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COMMENT CONCERNING THE EVALUATION OF THE CABIBBO-RADICATI SUM RULE

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The Gilman-Schnitzer evaluation of the Cabibbo-Radicati sum rule is discussed. It is argued that the SU(2) generalization of the $\Delta N\pi\gamma$ contact term has to be included in the evaluation. Doing so considerably *decreases* (15% rather than 8% discrepancy) the agreement of the sum rule with the data.

1. Introduction

The sum rule of Cabibbo and Radicati [1] follows from assuming that the equal-time commutators of the time components of the isovector currents have no Schwinger terms. In ref. [2] the sum rule has been evaluated and agreement with the data has been found. In the present note, we consider a further effect which affects the agreement and makes it considerably worse.

The sum rule reads

$$\langle \tfrac{1}{6} r_N^2 \rangle^V - \frac{1}{2}\left(\frac{\mu_p - \mu_n}{2M}\right)^2 = \frac{1}{4\alpha\pi^2} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} (\sigma_{tot}^{\gamma^- p}(\nu) - \sigma_{tot}^{\gamma^+ p}(\nu)), \tag{1}$$

with [2]

$$\tfrac{1}{6} m_\pi^2 \langle r_N^2 \rangle^V = 0.066, \qquad \tfrac{1}{2} m_\pi^2 \left(\frac{\mu_p - \mu_n}{2M}\right)^2 = 0.059.$$

The cross sections $\sigma_{tot}^{\gamma^\pm p}$ are the total cross sections for the scattering of charged photons (defined as zero-mass charged ρ) off a proton target.

In evaluating the sum rule the authors of ref. [2] have divided the integration region into three parts. They use photoproduction data and the assumption of dominance of the E_0^+ and M_1^+ multipoles between threshold ν_0 and $\nu = 500$ MeV. This region includes the Δ resonance and yields

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$$\frac{m_\pi^2}{4\alpha\pi^2}\int_{\nu_0}^{500}\frac{d\nu}{\nu}(\sigma_{tot}^{\gamma^- p}(\nu)-\sigma_{tot}^{\gamma^+ p}(\nu)) = 0.016 - 0.028 = -0.012. \tag{2}$$

(The -0.028 is the Δ resonance contribution and the 0.016 comes from the non-resonant s-wave.)

In order to compute the contribution from $\nu = 500$ MeV to $\nu = 800$ MeV the authors of ref. [2] apply two different methods. Firstly, they use saturation by the N**(1520) and N***(1690). We are concerned here with the second method. That method consists in arguing that

$$\frac{m_\pi^2}{4\pi^2\alpha}\int_{500}^{\infty}\frac{d\nu}{\nu}(\sigma_{tot}^{\gamma^- p}(\nu)-\sigma_{tot}^{\gamma^+ p}(\nu)) = \frac{1}{\pi f_\rho^2}\int_{500}^{\infty}\frac{d\nu}{\nu}(\sigma_{tot}^{\pi^- p}(\nu)-\sigma_{tot}^{\pi^+ p}(\nu)), \tag{3}$$

and taking the integral on the r.h.s. from the measured πN data up to $\nu = 800$ MeV. In this way ref. [2] finds

$$\frac{m_\pi^2}{4\alpha\pi^2}\int_{500}^{\infty}\frac{d\nu}{\nu}(\sigma_{tot}^{\gamma^- p}(\nu)-\sigma_{tot}^{\gamma^+ p}(\nu)) = 0.015. \tag{4}$$

(The first method has yielded 0.016 for this.)

Contributions from the region above $\nu = 800$ MeV are neglected. Thus, ref. [2] has altogether

$$0.066 = \tfrac{1}{2}\, m_\pi^2\, \langle r_N^2\rangle^V \overset{!}{=} 0.059 + 0.016 - 0.028 + 0.015 = 0.061. \tag{5}$$

There is agreement within 8%.

It is our point in the present note that eq. (3) is unreliable and that the resulting effect considerably influences the agreement. To this we turn next.

2. Photoproduction of two pions

It is well-known [3] that the π^0N and γN total cross section are very similar (fig. 1). In fact, one has

$$\sigma_{tot}^{\gamma N}(\nu) = \frac{1}{2\times 220}(\sigma_{tot}^{\pi^+ p}(\nu)+\sigma_{tot}^{\pi^- p}(\nu)), \tag{6}$$

with an accuracy to be read off fig. 1. Since the contribution of the isoscalar photon to $\sigma_{tot}^{\gamma p}(\nu)$ amounts to about 15% of the total cross section it has been argued that there is a more basic and more exact proportionality connecting the (isovector) zero-mass pion with the isovector (zero-mass) photon γ^V [3,4],

$$\sigma_{tot}^{\pi^0 p}(\nu) \equiv \tfrac{1}{2}(\sigma_{tot}^{\pi^+ p}(\nu, m_\pi = 0) + \sigma_{tot}^{\pi^- p}(\nu, m_\pi = 0)) = \tfrac{270}{2}(\sigma_{tot}^{\gamma^+ p}(\nu)+\sigma_{tot}^{\gamma^- p}(\nu))$$
$$\equiv \sigma_{tot}^{\gamma^V p}(\nu). \tag{7}$$

A successful calculation [4] of the proportionality constant 270 from a low-

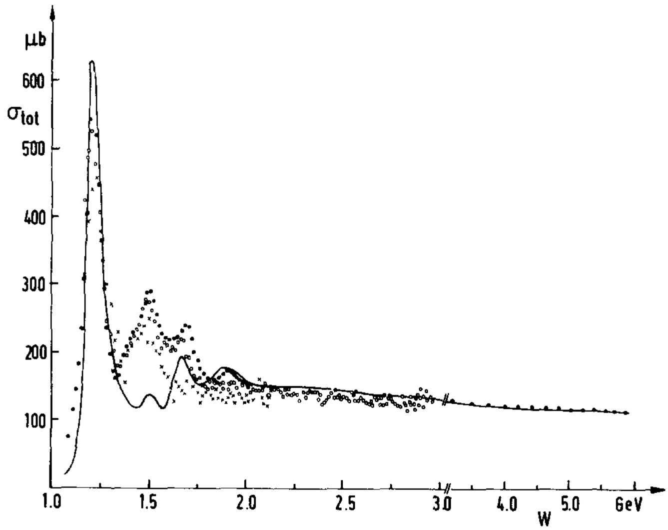

Fig. 1. The left- and right-hand side of eq. (6). The hadronic total cross section is denoted by the full line. The circles (open and closed) are γp data; the crosses denote γn data. For the origin of the data, see the references given in ref. [4].

energy limit supports eq. (7) as compared to eq. (6). Except for threshold, where mass effects have to be considered, there is only one region where eq. (7), combined with approximate equality of $\sigma_{\mathrm{tot}}^{\gamma^{\mathrm{V}}\mathrm{p}}(\nu)$ and $\sigma_{\mathrm{tot}}^{\gamma\mathrm{p}}(\nu)$ fails: immediately to the right of the Δ-resonance region where 1.3 GeV $\lesssim W \equiv \sqrt{s} \lesssim$ 1.6 GeV or 430 MeV $\lesssim \nu \lesssim$ 890 MeV. If one subtracts [3–5] the contribution σ_{c} of the 2π production from $\sigma_{\mathrm{tot}}^{\gamma^{\mathrm{v}}\mathrm{p}}$ one finds with high accuracy

$$\sigma_{\mathrm{tot}}^{\pi^0\mathrm{p}}(\nu) = 270\,(\sigma_{\mathrm{tot}}^{\gamma^{\mathrm{V}}\mathrm{p}}(\nu) - \sigma_{\mathrm{c}}(\nu)) \tag{8}$$

for ν away from threshold. Furthermore, the experimental $\sigma_{\mathrm{c}}(\nu)$ is almost completely given [3,5,6] by the contact term of $\Delta\mathrm{N}\gamma\pi$ (fig. 2), which is required by gauge invariance. Of course, no analogous term would be present in a Born term model for $\pi\mathrm{p} \to 2\pi\mathrm{N}$. Thus, as an experimental fact with some theoretical appeal, the $\gamma\pi$ analogy for π^0p and γ^{V}p is correct for the *photon cross section minus contact term contribution.*

We now should like to discuss eq. (3). This relation follows from the simple quark rule of Lipkin and Scheck [7] which implies *

$$\sigma_{\mathrm{tot}}^{\pi^{\pm}\mathrm{p}}(\nu) = \frac{4\pi\alpha}{f_\rho^2}\,\sigma_{\mathrm{tot}}^{\gamma^{\pm}\mathrm{p}}(\nu) \tag{9}$$

* Footnote see next page.

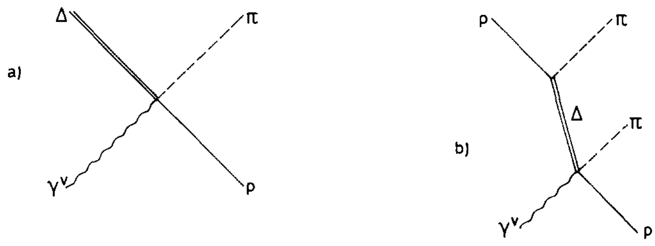

Fig. 2. (a) The $\Delta N\gamma\pi$-contact term. (b) Diagram contributing to $\gamma p \to 2\pi N$.

for "large" ν. In ref. [2], the rule has been assumed starting from $\nu = 500$ MeV. It is well known (e.g. ref. 8) that the numerical factors in eqs. (7) and (9) approximately agree. Thus eqs. (9) yield eq. (6) and thus disagree with experiment for 1.3 GeV $\lesssim W \lesssim$ 1.6 GeV. In order to obtain the difference $\sigma_{tot}^{\gamma^+p}(\nu) - \sigma_{tot}^{\gamma^-p}(\nu)$ from the πN cross sections we have to know how the discrepancy is distributed between $\sigma_{tot}^{\gamma^+p}(\nu)$ and $\sigma_{tot}^{\gamma^-p}(\nu)$. In effect, ref. [2] assumes an equal amount of discrepancy in both cross sections. That is without foundation. We will assume the discrepancy for 1.3 GeV $\lesssim W \lesssim$ 1.6 GeV to be given by the SU(2) Yang-Mills generalization of the contact term in the individual $\gamma^\pm$p cross sections **. In doing so, we have to replace $\partial_\mu \pi$ in the derivative $\Delta N\pi$ interaction, $\Delta^\mu \gamma_5 \tau N \partial_\mu \pi$, by the covariant derivative. This yields

$$\sum_{a,b,c=1}^{3} \epsilon_{abc}\, \bar{\Delta}^\mu \gamma_5 \tau^a N \rho_\mu^b \pi^c$$

as interaction term. Upon constructing the contribution to $\sigma_{tot}^{\pi^0 p}$ we find eq. (8). According to the interaction written above, mesons and baryons couple via isospin 1. This is violated [3] by about 15%. It is now easy to show that the contact term contributes $\frac{5}{4}\sigma_c$ to $\sigma_{tot}^{\gamma^+p}$ and $\frac{3}{4}\sigma_c$ to $\sigma_{tot}^{\gamma^-p}$. Thus comparing γp to πp we have to leave out the contact term and obtain

* In this footnote, we would like to comment on the *a priori* possible validity of the γ-π analogy for zero-mass pions of both charges at all energies, i.e.

$$\sigma_{tot}^{\pi^+p}(\nu, m_\pi = 0) - \sigma_{tot}^{\pi^-p}(\nu, m_\pi = 0) = 270\, (\sigma_{tot}^{\gamma^+p}(\nu) - \sigma_{tot}^{\gamma^-p}(\nu))$$

in addition to eq. (7). Since precisely the above πN cross section also occurs in the Adler-Weisberger relation we obtain from assuming validity of eq. (1) and combining the two sum rules,

$$\tfrac{1}{6} m_\pi^2 \langle r_N^2 \rangle^V = \tfrac{1}{2} m_\pi^2 \left(\frac{\mu_p - \mu_n}{2M}\right)^2 + \frac{1}{270\alpha}\left(\frac{m_\pi}{m_N}\right)^2 \frac{g_{\pi N}^2}{8\pi}\left(\frac{1}{r_A^2} - 1\right).$$

Numerically, there is a 50% discrepancy: 0.066 = 0.059 − 0.027 = 0.032. Leaving out the contact term (which might be included in the mass exptrapolation) makes the disagreement worse. The implication of this will depend on the experimental status of Cabibbo-Radicati.

** See ref. [9] for a review.

$$\sigma_{\text{tot}}^{\gamma^- p}(\nu) - \sigma_{\text{tot}}^{\gamma^+ p}(\nu) = \tfrac{1}{270}(\sigma_{\text{tot}}^{\pi^- p}(\nu) - \sigma_{\text{tot}}^{\pi^+ p}(\nu)) - \tfrac{1}{2}\sigma_c(\nu). \tag{10}$$

Therefore, integrating over the region of discrepancy, we obtain

$$-\frac{m_\pi^2}{8\pi^2\alpha}\int \frac{d\nu}{\nu}\,\sigma_c(\nu)$$

as additional contribution to the right-hand-side of (5). We take the integral from eq. (8) and the observed cross sections to obtain, integrating numerically, $m_\pi^2 \int_{420}^{890} (d\nu/\nu)\,\sigma_c(\nu) = 0.004 \times 8\pi^2\alpha$. Thus we have

$$0.066 = \tfrac{1}{6}\, m_\pi^2 \,\langle r_N^2\rangle^V \stackrel{!}{=} 0.061 - 0.004 = 0.057. \tag{11}$$

It is the main point of the present note to call attention to this possible discrepancy of about 15%. Any more definite conclusion would require additional tests. Firstly, the Stichel-Scholz model [3,6] for ρ^+p and ρ^-p can be completely worked out. The contact term, which is known to dominate 2π production by ρ^0, in that model then also dominates [10] the separate cross sections for 2π production by ρ^+ and ρ^-. Therefore, our prescription practically amounts to leaving out the total 2π production cross section of that model before comparing $\gamma^\pm$ to $\pi^\pm$. (The reader might notice that this is an obviously gauge invariant formulation of our prescription.)

Secondly, we recall (remark after eq. (4)) that Gilman and Schnitzer had found consistency of resonance saturation and the results of applying eq. (3) already for $1.3\ \text{GeV} \lesssim W \lesssim 1.6\ \text{GeV}$. Their analysis of 1966 can be reconsidered with improved data. Devenish [11], using the analysis of ref. [12], finds this way that the resonance-saturated integral in eq. (4) should be

$$\frac{m_\pi^2}{4\alpha\pi^2}\int_{500}^{\infty} \frac{d\nu}{\nu}(\sigma_{\text{tot}}^{\gamma^- p}(\nu) - \sigma_{\text{tot}}^{\gamma^+ p}(\nu)) = 0.013, \tag{12}$$

rather than 0.016, with large uncertainties in both directions. His correction of the resonance saturation, albeit much smaller, goes in the direction of our result. His final result would be $\frac{1}{6} m_\pi^2 \langle r_N^2\rangle^V = 0.059$ (as compared to our 0.057).

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