormous in the color case, where the final hadron state may be as light as 550 MeV $[\eta(549)]$. Radiative decay widths in the hidden charm case may thus be of the usual magnitude of first-order electromagnetic transitions (i.e., <1 MeV). In the color case, simple estimates from $\omega\to\pi^0\gamma$ lead to widths of the order of magnitude of 15 MeV (from phase space), and an identification of the new extremely narrow states with colored states seems to face severe problems right from the beginning.[6]

On the other hand, the $SU(3)\times SU(3)^c$ Han-Nambu model[4] has attractive features, as, e.g., integral charges of the basic quark triplets, no spin-statistics problem for baryons, and it also yields an asymptotic value of $R\equiv\sigma_{e^+e^-\to h}/\sigma_{\mu^+\mu^-}=4$ quite consistent with data for $\sqrt{s}\cong 4$ to 5 GeV. A more detailed discussion of the vector-meson phenomenology and of the width problem in particular thus seems clearly worthwhile. Subsequently we will show that *the radiative decays of colored vector mesons may be estimated more convincingly by relating them to experimentally known 2γ decays of ordinary (color-singlet) mesons, i.e.,* $\pi^0,\eta,X^0\to 2\gamma$. From our analysis of these decays we conclude that a *strong suppression of the radiative decays of the new particles should in fact be expected* in contrast to the above-mentioned estimate. However, considerations of different $1^c8^c8^c$ hadronic couplings show that such a suppression of $1^c8^c8^c$ couplings cannot be universally valid.

Specifically, in Sec. II we will briefly discuss different possibilities for the $SU(3)^c$ structure of the electromagnetic current and their consequences concerning number and photon coupling strengths of the new states. We will *tentatively identify the states at 3.1 and 3.7 GeV with colored ω and ϕ vector mesons,* respectively. In Sec. III

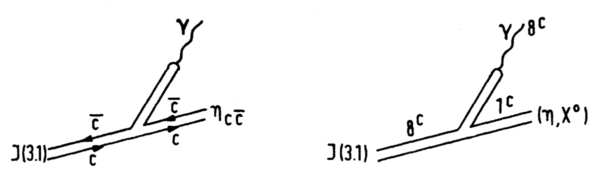

FIG. 1. Photon emission in the charm and color cases.

our arguments on the widths of the new vector mesons are presented. Further consequences of the color interpretation for the value of R via q^2 duality and the *spectrum of vector mesons to be expected* will be pointed out in Sec. IV. A few concluding remarks are collected in Sec. V.

II. COLOR STRUCTURE OF THE ELECTROMAGNETIC CURRENT, TENTATIVE ASSIGNMENT, AND COUPLING STRENGTHS OF THE NEW PARTICLES

In the Han-Nambu $SU(3)\times SU(3)^c$ model,[4] mesons and baryons are built up from three basic triplets $(\mathcal{P}, \mathcal{N}, \lambda)_i$, where the color index i runs over 1, 2, 3. The charges of the nine basic states take integral values 1, 0, −1 (2, 5, and 2 times, respectively), and ordinary hadrons are supposed to be $SU(3)^c$ singlet states. The electromagnetic current contains two pieces

$$J_\mu = J_\mu(8, 1^c) + J_\mu(1, 8^c)\,. \qquad (1)$$

The first term is well known from ordinary SU(3) symmetry. It transforms as the U-spin scalar component of the SU(3) octet and is a singlet in $SU(3)^c$. The second term in (1) supplements the Gell-Mann–Zweig third-integral charges due to the first term in (1) in such a way that the final charges of the basic triplets take the mentioned integral values. The $(1, 8^c)$ structure of the second term in (1) follows rather uniquely from the requirement[7] that the charges of the ordinary (color-singlet) hadrons come out correctly. There is freedom, however, as to which octet operator is actually chosen in color space. Physical properties of the color-octet hadron spectrum of states, e.g. the number of vector mesons coupled to the photon and their coupling strengths, sensitively depend upon the choice taken, which choice may thus eventually be confronted with experiment.

In the following, let us restrict ourselves in (1) to operators from the color octet which are diagonal in color space. Physically this restriction corresponds to assuming that the photon is a color neutral and thus cannot change the color of a particle [i.e., it cannot convert, e.g., a type-1 (red) into a type-2 (green) quark]. Excluding thus off-diagonal terms, the number of possibilities for the 8^c current operator is rather limited.

In fact, if the three basic triplets are chosen to be in the $(3, 3^*)$ representation of $SU(3)\times SU(3)^c$, only one possibility remains, namely the U-spin scalar in color space, U^c. The current with respect to $SU(3)^c$ then looks just the same (apart from a sign from $3\to 3^*$), as with respect to ordinary SU(3). For subsequent use let us note the explicit expression of the charge operator, which is then given by

$$Q = U\times 1^c + 1\times U^c$$
$$= \frac{1}{3}\left[\begin{pmatrix} 2 & & \\ & -1 & \\ & & -1\end{pmatrix}\times\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1\end{pmatrix} + \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1\end{pmatrix}\times\begin{pmatrix} -2 & & \\ & 1 & \\ & & 1\end{pmatrix}\right], \qquad (2)$$

where in each term the matrix on the left-hand side acts in ordinary SU(3), while the matrix on the right-hand side transforms in color space. U equals $U = I_3 + \frac{1}{2}Y$ $(U^c = I_3^c + \frac{1}{2}Y^c)$, and I_3 and Y denote the third component of isospin (color isospin) and hypercharge (color hypercharge), respectively.

If the basic triplets are chosen[8] to transform according to the (3, 3) representation of $SU(3)\times SU(3)^c$, two different choices are possible for the 8^c piece of J_μ. These correspond to a color-I-spin and a color-V-spin scalar, i.e., color hypercharge, Y^c, and color V spin, V^c, respectively. Explicitly, U^c in (2) has to be replaced by either

$$Y^c = \frac{1}{3}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2\end{pmatrix} \qquad (3)$$

or

$$V^c = \frac{1}{3}\begin{pmatrix} 1 & & \\ & -2 & \\ & & 1\end{pmatrix}. \qquad (4)$$

Both the U^c and Y^c choices are discussed in Ref. 9. The V^c choice does not seem to appear in the literature. The different choices of J_μ reflect themselves in the charge assignment within the spectrum of states and especially in the number of vector mesons coupled to the photon and in the magnitude of the coupling constants, to which properties we will turn next.

As the *color singlet piece of the current* (1) is a pure octet operator with respect to ordinary SU(3), the $SU(3)^c$ singlet states coupled to the photon must be neutral octet states with respect to ordinary SU(3) (ω_8 and ρ^0). As is well known, because of singlet octet (ω_1, ω_8) mixing

$$(1, 1^c) + (8, 1^c),$$

mixing

there are actually three ordinary vector mesons coupled to the photon, ρ^0, ω, and ϕ. In order to distinguish them from possible color-octet states, these well-known vector mesons may conveniently be denoted by (ρ^0, ω_1^c), (ω, ω_1^c), and (ϕ, ω_1^c), respectively. [The conventional notation is thus used also within $SU(3)^c$ multiplets, in order to denote the color quantum numbers of the particles.] Explicitly, the $SU(3)\times SU(3)^c$ wave function of these particles may be written as

$$(\rho^0, \omega_1^c) = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \times \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix},$$

$$(\omega, \omega_1^c) = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \times \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \qquad (5)$$

$$(\phi, \omega_1^c) = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 & & \\ & 0 & \\ & & -1 \end{pmatrix} \times \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix},$$

where ideal mixing (i.e., ϕ contains λ quarks only) has been assumed.

From (1) and (2), *the color-octet part of the current* is a singlet in ordinary SU(3), and thus a color-octet version of the ordinary ρ^0, and likewise of ω_8, does not couple to the photon. Depending on the color-octet structure of J_μ, there can be two ω_1 color-octet states (with color isospin $I_c = 0$ and $I_c = 1$, respectively) coupled to the photon: the states (ω_1, ω_8^c) and (ω_1, ρ^{0c}). We conjecture ω_1-ω_8 mixing to be present also for color-octet states,

$$(1, 8^c) + (8, 8^c).$$

mixing

If we, moreover, assume this mixing to be ideal, then ω and ϕ will have color-octet recurrences, which are coupled to the photon (ω_8^c and ρ^{0c}) and which predominantly consist of nonstrange and strange quarks, respectively. The *vector mesons thus allowed to couple to the photon* according to the $(8, 1^c) + (1, 8^c)$ structure of the electromagnetic current are collected in Table I. There is a maximum number of four additional (colored) vector mesons with direct electromagnetic coupling.

The vector-meson-photon couplings γ_V^{-2} are defined by the current matrix element

$$\langle 0 | J_\mu(0) | V\rangle = \frac{m_V^2}{2\gamma_V}\epsilon_\mu, \qquad (6a)$$

and are related to the width by

$$\Gamma_{V\to e^+e^-} = \frac{\alpha^2}{12}\left(\frac{\gamma_V^2}{4\pi}\right)^{-1} m_V. \qquad (6b)$$

For a local current, $J_\mu(x) = Z[\overline{\psi}(x), \gamma_\mu Q\psi(x)]$, and in a bound-state model for the vector mesons this matrix element is expressed by[10]

$$\frac{m_V^2}{2\gamma_V}\epsilon_\mu = (2\pi)^{-4}\,\mathrm{tr}\left(Z\gamma_\mu \int \chi(k, P)d^4k\, QV_{SU(3)\times SU(3)^c}\right)$$

and consists of a product of two terms:

(a) the $SU(3)\times SU(3)^c$ internal-symmetry structure of the vector meson V [Eq. (5) and Table I] and the electromagnetic current [Eqs. (2), (3), or (4)];

(b) the configuration-space wave function at zero distance, $\mathrm{tr}(\gamma_\mu \int \chi(k, P)d^4k)$. The first term is simply calculated from

$$\langle Q_V\rangle \equiv \mathrm{tr}(QV_{SU(3)\times SU(3)^c}), \qquad (7)$$

and the results thus obtained for the different models are collected in Table II. These numbers give the relative coupling strengths of vector mesons, if the dynamical term (b) is assumed to be $SU(3)\times SU(3)^c$ invariant. Empirically this holds in good approximation for the well known 9 : 1 : 2 ratio of $\gamma_\rho^{-2} : \gamma_\omega^{-2} : \gamma_\phi^{-2}$. We may thus conjecture that the *relative couplings of the colored states, as given in Table II,* are correct, while the absolute magnitude may be different for the color-octet mesons because of symmetry breaking in

TABLE I. $SU(3)\times SU(3)^c$ selection rules for vector-meson–photon couplings.

SU_3 \ SU_3^c	ω_1^c	ω_8^c $(I^c = 0, Y^c = 0)$	ρ^{0c} $(I^c = 1, I_3^c = 0, Y^c = 0)$
ω_1		$(\omega_1, \omega_8^c) <$ (ω, ω_8^c), (ϕ, ω_8^c)	$(\omega_1, \rho^{0c}) <$ (ω, ρ^{0c}), (ϕ, ρ^{0c})
ω_8	$\omega_8 <$ $\omega \equiv (\omega, \omega_1^c)$, $\phi \equiv (\phi, \omega_1^c)$		
ρ^0	$\rho^0 \equiv (\rho^0, \omega_1^c)$		

the dynamical part.

Let us next try to assign tentatively the observed states $J(3.1)$ and $\psi(3.7)$ to colored vector mesons. Since colored ρ^0 mesons do not couple to the photon, both J and ψ have to be isospin-0 states. Only two narrow states have been found up to now, and Table II thus suggests the *I^c-spin scalar model (3),* leading to only two additional new ground-state vector mesons coupled to the photon. If we associate the lower-mass state, J, with (ω, ω_8^c) and the higher one, $\psi(3.7)$, with (ϕ, ω_8^c), i.e., $J(3.1) \equiv (\omega, \omega_8^c)$, $\psi(3.7) \equiv (\phi, \omega_8^c)$, the leptonic decay widths of these particles should be in the ratio of $2m_J/m_\psi \cong 1.7$ [see Table II and Eq. (6b)], which indeed seems in agreement with experiment,[1,2,11]

$$\Gamma_{e^+e^-}(J) \cong 5.5 \pm 0.5 \text{ keV}, \quad \Gamma_{e^+e^-}(\psi) \cong 3 \text{ keV},$$

$$\gamma_J \cong 5.6 \pm 0.3, \quad \gamma_\psi \cong 8.3.$$

The hadronic decay products of the lower-mass state should then be ω-like, while the upper state should give rise to a large fraction of strange particles. The cascade decay $\psi(3.7 \to J(3.1) +$ hadrons is strongly suppressed by Zweig's rule, because $\psi(3.7)$ thus consists of strange quarks while J contains nonstrange ones only. Cascading is present insofar as the ω_1-ω_8 [ordinary SU(3)] mixing differs from the ideal one.

The ratio of photon couplings and the suppression of strong cascading remain unchanged, if J and ψ are assigned to colored ω and ϕ with $I^c = 1(\rho^{0c})$, in the *U^c-spin scalar model:* $J(3.1) \equiv (\omega, \rho^{0c})$, $\psi(3.7) \equiv (\phi, \rho^{0c})$. The two additional colored vector mesons required in this model have not been seen so far. They could be broader states, if we would allow for color octet-singlet $\omega_8^c - \omega_1^c$ mixing. The observed $I^c = 1$ states, however, would clearly have a small width if I^c is assumed conserved in strong interactions, much in analogy to ordinary isospin.

III. ARE THE RADIATIVE DECAYS SUPPRESSED?

Having thus concluded that the interpretation of $J(3.1)$ and $\psi(3.7)$ as colored ω and ϕ is reasonable as regards the ratios of their photon couplings and the suppression of cascading, let us next turn to what seems to be a central problem for the color interpretation, the question of the magnitude of the radiative decay widths. Because of the general selection rules for decays of color-octet states (8^c)

$$\begin{aligned} &8^c \not\to 1^c + 1^c, \\ &8^c \to 1^c + 8^c, \\ &8^c \to 8^c + 8^c, \end{aligned} \tag{8}$$

we may have the radiative transitions

$$8^c \to 1^c + \gamma, \tag{9a}$$

$$8^c \to 8^c + \gamma, \tag{9b}$$

where γ now stands for a color-octet photon and 1^c and 8^c denote color-singlet and color-octet hadrons, respectively. (A 2γ decay is forbidden for 1^{--} vector mesons.) Since the 8^c photon as well as $J(3.1)$ and $\psi(3.7)$ have zero isospin in the color scheme, typical radiative decays are

$$\begin{aligned} J(3.1), \psi(3.7) &\to \eta + \gamma, \\ &\to X^0 + \gamma, \end{aligned} \tag{10}$$

while the $\pi^0\gamma$ decay is forbidden.

Phase space is small for (9b), if the 8^c pseudoscalars are assumed to have masses comparable to the masses of the new particles, and radiative widths may therefore be sufficiently small to be consistent with experiment. Simple estimates[12] for reactions (9a), however, as mentioned in the Introduction, yield widths which are larger by roughly two orders of magnitude than the observed extremely narrow widths of the new particles. Indeed, from

$$\Gamma(V \to P + \gamma) = \alpha g^2 \frac{p_{\text{c.m.}}{}^3}{3}, \tag{11}$$

where P denotes pseudoscalar meson, one obtains a width for $J(3.1) \to \eta\gamma$ of the order of 15 MeV, if one simply inserts for the coupling g the value obtained from the $\omega \to \pi^0\gamma$ decay ($\Gamma_{\omega\pi^0\gamma} \cong 0.9$ MeV) corrected by a Clebsch-Gordan coefficient. Such a large width would obviously exclude the color interpretation. Evidence will be presented, however, in what follows which shows that the above estimate may in fact be misleading. By analyzing the decay of the $\eta(549)$ meson (which can decay into 8^c photons) we will see that there is empirical support for the hypothesis that the *coupling between two color-octet vector mesons and a color-singlet pseudoscalar meson is much smaller* than suggested by the above reasoning based on the coupling between three color-singlet states (ω, π^0, γ).

TABLE II. Photon–vector-meson couplings $\langle Q_V \rangle$ in various $SU(3) \times SU(3)^c$ models.

SU_3 \ SU_3^c		$I^c = 0$		$U^c = 0$		$V^c = 0$	
	ω_1^c	ω_8^c	ρ^{0c}	ω_8^c	ρ^{0c}	ω_8^c	ρ^{0c}
ρ^0	$\frac{3}{\sqrt{6}}$	0	0	0	0	0	0
ω	$\frac{1}{\sqrt{6}}$	$\frac{2\sqrt{2}}{\sqrt{6}}$	0	$\frac{-\sqrt{2}}{\sqrt{6}}$	$\frac{-\sqrt{6}}{\sqrt{6}}$	$\frac{-\sqrt{2}}{\sqrt{6}}$	$\frac{\sqrt{6}}{\sqrt{6}}$
ϕ	$\frac{\sqrt{2}}{\sqrt{6}}$	$\frac{-2}{\sqrt{6}}$	0	$\frac{1}{\sqrt{6}}$	$\frac{\sqrt{3}}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\frac{-\sqrt{3}}{\sqrt{6}}$

From the $SU(3)\times SU(3)^c$ structure of π^0, η_8, and η_1 and the electromagnetic current (1), one obtains for the couplings

$$g_{\pi^0\gamma\gamma}=\frac{1}{\sqrt{6}}g^{1^c},$$
$$g_{\eta_8\gamma\gamma}=\frac{1}{(6\times3)^{1/2}}g^{1^c}, \qquad (12)$$
$$g_{\eta_1\gamma\gamma}=\frac{2\sqrt{2}}{(6\times3)^{1/2}}(g^{1^c}+g^{8^c}),$$

where g^{1^c} and g^{8^c} denote the reduced couplings of a color-singlet hadron to two color-singlet and two color-octet photons, respectively. Because of mixing, the η and X^0 couplings are related to the couplings (12) by

$$g_{\eta\gamma\gamma}=g_{\eta_8\gamma\gamma}\cos10^\circ+g_{\eta_1\gamma\gamma}\sin10^\circ,$$
$$g_{X^0\gamma\gamma}=-g_{\eta_8\gamma\gamma}\sin10^\circ+g_{\eta_1\gamma\gamma}\cos10^\circ, \qquad (13)$$

and the η and X^0 decay widths are given by

$$\Gamma(P\to\gamma\gamma)=\frac{\alpha^2\pi}{4}m_P{}^3g_{P\gamma\gamma}{}^2\,. \qquad (14)$$

From the experimental π^0, η, X^0 widths according to (12), (13), and (14) one may thus (at least in principle) derive a bound on the coupling g^{8^c} between a color-singlet hadron and two color-octet photons.

In Table III two extreme alternatives for this coupling are compared with experiment,[13,14] namely the assumptions $g^{8^c}\equiv0$ and $g^{8^c}\equiv g^{1^c}$. One observes that $g^{8^c}\equiv g^{1^c}$ is incompatible with the experimental value[13] of the η width, whereas $g^{8^c}\ll g^{1^c}$ is in good agreement with the data. Thus the coupling of a 1^c pseudoscalar hadron to two color-octet photons (8^c) seems suppressed. This result may suggest the $8^c\to1^c+\gamma(8^c)$ transition (10) to be strongly suppressed, also relative to what one estimates from the $1^c\to1^c+\gamma(1^c)$ type transition, $\omega\to\pi^0\gamma$.

This latter conclusion may be more explicitly arrived at, if the $\eta\gamma\gamma$ decay is related to the $J(3.1)\to\eta\gamma$ decay via $J(3.1)$ dominance in the manner of Gell-Mann–Sharp-Wagner[15] (Fig. 2). From

$$g^{8^c}_{\eta\gamma\gamma}=\sum_{V=J,\psi}\frac{1}{2\gamma_V}g_{V\eta\gamma}$$
$$=\sum_{V=J,\psi}\frac{1}{4\gamma_V{}^2}g_{V\eta V} \qquad (15)$$

we infer that the smallness of $g^{8^c}_{\eta\gamma\gamma}$ should in fact also yield a suppression of $g_{J\eta\gamma}$, which coupling is responsible for the $J(3.1)\to\eta\gamma$ decay. Putting experimental errors aside for the moment, we may be bold enough to conclude: *If the colored-triplet model is correct at all, then from the above analysis of the η decay a suppression of the decay of the colored vector mesons does not seem so surprising after all and could have been anticipated (at least qualitatively) prior to the experimental data.*

Let us add at this point a quantitative estimate of the $1^c8^c8^c$ couplings $g_{J\eta\gamma}$ and $g_{J\eta J}$. While a quantitative statement on $g^{8^c}_{\eta\gamma\gamma}$ from the η decay beyond the above conclusion, $g^{8^c}\ll g^{1^c}$, seems unfeasible, we may at least check the consistency of the $\eta, X^0\to\gamma\gamma$ and $J(3.1)\to\eta\gamma, X^0\gamma$ decays. From the total width $\Gamma_{tot}(J)\cong100$ keV, one estimates [with $\Gamma(J\to X^0\gamma)\cong2\Gamma(J\to\eta\gamma)$] $g_{JX^0\gamma}<0.1\ \mathrm{GeV}^{-1}$ and (with $\gamma_J=5.6$) $g^{8^c}_{X^0\gamma\gamma}<0.015\ \mathrm{GeV}^{-1}$. From this value we obtain with the help of Eqs. (12) and (13) $g^{8^c}<0.02\ \mathrm{GeV}^{-1}$, which has to be compared with the corresponding color-singlet coupling, $g^{1^c}=0.68\ \mathrm{GeV}^{-1}$. One concludes that the ratio of the color-octet to color-singlet couplings must be less than 3% in consistency with our earlier conclusion (from $\eta\to\gamma\gamma$) that $g^{8^c}\ll g^{1^c}$. Likewise from (15) we infer $\sqrt{3}\,g_{J\eta J}<1.2\ \mathrm{GeV}^{-1}$, to be compared with the $\rho^0\omega\pi^0$ coupling, $g_{\rho^0\omega\pi^0}\cong14\ \mathrm{GeV}^{-1}$. From our analysis of the η and the $J(3.1)$ decays one may thus be led to conjecture a strong suppression of the hadronic couplings,

$$g_{P^{1^c}V^{8^c}V^{8^c}}/g_{P^{1^c}V^{1^c}V^{1^c}}<9\%\,,$$

TABLE III. Comparison with experiment (see Refs. 13 and 14) of the $\gamma\gamma$ couplings for two extreme alternatives for g^{8^c}.

	π^0	η	X^0
$\Gamma^{\mathrm{exp}}_{\gamma\gamma}$	7.8 ± 0.9 eV	324 ± 46 eV	<22 keV
g^{exp} [GeV^{-1}]	0.275 ± 0.016	0.216 ± 0.015	<0.78
g^{th} [GeV^{-1}] ($g^{8^c}\equiv0$)	input	0.234 ± 0.014	0.415 ± 0.024
g^{th} [GeV^{-1}] ($g^{8^c}\equiv g^{1^c}$)	input	0.312 ± 0.018	0.857 ± 0.05

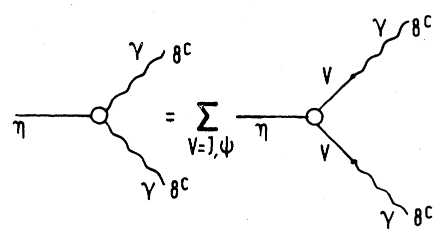

FIG. 2. J, ψ dominance of the $\eta\to\gamma^{8^c}\gamma^{8^c}$ amplitude.

as the dynamical origin for the strongly suppressed radiative decays of the new particles.

From the above analysis it may be tempting to quite generally postulate a suppression of the $1^c8^c8^c$ relative to the $1^c1^c1^c$ couplings. One immediately convinces oneself, however, that at least some of the $1^c8^c8^c$ couplings have to be large, provided simple vector-meson dominance arguments are again assumed to hold. In fact, for a color-neutral π^+ meson $(\pi^+, \pi^{0c}) = (1/\sqrt{2})(\bar{n}_1 p_1 - \bar{n}_2 p_2)$ the color-octet charge vanishes and only the color-singlet photon couples. Simple $\rho^0(770)$ dominance combined with universality of the electric charge requires

$$g_{\rho\,\pi^c\pi^c}/2\gamma_\rho = 1\,.$$

The V^{1^c}-P^{8^c}-P^{8^c} coupling would thus be expected to be of the same magnitude as the V^{1^c}-P^{1^c}-P^{1^c} type $\rho\pi\pi$ coupling. If the three-triplet model is correct at all, we have to conclude at this stage that the underlying dynamics is complicated.

IV. THE VALUE OF R AND THE RECURRENCES OF $J(3.1)$ AND $\psi(3.7)$

Recently it has been suggested by two of us[16] that the new particles J and ψ set the scale for e^+e^- annihilation into hadrons of a new hadronic degree of freedom quite independently of the color or charm interpretation. In what follows, we wish to supplement recent considerations by a detailed discussion of the ratio R in the light of the color interpretation as presented in the previous sections.

With "new duality,"[17] R is given by[16,18]

$$R = \sum_V R_V = \frac{3\pi}{4}\sum_V \frac{1}{\gamma_V^2/4\pi}\,\frac{m_V^2}{\Delta m_V^2}\,\Theta\left(s - \left(m_V^2 - \frac{\Delta m_V^2}{2}\right)\right), \qquad (16)$$

where the sum over V runs over ρ^0, ω, ϕ and over additional (ground-state) vector mesons corresponding to the coupling of the photon to new types of hadronic matter, i.e., $J(3.1)$ and $\psi(3.7)$ in the color scheme.

Below the color threshold, determined by the mass of $J(3.1)$, the ratio R from (16) is given by the couplings of the photon to ρ^0, ω, ϕ. With a Veneziano-type mass spectrum, $m_n^2 = m_\rho^2(1+2n)$, $n = 0, 1, 2, \ldots$, we have $\Delta m_V^2 \equiv \alpha'^{-1} = 2m_\rho^2 \cong 1.2$ GeV^2, and consequently (with $\gamma_\rho^2/4\pi \cong 0.64$) $R_{\rho^0,\omega,\phi} \cong 2.5$. This value for R agrees surprisingly well with the value determined from the sum of the squared quark charges,[4,19]

$$R^{(1^c)} = \sum_i (Q_i^{(8,1^c)})^2 = 2\,,$$

where below the color threshold only color-singlet states are produced.

It is thus tempting to require consistency between the value of R obtained via "new duality" and the value of R obtained from the sum of the squared charges of the constituents to also hold for the contribution due to the production of colored states. From[4] $R = R^{(1^c)} + R^{(8^c)} = \sum_i Q_i^2 = 4$ we thus have

$$R^{(8^c)} = R_J + R_\psi = 2\,, \qquad (17)$$

which equation may be used to *calculate the a priori unknown level spacing* $\Delta m_V^2(8^c)$ *for the recurrences of the color-octet states*. From the experimental e^+e^- width,[11] $\Gamma^J_{e^+e^-} = 5.5 \pm 0.5$ keV, one obtains the photon coupling $\gamma_J^2/4\pi = 2.5 \pm 0.25$, which yields

$$\Delta m_V^2(8^c) = \frac{3\pi}{8}\,\frac{1}{\gamma_J^2/4\pi}\,(m_J^2 + \tfrac{1}{2}m_\psi^2) = 7.8 \pm 0.7\ \text{GeV}^2\,, \qquad (18)$$

where the predicted ratio $\gamma_J^{-2} : \gamma_\psi^{-2} = 2 : 1$ from Table II has been used in addition. With the calculated value for the level spacing[20,21] the ratios R_J and R_ψ are found to be $R_J = 1.2 \pm 0.2$ and $R_\psi = 0.8 \pm 0.1$, in good agreement with experiment (see Fig. 3).

If the above value for $\Delta m_V^2(8^c)$ is taken literally

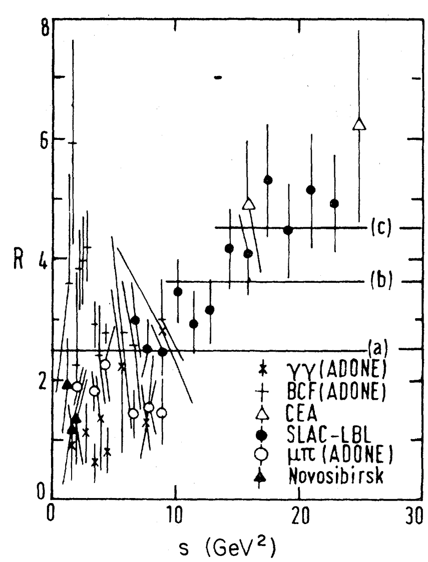

FIG. 3. R from "new duality," showing the contributions to R dual to ρ^0, ω, ϕ [curve (a)] and the effect of adding the contributions dual to $J(3.1)$ [curve (b)] and to $\psi(3.7)$ [curve (c)]. Figure from Ref. 22.

as the level spacing of a spectrum of daughters of J and ψ, or of radially excited quark-antiquark bound states, *one predicts two series of colored vector mesons* $[m_n^2 = m_{J,\psi}^2 + \Delta m_V^2(8^c)\times n$, $n = 0, 1, 2, \ldots]$ with the masses given in Table IV. It is quite clear from the table that the predicted levels involve experimental errors as well as a theoretical uncertainty as regards the exact validity of the "*strong new duality*" *requirement, which says that the magnitude of R determined by the prominent low-lying vector mesons coincides*[23] *with the value of R determined from the squares of the constituent-quark charges.* Nevertheless, the level spectrum may serve as a guide for further experimental searches. Because of the huge number of predicted colored-meson states in the three-triplet model, narrow widths should not be expected, however, for the higher-mass recurrences, as many decay channels open up.

V. SUMMARIZING AND CONCLUDING REMARKS

Let us thus briefly summarize the main points which have been made and at the same time add a few comments on topics not discussed in the previous sections.

From Sec. II within the three-triplet model, one expects (Table I) either two (I^c-scalar model) or four (U^c-scalar model) additional (color-octet) vector mesons with direct photon couplings. These are colored versions (color-neutral members of the color octet) of the ordinary ω and ϕ mesons. Their relative couplings (Table II) fulfill $\gamma_{(\omega,8^c)}{}^{-2} : \gamma_{(\phi,8^c)}{}^{-2} = 2:1$. The new particles $J(3.1)$ and $\psi(3.7)$ have been tentatively identified[24] with colored ω and colored ϕ, respectively. The relative photon couplings (leptonic widths $\Gamma_{e^+e^-}$) of these two states are thus correctly predicted from $SU(3)\times SU(3)^c$ symmetry; the absolute values, relative to the $\rho^0(1^c)$ photon coupling, show an $SU(3)\times SU(3)^c$ symmetry-breaking effect, the magnitude of which depends upon the $SU(3)^c$ structure chosen for the electromagnetic current.

TABLE IV. Recurrences of $J(3.1)$ and $\psi(3.7)$ in the color scheme.

n	$SU(3)\times SU(3)^c$ structure	mass (GeV)	e^+e^- width (keV)
0	$(\omega,\omega_8^c)\equiv J$	3.105 (input)	5.5 ± 0.5 (input)
1	$(\omega,\omega_8^c)'\equiv J'$	4.18 ± 0.08	4.0 ± 0.5
2	$(\omega,\omega_8^c)''\equiv J''$	5.03 ± 0.13	3.4 ± 0.4
...	...	...	...
0	$(\phi,\omega_8^c)\equiv\psi$	3.695 (input)	3.3 ± 0.3
1	$(\phi,\omega_8^c)'\equiv\psi'$	4.63 ± 0.08	2.6 ± 0.3
2	$(\phi,\omega_8^c)''\equiv\psi''$	5.41 ± 0.13	2.3 ± 0.2

As regards the analysis[25] of the $\eta\to\gamma\gamma$ decay in Sec. III, a pessimist (as regards the relevance of the three-triplet model) may stress that the η decay width just shows that there is no color-octet current contribution, and that the new particles have thus quite obviously nothing to do with the Han-Nambu model. He may add that the suppression of certain couplings ($P^{1^c}V^{8^c}V^{8^c}$, as inferred from $\eta\to\gamma\gamma$), while others seem to be required to have normal strength ($V^{1^c}P^{8^c}P^{8^c}$), looks artificial to him. An optimist may answer at this point that the consistency between the η decay and the narrow widths of the new particles is encouraging to him, and that nothing prevents the dynamics behind the couplings from being more complicated. The realist may wish to wait for further crucial experiments. An answer to the question of whether the missing neutral energy around 4 GeV is due to (color-octet) photons may be decisive already.

Requiring "strong new duality," i.e., consistency between the value of R calculated from the quark charges and the value obtained from the photon couplings to the prominent low-lying vector mesons, in Sec. IV, we have predicted a level spacing $\Delta m_V^2(8^c)\cong 8\ \text{GeV}^2$, dramatically different from the well-known level spacing of 1^c states $\Delta m_V^2(1^c)\cong 1.2\ \text{GeV}^2$. The spectrum of daughter states to be expected from $\Delta m_V^2(8^c)\cong 8\ \text{GeV}^2$ has been listed in Table IV.

We have not attempted in this paper to give complete systematics and production characteristics of all the additional 72 colored spin-0, spin-1, etc. states to be expected in the Han-Nambu model. Let us, however, add a few rather obvious comments. Since we have assumed the ω_1-ω_8 mixing (in ordinary SU_3) to be the same for the 8^c versions of $\omega(783)$ and $\phi(1019)$, we can use the mass formulas valid for ideal mixing,

$$m_\rho^2 = m_\omega^2\,,\qquad 2m_{K^*}^2 = m_\omega^2 + m_\phi^2\,, \tag{19}$$

to predict the masses of the color-neutral partners of $J(3.1)\equiv(\omega,\rho^{0c}$ or $\omega_8^c)$ and $\psi(3.7)\equiv(\phi,\ \rho^{0c}$, or $\omega_8^c)$, namely $m((\rho,\ \rho^{0c}$, or $\omega_8^c)) = 3.1$ GeV and $m((K^*,\ \rho^{0c}$, or $\omega_8^c)) = 3.41$ GeV. The masses of, e.g. (ω, K^{*c}) are probably not degenerate with the (ω,ρ^{0c}) because of the *a priori* unknown symmetry breaking in color space.

Finally, let us add a brief remark on the decays of especially the recurrences J' and ψ' of the colored ω and ϕ mesons. Typical decay modes are, e.g.,

$$J'(4.18) \to (\rho^0, 8^c) + \pi^0 , \quad (\rho^0, 8^c) \searrow \pi^0 + \gamma^c ,$$

$$J'(4.18) \to (\rho^+, 8^c) + \pi^- , \quad (\rho^+, 8^c) \searrow \pi^+ + \gamma^c ,$$

$$\psi'(4.6) \to (K^{*+}, 8^c) + K^- , \quad (K^{*+}, 8^c) \searrow K^+ + \gamma^c ,$$

and thus involve many direct photons, which would have to be responsible for the missing neutral energy,[22] if the color option is correct.

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[18]In a more sophisticated approach the Θ function in Eq. (16) should be smoothed out by an appropriate threshold factor. If $\sigma(e^+e^- \to h)$ is built up from a discrete set of vector mesons, Δm_V^2 in (16) is identical with the level spacing. For the case of a continuum in addition to the low-lying vector mesons V, Δm_V^2 corresponds to the distance beyond m_V^2 at which continuum production with the quantum numbers of V sets in.

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[20]A large spacing, $\Delta m_V^2 = 11\ \mathrm{GeV}^2$, has been predicted in Ref. 21. The argument was based on the dynamical quark model, in which the leptonic decay width of $J(3.1)$ requires the wave function to be concentrated at smaller distances than for normal hadrons, thus leading to a larger level spacing of the bound states. Δm_V^2 was obtained from the zero-point energy (mass of the ground state) in the four-dimensional oscillator model.

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[23]A related requirement for the $\pi^0 \to 2\gamma$ decay has been discussed by P. G. O. Freund and S. Nandi, Phys. Rev. Lett. 32, 181 (1974).

[24]While this paper was being completed, we received the papers of B. Stech [Heidelberg report, 1974 (unpublished)] and S. Y. Tsai [Nihon Univ. report, 1974 (unpublished)]. In contrast with our work, these authors assign both new particles to pure singlet state with respect to ordinary SU(3) (with $I^c = 1$ and 0, respectively). Our assumption of ideal mixing also for colored states seems very natural, however, within the quark model. Moreover, in the assignment to SU(3) singlets, the cascade decay $\psi(3.7) \to J(3.1) + 2\pi$ would involve an $I^c = 1 \to I^c = 0$ transition. Simple estimates for such an electromagnetic (?) mixing effect from $\rho\omega$ mixing, taking into account the mass differences between $\psi(3.7)$ and $J(3.1)$, lead to widths for $\psi(3.7) \to J(3.1)$ + hadrons of the order of less than 1 keV, i.e., much smaller than the 20% of the total width reported from the experiments.

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