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A DISPERSION THEORETICAL CALCULATION OF INELASTIC CONTRIBUTIONS TO THE NUCLEON ISOVECTOR FORM FACTORS

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We calculate the contribution of the $\pi\pi$, $\pi^0\omega$ and $\rho\epsilon$ intermediate states to the isovector nucleon form factors $F_i^V(q^2)$ in the space- and time-like region using dispersion relations together with unitarity. In the space-like region good agreement with the existing data is found, i.e. we are able to explain the difference in the behaviour of F_1^V which decreases like $1/q^2$ and F_2^V which shows a dipole-like fall-off for $q^2 < 0$. This is due to the $\pi^0\omega$ contribution which is rather small for F_1^V but gives a large negative contribution in the spectral function for F_2^V. Also the anomalous magnetic moment is found to be dominated by the $\pi\pi$ and $\pi^0\omega$ contributions. In the time-like region beyond the $\mathrm{N\bar{N}}$ threshold our method is not sufficient. First, we neglected the $\mathrm{N\bar{N}}$ intermediate state which certainly is important in this region. Second, the existing data of the various meson form factors in the time-like region and the necessary coupling constants are not known with sufficient accuracy.

1. Introduction

The calculation of the nucleon form factors using dispersion relations has first been tried by Chew et al. [1] and Federbush et al. [2]. These two groups estimated the contribution of the 2π intermediate state and showed that the electromagnetic structure of the nucleon is strongly influenced by the structure of the pion, i.e. the pion form factor. In 1960 Frazer and Fulco [3] calculated the πN amplitudes in the t-channel which are related to the nucleon form factors *via* unitarity and were able to make a qualitative prediction for the existence of the ρ-meson. In 1968 Höhler et al. [4] used the then more complete experimental information of the πN scattering and the pion form factor and calculated again the form factors G_E^V, G_M^V following the work of Frazer and Fulco. In an earlier work [5] we repeated the calculation of the 2π contribution and showed that it is not sufficient to explain the behaviour

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of the isovector form factors $F^{\mathrm{v}}_{1,2}$. This result was confirmed by Höhler and Pietarinen [6] recently. In this work we extend our earlier work in two directions: As a first point we try to calculate the contribution of higher-mass intermediate states and as a second step we evaluate the dispersion relation in the time-like region. This is particularly interesting because in the near future results on $e^+e^- \to \bar{n}n, \bar{p}p$ from the existing storage rings will become available [7].

The calculation of F^{v}_i instead of $G^{\mathrm{v}}_E, G^{\mathrm{v}}_{\mathrm{M}}$ ensures that the threshold constrains $G^{\mathrm{v}}_{\mathrm{E}}(4M^2) = G^{\mathrm{v}}_{\mathrm{M}}(4M^2)$ are automatically fulfilled. As is well known, because of G-parity conservation, only intermediate states with an even number of pions contribute to the isovector part of the form factors. These states must have the quantum numbers of the photon and besides the two-pion P-state we consider 4π states with $J = 1, P = -1, G = +1$ in a quasi-two-body approximation. This is justified through the existence of strong resonances in the two- and three-pion channel like $\rho(I = J = 1)$, $\omega(I = 0, J = 1)$ and $\epsilon(I = J = 0)$. Therefore we consider the $\pi^0\omega$ and $\rho\epsilon$ intermediate states which should give a good approximation for the four-pion states. The inclusion of the $\mathrm{N\bar{N}}$ states $^3\mathrm{S}_1$ and $^3\mathrm{D}_1$ would lead to a highly complicated system of coupled integral equations which are not trivial to solve and whose solution lies beyond the scope of this paper.

Other intermediate states like $\pi\mathrm{A}_1$, $\pi\mathrm{A}_2$, $\mathrm{K\bar{K}}$, $\mathrm{K^*\bar{K}}$, $\mathrm{K^*\bar{K}^*}$, $\rho^+\rho^-$ etc. might also contribute but all of these have not been seen yet in e^+e^- annihilation experiments, whereas there seems to be evidence that $\pi\omega$ and $\rho\epsilon$ are present [29,31]. $e^+e^- \to \mathrm{K\bar{K}}$ is very small above threshold. So far, none of the higher intermediate states have been considered. Therefore we consider our calculation of the $\pi^0\omega$ and $\rho\epsilon$ contribution more as a first step towards a more systematic study of all these intermediate states. Furthermore from the experimental information about $e^+e^- \to 4\pi$ it seems that $\rho\epsilon$ and $\pi^0\omega$ are the dominant states [29].

The outline of the paper is as follows. In sect. 2 we discuss the general formalism. We start with the dispersion relation for the nucleon form factors and investigate the t-channel helicity amplitudes for $\pi\pi$, $\pi^0\omega$, $\rho\epsilon \to \mathrm{N\bar{N}}$, i.e. we give the connection to the invariant amplitudes and determine the analytic structure of the $J = 1$ helicity partial-wave amplitudes. This allows us to derive a dispersion relation for these amplitudes which we solve with the help of the N/D method. Unitarity then gives the connection to the imaginary part of the nucleon form factors. In sect. 3 we describe the details of the numerical evaluation and the experimental input used in the calculation. We also discuss our assumptions and models for the form factors F_π, $F_{\pi\omega\gamma}$, $F^{1,2}_{\rho\epsilon\gamma}$. In sect. 4 we present and discuss our results and we make some concluding remarks in sect. 5.

2. General formalism

The framework of our calculations is similar to our earlier work [5] and to the work of Frazer and Fulco [3]. Our starting point is the usual assumption that the form factors fulfill a dispersion relation:

$$F_i^{\mathrm{v}}(t) = F_i^{\mathrm{v}}(0) + \frac{t}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, F_i^{\mathrm{v}}(t')}{t'(t'-t)}\, \mathrm{d}t' \,. \tag{1}$$

The imaginary part is connected with the strong interaction scattering processes through the unitarity relation which reads in the helicity formalism [8]:

$$\mathrm{Im}\, \Gamma^{\lambda_c,\lambda_d} = |q| \sum_{\lambda_a,\lambda_b} \Gamma^{\lambda_a,\lambda_b^*} \langle \lambda_c \lambda_d | T^{J=1} | \lambda_a \lambda_b \rangle \,. \tag{2}$$

The connection of the helicity form factors $\Gamma^{\lambda_a,\lambda_b}$ with the invariant form factors is given in [9]:

$$\Gamma^{1/2,1/2} = -2M\left(F_1^{\mathrm{v}} + \frac{t}{4M^2} F_2^{\mathrm{v}}\right) = -2M\, G_{\mathrm{E}}^{\mathrm{v}} \,,$$

$$\Gamma^{1/2,-1/2} = \sqrt{2t}\,(F_1^{\mathrm{v}} + F_2^{\mathrm{v}}) = \sqrt{2t}\, G_{\mathrm{M}}^{\mathrm{v}} \,. \tag{3}$$

We start our investigation of the t-channel helicity amplitudes $\langle \lambda_c \lambda_d | T^{J=1} | \lambda_a \lambda_b \rangle$ with the decomposition of the matrix element M_{fi} into invariant amplitudes A_i which reads in the t-channel center of mass system:

$$M_{\mathrm{fi}} = \bar{u}(p_2, m') \sum_i A_i(s,t) M_i v(p_2, m) \,. \tag{4}$$

The normalization is as usual. The notation and kinematics are given in ref. [14] which is an extended version of this paper. For $\pi\pi \to \mathrm{N\bar{N}}$ the two invariants are well known while for the other channels there are six invariant amplitudes which we choose similar to the Dennery amplitudes [10] for electroproduction of pions. The t-channel helicity amplitudes can be expressed as functions of the invariant amplitudes. This is easily done by evaluating eq. (4) in the t-channel c.m.s. for specific helicities of the four particles. The results of the three channels are given in ref. [14].

The next step is the assumption of a fixed-t dispersion relation for the invariant amplitudes A_i:

$$A_i(s,t) = \mathrm{Res}\, A_i \left[\frac{1}{M^2 - s} \pm \frac{1}{M^2 - u}\right] + \frac{1}{\pi} \int_{(M+m_\pi)^2}^{\infty} \mathrm{d}s'\, \mathrm{Im}\, A_i(s',t) \left[\frac{1}{s'-s} \pm \frac{1}{s'-u}\right] \tag{5}$$

The crossing behaviour of the A_i is determined from the crossing of the covariants M_i. The residues of the nucleon intermediate state are as given in table 1.

In the unitarity relation for $\mathrm{Im}\, F_i^{\mathrm{v}}$ we need only the $J = 1$ partial-wave helicity amplitudes. The projection is defined as usual.

Table 1

$\pi\pi$:	$\mathrm{Res}\,A^{(\pm)} = 0$	even: $A^{(+)}, B^{(-)}$	
	$\mathrm{Res}\,B^{(\pm)} = g^2$	odd: $A^{(-)}, B^{(+)}$	
$\pi^0\omega$:	$\mathrm{Res}\,A_1 = -g g^{\mathrm{V}}_{\omega\mathrm{NN}}$		
	$\mathrm{Res}\,A_2 = 2\,\mathrm{Res}\,A_5 = \dfrac{2}{t - m_\pi^2} g g^{\mathrm{V}}_{\omega\mathrm{NN}}$		even: A_1, A_2, A_4
	$\mathrm{Res}\,A_3 = \mathrm{Res}\,A_4 = -g \dfrac{g^{\mathrm{T}}_{\omega\mathrm{NN}}}{2M}$		odd: A_3, A_5, A_6
	$\mathrm{Res}\,A_6 = 0$		
$\rho\epsilon$:	$\mathrm{Res}\,A_1 = -g_{\epsilon\mathrm{NN}}(g^{\mathrm{V}}_{\rho\mathrm{NN}} - g^{\mathrm{T}}_{\rho\mathrm{NN}})$		
	$\mathrm{Res}\,A_2 = 2\,\mathrm{Res}\,A_5 = \dfrac{2g_{\epsilon\mathrm{NN}} g^{\mathrm{V}}_{\rho\mathrm{NN}}}{t - m_\epsilon^2}$		even: A_1, A_2, A_3, A_6
	$\mathrm{Res}\,A_3 = \mathrm{Res}\,A_4 = g_{\epsilon\mathrm{NN}} \dfrac{g^{\mathrm{T}}_{\rho\mathrm{NN}}}{2M}$		odd: A_4, A_5
	$\mathrm{Res}\,A_6 = 0$		

We then express the helicity amplitudes through the invariant amplitudes and insert the fixed-t dispersion relation. Using the Legendre functions of the second kind we get the well-known representation for the $\pi\pi \to \mathrm{N\bar{N}}$ $J = 1$ t-channel helicity amplitudes. The explicit formulas for $\pi\pi \to \mathrm{N\bar{N}}$, $\pi\omega \to \mathrm{N\bar{N}}$, $\rho\epsilon \to \mathrm{N\bar{N}}$ can be found in [14].

The analytical structure of these amplitudes is easily derived from the properties of the Legendre functions $Q_J(t)$. They have branch points at $z = 1$ and a cut on the real axis between these points. Using this we obtain the singularities in the t-plane resulting from the nucleon exchange in the s- and u-channel and from the possible higher-mass intermediate states in these channels. The resulting cut structure from the dynamical singularities is shown in fig. 1.

We obtain no complex cut for the $\rho\epsilon \to \mathrm{N\bar{N}}$ amplitudes because we perform our calculations with equal ρ and ϵ masses which seems to be justified experimentally ($m_\rho = 0.775, m_\epsilon = 0.7$–$1$ GeV). After having determined the analytic structure of the amplitudes we are able to write a dispersion relation of the kind:

$$\mathrm{Re}\, f^{t^1}_{\lambda_1 \dots \lambda_4} = \frac{1}{\pi} \int_{\text{left-hand cut}} \frac{\mathrm{Im}\, f^{t^2}_{\lambda_1 \dots \lambda_4}(t')}{t' - t}\, \mathrm{d}t'$$

$$+\frac{1}{\pi}\int_{(m_1+m_2)^2}^{\infty}\frac{\operatorname{Im} f_{\lambda_1\ldots\lambda_4}^{t^2}(t')}{t'-t}\,\mathrm{d}t' . \tag{6}$$

The t-channel helicity amplitudes $f_{\lambda_1\ldots\lambda_4}^{t^{J=1}}$ are directly related to the nucleon form factors $G_{\mathrm{E,M}}$ through the unitarity relation. We use certain linear combinations of these amplitudes which we denote $\Gamma_i^{(a,b)}$. They can be found in ref. [14] and are related to the form factors F_i^{V} in the following way:

$$\operatorname{Im} F_i^{\mathrm{V}}(t) = \rho_1^{(\pi\pi)}(t)F_\pi^*(t)\Gamma_1^{(\pi\pi)} + \rho_1^{(\pi^0\omega)}F_{\pi\omega\gamma}^*\Gamma_1^{(\pi^0\omega)}$$

$$+\,\rho_1^{(\rho\epsilon)}[\Gamma_{(\rho\epsilon)}^{00*}\Gamma_1^{(\rho\epsilon)} + \Gamma_{(\rho\epsilon)}^{01*}\Gamma_2^{(\rho\epsilon)}] . \tag{7}$$

The $\rho^{(\pi\pi)}$, $\rho^{(\pi^0\omega)}$, $\rho^{(\rho\epsilon)}$ denote phase space factors; the $\Gamma_i^{(\pi\pi),(\pi^0\omega),(\pi\epsilon)}$ are the t-channel helicity amplitudes and the $\Gamma_{(\rho\epsilon)}^{00}$, $\Gamma_{(\rho\epsilon)}^{01}$ are the helicity form factors of the mesons.

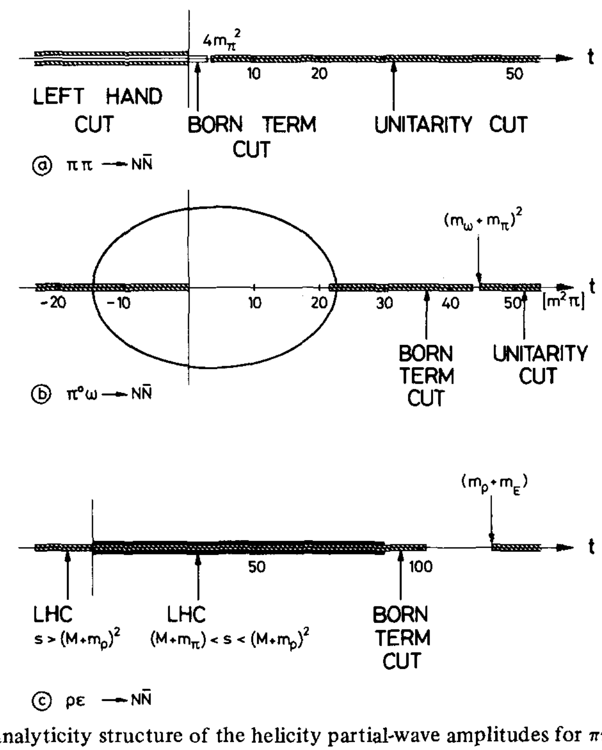

Fig. 1. t-plane analyticity structure of the helicity partial-wave amplitudes for $\pi\pi$, $\pi^0\omega$, $\rho\epsilon \to \mathrm{N\bar{N}}$.

For the evaluation of eqs. (6) and (7) we need the connection of the helicity form factors with the invariant ones which are free of kinematical singularities. We then have to construct models for F_π, $F_{\pi\omega\gamma}$, $F^{12}_{\rho\epsilon\gamma}$ and have to compare these models with the experimental information which exists as data for the invariant form factors.

For the evaluation of (6) we need the imaginary part of the amplitudes on the cut. For a part of the left-hand cut $\mathrm{Im}\, f^t_{\lambda_N\lambda_{\bar{N}}\lambda_a\lambda_b}$ is given if we insert experimental data of the crossed channel scattering processes. The right-hand cut can be removed by an N/D method using information about the phase of the amplitude from unitarity. The best method to do this would be a multi-channel N/D method as described in [11]. As a first approximation we consider only the channels $\pi\pi$, $\pi^0\omega$, $\rho\epsilon \to \mathrm{N\bar{N}}$ and the elastic processes $\pi\pi \to \pi\pi$, $\pi^0\omega \to \pi^0\omega$, $\rho\epsilon \to \rho\epsilon$. The inclusion of other channels would lead to a highly complicated system of coupled integral equations.

The simplified N/D method works as follows: As usual from unitarity we conclude that the phase of the $\pi\pi \to \mathrm{N\bar{N}}$ amplitudes must be the same as the phase of the pion form factor for $4m_\pi^2 \leqslant t \leqslant 16m_\pi^2$. Writing

$$\Gamma_i^{(\pi\pi)} = \frac{N(t)}{D(t)}\,,$$

we can construct $D_i(t)$ between $4m_\pi^2$ and $16m_\pi^2$ from the phase:

$$D(t) = \frac{1}{F_\pi(t)} = \exp\left\{-\frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\delta_1^1(t')}{t'(t'-t-i\epsilon)}\,\mathrm{d}t'\right\}. \tag{8}$$

Neglecting inelastic channels we can write down a dispersion relation for $J_i = \Gamma_i^{(\pi\pi)}$. $D_i(t)$ which has only a left-hand cut:

$$\mathrm{Re}\, J_i(t) = \frac{1}{\pi}\int_{-\infty}^{a} \frac{\mathrm{Im}\,\Gamma_i(t')\,\mathrm{Re}\, D(t')}{t'-t}\,\mathrm{d}t'\,. \tag{9}$$

The unitarity relation then reads for the $\pi\pi$-contribution:

$$\mathrm{Im}\, F_i^{\mathrm{V}} = -\frac{eq^3}{2E}\,|F_\pi(t)|^2 J_i(t)\,. \tag{10}$$

This method is justified for the $\pi\pi$ channel because the inelastic contributions to F_π are shown to be small experimentally. For the other channels we try two methods of evaluation. The first one is identical to the method described above for the $\pi\pi$ channel, i.e. we consider only the elastic $\pi^0\omega \to \pi^0\omega$, $\rho\epsilon \to \rho\epsilon$ scattering and assume that these processes are dominated by resonances $\rho'(1250)$ in the $\pi^0\overline{\omega}$ case and $\rho''(1600)$ in the $\rho\epsilon$ channel. Though it is experimentally not clear whether these resonances exist, we tried another method of evaluation which we call method 2. In this method we neglect in eq. (6) the right-hand cut, which has the physical meaning that we consider only the two-pion contribution to the $\pi^0\omega$ and $\rho\epsilon$ scattering. Here we assume that this two-pion background is the dominant contribution and that no resonance exists in the $\pi^0\omega$ and $\rho\epsilon$ channels.

The models for the form factors $F_{\pi\omega\gamma}, F^{1,2}_{\rho\epsilon\gamma}$ are given in sect. 3 together with the details of the evaluation of the dispersion relation and the experimental input.

3. Numerical evaluation

The starting point is the dispersion relation which contains only the left-hand cut (eq. (9)). Because we know the Born term exactly for the whole t-range we rewrite the dispersion relation with the help of the decomposition

$$J_i = J_{i\mathrm{Born}} + \widetilde{J}_i = (\Gamma_{i\mathrm{B}} + \widetilde{\Gamma}_i)D(t)\ ,$$

$$J_i = J_{i\mathrm{Born}} + \frac{1}{\pi}\int\limits_{\mathrm{LHC}} \frac{\mathrm{Im}\,\widetilde{\Gamma}_i(t')\,\mathrm{Re}\,D(t')}{t'-t}\,\mathrm{d}t' + \frac{1}{\pi}\int\limits_{(m_a+m_b)^2}^{\infty} \frac{\mathrm{Im}\,D(t')\,\mathrm{Re}\,\Gamma_{i\mathrm{B}}(t')}{t'-t}\,\mathrm{d}t' \tag{11}$$

Although we have neglected the inelastic contributions from unitarity on the right-hand cut the remaining integral can not be evaluated exactly because we do not know $\mathrm{Im}\,J_i$ on the whole left-hand cut. From experimental information on the crossed channel scattering processes we can calculate $\mathrm{Im}\,J_i$ and $\mathrm{Re}\,J_i$ on a part of the cut which will be given later. On this part we can also calculate the influence of the unknown distant cut and define a discrepancy function:

$$\Delta_i = \mathrm{Re}\,J_i(t) - \frac{1}{\pi}\int\limits_{-75m_\pi^2}^{0} \frac{\mathrm{Im}\,J_i(t')}{t'-t}\,\mathrm{d}t'\ , \tag{12}$$

This discrepancy function has then to be extrapolated to the time-like region which is of interest to us. For the $\pi\pi$ channel this method is well known and very reliable [5,12,13]. The connection with the measured phase shifts and inelasticities is given in [14] and the part of the left-hand cut for which the s-channel data can be extrapolated *via* a Legendre polynomial expansion is also given by Frazer and Fulco [3] namely $-26m_\pi^2 \leqslant t \leqslant 3.98\ m_\pi^2$. It has been shown recently [15] that the truncated expansion even converges for larger negative t-values up to $-75m_\pi^2$.

For our calculation we use the phase shifts from CERN [16] which are available for energies up to $s = 4.8\ \mathrm{GeV}^2$. Beyond this value we parametrize the amplitudes $A^{(-)}, B^{(-)}$ with the help of a Regge fit by Barger and Phillips [17].

For the $\pi^0\omega$ channel no phase shifts are available. Apart from the Born term we made the assumption that the invariant amplitudes $A_1 \ldots A_6$ are dominated by the t-channel exchange of the ρ-trajectory in the whole kinematical s-region. The result of the usual reggeization procedure is

$$A_1 = g_{\omega\rho\pi}g^{\mathrm{v}}_{\rho\mathrm{NN}}\frac{\xi^J(\alpha)\alpha}{\Gamma(\alpha+1)}\left(\frac{s-u}{2s_0}\right)^{\alpha-1}\ , \qquad A_3 = A_6 = 0\ ,$$

$$A_4 = -g_{\omega\rho\pi} \frac{g^{\mathrm{T}}_{\rho\mathrm{NN}}}{2M} t\, \xi^J(\alpha)\alpha \left(\frac{s-u}{2s_0}\right)^{\alpha-1} ,$$

$$A_2 = -\frac{t - m_\pi^2 + m_\omega^2}{t(t-m_\pi^2)} A_1 , \qquad A_5 = \frac{s-u}{2t(t-m_\pi^2)} A_1 . \tag{13}$$

As experimental input we use the knowledge of the following coupling constants which are determined from different experiments with various theoretical methods. The numbers given below are taken from [18] and are partly not very well determined *,

$$\frac{g^{\mathrm{v}\,2}_{\omega\mathrm{NN}}}{4\pi} = 11.0 , \qquad g^{\mathrm{T}}_{\omega\mathrm{NN}} = 0.1\, g^{\mathrm{v}}_{\omega\mathrm{NN}} ,$$

$$\frac{g^2_{\omega\rho\pi}}{4\pi} = 23.2\ \mathrm{GeV}^{-2} , \qquad (\text{ref. [19]}) ,$$

$$\frac{g^{\mathrm{v}\,2}_{\rho\mathrm{NN}}}{4\pi} = 2.2 , \qquad g^{\mathrm{T}}_{\rho\mathrm{NN}} = 6.6\, g^{\mathrm{v}}_{\rho\mathrm{NN}}\ (\text{ref. [13]}) . \tag{14}$$

We remark that also the nearby part between $-1\ \mathrm{GeV}^2 \leqslant t \leqslant 0$ of the left-hand cut is described by the Regge contribution. Here we used the simple assumption that the residues are not t-dependent in this region. We have neglected also the complex cut and have tried to take this contribution into account by the discrepancy method.

In the $\rho\epsilon$ channel the cut is caused by two different sets of intermediate states. The physical states with $s > (M + m_\rho)^2$ give rise to a cut from $-\infty$ to 0 while from the unphysical states we obtain a cut on the real axis from 0 to approximately 1.7 GeV^2.

For the first part of the cut we work again with a Regge ansatz for the invariant amplitudes. The result for ρ-exchange is:

$$A_2 = -\frac{g^{\mathrm{T}}_\rho}{2M} \frac{1}{t-m^2} [g^{(1)}_{\epsilon\rho\rho} m^2 + g^{(2)}_{\epsilon\rho\rho} \tfrac{1}{2}(t - 2m^2)] \frac{\xi^J(\alpha)\alpha}{\Gamma(\alpha+1)} \left(\frac{s-u}{2s_0}\right)^{\alpha-1} ,$$

$$A_3 = 2A_6 = g^{(2)}_{\epsilon\rho\rho}(g^{\mathrm{v}}_\rho - g^{\mathrm{T}}_\rho) \frac{\xi^J(\alpha)\alpha}{\Gamma(\alpha+1)} \left(\frac{s-u}{2s_0}\right)^{\alpha-1} ,$$

$$A_1 = A_4 = A_5 = 0 . \tag{15}$$

For the second part of the cut we make use of the existing data for $\pi\mathrm{N} \to \pi\pi\mathrm{N}$

* Höhler [37] recently calculated $g^{\mathrm{v}}_{\omega\mathrm{NN}}$. The influence of his result $g^2_{\omega\mathrm{NN}/4\pi} = 24 \pm 6$ is discussed in sect. 4.

[20] which are analyzed in terms of quasi-two-body final states ϵN, ρN, $\pi\Delta$ [21]. The coupling of these states to the N* resonances are given in [21] and this enables us to extrapolate the data to the s-channel process $\epsilon N \to \rho N$ which is done with the help of the crossing matrix [23].

The experimental information for the D-function in the $\pi\pi$ channel is given by the data for $F_\pi(t)$ which have been determined from different experiments [24–26]. The fit we used and which is plotted in fig. 2 was taken from Pisut and Roos [27] and agrees with the Gounaris-Sakurai ρ-dominance fit. For comparison we plotted a second fit to F_π with a $\rho + \rho'$ dominance model constructed by Bramon [28]. As mentioned above inelastic contributions and possible new vector mesons $\rho'(1250)$, $\rho''(1600)$ may cause deviations from the simple ρ-dominance model above 1 GeV^2. We have not included these contributions in our N/D calculation because we are only interested in the elastic part of the pion form factor from consistency arguments.

The $\pi^0\omega$ form factor $F_{\pi\omega\gamma}$ is not very well known. We represent the form factor by a $\rho'(1250)$ vector meson dominance model which is justified by the data for $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ [29]:

$$F_{\pi\omega\gamma}(t) = \frac{g_{\rho'\omega\pi}}{g_{\rho'}} \frac{m_{\rho'}^2}{m_{\rho'}^2 - t - i m_{\rho'}\Gamma_{\rho'}} \,. \tag{16}$$

We use $g^2_{\omega\rho'\pi/4\pi} = 7.87$ GeV^{-2}, $m_{\rho'} = 1.25$ GeV, $\Gamma_{\rho'} = 0.12$ GeV, $g_{\rho'}$ was determined by a fit to the data (fig. 3). The connection to the helicity form factors is given in [9].

Our second method of evaluation was done with $D(t) = 1$ and a simple ρ-dominance model for $F_{\pi\omega\gamma}$ which corresponds to the $\pi\pi$ contribution of $\pi^0\omega \to \pi^0\omega$ scattering.

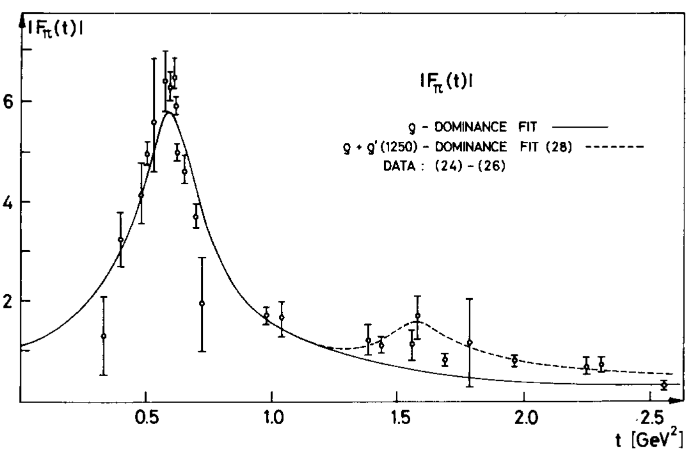

Fig. 2. Pion form factor in the time-like region.

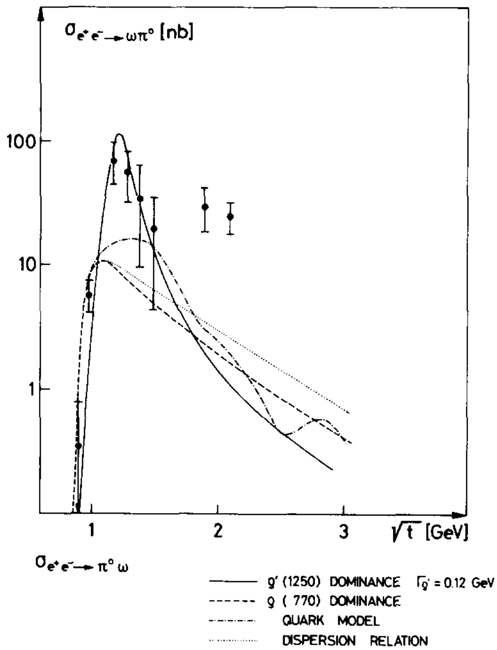

Fig. 3. Total cross-section data for $e^+e^- \to \pi^0\omega$ and comparison with different models.

We also show the results of two other calculations of $F_{\pi\omega\gamma}$ [19,30] in fig. 3 which however do not give better results for $\sigma(e^+e^- \to \pi^+\pi^-\pi^0\pi^0)$ either.

The only measurements which are available for the $\rho\epsilon$ channel are the data for $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ [31]. They show a peak at $t = 1.6^2$ GeV^2 which has also been observed in photoproduction $\gamma p \to p\pi^+\pi^-\pi^+\pi^-$ and which can be interpreted as a new vector meson $\rho''(1600)$. We therefore try a vector meson dominance ansatz for $F^{1,2}_{\epsilon\rho\gamma}$. The invariant form factors are defined as in [9] and the connection to the invariant factors $F^{1,2}_{\epsilon\rho\gamma}$ has also been given there.

The parameters are determined from experiment [31]:

$$m_{\rho''} = 1.56 \text{ GeV}\,, \qquad \Gamma_{\rho''} = 0.35 \text{ GeV}\,, \qquad g_{\rho''} = g_\rho \frac{m_{\rho''}}{m_\rho}\,.$$

One of the unknown coupling constants $f^{1,2}_{\epsilon\rho\rho'}$, can be evaluated from the width $\Gamma_{\rho''\to\rho\epsilon}$ the other was parametrized by $f^1_{\epsilon\rho\rho''} = x f^2_{\epsilon\rho\rho''}$. We tried also another ansatz for $F^{1,2}_{\epsilon\rho\gamma}$ which was recently published by Kramer et al. [32] but the results again showed the strong dependence on the unknown parameter x. Similar to the previous $\pi^0\omega$ case we calculated the nonresonant $\rho\epsilon$ contribution with $D(t) = 1$.

The results are discussed in the following section.

4. Results

We start the discussion of our calculation with the results for the $\pi\pi$ channel. The amplitudes J_1, J_2 are well determined on the known part of the left-hand cut and the extrapolation to the time-like region is reliable up to 1 GeV^2. (See for comparison [12–14, 35].) We obtain the contribution to Im F_i^{V} by multiplying these amplitudes with the squared pion form factor.

Taking only the $\pi\pi$ channel into account we obtain with the help of the dispersion relation for the form factors (eq. (1)) the results for $F_i^{\mathrm{V}}(t)$ in the space-like region, where experimental results from electron scattering are available [33]. We found the result that for small $|t| < 9$ GeV^2 the $\pi\pi$ contribution to F_1 agrees well with the data which show a pole-like behaviour. This behaviour is obviously saturated by the ρ-pole alone. From an unsubtracted dispersion relation we obtain for the isovector part of the nucleon charge $F_1^{\mathrm{V}}(0) = 0.512$, compared to $F_1^{\mathrm{V}}(0) = 0.5$ due to the normalization of the proton and neutron charge.

For the anomalous isovector magnetic moment we obtain from the $\pi\pi$ channel $F_2^{\mathrm{V}}(0) = 2.53$ instead of the experimental value 1.853. This indicates that higher-mass intermediate states may play an important role as will be discussed on the following pages.

In the $\pi^0\omega$ case the amplitudes are certainly not better determined than the $\pi\pi$ contribution in this region since only a few experimental data are available and since the coupling constants and other parameters which we used in the calculations are not too accurately known. Therefore we can hope to determine only the order of magnitude and the sign of the $\pi^0\omega$ contribution to Im F_i^{V}. We found the following significant results: The resonant and the nonresonant contribution to Im F_1^{V} are both small in accordance with our conclusions from the $\pi\pi$ channel. The results for Im F_2^{V} go along with the results for the $\pi\pi$ channel too, i.e. we find a large negative dip with both methods, the N/D solution and the calculation without a resonance. The magnitude of the dip is strongly dependent on various parameters, especially on $g_{\omega\mathrm{NN}}^{\mathrm{V}}$ and on $\Gamma_{\rho'}$. Höhler [34] recently calculated the ω coupling constant to be $g_{\omega\mathrm{NN}/4\pi}^2 = 24 \pm 6$ which is twice as large as the value 11.0 we used for our calculations. This would change the magnitude of the dip by about 50%. Also the unknown width of the $\rho'(1250)$ plays an important role for the magnitude of the dip in the N/D calculation. To give precise values for the $\pi^0\omega$ contribution to the spectral functions we need to know the above mentioned parameters more exactly and we need more information on the $\pi^0\omega$ scattering phase, i.e. whether the phase is dominantly elastic or inelastic.

In the $\rho\epsilon$ case we start again with the calculation of the t-channel helicity amplitudes. Our extrapolation distance from the left-hand cut to the region $t > (m_\rho + m_\epsilon)^2$ is not as large as in the $\pi^0\omega$ case but the determination of the spectral functions suffers from the fact that we can only determine one of the two relevant coupling constants $g_{\epsilon\rho\rho'}^{1,2}$ (resp. $g_{\epsilon\rho\rho}^{1,2}$ for method 2). This leads to a strong dependence on the parameter $x = g_{\epsilon\rho\rho}^1/g_{\epsilon\rho\rho}^2$. We have tried to fix the parameter x in a way which gives a reasonable result for the spectral functions. This leads to $x = -0.35$. If we calculate the cross section $e^+e^- \to 2\pi^+ 2\pi^-$ with this value of the coupling constants we find reasonable agreement with the existing data [31].

Kramer et al. [32] recently investigated the vertex $\gamma \to \rho\pi^+\pi^-$ with a dispersion relation method and showed that naive vector dominance extrapolations to time-like t are rather doubtful. Nevertheless it gives the right magnitude for the cross section $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ so that we can expect that our calculation gives the right order of magnitude for the $\rho\epsilon$ contributions to Im F_i^v. Quantitatively we can say that these contributions are small compared to the $\pi\pi$ channel and to the $\pi\omega$ part of F_2^v in both cases. Neither the resonant nor the background contribution gives a clear effect in the spectral functions. Therefore we conclude that the behaviour of Im $F_i^v(t)$ for t beyond the $N\bar{N}$ threshold is only poorly determined. To decide whether we must improve our calculations we first have to estimate the magnitude of the $N\bar{N}$ contribution. In [14] we showed that from unitarity arguments the $N\bar{N}$ state cannot be neglected. Höhler [34] [36] found a considerable contribution to Im F_i^v beyond the threshold by fitting the data in the space-like region up to SLAC energies with effective poles. This shows that we must include the $N\bar{N}$ state to obtain reliable results for F_i^v for time-like t. Of course other intermediate states like πA_1, πA_2, KK etc. might contribute but our expectation is that these contributions are comparable to the $\rho\epsilon$ contribution. This means that they will have no significant effect in the space-like region. For this region up to $-t = 9$ GeV^2 the $\pi\pi$ and $\pi\omega$ contributions are the dominant ones and are sufficient to explain the behaviour of $F_i^v(t)$.

Adding up the three channels $\pi\pi$, $\pi^0\omega$, $\rho\epsilon$ we get the spectral functions Im F_i^v shown in fig. 4. Our results for the isovector part of the charge and the anomalous magnetic moment are given in table 2.

From table 2 we can see that the normalisations of the form factors $F_{1,2}^v(0)$ are well explained in terms of the $\pi\pi$, $\pi^0\omega$ and $\rho\epsilon$ intermediate states. Therefore contributions of higher-intermediate states must be small or the spectral functions must be oscillating.

We can conclude that the $\pi^0\omega$ state gives a considerable contribution to Im F_2^v

Table 2

	$F_1^v(0)$		$F_2^v(0)$	
	N/D	Method 2	N/D	Method 2
$\pi\pi$	+0.5123		+2.53	
$\pi^0\omega$	+0.0114	−0.023	−0.349	−0.559
$\rho\epsilon$	+0.022	−0.027	+0.134	−0.024
$\pi\pi + \pi^0\omega + \pi\epsilon$	+0.546	+0.462	+2.304	+1.985

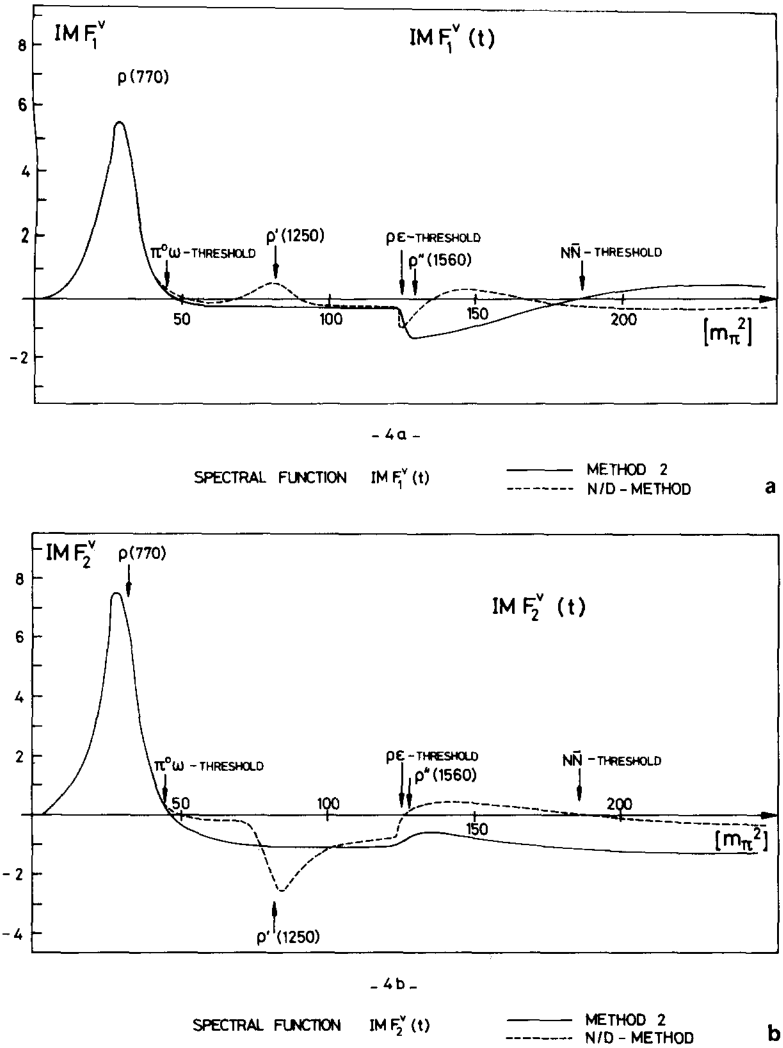

Fig. 4. Spectral functions Im F_i^V obtained with the two different methods.

which explains the dipole-like behaviour of F_2^V in the space-like region. If we compare our results with the data for $t < 0$ we obtain the result that the inclusion of $\pi^0\omega$ and $\rho\epsilon$ improves the results for F_2^V. For F_1^V we get only small corrections to the dominating $\pi\pi$ state. The influence of higher-mass states may become important in the space-like region at very large negative t-values and of course in the time-like region.

5. Summary and conclusions

We have tried to calculate the t-channel helicity amplitudes in the time-like region for the processes $\pi\pi$, $\pi^{\rho}\omega$, $\rho\epsilon \to \mathrm{N\bar{N}}$ with dispersion relation techniques. Including experimental data of the crossed channel processes and using the unitarity relation we have obtained the spectral functions of the nucleon form factors $\mathrm{Im}\, F_i^{\mathrm{V}}(t)$. Only the $\pi\pi \to \mathrm{N\bar{N}}$ amplitudes are reliably determined. For the other channels we have estimated the magnitude of the amplitudes using models and approximations instead of inserting data which do not exist yet. For an improved calculation of $\mathrm{Im}\, F_i^{\mathrm{V}}$ in particular for large t we would need:

(i) experimental information in the $\pi^0\omega$ and $\rho\epsilon$ channel, for example $\Gamma_{\rho'}$, $g^{\mathrm{V}}_{\omega\mathrm{NN}}$ and especially $x = g^1_{\epsilon\rho\rho}/g^2_{\epsilon\rho\rho}$;

(ii) to include $\mathrm{N\bar{N}}$ and possible other intermediate states. It is easy to see that this leads to a complicated system of coupled integral equations whose solution would require a large amount of computer time.

As a result of our simplified calculation we can say that we are able to explain the space-like behaviour of F_i^{V} in the limited range of $|t|$ up to $|t| = 9\ \mathrm{GeV}^2$ but are not able to make a reliable prediction for the time-like region. We found that the structure of F_1^{V} is dominantly determined by the 2π contribution whereas F_2^{V} is dominated by 2π and $\pi^0\omega$. Other contributions do not play an important role in the space-like region. For example the $\rho\epsilon$ state gives only corrections of the order of 5%.

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