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# Collinearity angle distribution in $e$-$\mu$ events

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We present an analytical formula for the collinearity angle distribution of $e$ and $\mu$ in the process $e^+e^- \to L^+L^-$ followed by heavy-lepton decay $L \to \nu_L + l + \bar{\nu}_l$, $l = e$ or $\mu$. Both $V-A$ and $V+A$ couplings are included, as is a mass for the $\nu_L$. At high energies we find that the collinearity angle distribution in $\cos\theta$ becomes a function of the single scaling variable $x = \beta^2\gamma^2(1-\cos\theta)/2$, where $\beta$ and $\gamma$ are heavy-lepton boost parameters from the rest frame to the $e^+e^-$ laboratory frame. This scaling in $x$ may be compared with the scaling in $p_T{}^2$ in hadron physics, and it holds independently of the detailed coupling scheme. A violation of this scaling is a signal of the appearance of heavier leptons.

Perl *et al.*[1] recently reported the observation of $e$-$\mu$ events in electron-positron annihilation. The most plausible interpretation of these events is that they are due to a heavy lepton[2] $L$ with a mass of around 1.8 GeV and an associated neutrino $\nu_L$.[1,3] One of the dynamical variables which characterize the $e$-$\mu$ events is the so-called collinearity angle distribution.[1,4] However, an analysis of the actual experimental data is complicated because of various kinematical cuts involved. The momentum cut, for example, significantly modifies the collinearity angle distribution.[3]

Before discussing the detailed effects of various kinematical cuts in the experimental data, it may be useful to study the information contained in the collinearity angle distribution. In this respect it is certainly worthwhile to have an analytical formula of the collinearity angle distribution for a somewhat idealized experimental setup, namely, no energy and angular cutoffs. In this paper we present such a formula for the sequential heavy-lepton production (see Fig. 1). Unlike the ordinary sequential heavy-lepton scheme,[2] where the weak and electromagnetic interactions act on $e$, $\mu$, and $L$ universally, we allow $V+A$ as well as $V-A$ currents for the heavy lepton. The associated neutrino may also have a nonvanishing mass.

The amplitude corresponding to Fig. 1 can be readily evaluated by the standard method or, better, by the method utilized by Tsai,[4] which allows us to identify the spin-alignment term directly. This spin-alignment term represents one of the dynamical effects which allow us to distinguish $V-A$ and $V+A$ coupling for a heavy lepton. In the six-body phase-space reduction, the kinematical variables of the neutral particles may be first integrated over. One can then integrate over the directions of the heavy lepton with all other variables *relatively* fixed in the heavy-lepton frame. Equivalently, the collinearity angle distribution without any angular cutoffs can be evaluated by first taking the average over the directions of the *incident* electrons with the final-state configuration fixed.[3] In this way the six-body phase space is reduced to the following essential part (see Fig. 1 for the definition of momenta):

$$d\Phi \equiv \frac{(s-m^2)(s'-m^2)}{ss'}\,\frac{d^3p}{2p_0}\,\frac{d^3q}{2q_0}\,\delta((P-p)^2-s)\times\delta((P'-q)^2-s')ds\,ds'. \tag{1}$$

The normalized distribution for $V-A$ and $V+A$ couplings can now be written as

$$d\Gamma = \frac{1}{N}(T_1+T_2)d\Phi, \tag{2}$$

where the matrix elements $T_1$ and $T_2$ depend on the coupling scheme.

(*a*) *$V-A$ current.*

$$T_1 = \left[(M^2-m^2)\left(1+\frac{m^2}{s}\right)+2s-\frac{2M^2m^4}{s^2}\right]\left[(M^2-m^2)\left(1+\frac{m^2}{s'}\right)+2s'-\frac{2M^2m^4}{(s')^2}\right]\left(\frac{Q^2+2M^2}{2M^2}\right)(P\cdot p)(P'\cdot q), \tag{3}$$

$$T_2 = \left[(M^2+m^2)\left(1+\frac{m^2}{s}\right)-2s-\frac{2M^2m^4}{s^2}\right]\left[(M^2+m^2)\left(1+\frac{m^2}{s'}\right)-2s'-\frac{2M^2m^4}{(s')^2}\right]\times\left[M^2(p\cdot q)-(P\cdot p)(P\cdot q)-(P'\cdot p)(P'\cdot q)+\left(\frac{Q^2-2M^2}{2M^2}\right)(P\cdot p)(P'\cdot q)\right]. \tag{4}$$

(*b*) *V+A current*

$$T_1=36(s-m^2)(s'-m^2)\left(\frac{Q^2+2M^2}{2M^2}\right)(P\cdot p)(P'\cdot q), \tag{5}$$

$$T_2=36(s-m^2)(s'-m^2)\left[M^2(p\cdot q)-(P\cdot p)(P\cdot q)-(P'\cdot p)(P'\cdot q)+\left(\frac{Q^2-2M^2}{2M^2}\right)(P\cdot p)(P'\cdot q)\right], \tag{6}$$

and the common normalization factor is given by

$$N=\left(\frac{\pi M^6}{8}\right)^2\left(\frac{Q^2+2M^2}{2M^2}\right)$$
$$\times[(1-\epsilon^4)(1-8\epsilon^2+\epsilon^4)-24\epsilon^4\ln\epsilon]^2. \tag{7}$$

In Eqs. (1)–(7)

$s$ = invariant mass for the $\nu_L\bar{\nu}_\mu$ system,
$s'$ = invariant mass for the $\bar{\nu}_L\nu_e$ system; (8)

$M$ = mass of the heavy lepton $L$,
$m$ = mass of the neutrino $\nu_L$; (9)

$Q^2=4E^2$ with $E$ the energy of the incident electron; (10)

$\epsilon\equiv m/M$; (11)

and $p$ and $q$ stand for the momenta of the muon and electron, respectively. In Eq. (2), $T_2$ stands for the spin-alignment term. The natural phase-space boundaries are provided by

$$m^2\leqslant s\leqslant M^2 \text{ and } m^2\leqslant s'\leqslant M^2. \tag{12}$$

For simplicity we neglect the electron and muon masses. In this case the energy and angular integrations factorize. The integration over energy variables may be performed by using the δ functions in (1). One can then perform the integration over $s$ and $s'$ within the natural boundaries (12). The final step is the integration over the angular variables with the fixed collinearity angle $\theta$,

$$\cos\theta\equiv-(\vec{p}\cdot\vec{q})/|\vec{p}|\,|\vec{q}|. \tag{13}$$

One finally obtains the normalized collinearity angle distribution for $0\leqslant\theta\leqslant\pi$

$$\frac{d\Gamma}{d\cos\theta}=\frac{\gamma^2}{2}\left\{F_1(x,\gamma^2)+\eta(\epsilon)\left[F_1(x,\gamma^2)-\frac{2}{2+1/\gamma^2}F_2(x,\gamma^2)+\frac{1+\cos\theta}{2+1/\gamma^2}F_3(x,\gamma^2)\right]\right\}, \tag{14}$$

where

$$F_1(x,\gamma^2)=\frac{1}{16}\left\{\left[\frac{3}{(1+x)^2}-\frac{2}{\gamma^2}\frac{1}{(1+x)}\right]L(x)+\frac{12}{(1+x)^2}+\frac{(1-1/\gamma^2)}{x(1+x)}[L(x)-4]\right\}, \tag{15}$$

$$F_2(x,\gamma^2)=\frac{1}{32}\left\{\left[\frac{15}{(1+x)^3}-\frac{12}{\gamma^2}\frac{1}{(1+x)^2}\right]L(x)+\frac{60}{(1+x)^3}+\frac{16-24/\gamma^2}{(1+x)^2}+\frac{3(1-1/\gamma^2)}{x(1+x)^2}[L(x)-4]\right\}, \tag{16}$$

$$F_3(x,\gamma^2)=\frac{1}{256}\left\{3\left[\frac{35}{(1+x)^4}-\frac{(8/\gamma^2)(6-1/\gamma^2)}{(1+x)^3}+\frac{8}{\gamma^4}\frac{1}{(1+x)^2}\right]L(x)+20\left[\frac{21}{(1+x)^4}-\frac{2(1+4/\gamma^4)}{(1+x)^3}\right]\right.$$
$$\left.+3\left[\frac{(10-8/\gamma^2)(1-1/\gamma^2)}{x(1+x)^3}+\frac{3(1-1/\gamma^2)^2}{x^2(1+x)^2}\right][L(x)-4+\tfrac{8}{3}x]\right\}, \tag{17}$$

with

$$L(x)\equiv 2\ln\{1+2x+2[x(1+x)]^{1/2}\}/[x(1+x)]^{1/2}, \tag{18}$$

$$x\equiv\tfrac{1}{2}\beta^2\gamma^2(1-\cos\theta), \tag{19}$$

and $\beta$ and $\gamma$ are the Lorentz factors for the heavy lepton ($\gamma\equiv E/M$). The parameter $\eta(\epsilon)$ in (14) is a function of the mass ratio, $\epsilon=m/M$, and it characterizes the structure of the heavy-lepton current[5]

$$\eta(\epsilon)=1 \text{ for } V+A, \tag{20}$$

$$\eta(\epsilon)=\frac{1}{9}\left[\frac{(1-\epsilon^2)(1-11\epsilon^2-47\epsilon^4-3\epsilon^6)-12\epsilon^4(3+2\epsilon^2)\ln\epsilon}{(1-\epsilon^4)(1-8\epsilon^2+\epsilon^4)-24\epsilon^4\ln\epsilon}\right]^2 \text{ for } V-A. \tag{21}$$

Equation (21) is plotted in Fig. 2. $\eta=0$ corresponds to the vanishing spin-alignment effects. We emphasize that formula (14) is valid for arbitrary mass values of $L$ and $\nu_L$. The only constraint is that the energy cutoff, if any, is kept small and the counter covers (approximately) $4\pi$ angles.

At threshold $\gamma = 1$ (and $\beta = 0$), (14) becomes

$$\frac{d\Gamma}{d\cos\theta} = \tfrac{1}{2}[1 + \tfrac{1}{3}\eta(\epsilon)\cos\theta], \tag{22}$$

which is a generalization of the formula given by Tsai.[4] In the forward direction

$$\left(\frac{d\Gamma}{d\cos\theta}\right)_{\theta=0} = \frac{\gamma^2}{30}\left\{(20 - 5/\gamma^2) + \eta(\epsilon)\frac{2\gamma^2}{(2\gamma^2+1)}[8 - 1/\gamma^2 + 1/(2\gamma^4)]\right\}, \tag{23}$$

which shows that the $\cos\theta$ distribution has a sharp peak around $\theta \simeq 0$ at high energies. The height of the peak is different for $V-A$ and $V+A$ currents. Another interesting result is obtained at large values of $\gamma^2$, where (14) can be rewritten as

$$\beta^2 \frac{d\Gamma}{dx} \cong F_1(x,\infty) + \eta(\epsilon)[F_1(x,\infty) - F_2(x,\infty) + F_3(x,\infty)]. \tag{24}$$

The right-hand side is a *universal* function of $x$ characterized by the parameter $\eta(\epsilon)$ which contains the dynamical information. Equation (24) shows that the $\beta^2 d\Gamma/dx$ distribution becomes "scale invariant" at high energies in terms of the $x$ variable (this scaling starts around $\gamma \simeq 2$ and works very well above $\gamma \simeq 3$). Thus one can compare the data from various values of $\gamma$ corresponding to the same $x$. The scaling formula (24) in the small-$x$ region is plotted in Fig. 3. For the $V-A$ current we used $\eta(0) = \frac{1}{9}$ (see also Fig. 2). Figure 3 shows that a discrimination between $V-A$ and $V+A$ may be possible even at high energies. The definition of $x$ in (19) and Fig. 3 indicates that the major part of the collinearity angle distribution (i.e., about 70%) is concentrated in $0 \le x \lesssim 2$, namely[6]

$$\theta \lesssim 3/\beta\gamma \approx 2[(M/2)/(|\vec{P}|/3)], \tag{25}$$

or $\theta \lesssim 20°$ at $\gamma \approx 10$, which may be attainable at the next generation of colliding machines[7] if the heavy-lepton mass is not large.[1] The $\cos\theta$ distribution at lower energies is more sensitive to the energy cutoff[3] and varying values of $\epsilon$, and Eq. (14) is not quite adequate for a quantitative analysis of the existing data.[1] However, a numerical analysis at lower energies indicates that a $V+A$ current still gives rise to more events at smaller values of $\theta$ compared with a $V-A$ current for the identical values of $\epsilon$ and cutoff energy. See also the threshold formula (22).

Finally several comments are in order.

(i) At high energies the incident electrons are usually (transversely) polarized. This polarization may modify the collinearity angle distribution at a fixed outgoing angle of the muon (or electron) with respect to the incident beam direction. However, if one assumes a $4\pi$ counter[8] and integrates over the directions of the outgoing muon (or electron) the effects of the polarization of incident particles are smeared and the collinearity angle distribution is still correctly given by our formula (14), which is based on an unpolarized incident beam. This statement, which is valid in the one-photon approximation, can be most easily understood by observing that the collinearity angle distribution without any angular cutoff can be evaluated by first taking the average over the directions of the *incident* electrons with the final-state configuration fixed.[3]

(ii) For a finite energy cutoff, the collinearity angle distribution (14) is modified. One of the modifications is a strong suppression of the $\cos\theta$ distribution at large values of $\theta$ compared with formula (14). Another important modification

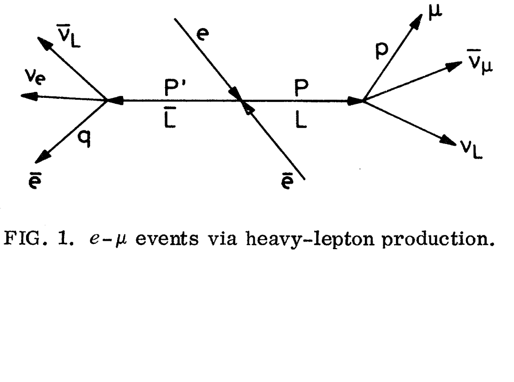


FIG. 1. $e$-$\mu$ events via heavy-lepton production.

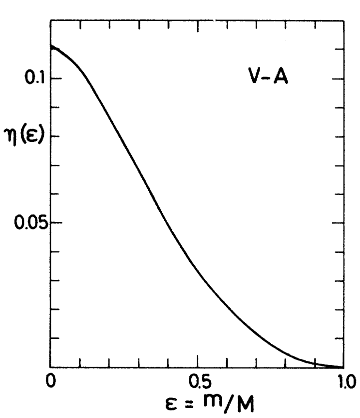


FIG. 2. The parameter $\eta(\epsilon)$ for a $V-A$ current given by Eq. (21).

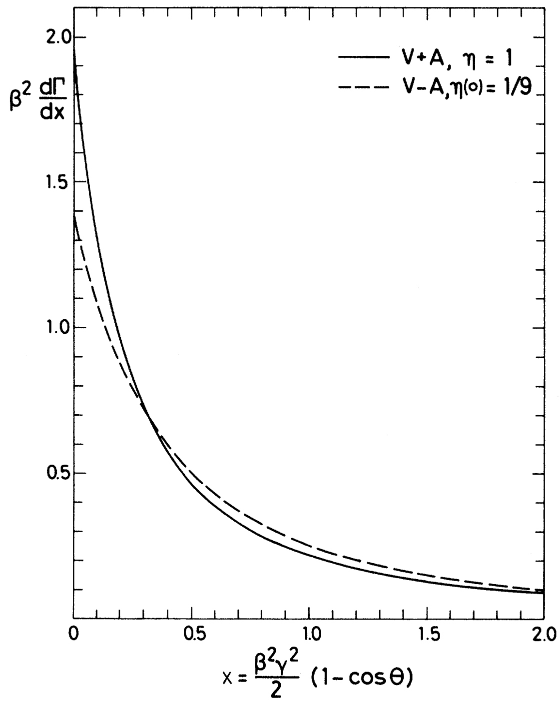


FIG. 3. The behavior of the scale-invariant distribution given by Eq. (24) in the small-$x$ region, where the bulk of the $x$ distribution (i.e., about 70%) is concentrated.

arises from the different energy spectrum of the muon (or electron) which depends on $V-A$ or $V+A$ coupling assumed for the heavy lepton.[9] In other words, the $T_1$ terms in Eqs. (3) and (5) also give rise to different collinearity angle distributions for a finite energy cutoff, in addition to the spin alignment term $T_2$. In this respect it should be noted that the distribution (14), which is valid when one does not impose any significant energy cutoff, corresponds to the pure phase-space distribution given by Eq. (1) when the spin alignment effects vanish, namely, $\eta=0$.

In conclusion the exact formula (14) combined with the $x$ variable will provide a convenient basis for the analysis of collinearity angle distributions at high energies. A more detailed discussion of the effects of the finite energy cutoff and varying values of $\epsilon$ will be given elsewhere.

*Note added.* After completing the present work, a related work by S. Y. Park and A. Yildiz [Harvard report (unpublished)] came to our attention. Their result agrees well with ours at threshold $\beta=0$.

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[1]M. L. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975); M. L. Perl, Lectures on Electron-Positron Annihilation, Part II, SLAC Report No. SLAC-PUB-1592, 1975 (unpublished).

[2]A review of earlier works on heavy leptons is found in M. L. Perl and R. Rapidis, SLAC Report No. SLAC-PUB-1496, 1974 (unpublished). For earlier experimental searches for $e$-$\mu$ events, see S. Orito *et al.*, Phys. Lett. 48B, 165 (1974), and references therein.

[3]K. Fujikawa and N. Kawamoto, contribution to the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford University, 1975 (unpublished); Phys. Rev. Lett. 35, 1560 (1975).

[4]Y. S. Tsai, Phys. Rev. D 4, 2821 (1971). A formula which is valid at low energies with massless neutrinos is found in S. Kawasaki, T. Shirafuji, and S. Y. Tsai, Prog. Theor. Phys. 49, 1656 (1973).

[5]A generalization of Eq. (14) to an arbitrary combination of $V$ and $A$ currents for the heavy lepton, $\sin\alpha(V-A)+\cos\alpha(V+A)$ with $-\pi/2\le\alpha\le\pi/2$, can be readily made. One should just replace $\eta(\epsilon)$ by

$$\eta(\epsilon,\alpha)=\left[\frac{\cos^2\alpha\, A(\epsilon)+\sin^2\alpha\, B(\epsilon)+\sin 2\alpha\, C(\epsilon)}{A(\epsilon)+\sin 2\alpha\, C(\epsilon)}\right]^2,$$

where

$$A(\epsilon)=(1-\epsilon^4)(1-8\epsilon^2+\epsilon^4)-24\epsilon^4\ln\epsilon,$$

$$B(\epsilon)=\tfrac{1}{3}[(1-\epsilon^2)(1-11\epsilon^2-47\epsilon^4-3\epsilon^6)-12\epsilon^4(3+2\epsilon^2)\ln\epsilon],$$

$$C(\epsilon)=-2\epsilon[(1-\epsilon^2)(1+10\epsilon^2+\epsilon^4)+12\epsilon^2(1+\epsilon^2)\ln\epsilon].$$

If one assumes a $V+A$ current for the ordinary leptons $e$ and $\mu$ when they couple to the heavy lepton, the parameter $\eta(\epsilon,\alpha)$ is replaced by $\eta(\epsilon,\pi/2-\alpha)$ and the role of $V-A$ and $V+A$ couplings for the heavy lepton in the collinearity angle distribution is interchanged. If one takes a completely arbitrary four-fermion coupling scheme including $S$, $P$, $T$, and derivative couplings, the expression for $\eta$ becomes more complicated. However, formula (14) still retains its form with this modified parameter $\eta$, which is always limited within $|\eta|\le 1$ by the positivity constraint on the heavy-lepton decay probability.

[6]The average muon (or electron) momentum is about one third of the heavy-lepton momentum at high energies, and the maximum transverse momentum of the muon (or electron) with respect to the heavy-lepton direction is $M/2$. Thus Eq. (25) is physically reasonable. The scaling in $x$ may be compared with the scaling in $p_T{}^2$ in hadron physics. At high energies (i.e., $\gamma\gtrsim 5$), the finite muon mass modifies our formula (24) in the

large-$x$ region, $x \gtrsim 10$, if the heavy-lepton mass is relatively small. However, more than 90% of the $x$ distribution is concentrated in the small-$x$ region, $x \lesssim 10$, and the effects of the finite muon mass can be safely neglected even at large $\gamma^2$ except for the very large values of the parameter $\epsilon$, $\epsilon \approx 1$.

[7]At high energies the identification of electrons and muons becomes easier, and the noncoplanarity cutoff may not be a major obstacle to measure small $\theta$. We thank S. Orito for a comment on this point.

[8]An (approximate) $4\pi$ detector will after all be required in future experiments if one wants to make sure that there are no particles other than $e$, $\mu$, and neutrinos in the final state. Our formula (14) is valid for any noncollinearity cutoff with a $4\pi$ detector.

[9]This difference in the energy spectrum is well known for the case of muon decay. See, for example, J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), p. 263.