# Comment on the phases of inelastic partial waves and their comparison with the quark model, the current to constituent quark transformation, $SU(6)_W$, and vector dominance

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It is shown that the phases of inelastic $\pi N$ partial-wave amplitudes can be absolutely determined, using results from photoproduction. The signs obtained for the resonant amplitudes confirm the agreement with the relativistic quark model, the Melosh transformation, the broken $SU(6)_W$ scheme, and vector dominance in the comparison with photoproduction.

Recently, from a partial-wave analysis of the reaction $\pi N \to \pi\pi N$ the resonance amplitudes in the reactions

$$\pi N \to N^* \to \pi\Delta(1236), \quad (1)$$

and

$$\pi N \to N^* \to \rho N \quad (2)$$

have been obtained.[1] The signs of these SU(3)-inelastic amplitudes are of particular significance. They are characteristic of the quark-model state assignments of the $N^*$'s and allow stringent tests of symmetries, of the transformation from current to constituent quarks, and of vector-meson dominance by comparison with photoproduction.[2] In these tests, however, one had an over-all ± sign ambiguity in the amplitudes of (1) and (2), and the same ambiguity in the comparison between (2) and photoproduction. We resolve these ambiguities [i.e., we determine the phase of the resonant amplitudes of (1) and (2) in an absolute sense], with the help of measurements of the photoproduction reaction

$$\gamma N \to \pi\Delta. \quad (3)$$

We first have to adopt a convention for the (unobservable) relative phase between the ground state [8] and [10] baryon states. We do this by defining the $NN\pi$ and $\Delta N\pi$ coupling constants, $g$ and $g^*$, to have identical signs.[3] This convention implies the customary choice of phases of the quark-model wave functions (as used, e.g., in Ref. 4), if pionic transitions are calculated from matrix elements of the divergence of the axial quark current.

With this convention the Born terms for the reactions $\gamma N \to \pi N$ and $\gamma N \to \pi\Delta$ have well-defined relative signs. Therefore, observation of an interference of any $s$-channel resonance with the Born terms in both of these reactions will tell us the relative sign of the amplitudes for decay of this resonance into $\pi N$ and $\pi\Delta$. Since the $\pi N \to \pi N$ signs are fixed by the optical theorem, the sign of the resonance amplitude in reaction (1), and with it also the signs of all the interfering amplitudes in reactions (1) and (2), are then unambiguously determined.

Results from our measurement at DESY of reaction (3) in the charge state $\pi^-\Delta^{++}$, in the energy region $1.3 < \sqrt{s} < 1.8$ GeV, are shown in Fig. 1.[5] The interference pattern observed can be uniquely attributed to the $D_{13}(1520)$ interfering with the (large) electric Born partial wave in the $J^P = \frac{3}{2}^-$, final $S$-wave state.[6] We calculated this gauge-invariant Born partial wave as well as the corresponding one ($J^P = \frac{3}{2}^-$, final $D$ wave) for the reaction $\gamma p \to \pi^+ n$; they turn out to have the same sign. From the analysis of single-pion photoproduction[7] it is clearly established that the $D_{13}(1520)$ resonance interferes with the electric Born term in the reaction $\gamma p \to \pi^+ n$ with the same relative sign as in Fig. 1. Dividing out the isospin factors ($1/\sqrt{2}$ and $1/\sqrt{3}$, respectively[8]) the amplitude for the reaction

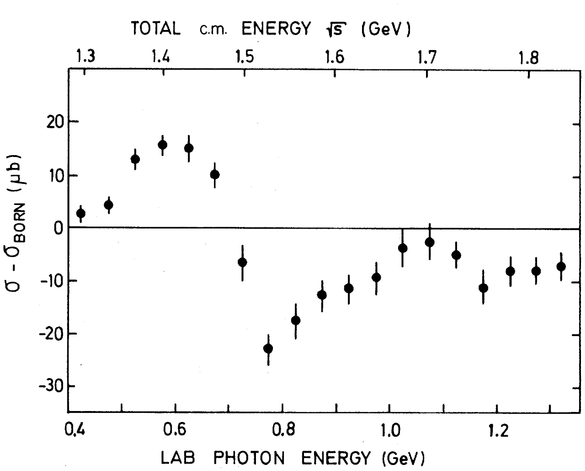


FIG. 1. Total cross section $\sigma(\gamma p \to \pi^-\Delta^{++})$ with the electric Born cross section subtracted (see Ref. 5).

$\pi N \to D_{13}(1520) \to \pi\Delta(l=0)$

is therefore found to have sign "up" (Im $T>0$ at resonance). With this established, the phases of all the mutually interfering amplitudes measured for the reactions (1) and (2) are absolutely calibrated. They are summarized in Table I.[9]

The comparison with the theoretical signs is now unique. In the quark-model calculations by Moorhouse and Parsons[10] the phases were chosen in accord with our convention on $g$ and $g^*$ so that the results can be directly compared. In fact, all their *unambiguously* defined signs agree with the experimental ones. The phase convention adopted in the predictions from broken $SU(6)_W$, and from the current to constituent quark transformation, of Refs. 11 and 12 amounts to an additional over-all factor $-1$ in the amplitudes for reaction (1). Removing it we again obtain agreement in absolute phase.

We proceed to a comparison of the amplitudes for the reaction $\pi N \to \rho_{\text{trans}} N$ with the photoproduction amplitudes for isovector $\gamma N \to \pi N$, to check whether $\rho$ dominance holds. We now no longer[3] consider the $\Delta$ a stable particle but include its decay amplitude. We note that the phase of the amplitude for the reaction $\gamma N \to \pi\pi N$ via an intermediate $D_{13}(1520) \to \pi\Delta(l=0)$ is absolutely determined independently of any assumptions, since the electric Born term with which it interferes depends on the product $eg^{*2}$ of coupling constants and is therefore absolutely known. It then follows from our discussion above that the sign of the amplitude for $\pi N \to \pi\pi N$ via this same intermediate resonant state is determined relative to sign $g$ (as the relative sign of the resonance decay amplitudes into the $\pi N$ and $\gamma N$ channels is defined relative to sign $g$ which occurs in the $\gamma N \to \pi N$ Born term). The same holds then for all the resonant amplitudes for the processes

$$\pi N \to \begin{bmatrix} \pi\Delta \\ \rho N \end{bmatrix} \to \pi\pi N. \tag{4}$$

Converting from an amplitude for

$\pi N \to \rho^0_{\text{trans}} N \to \pi_1 \pi_2 N$

to a photoproduction amplitude using $\rho$ dominance involves a factor

$$\frac{em_\rho^2}{2\gamma_\rho g_{\rho\pi\pi}} = \frac{em_\rho^2}{g^2_{\rho\pi\pi}}$$

and replacement of the $\rho^0$ polarization vector $(q_{\pi_1} - q_{\pi_2})_\mu$ by $\epsilon_\mu^{(\gamma)}$; here $\pi_1$ ($\pi_2$) has charge $+e$ ($-e$). In the partial-wave analysis[1,13] of reaction (4) the $\rho^0$ polarization (and helicity) states and the isospin Clebsch-Gordan factors were defined such that $\pi_1$ is the $\pi^+$, thus $e = +|e|$. The phases of the *predicted* photoproduction amplitudes are therefore also relative to sign $g$. On the other hand, the *measured* photoproduction phases[7] are also relative to sign $g$ as contained in the Born term; thus this unknown sign drops out from the comparison. $\rho$ dominance therefore requires absolute

TABLE I. Signs of the residues of the resonance amplitudes in the reactions $\pi N \to \pi\Delta$ and $\pi N \to \rho N$ (see Refs. 1 and 2). A question mark implies sign not determined as reliably as the others.

| $q$ multiplet | Resonance | $\pi\Delta$ lower $l$ | $\pi\Delta$ higher $l$ | $\rho N$ [a] $S=\frac{1}{2}$ | $\rho N$ [a] $S=\frac{3}{2}$, lower $l$ |
|---|---|---|---|---|---|
| $\{70, 1^-\}$ | $D_{13}(1520)$ | + | + | | + |
| | $S_{11}(1520)$ | | | (+ ?) | |
| | $S_{31}(1640)$ | + | | (− ?) | |
| | $D_{33}(1670)$ | − | (− ?) | | |
| | $D_{15}(1670)$ | − | | | |
| | $D_{13}(1700)$ | + | (− ?) | | (− ?) |
| | $S_{11}(1700)$ | − | | | |
| $\{56, 2^+\}$ | $F_{15}(1688)$ | + | (− ?) | | + |
| | $P_{13}(1700)$ | | | − | |
| | $F_{37}(1930)$ | − | | | (− ?) |
| | $F_{35}(1880)$ | | − | | − |
| $\{56, 0^+\}_{n=2}$ | $P_{11}(1470)$ | − | | (− ?) | |
| | $P_{33}(1700)$ | − | | | |
| $\{56, 0^+\}_{n=4}$ | $P_{11}(1700)$ | (− ?) | | (+ ?) | |

[a] $S$ is the total spin of $\rho$ and $N$, $\vec{S} = \vec{S}_\rho + \vec{S}_N$. The $lS$ states are defined as in Jacob and Wick, Ann. Phys. (N.Y.) 7, 404 (1959) (and in Ref. 13); in the isospin Clebsch-Gordan coefficients the baryon-first ordering is used.

agreement of the signs of corresponding amplitudes. Table 4 of Ref. 2 in fact shows such an agreement for all the well-determined signs.

Thus, the determination of the absolute phase of the inelastic $\pi N$ partial-wave amplitudes confirms the agreement already noted in the relative phases for different resonances, with the predictions from the relativistic quark model, broken $SU(6)_W$, and the Melosh transformation, as well as from vector dominance for the relation between $\rho$ and $\gamma$ transitions.

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[1] D. J. Herndon, R. Longacre, L. R. Miller, A. H. Rosenfeld, G. Smadja, P. Söding, R. Cashmore, and D. W. G. S. Leith, Phys. Rev. D 11, 3183 (1975); A. H. Rosenfeld *et al.*, Phys. Lett. 55B, 486 (1975).

[2] R. J. Cashmore, D. W. G. S. Leith, R. S. Longacre, and A. H. Rosenfeld, Nucl. Phys. B92, 37 (1975).

[3] $g$ and $g^*$ are defined by the interactions $i\sqrt{2}g\bar{\psi}_p\gamma_5\psi_n\Phi$ and $-(g^*/m_\pi)\bar{\psi}^\mu_\Delta\psi_p\partial^\mu\Phi$, describing $\pi^+n\to p$ and $\pi^+p\to\Delta^{++}$, respectively. Our metric for four-vectors is $p^2=p^\mu p^\mu=-m^2$. *The $\Delta$ is here considered a stable particle*. The minus sign in front of $g^*$ is introduced because $\Phi$ (the $\pi^+$) is a state $-|I=1, I_3=1\rangle$ in isospin space.

[4] R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).

[5] D. Lüke and P. Söding, *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, New York, 1971), Vol. 59; D. Lüke, DESY Internal Report No. F1-72/7, 1972 (unpublished).

[6] Experimental constraints on the photon couplings of the resonances (Ref. 7), as well as the observed angular distributions and helicity density matrix elements of the $\Delta(1236)$ in reaction (3) (see Ref. 5), rule out a significant contribution of the $P_{11}(1470)$ or of any other resonance to this pattern. In the $J^P=\frac{3}{2}^-$ electric Born amplitude the final-state $S$ waves dominate strongly over the $D$ waves. The $D_{13}(1520)$ resonance circle appears somewhat rotated, with the phase advanced relative to the Born term; a similarly rotated phase is observed in the reaction $\pi N\to D_{13}(1520)\to\pi\Delta(l=0)$.

[7] G. Knies, R. G. Moorhouse, and H. Oberlack, Phys. Rev. D 9, 2680 (1974); W. J. Metcalf and R. L. Walker, Nucl. Phys. B76, 253 (1974); R. C. E. Devenish, D. H. Lyth, and W. A. Rankin, Phys. Lett. 52B, 227 (1974).

[8] We always use the baryon-first ordering in the isospin Clebsch-Gordan coefficients. Note also that there is a minus sign in the $\pi^+$ isospin state (cf. Ref. 3).

[9] It so happens that in the Argand diagrams of Ref. 1 the phases were (accidentally) given correctly, if one takes into account the proper signs to convert to the baryon-first isospin convention.

[10] R. G. Moorhouse and N. H. Parsons, Nucl. Phys. B62, 109 (1973).

[11] D. Faiman and J. Rosner, Phys. Lett. 45B, 357 (1973).

[12] F. J. Gilman, in *Experimental Meson Spectroscopy—1974*, proceedings of the Boston Conference, edited by D. A. Garelick (A.I.P., New York, 1974), p. 369; F. J. Gilman, M. Kugler, and S. Meshkov, Phys. Rev. D 9, 715 (1974).

[13] D. J. Herndon, P. Söding, and R. J. Cashmore, Phys. Rev. D 11, 3165 (1975).