TWO-COMPONENT PICTURE OF e^+e^- INCLUSIVE PROCESSES

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From the hypothesis that the characteristics of hadronic production in e^+e^- are those seen in transverse distributions of hadron–hadron collisions we are led to consider the inclusive distributions in e^+e^- annihilation as composed of two contributions, a cluster component which contains the e^+e^- resonances and a parton component. The decay of the cluster is described by the same production radii as those observed in hadron–hadron collisions. The parton component obeys asymptotically Bjorken scaling and the approach to scaling is set by a mass of 1.2 GeV. Within this picture we give a simple explanation of the energy dependence of the average charged energy and the multiplicity.

In addition to the new particles and the measurement of the total cross-section up to the c.m. energy W of 7.4 GeV the new e^+e^- machines have provided us with new data on inclusive distributions, multiplicities and average energies [1]. To explain these data several models have been proposed. These models fall roughly into two classes: the quark-parton picture and the cluster or thermodynamical picture [2]. The parton model leads to the idea of Bjorken scaling [3] which is not met with the data for the scaling variable $2p/W$ (p = c.m. momentum of the observed particles) of less than 0.4 [1]. The cluster or thermodynamical models fail to reproduce the observed large p behaviour of the inclusive distributions. Also none of the proposed models give a satisfactory explanation of the observed behaviour of the average energy as a function of W.

In this paper we propose to picture the inclusive e^+e^- data as composed of two components, a cluster component and a scaling parton component. The basic idea is here that the characteristics of hadronic production in e^+e^- are those seen in *transverse* distributions of hadron–hadron collisions.

The structure of $p_\perp$-distributions for production of a hadron, h, by the inclusive process $p + p \to h$ + anything is at the ISR of the following form at longitudinal momentum $p_\parallel = 0$ (90°) in the c.m. system [4]

$$E\frac{d\sigma}{d^3p}\bigg|_{p_\parallel=0} = C(M)\exp(-2rp_\perp) + \frac{f(x)}{(p_\perp^2+m^2)^N}. \quad (1)$$

The scaling variable is $x = 2p_\perp/M$ and M is the missing mass. The choice of the variables $(p_\parallel, p_\perp, M)$ is motivated by the study in refs. [5, 6] of the structure of inclusive and exclusive hadron–hadron collisions in the transverse position plane (impact parameter plane). This study leads to the assignment of a production radius, r_h, to each type of hadron, h. These production radii are for the pions, kaons and nucleons

$$r_\pi \approx 3.0\ \mathrm{GeV}^{-1}; \quad r_k \approx 2.5\ \mathrm{GeV}^{-1}; \quad r_N \approx 2.0\ \mathrm{GeV}^{-1}. \quad (2)$$

They are independent of $p_\parallel$ and they describe the $p_\perp$-distributions of both inclusive and exclusive two-to-two hadron–hadron collisions at not too large values of $p_\perp$. The coefficient $C(M)$ is a slowly varying function of M and for practical purposes we have of course $M \approx W$, where W is the total c.m. energy. The form of the first component in eq. (1) is similar to that given in statistical or thermodynamical models [2]. We therefore expect this contribution to contain the direct channel resonances. In the second component of eq. (1) N is an integer of the order of $N \approx 4$ and m is a mass parameter which falls in the range $1.0 \lesssim m \lesssim 1.4$ GeV for the production of pions, kaons and nucleons. The function $f(x)$ is over a limited x-range of the form

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$$f(x) \approx (1-x)^{\gamma} \approx \exp(-\gamma x) \qquad (3)$$

with $\gamma \approx 12$. The general form of this component is predicted by the constituent interchange model of ref. [7]. It is clear that this part of the cross-section is dominating at large values of $p_{\perp}$ and M^*. Carrying over these general ideas to the production of hadrons in e^+e^- annihilation there is one obvious change. Since e^+e^- annihilation is thought to proceed via one photon exchange we are lacking the preferred direction which we had in hadron–hadron collisions. We therefore expect that the qualitative features of the p-distributions (p = total c.m. momentum) in e^+e^- annihilation should be similar to those of the $p_{\perp}$-distributions in hadron–hadron processes. Since the cluster part presumably is of a statistical nature we expect this to be isotropic whereas the parton component could have an angular dependence consistent with jets [10]. In conclusion the general expectation is that the p-distributions in e^+e^- annihilation have the form of eq. (1), where $p_{\perp}$ is, however, replaced by p. First we have to show that the invariant e^+e^- inclusive distributions contain at low values of c.m. energy W indeed a large cluster component with the same radii for π, K and p as found in hadron–hadron collisions. In fig. 1 the inclusive distributions on the $\psi(3.1)$ resonance into π, K and $\bar{p}$ measured by the DASP group [1] † are given together with the curves $\exp(-2r_{h}p)$ with the radii of eq. (2). It is obvious that the data are excellently described by the cluster component

$$E_{h} \frac{d\sigma_{cluster}}{d^3p}(3.1 \text{ GeV}) = C\exp(-2r_{h}p) \qquad (4)$$

and we emphasize with the *same radii* as found in hadron–hadron collisions. We take this as strong evidence for the existence of a cluster component in e^+e^- annihilation and we can try to explain the inclusive SPEAR data [1] in the whole energy region between 3 and 7.4 GeV. Unfortunately the SPEAR group has made no particle separation and we therefore have to use an average radius $\langle r \rangle$, $r_p \leqslant \langle r \rangle \leqslant r_{\pi}$ in order to describe the *charged particle* inclusive distributions. With the choice $\langle r \rangle = 2.5 \text{ GeV}^{-1}$ we compare the experimental data ‡ displayed in fig. 2 with the cluster component

$$E\frac{d\sigma_{cluster}}{d^3p} = C(W)\exp(-2\langle r \rangle p) \qquad (5)$$

The strength $C(W)$ is obtained by normalization to the experimental data at $p = 0.3$ GeV. It is seen that the low p data are nicely described by (5) even at the highest energy, $W = 7.4$ GeV ‡. The function $C(W)$ is shown in fig. 3 and compared with $\sigma_{tot}(W)$, from where we obtain

$$C(W) \approx (10 \text{ GeV}^{-2})\sigma_{tot}(W). \qquad (6)$$

* As a side remark we note that application of the correspondence principle of Bjorken and Kogut [8] implies that the exclusive $p_{\perp}$-distribution (low missing mass) should be dominated by the radius contribution over a much larger $p_{\perp}$ range than the inclusive $p_{\perp}$-distributions. This is consistent with the data [9].

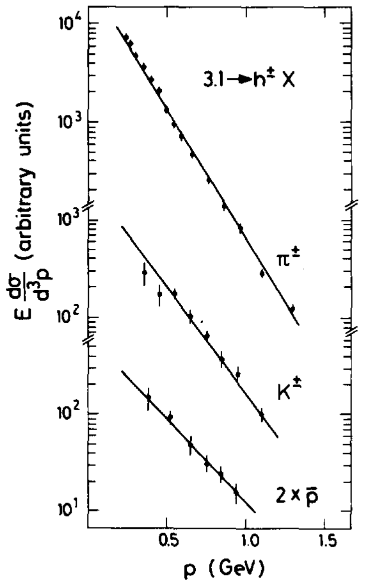

Fig. 1. Invariant cross-sections for inclusive production of $\pi^{\pm}$, $K^{\pm}$ and $\bar{p}$ at W = 3.1 GeV. The full lines represent the cluster contribution, eq. (4), with the slopes given by eq. (2). Data from DASP [1].

† These are the only known inclusive e^+e^- data where the particle separation has been done.

‡ We neglect mass corrections from particles heavier than the pion and approximate the energy by $E = \sqrt{p^2 + m_{\pi}^2}$.

‡ The form (5) is only expected to work for $p \gtrsim 0.2$ GeV, see refs. [5, 6].

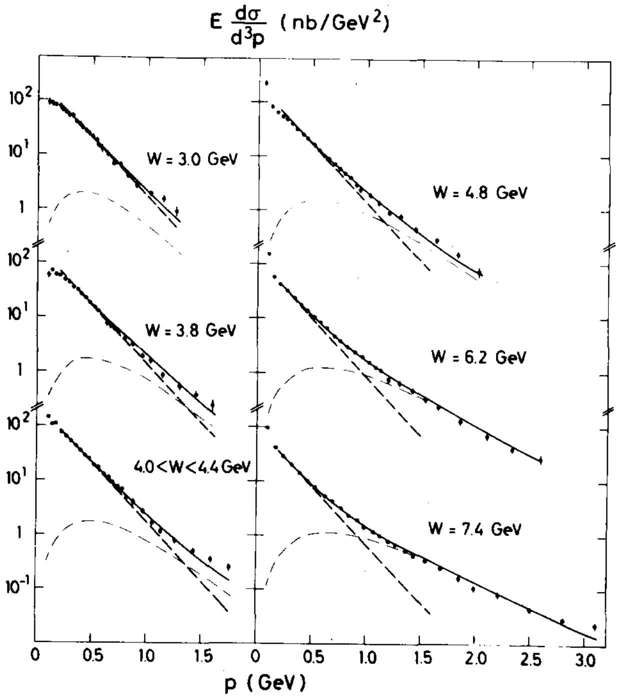

Fig. 2. Invariant cross-sections for inclusive charged particle production. The dashed line gives the cluster contribution, eq. (5), the dot-dashed line the parton contribution, eq. (7), and the full line the sum of these two contributions. Data from SPEAR [1].

It is clearly seen from the peak in $C(W)$ near 4.1 GeV that the cluster component includes the resonance contributions which is just what we expect if our physical picture is correct, where the cluster component describes the isotropic decay of fireballs.

Having fixed the cluster component, we can *calculate* the remaining part of the cross-section, which should then be the parton component $E\,\mathrm{d}\sigma_{\text{parton}}/\mathrm{d}^3p$ and similar to the second term of eq. (1). If this term obeys Bjorken scaling for $p \to \infty$ we have $N = 2$ and in fig. 4 we plot $(p^2+m^2)^2 x^{-2} E\,\mathrm{d}\sigma_{\text{parton}}/\mathrm{d}^3p$ as a function of the scaling variable $x = 2p/W$ *. We return below to the significance of the extraction of the factor x^{-2}. The mass parameter m is chosen as $m = 1.2$ GeV, i.e., in the middle of the range required by the various particles at ISR. We see that the plotted quantity does indeed scale (within rather large errors at large x) and it is furthermore well approximated by an exponential, cf. eq. (3). This means that ($s = W^2$)

$$s\,\frac{\mathrm{d}\sigma_{\text{parton}}}{\mathrm{d}x} \approx D\left(\frac{p^2}{p^2+m^2}\right)^2 \frac{\exp(-ax)}{x} = D\,\frac{x^3}{(x^2+4m^2/s)^2}\exp(-ax) \qquad (7)$$

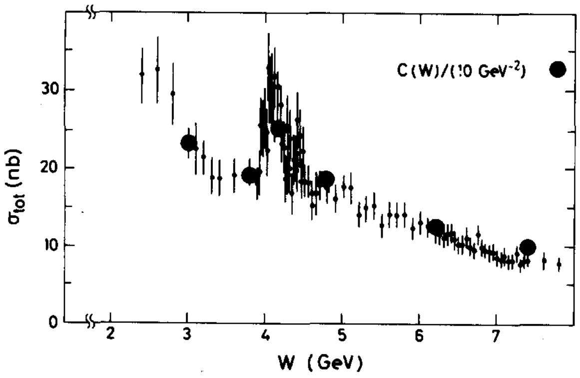

Fig. 3. The strength, $C(W)$, of the cluster component, eq. (5), compared with $\sigma_{\text{tot}}(W)$. Data from SPEAR [1].

* No reliable error estimate is possible with the available data for $x \lesssim 0.3$. In this region we show therefore no errors and include only those points that are well determined by our procedure.

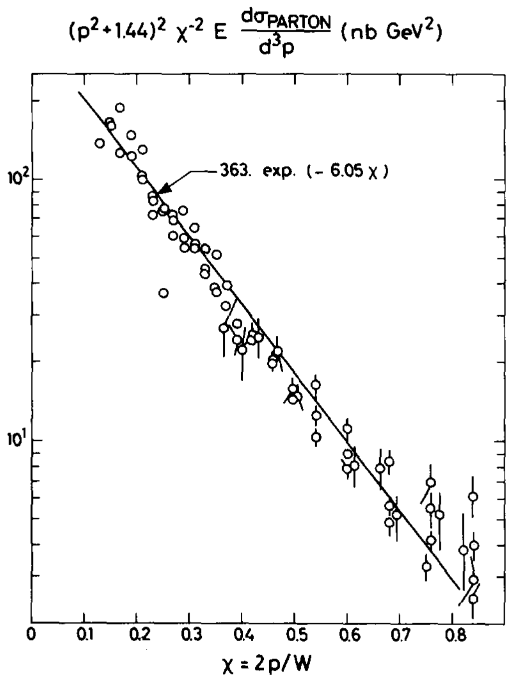

Fig. 4. The scaling parton cross-section. Data from the energies W = 3.8, 4.2, 4.8, 6.2, 7.4 GeV.

where $D = 1.83 \times 10^4$ nb GeV^2 and $a = 6.05$. Here it is seen that the parton component asymptotically obeys Bjorken scaling where the approach to scaling is set by the mass m.

To summarize the results so far we have shown that the e^+e^- inclusive p-distributions can be described by a sum of two terms, a cluster component and a parton component. The full line in fig. 2 represents the sum of these two contributions as given by eqs. (5, 7).

Having explained the inclusive distributions we should be able to understand quantities like the average energy per charged particle, $\langle E_{ch}\rangle$, and the charged multiplicity, $\langle n_{ch}\rangle$. In order to do that we evaluate the sum rules

$$\sum_h \int d^3p E_h \frac{d\sigma_h}{d^3p} = \langle n_{ch}\rangle\langle E_{ch}\rangle\sigma_{tot} \quad (8)$$

and

$$\sum_h \int d^3p \frac{d\sigma_h}{d^3p} = \langle n_{ch}\rangle\sigma_{tot} \quad (9)$$

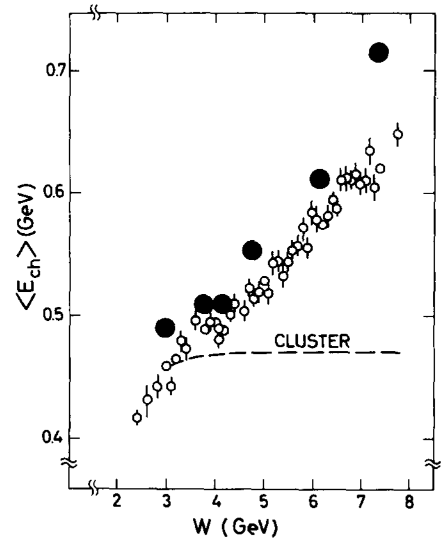

Fig. 5. The average energy per charged particle compared with the sum rule estimate (•) eqs. (8, 9). Also shown is the result with only the cluster contribution (---). Data from SPEAR [1]

where the sum goes over all charged particles, h. Strictly speaking we cannot evaluate these sum rules since we know only the cross-sections as averaged over all charged particles. Nevertheless we should be able to trace *qualitative* structures by inserting the averaged cross-sections in the integrals. In evaluating (8, 9) we neglect mass corrections from particles heavier than the pion and we use relation (6). This introduces the biggest error at the highest energies, where heavy particles are expected to play a relatively bigger role. The results for $\langle E_{ch}\rangle$ and $\langle n_{ch}\rangle$ are shown in figs. 5 and 6. First we discuss $\langle E_{ch}\rangle$. In fig. 5 we also show the average energy as obtained only by the cluster cross-sec-

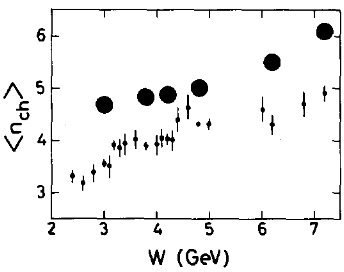

Fig. 6. The average charged multiplicity compared with the sum rule estimates (•), eqs. (9). Data from SPEAR [1].

tion, eq. (5). It is seen that the increase in $\langle E_{ch} \rangle$ with energy comes from the parton contribution, eq. (7). The structure around $W = 4.2$ GeV is explained as follows: the cluster cross-section becomes relatively large in this resonance region, whereas of course the parton cross-section is smooth as a function of energy. Since the cluster cross-section by itself has a smaller average energy than the parton component we obtain a break in the rise of $\langle E_{ch} \rangle$ near $W = 4.1$ GeV. The averaged charged multiplicity $\langle n_{ch} \rangle$ on fig. 6 shows no significant structures as a function of W. Within our model and with eq. (6) it is, however, clear that the rise with W in $\langle n_{ch} \rangle$ is due to the parton contribution and the factor x^{-1} in eq. (7) leads asymptotically for $W \to \infty$ to a logarithmic increase of $\langle n_{ch} \rangle$ if σ_{tot} scales.

In conclusion we have shown that the inclusive p-distributions in e^+e^- annihilation are qualitatively similar to $p_\perp$-distributions of final states in hadron–hadron collisions. They can be described by two components, a cluster component and a parton component. The p-dependence of the cluster component is controlled by the *same* radii as those we observe in hadron–hadron scattering and the strength of this component as a function of W is proportional to σ_{tot}, thus supporting the idea that the resonances are contained in the cluster contribution. The parton component has a form similar to the large $p_\perp$ component observed in the inclusive $p_\perp$-distributions at the ISR. It obeys Bjorken scaling for $p \to \infty$ and the approach to scaling is controlled by one mass parameter. Within this picture we have given a simple explanation of the W-dependence of the average charged energy and multiplicity.

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